

Year 2013

VCE

Specialist Mathematics

Trial Examination 1



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- Teachers and students are reminded that for the purposes of school requirements and external assessments, students must submit work that is clearly their own.

Victorian Certificate of Education 2013

STUDENT NUMBER

Figures
Words

Letter

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SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

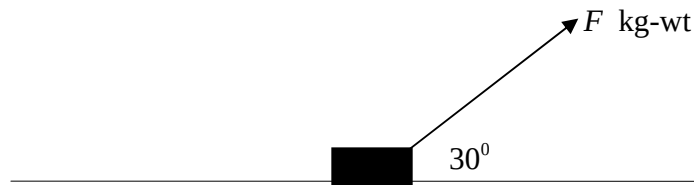
Question 1

4 marks

Find an antiderivative of $\frac{3x-5}{\sqrt{25-9x^2}}$, for $x \in (-b, b)$, stating the value of b .

Question 3

4 marks



A box of mass 5 kg is on a horizontal plane. A force of magnitude F kg-wt acting at an angle of 30° to the horizontal is applied to the box. The coefficient of friction between the box and the plane is $\frac{\sqrt{3}}{3}$. Find a range of values of F , so that the box moves with constant acceleration.

Question 4

3 marks

Consider the equation $P(z) = z^3 + z^2 + bz - 24$, $z \in \mathbb{C}$. Given that $z = 2\sqrt{2}\text{cis}\left(\frac{3\pi}{4}\right)$ is a root of the equation $P(z) = 0$, find the value of the real constant b and state all the roots.

Question 5

2 marks

If $\cos(\theta) = \frac{2}{3}$, where $0 < \theta < \frac{\pi}{2}$ and $z = 3\text{cis}(\theta)$ find the complex number z^2 in $a + bi$ form.

Question 6

4 marks

$OABC$ is a parallelogram in which P is the midpoint of OA . D is a point on PC such that $d(PC) = 3d(PD)$. Prove that O, D and B are collinear.

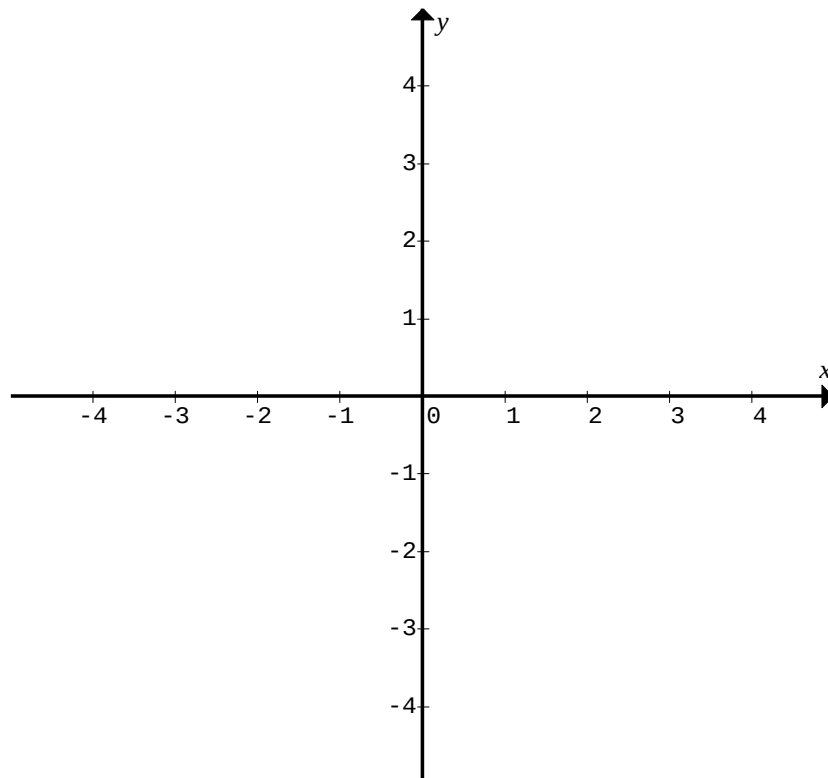
Question 7

1 + 2 + 2 = 5 marks

a. Show that $\cos(3A) = 4\cos^3(A) - 3\cos(A)$

- b.** The position vector of a particle at time t , is given by
 $\vec{r}(t) = 2\cos(t)\vec{i} + 4\cos(3t)\vec{j}$ for $t \geq 0$.
- i.** Find the cartesian equation of the path followed by the particle.

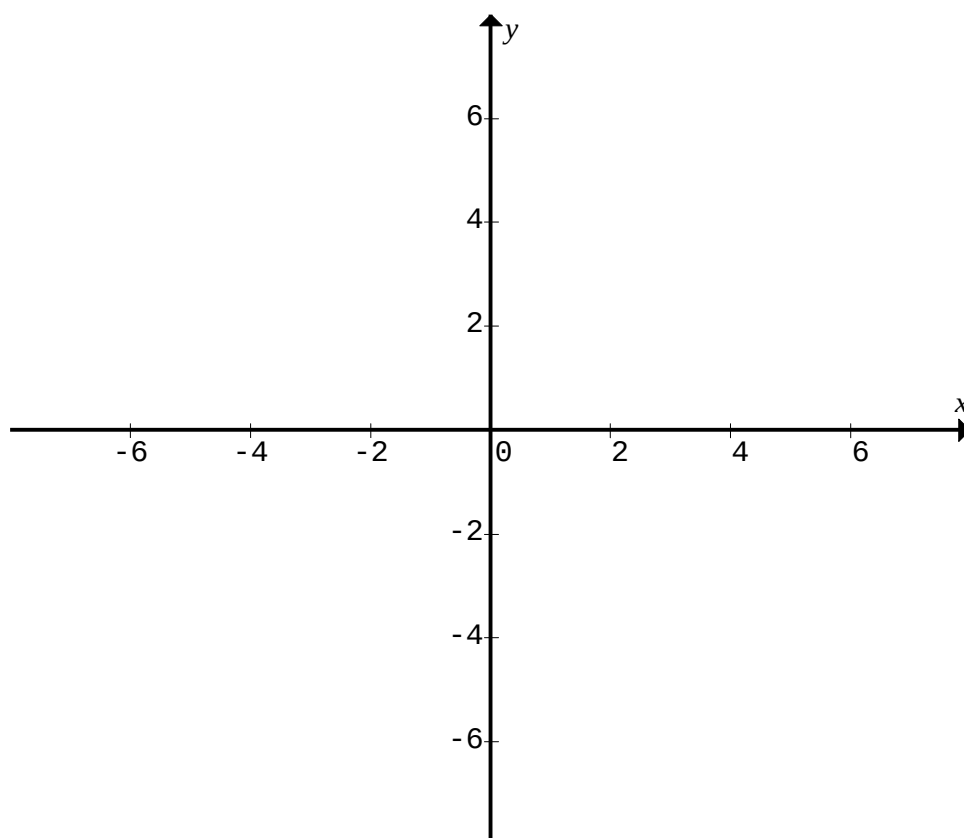
- ii.** Sketch the path followed by the particle on the axes below, clearly state the coordinates of all axial intercepts and turning points.



Question 9

3 + 1 = 4 marks

- a. On the axes below, sketch the graph with equation $x^2 + 4x - 4y^2 = 0$.
State all intercepts with the coordinate axes and give the equations of any asymptotes.

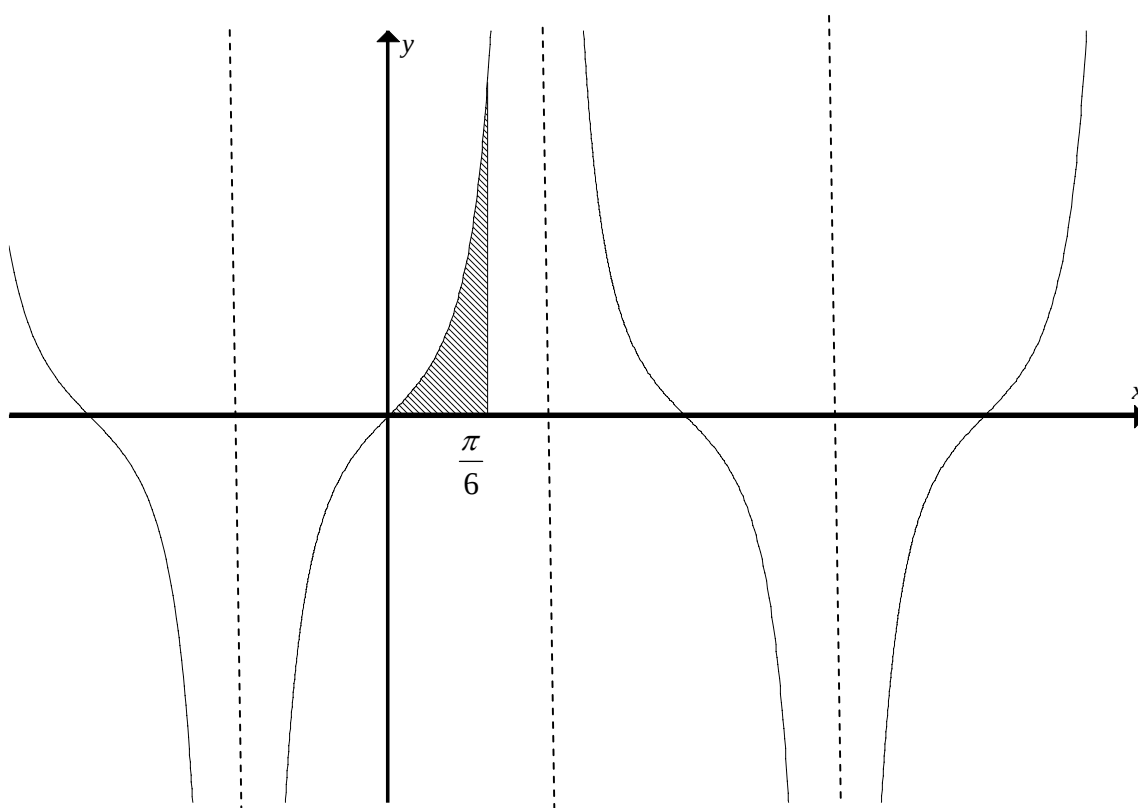


- b. Find the gradient of the curve $x^2 + 4x - 4y^2 = 0$, in terms of both x and y .

Question 10

1 + 3 + 2 = 6 marks

Part of the graph with equation $y = \tan(2x)\sec(2x)$ is shown below.



- i.** Find the general solution for the equations of the asymptotes on the graph of $y = \tan(2x)\sec(2x)$.

- ii.** Find the shaded area, that is the area bounded by the graph of $y = \tan(2x)\sec(2x)$, the x-axis and the line $x = \frac{\pi}{6}$.

- iii. When the shaded area is rotated about the x -axis, it forms a volume of revolution. Find the resulting volume formed.

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of triangle: $\frac{1}{2}bc \sin(A)$

sine rule: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$-\pi < \operatorname{Arg}(z) \leq \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{dr}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$$

Mechanics

momentum: $\underline{p} = m\underline{v}$

equation of motion: $\underline{R} = m\underline{a}$

sliding friction: $F \leq \mu N$

constant (uniform) acceleration:

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u + v)t$$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2}v^2 \right)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \quad \text{for } x > 0$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

END OF FORMULA SHEET