

2010 Trial Examination

STUDENT NUMBER

Figures

Words

Letter

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MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
8	8	40
		Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 8 pages.
- Working space is provided throughout the book.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

- a. Let $y = e^{-2x} \sin(3x - 1)$. Find $\frac{dy}{dx}$.

2 marks

- b. Let $f(x) = \frac{e^{3x}}{x}$. Evaluate $f'(1)$.

3 marks

Question 2

- a. Find an anti-derivative of $e^{3\pi x-1}$ with respect to x .

2 marks

- b. If $\int_m^4 \frac{3}{9-2x} dx = 1$, find m for $m < 4$.

3 marks

TURN OVER

Question 3

Consider the transformations $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ of the plane defined by $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$.

- a.** Describe the type of transformation.

2 marks

- b.** Find the image, y' , of the curve with equation $y = \cos(x)$ under T .

2 marks

- c.** Solve the equation $y' = 0$ for $-\pi \leq x' \leq \pi$.

3 marks

- d. Hence, sketch the graph of y' against x' for $-\pi \leq x' \leq \pi$, showing all axial intercepts.

2 marks

Question 4

Sand is flowing into an hourglass at a rate of 8 m^3 per minute and forms a right circular cone which has a height equal to three times its radius.

Find the exact rate at which the height of the pile is increasing when the height is 2 m.

3 marks

TURN OVER

Question 5

Let $f: R \setminus \{1\} \rightarrow R$ where $f(x) = \frac{1}{(x-1)^2} + 2$.

- a. Find the maximal domain for which $f(x)$ is a one-one function and $x > 1$.

1 mark

- b. Hence, find f^{-1} , the inverse function of f for this maximal domain.

3 marks

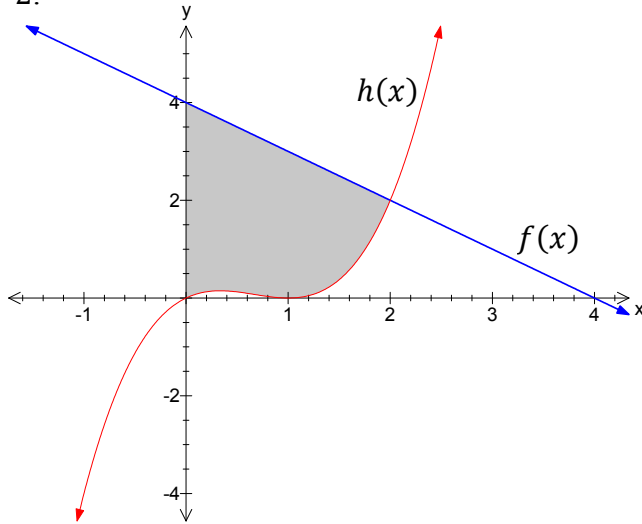
Question 6

Use the relationship $f(x + h) \approx f(x) + hf'(x)$ for a small negative value of h , to find an approximate value for $\sqrt[4]{15.92}$.

4 marks

Question 7

The graphs of $f(x) = -x + 4$ and $h(x) = x(x - 1)^2$ are shown below. The graphs intersect at $x = 2$.



- a. Find the exact area of the shaded region.

3 marks

- b. Find the equation of the normal to the graph of h where it crosses the y - axis.

2 marks

TURN OVER

Question 8

Let $p(x) = \frac{1}{24}(7 - 2x)$ where $x \in \{-1, 0, 1, 2\}$.

- a.** State with reasons why $p(x)$ is a probability function.

2 marks

- b.** Let X be a discrete random variable with the probability function from part a. Find:

- i.** the mean of the discrete random variable X .

1 mark

- ii.** $P(X > -1 | X \leq 1)$

2 marks

END OF QUESTION AND ANSWER BOOK