

Year 2013

VCE

Specialist Mathematics

Trial Examination 2



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Victorian Certificate of Education

2013

STUDENT NUMBER

Figures
Words

Letter

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SPECIALIST MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes

Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 32 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

The graphs of $\frac{(x+a)^2}{a^2} + \frac{(y+a)^2}{b^2} = 1$ and $\frac{(y+a)^2}{b^2} - \frac{(x+a)^2}{a^2} = 1$, where a and b are non-zero real constants,

- A. do not intersect.
- B. touch at the two points $(-2a, -a)$ and $(0, -a)$.
- C. touch at the two points $(-a, b-a)$ and $(-a, -(a+b))$.
- D. intersect at the three points.
- E. intersect at four points.

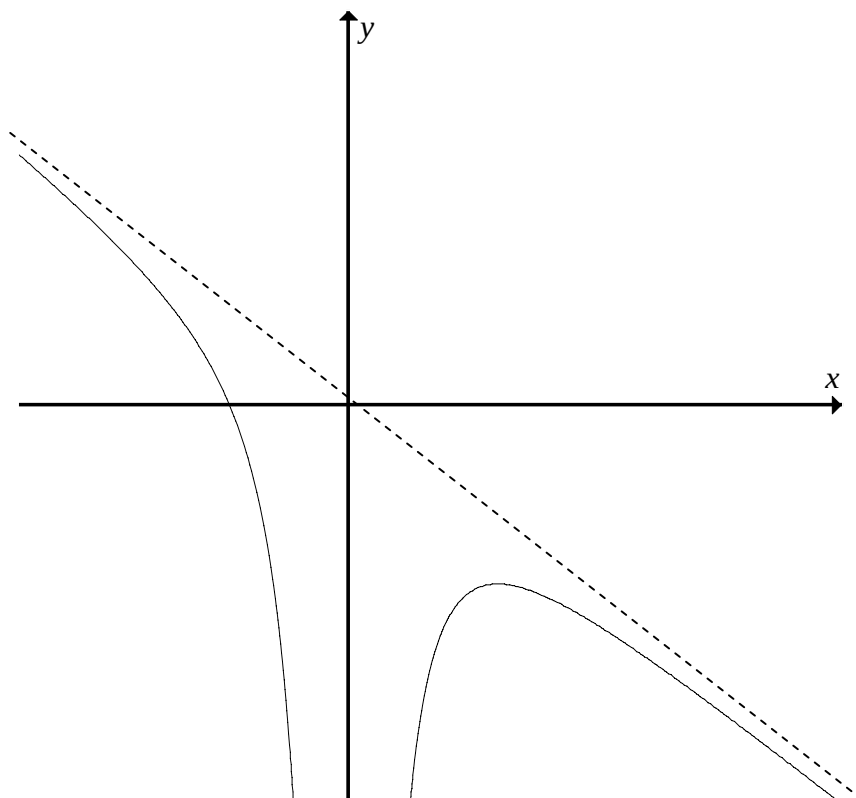
Question 2

The graph of $y = \frac{1}{ax^2 + b(1-a)x - b^2}$ where a and b are non-zero real constants, has asymptotes at

- A. $x = -b$ and $x = \frac{b}{a}$ only.
- B. $x = b$ and $x = -\frac{b}{a}$ only.
- C. $x = -b$, $x = \frac{b}{a}$ and $y = 0$.
- D. $x = b$, $x = -\frac{b}{a}$ and $y = 1$.
- E. $x = b$, $x = -\frac{b}{a}$ and $y = 0$.

Question 3

The graph of $y = \frac{ax^n + b}{x^2}$ is shown below.



Then

- A. $n = 3, a > 0, b < 0.$
- B. $n = 3, a > 0, b > 0.$
- C. $n = 3, a < 0, b < 0.$
- D. $n = 3, a < 0, b > 0.$
- E. $n = 2, a < 0, b > 0.$

Question 4

The graph of $y = a \sin^{-1}(bx - c) + d$ has both the domain and range equal to $[0, \pi]$. Then

- A. $a = c = 1, b = d = \frac{\pi}{2}$
- B. $a = c = 1, b = \frac{2}{\pi}, d = \frac{\pi}{2}$
- C. $a = c = 1, b = d = \frac{2}{\pi}$
- D. $a = c = -1, b = \frac{2}{\pi}, d = \frac{\pi}{2}$
- E. $a = c = -1, b = d = \frac{\pi}{2}$

Question 5

If $a + bi = r \operatorname{cis}(\theta)$ where a and b are positive real constants, then $-b - ai$ is equal to

- A. $-r \operatorname{cis}(\theta)$
- B. $-r \operatorname{cis}\left(\frac{\pi}{2} - \theta\right)$
- C. $r \operatorname{cis}(-\theta)$
- D. $r \operatorname{cis}(\pi - \theta)$
- E. $r \operatorname{cis}\left(\theta - \frac{3\pi}{2}\right)$

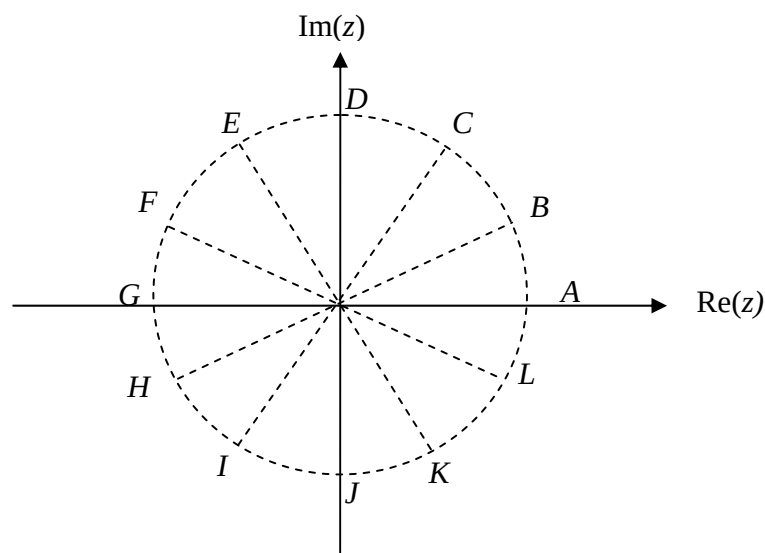
Question 6

The polynomial $P(z)$ has real coefficients. Two of the roots of the equation $P(z) = 0$ are $z = (a + b)i$ and $z = (a - b)i$, where a and b are non-zero real constants, and $|a| \neq |b|$. Then the minimum degree of the polynomial $P(z)$ is

- A. 2
- B. 3
- C. 4
- D. 5
- E. 6

Question 7

The diagram shows a circle of radius a on an argand diagram.



The roots of the equation $z^3 - ia^3 = 0$ where a is a positive real number, are given by

- A. D only.
- B. G, C and K.
- C. A, E and I.
- D. B, F and J.
- E. D, H and L.

Question 8

A cricket ball is hit from ground level at an angle of 30° to the horizontal, with a speed of 19.6 ms^{-1} . Then

- A. it takes one second to reach a maximum height of $2g\sqrt{3}$ metres.
- B. it takes two seconds to reach a maximum height of 4.9 metres.
- C. it takes two seconds to reach a maximum height of $2g\sqrt{3}$ metres.
- D. it takes one second to reach a maximum height of 4.9 metres and hits the ground 49 metres from the point of impact.
- E. it reaches a maximum height of 4.9 metres and hits the ground after two seconds $2g\sqrt{3}$ metres from the point of impact.

Question 9

A boy is standing on top of a two metre high fence. He jumps off the fence and when his hand is one metre above the ground, he throws a ball vertically upwards, with a speed of 4.9 ms^{-1} . The time taken in seconds, for the ball to hit the ground is closest to

- A. 1.79
- B. 1.17
- C. 1.15
- D. 0.50
- E. 0.17

Question 10

If b is a non-zero real number, and $z = x + yi$, then the set of points in the argand plane defined by $\{z : |z - b|^2 + |z + bi|^2 = 2b^2\}$ $z \in C$ represents a circle with

- A. centre $(b, -b)$ and radius $\sqrt{2}b$
- B. centre $(-b, b)$ and radius $\sqrt{2}b$
- C. centre $(b, -b)$ and radius $\frac{\sqrt{2}b}{2}$
- D. centre $\left(-\frac{b}{2}, \frac{b}{2}\right)$ and radius $\frac{\sqrt{2}b}{2}$
- E. centre $\left(\frac{b}{2}, -\frac{b}{2}\right)$ and radius $\frac{\sqrt{2}b}{2}$

Question 11

A particle is acted upon by two forces. One has a magnitude of $\sqrt{2}b$ newtons and acts in the direction $S 45^\circ E$, the other has a magnitude of $\sqrt{2}c$ newtons and acts in the direction $N 45^\circ E$, where b and c are non-zero real positive constants. The magnitude of the resultant force is equal to

- A. $\sqrt{2}(b + c)$
- B. $\frac{\sqrt{2}}{2}(b + c)$
- C. $2\sqrt{2(b^2 + c^2)}$
- D. $\sqrt{2(b^2 + c^2)}$
- E. $2\sqrt{b^2 + c^2}$

Question 12

A car of mass m kg is travelling on a level roadway. The engine exerts a constant propulsive force of F newtons and the total resistance to the motion of the car is kv newtons, where k is positive constant and v is its speed in m/s. The car moves from rest, and travels a distance of D metres until it obtains a speed of V m/s, in a time of T seconds.



Five students stated some relationships between the constants, m , V , k , F , D and T .

Alan stated that $mV = (F - kV)T$ Ben stated that $2mD = (F - kV)T^2$

Colin stated that $\frac{1}{2}mV^2 = (F - kV)D$ David stated that $D = \int_0^V \frac{mv}{F - kv} dv$

Edward stated that $T = \int_0^V \frac{m}{F - kv} dv$

Then

- A. Alan, Ben and Colin are all correct.
- B. Alan and Colin are both correct.
- C. Only Colin is correct.
- D. David and Edward are both correct.
- E. Only Edward is correct.

Question 13

A girl of mass 50 kg is standing in a lift. The reaction of the lift floor on the girl is equal to 60 kg-wt. Then the lift is moving

- A. with constant speed.
- B. down with an acceleration equal to 1.96 ms^{-2} .
- C. up with an acceleration equal to 1.96 ms^{-2} .
- D. down with an acceleration equal to 0.2 ms^{-2} .
- E. up with an acceleration equal to 0.2 ms^{-2} .

Question 14

The gradient of the normal to a curve at any point $P(x, y)$ is twice the square root of the gradient joining P and the origin. The coordinates on the curve satisfy the differential equation

A. $\frac{dy}{dx} + \sqrt{\frac{x}{4y}} = 0$

B. $\frac{dy}{dx} - \sqrt{\frac{x}{4y}} = 0$

C. $\frac{dy}{dx} - 2\sqrt{\frac{y}{x}} = 0$

D. $\frac{dy}{dx} + 2\sqrt{\frac{y}{x}} = 0$

E. $\frac{dy}{dx} + \frac{4y^2}{x^2} = 0$

Question 15

Two vectors $\underline{u}$ and $\underline{v}$ are such that $|\underline{u}| = |\underline{v}| = L$. The angle between the vectors $\underline{u}$ and $\underline{v}$ is θ . Then $|\underline{u} - \underline{v}|$ is equal to

A. 0

B. $2L$

C. $\sqrt{2}L$

D. $L \sin(\theta)$

E. $2L \sin\left(\frac{\theta}{2}\right)$

Question 16

Given the vectors $\underline{a} = \underline{i} - \underline{j} + \underline{k}$, $\underline{b} = 2\underline{i} + 2\underline{j} - \underline{k}$ and $\underline{c} = \frac{1}{3}(\underline{i} - 5\underline{j} + 4\underline{k})$

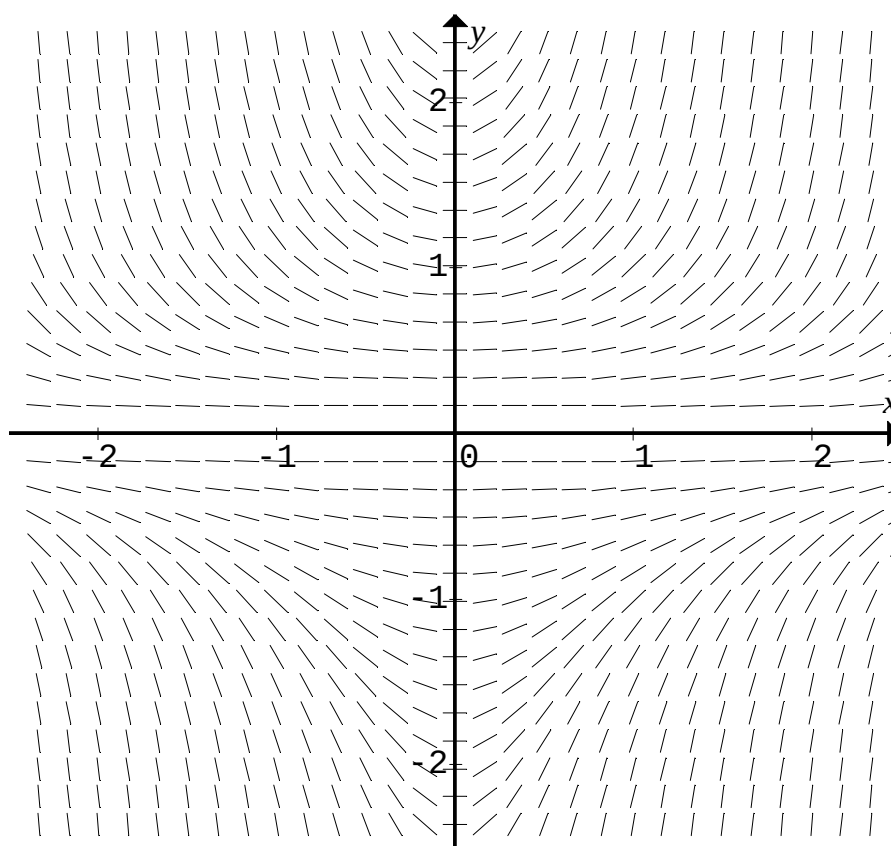
- A. Then the vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ are linearly dependant.
- B. The vector $\underline{c}$ is the scalar resolute of the vector $\underline{a}$ parallel to the vector $\underline{b}$.
- C. The vector $\underline{c}$ is the vector resolute of the vector $\underline{a}$ parallel to the vector $\underline{b}$.
- D. The vector $\underline{c}$ is the vector resolute of the vector $\underline{a}$ perpendicular to the vector $\underline{b}$.
- E. The vector $\underline{c}$ is parallel to the vector $\underline{a} + \underline{b}$.

Question 17

A tank initially holds 50 litres of a solution, in which 4 kilograms of salt has been dissolved. Another solution containing 4 kilograms of salt per litre is poured into the tank at a rate of 3 litres per minute. The well-stirred mixture leaves the tank at a rate of 2 litres per minute. If Q kilograms is the amount of salt in the tank a time t minutes, and c a constant, then

- A. $Q = 12(50 + t) + \frac{c}{50 + t}$
- B. $Q = 12(50 + t) - \frac{c}{50 + t}$
- C. $Q = 4(50 + t) + \frac{c}{(50 + t)^2}$
- D. $Q = 4(50 + t) - \frac{c}{(50 + t)^2}$
- E. $Q = 4(50 - t) - \frac{c}{(50 - t)^2}$

Question 18



The differential equation which best represents the above direction field is

- A. $\frac{dy}{dx} = y^2 x$
- B. $\frac{dy}{dx} = y x^2$
- C. $\frac{dy}{dx} = y^2 x^2$
- D. $\frac{dy}{dx} = \frac{x^2}{y}$
- E. $\frac{dy}{dx} = \frac{y^2}{x^2}$

Question 19

A moving particle has a velocity vector at a time t , given by $e^t \cos(t) \mathbf{i} + e^t \sin(t) \mathbf{j}$, for $t \geq 0$. Initially the particle is at the origin. The position vector is given by

- A. $e^t \sin(t) \mathbf{i} - e^t \cos(t) \mathbf{j}$.
- B. $e^t \sin(t) \mathbf{i} - (e^t \cos(t) - 1) \mathbf{j}$.
- C. $e^t (\cos(t) - \sin(t) - 1) \mathbf{i} + e^t (\cos(t) + \sin(t) - 1) \mathbf{j}$.
- D. $\frac{1}{2} [e^t (\cos(t) + \sin(t)) - 1] \mathbf{i} + \frac{1}{2} [e^t (\sin(t) - \cos(t)) + 1] \mathbf{j}$.
- E. $\frac{1}{2} e^t (\cos(t) + \sin(t)) \mathbf{i} + \frac{1}{2} e^t (\sin(t) - \cos(t)) \mathbf{j}$

Question 20

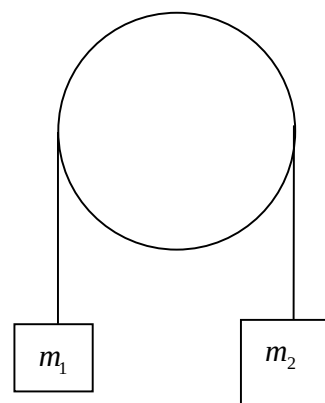
When Euler's method, with a step size of $\frac{1}{5}$, is used to solve the differential equation

$\frac{dy}{dx} = \tan(x)$ with $x_0 = 0$ and $y_0 = 0$, the value of y_3 would be given as

- A. $-\log_e(\cos(0.4))$
- B. $0.2(\tan(0.2) + \tan(0.4))$
- C. $0.2 \tan(0.6)$
- D. $0.2 \tan(0.4)$
- E. $0.2 \tan(0.3)$

Question 21

A light inextensible string passes over a smooth pulley, with particles of masses m_1 and m_2 kg, attached to each end of the string as shown in the diagram.



Which of the following is **false**?

- A. If $m_2 = 2m_1$ the mass m_2 moves downwards with an acceleration $\frac{g}{2} \text{ ms}^{-2}$.
- B. If $m_1 = m_2 = m$ the tension in the string is equal to mg newtons.
- C. If $m_1 = m_2$ both masses remain at rest.
- D. If $m_2 > m_1$ the mass m_2 moves downwards with an acceleration $\frac{(m_2 - m_1)g}{m_1 + m_2} \text{ ms}^{-2}$.
- E. If $m_1 \neq m_2$ the tension in the string is equal to $\frac{2m_1m_2}{m_1 + m_2} \text{ kg-wt}$.

Question 22

A particle of mass m kg is acted upon by a variable force, so that its velocity v m/s when the particle is x m from the origin is given by $v = e^{cx}$, where c is a non-zero real constant. The force acting on the particle when $x = \frac{1}{c}$, in newtons, is

- A. mc^2
- B. mec
- C. me
- D. me^2
- E. mce^2

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

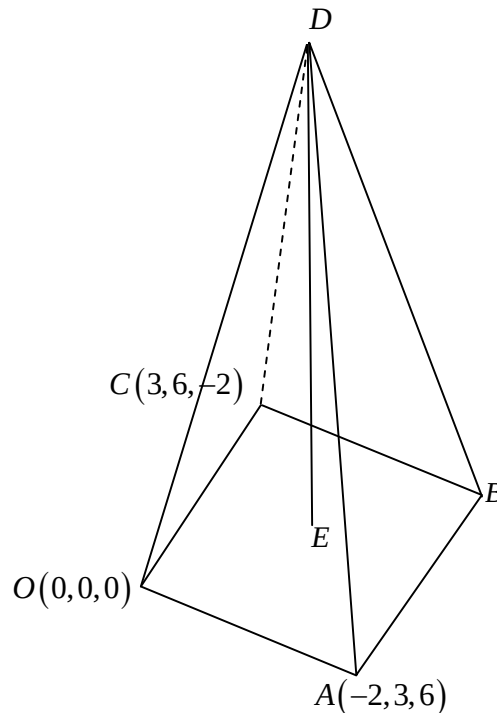
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

11 marks

$OABCD$ is a right pyramid, where O is the origin. The coordinates of the points are $A(-2, 3, 6)$, $C(3, 6, -2)$ and $D(x, y, z)$. The height of the pyramid is the length of ED , where E is a point on the base of $OABC$, such that E is the midpoint of OB .



- a. Given that $OABC$ is a square, find the coordinates of the point B .

2 marks

- b.** Find the coordinates of the point E .

1 mark

- c.** If the vector $\overrightarrow{ED}$ is perpendicular to $\overrightarrow{OE}$ show that $x + 9y + 4z = 49$.

3 marks

Question 2

13 marks

- a.** Given that $\sin\left(\frac{\pi}{8}\right) = \frac{1}{2}\sqrt{2-\sqrt{2}}$, show that $\cos\left(\frac{\pi}{8}\right) = \frac{1}{2}\sqrt{2+\sqrt{2}}$.

2 marks

- b.** Use a double angle formula to show that $\tan\left(\frac{\pi}{8}\right) = \sqrt{2}-1$.

2 marks

- c. If $\text{Arg}(1 - \sqrt{2} + i) = k\pi$, write down the value of k , and explain your result geometrically.

2 marks

Let $u = \sqrt{2 + \sqrt{2}} + i\sqrt{2 - \sqrt{2}}$

- d. Express u in exact polar form.

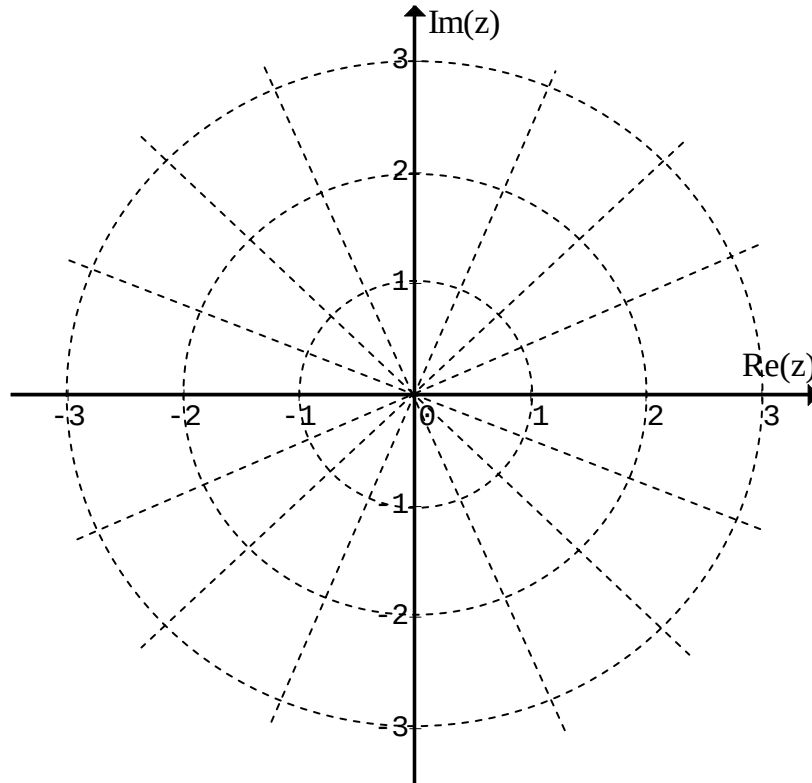
1 mark

- e. Find u^{10} giving your answer in both polar and rectangular form.

2 marks

- f.i.** On the argand diagram below, shade the region defined by
 $\{z : \text{Arg}(u^{10}) \leq z \leq \text{Arg}(u)\} \cap \{z : 1 \leq z \leq 2\}, z \in \mathbb{C}$

2 marks



- ii.** Find the area corresponding to the region sketched above in **f.i.**

1 mark

- g.** Find the value(s) of n , such that $\text{Im}(u^n) = 0$

1 mark

Question 3

11 marks

A train travelling along a straight line track between two stations has its velocity $v \text{ ms}^{-1}$ at a time t seconds, given by

$$v(t) = \begin{cases} a \sin\left(\frac{\pi t}{100}\right) & 0 \leq t < 50 \\ bt + c & 50 \leq t \leq 120 \\ \frac{32}{\pi} \cos^{-1}\left(\frac{t-120}{60}\right) & 120 < t \leq 180 \end{cases} \quad \text{where } a, b \text{ and } c \text{ are real constants.}$$

- a.** Over the time interval $[50, 120]$ the train travels a distance of 875 metres.

Show that $5950b + 70c = 875$.

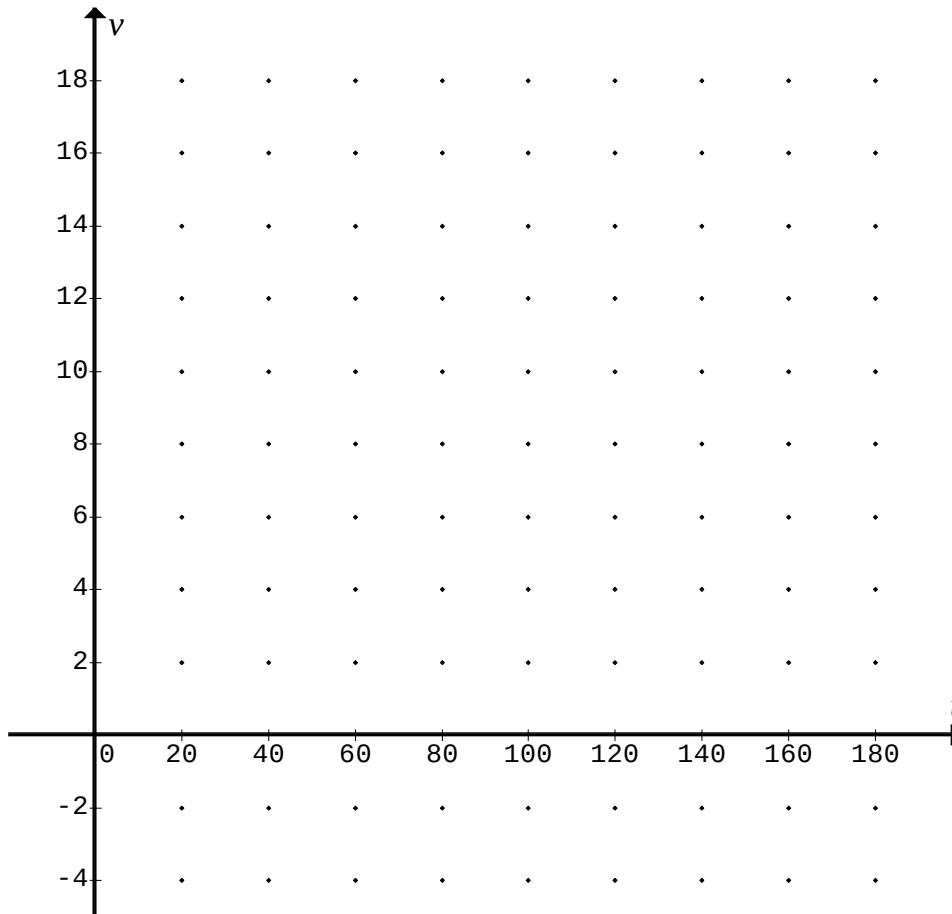
2 marks

- b.** Write down another two simultaneous equations, which can be used to solve for a , b and c , and hence shown that $a = 9$, $b = \frac{1}{10}$ and $c = 4$

2 marks

c. Sketch the velocity time graph on the axes below.

2 marks



d.i. Write down in terms of definite integrals the total distance between the two stations.

1 mark

ii. Find the total distance between the two stations correct to the nearest metre.

1 mark

- e.** Find the acceleration of the train after 150 seconds. Given your answer correct to three decimal places.

1 mark

- f.** The passengers on the train, are most comfortable when the train travelling between the station travels at speeds less than 30 km/hr. For what percentage of the journey are the passengers most comfortable?
Give your answer correct to one decimal place.

2 marks

Question 4

12 marks

A curve is defined by the parametric equations

$$x = \cos(t)(4\cos(t) - \sec(t))$$

$$y = \sin(t)(4\cos(t) - \sec(t)) \text{ for } 0 \leq t \leq \pi$$

- a.** Show that the curve can be expressed in cartesian form as

$$4x^2 - (x+1)(x^2 + y^2) = 0$$

2 marks

b.i. Show that $\frac{dy}{dx} = \frac{6x - 3x^2 - y^2}{2y(x+1)}$.

2 marks

ii. Express the gradient of the curve in terms of t only.

1 mark

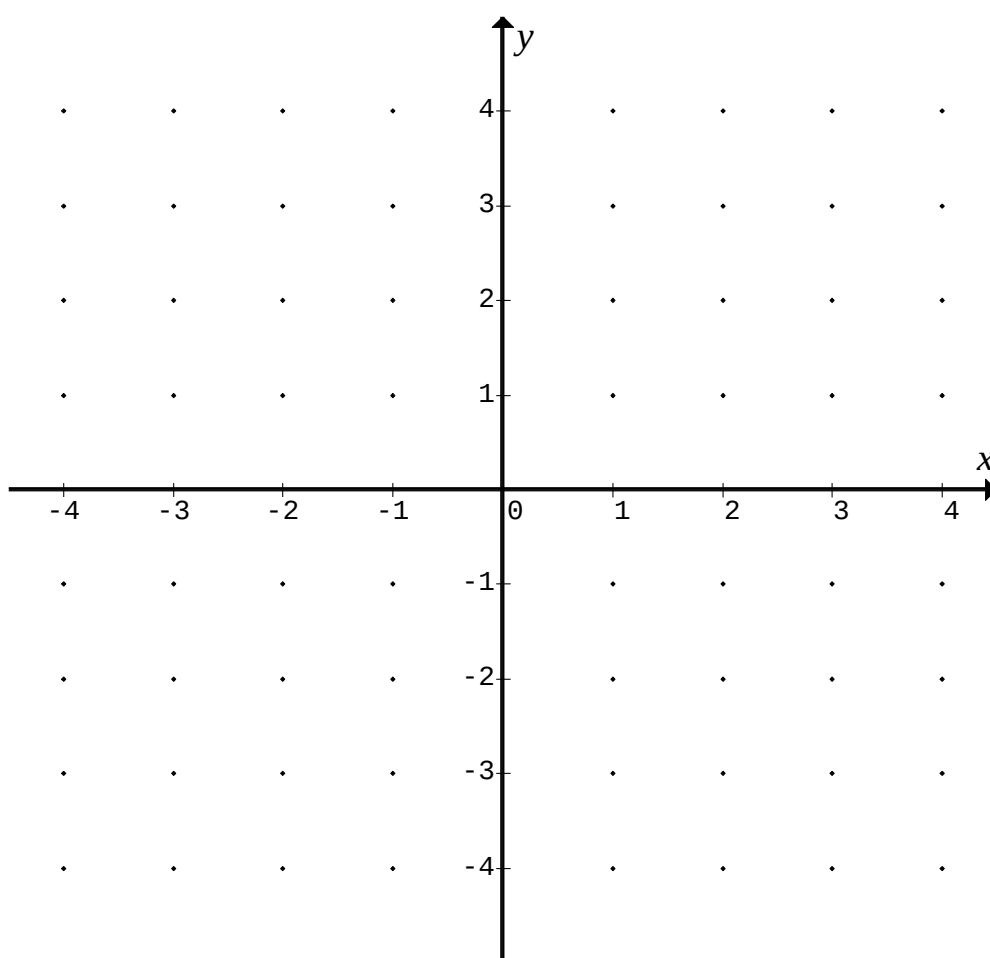
c. Sketch the curve defined by

$$x = \cos(t)(4\cos(t) - \sec(t))$$

$$y = \sin(t)(4\cos(t) - \sec(t)) \text{ for } 0 \leq t \leq \pi$$

labelling any asymptotes with their equations.

2 marks



The region bounded by the curve $4x^2 - (x+1)(x^2 + y^2) = 0$, the x -axis, $x = 0$ and $x = 3$ is rotated about the x -axis, to form a solid of revolution with volume V .

d.i. Show that $V = \pi \int_0^3 \frac{x^2(3-x)}{x+1} dx$

1 mark

ii. Using a suitable substitution, express V in the form

$V = \pi \int_a^b \left(pu^2 + qu + r + \frac{s}{u} \right) du$, where a, b, p, q, r and s are integers. 2 marks

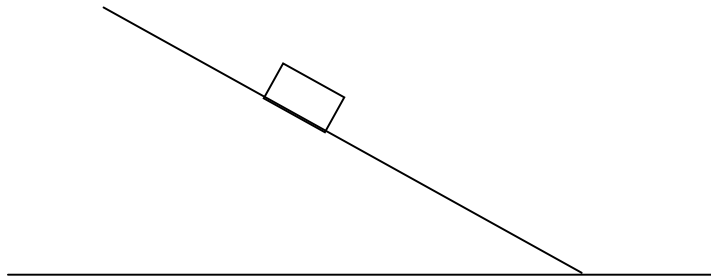
Question 5

11 marks

A block of mass m kg slides from rest down a plane inclined at an angle of 60° to the horizontal. The coefficient of friction between the block and the plane is μ .

a.i. On the diagram below, mark in and label all the forces acting on the block.

1 mark



ii. If a is the acceleration of the block in m/s^2 as it slides down the plane show that

$$a = \frac{g}{2}(\sqrt{3} - \mu).$$

2 marks

The block moves down the plane a distance of D metres and reaches a speed of $\frac{g}{4}$ m/s, in a time of T seconds.

b. If the coefficient of friction is constant and equal to $\mu = \sqrt{3} - 1$,

i. find the value of T .

1 mark

ii. find the value of D .

1 mark

- c. For this block and plane, the coefficient of friction between the block and the plane is not constant, but depends on the speed v m/s of the block as it moves down the plane. For this case when $\mu = \sqrt{3} - 1 - \frac{v^2}{5g}$,

- i. find the value of T .

3 marks

- ii. find the value of D .

3 marks

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of triangle: $\frac{1}{2}bc \sin(A)$

sine rule: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$-\pi < \operatorname{Arg}(z) \leq \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{dr}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$$

Mechanics

momentum: $\underline{p} = m\underline{v}$

equation of motion: $\underline{R} = m\underline{a}$

sliding friction: $F \leq \mu N$

constant (uniform) acceleration:

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u + v)t$$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2}v^2 \right)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \quad \text{for } x > 0$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

END OF FORMULA SHEET

ANSWER SHEET

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