

Year 2013

VCE

Mathematical Methods

Trial Examination 1



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Victorian Certificate of Education 2013

STUDENT NUMBER

Figures
Words

Letter

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MATHEMATICAL METHOD CAS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

2 + 2 = 4 marks

a. If $f(x) = \log_e(\cos(3x))$, find $f'\left(\frac{\pi}{18}\right)$.

b. If $y = \frac{\sin(4x)}{2x^2}$ and $\frac{dy}{dx} = \frac{g(x)}{x^3}$, find the function $g(x)$.

Question 2

3 marks

Consider the linear simultaneous equations

$$kx - 4y = 2$$

$$3x - (k + 4)y = k + 1$$

where k is a real constant.

- i. Find the value(s) of k , for which there is a unique solution.
- ii. Find the value(s) of k , for which there is no solution.

Question 3

3 marks

A certain curve has a gradient equal to $\frac{2}{\sqrt{4x+9}}$. Find the equation of the curve which passes through the origin.

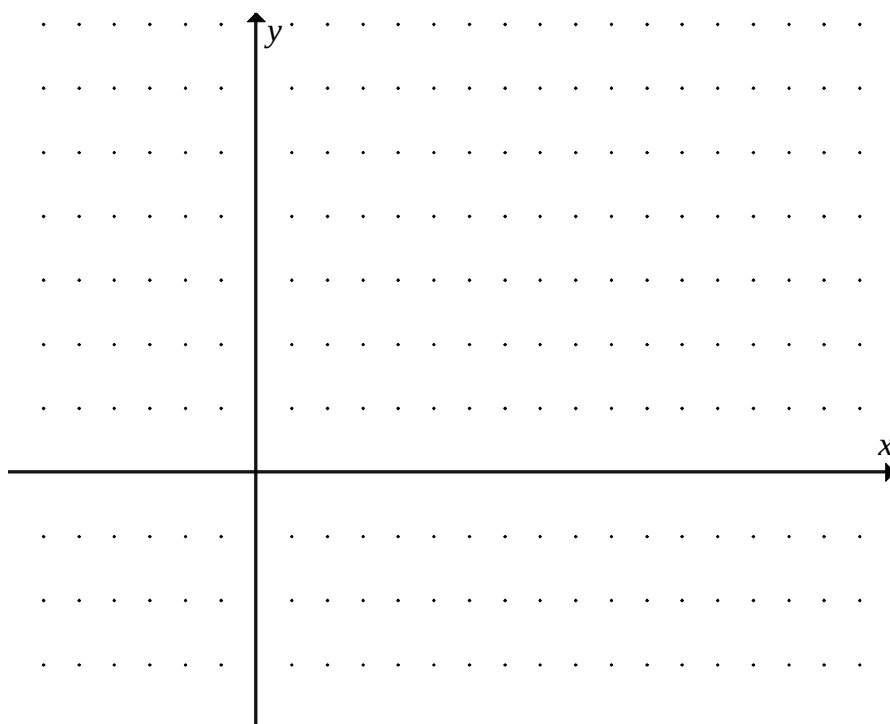
Question 4

2 + 3 = 5 marks

a. Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 1 - 2\cos\left(\frac{\pi x}{6}\right)$.

Find the general solution for the x -coordinates where the graph of $y = f(x)$ crosses the x -axis.

b. Sketch the graph of $g : [0, 12] \rightarrow \mathbb{R}$, $g(x) = 1 - 2\cos\left(\frac{\pi x}{6}\right)$, stating the amplitude, period and the range. Clearly label the axes scale, intercepts and the coordinates of any turning points and endpoints.



Question 5

3 marks

Solve the equation $\log_x 5 + \log_5 x = \frac{5}{2}$ for x .

Question 6

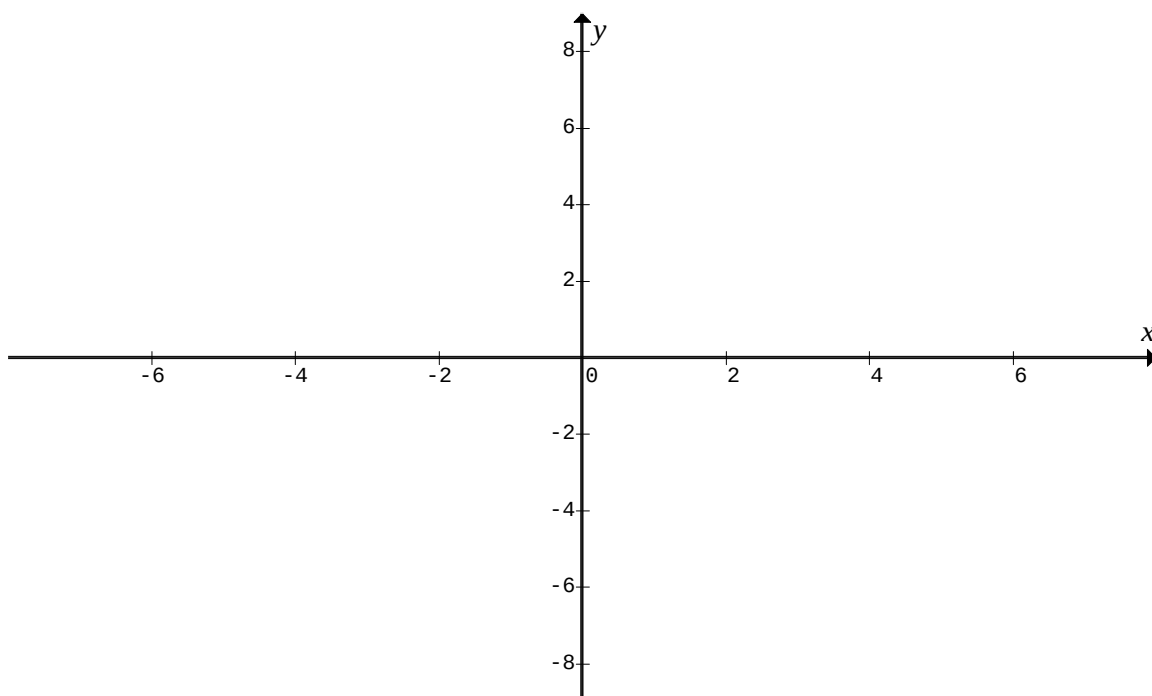
3 marks

The rule for function f is $f(x) = 3e^{-2x} - 4$. Find the inverse function f^{-1} .

Question 7

3 + 2 = 5 marks

- a. Sketch the graph of $y = 3 - \frac{12}{(x+2)^2}$ on the axes below, clearly indicating all axial cuts and the equation of any asymptotes, stating the maximal domain and range.



- b. Describe in words, giving scale factors, the transformations in a suitable order, required to sketch the graph of $y = 3 - \frac{12}{(x+2)^2}$ from the graph of $y = \frac{1}{x^2}$.

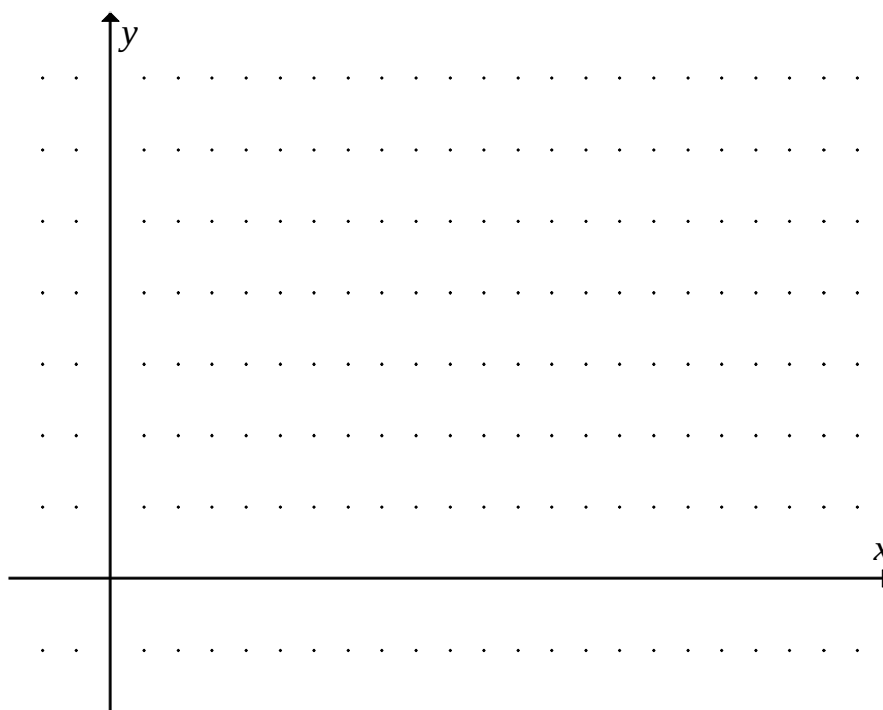
Question 8

2 + 1 = 3 marks

The probability distribution function for the continuous random variable X is given by

$$f(x) = \begin{cases} \frac{1}{16}(4 - |x - 4|) & \text{for } 0 \leq x \leq 8 \\ 0 & \text{elsewhere} \end{cases}$$

- a. Sketch the graph of $f(x)$ on the axes below.



- b. Find $\Pr(X < 2 \mid X < 6)$

Question 9

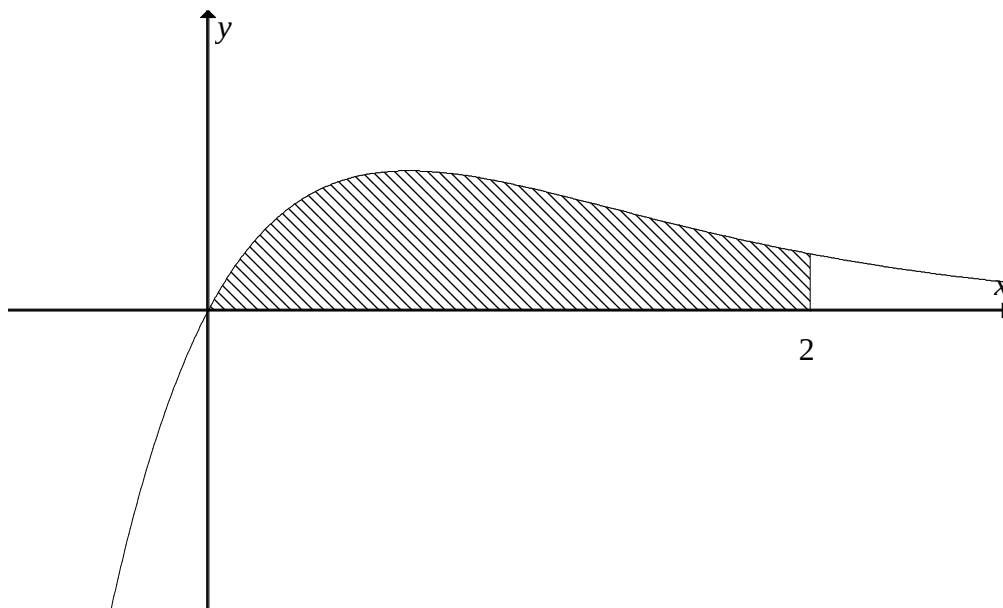
2 marks

Two independent events, A and B , are such that $\Pr(A) = \frac{1}{3}$ and $\Pr(B) = \frac{2}{5}$. If A' and B' denote the complements of A and B respectively, calculate $\Pr(A' \cup B')$.

Question 10

1 + 2 + 3 = 6 marks

The graph of the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = xe^{-2x}$ is shown below.



a. Find $f'(x)$.

- b.** Find the coordinates of the stationary point on the graph of $y = xe^{-2x}$.

- c.** Use your answer to **part a.** to find the area of the shaded region.

MATHEMATICAL METHODS CAS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of triangle: $\frac{1}{2}bc \sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h f'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Transition Matrices $S_n = T^n \times S_0$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$