

2013 VCE Further Mathematics Trial Examination 2 Suggested Answers



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- While every care has been taken, no guarantee is given that these answers are free from error. Please contact us if you believe you have found an error.

Core: Data analysis

Question 1

a.

Use calculator to get Mean = \$12786, Standard Deviation = \$3332

(2 marks)

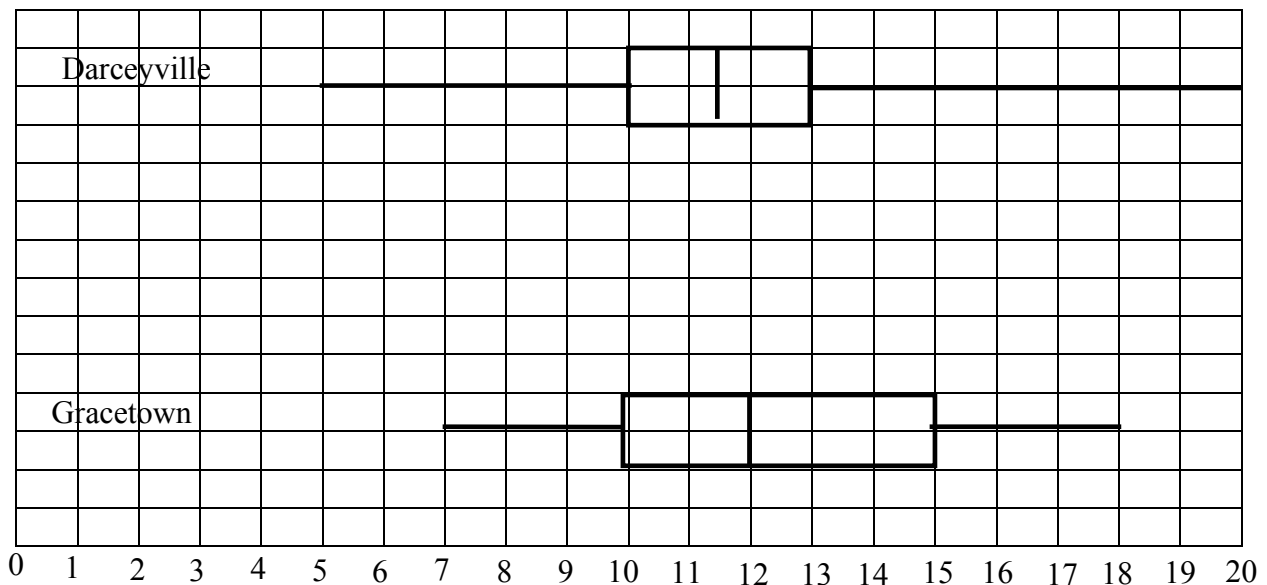
1 mark for mean and one mark for standard deviation.

b.

Gracetown has the higher average takings for the given period with a mean of \$12786 but Darceyville has the greater spread of \$3863

(1 mark)

c.



Obtain the 5 point summary from calculator

One mark for correct median and end points and one mark for correct lower and upper quartile.

(2 marks)

Question 2

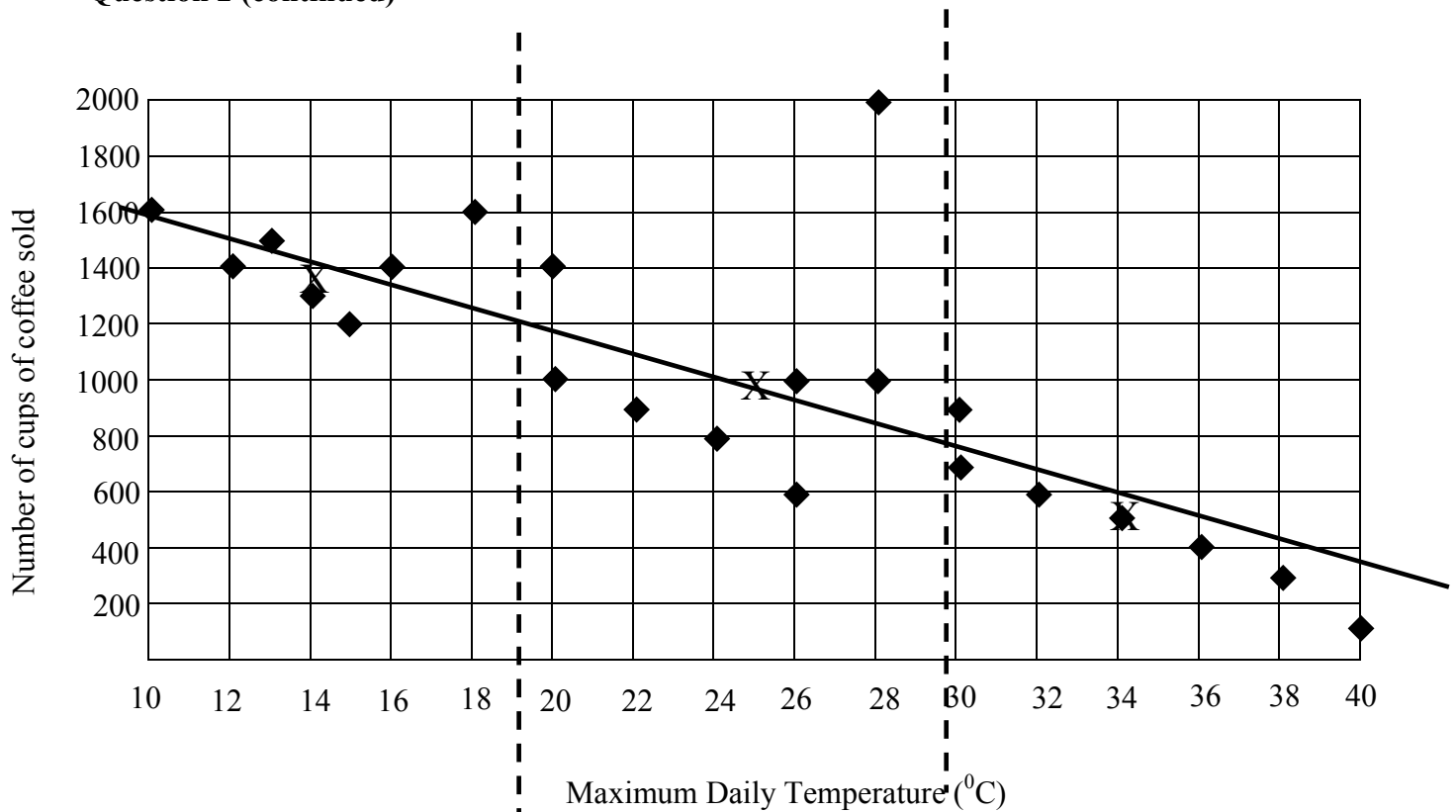
a.

The dependent variable is the number of cups of coffee.

(1 mark)

Core: Data analysis

Question 2 (continued)



b.

There are 22 points. Draw two vertical lines with 7 points in the first third 8 in the second third and 7 in the last third. The median of the first third is (14, 1400) and the median of the last third is (34, 500)

$$\text{gradient} = \frac{500 - 1400}{34 - 14} = -45$$

(1 mark)

c.

For every rise of 1°C in temperature, the average daily number of cups of coffee drops by 45.

(1 mark)

d.

Median of centre 8 points is (25, 1000). Put ruler on the two outer X's and drag it one third of the way towards the middle X

(1 mark)

e.

Because of the outlier (28, 2000)

(1 mark)

Core: Data analysis

Question 3

a. Make data headings temperature and coffee and enter data. Home – graph – enter temperature on X axis and coffee on Y axis. Menu – statistics – linear regression . This gives <i>Average number of cups of coffee</i> = $1394 - 24 \times \text{maximum daily temperature}$. (1 mark)	
b. Predicted number of cups of coffee = $1394 - 24 \times 27 = 746$ Actual number of cups of coffee = 800 Residual = $800 - 746 = 54$ (1 mark)	c. It is a strong negative relationship with $r = -0.93$ (1 mark)
d. Use linear regression on calculator to get $r^2 = 0.8687$ (1 mark)	e. 86.87% of the variation in the average number of cups of coffee sold per day at Gracetown can be explained by the variation in the temperature. (1 mark)

Module 1: Number patterns and applications

Question 1

a. i. $R_2 = 4 \times 2 = 8$ $R_3 = 4 \times 8 = 32$ $R_4 = 4 \times 32 = 128$ (1 mark)	a. ii. 4 (1 mark)
b. $R_4 = 92 - 3 = 89$ $R_3 = 89 - 3 = 86$ $R_2 = 86 - 3 = 83$ $R_1 = 83 - 3 = 80$ (1 mark)	c. <i>c</i> is the first term which = 10 $R_2 = 10a + b = 12$ $R_3 = 12a + b = 16$ Solve on calculator to get $a = 2$ $b = -8$ $c = 10$ <i>1 mark for each correct value.</i> (3 marks)

Module 1: Number patterns and applications

Question 2

a. Increase is \$1.20 per week. Week 4 is $\\$4.40 + \\$1.20 = \\$5.60$ (1 mark)	b. This is an arithmetic sequence. $t_n = a + (n - 1)d$ $t_{10} = 2 + (10 - 1) \times 1.2$ $t_{10} = 2 + (10 - 1) \times 1.2 = \\12.80 (1 mark)
c. $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 1.2) = \\268 (1 mark)	d. Use calculator to show that in week 22 he only has \$321.20, which is not enough, but in week 23 he has \$349.60, which is sufficient to buy the cricket bat. So he must save for 23 weeks. (1 mark)

Module 1: Number patterns and applications

Question 3

a. Each week she gives Mitch 2 times what she gave him the week before. In 9th week he gets $12 \times 2 = \\$24$ In 10th week he gets $24 \times 2 = \\$48$ (1 mark)	b. Amount saved by Mitch at end of 10 weeks $= \frac{10}{2} (4 + 9 \times 1.2) = \\74 Amount received from granny at end of 10 weeks $= \frac{3(2^5 - 1)}{2 - 1} = \\93 Total amount saved $= 74 + 93 = \\$167$ Amount to be saved $= 340 - 167 = \\$173$ (1 mark)
c. Use calculator to get grandmother would give \$96 in 11th week of his savings and \$192 in his 12th week of saving. In week 12, she would first give him more than \$100. (1 mark)	d. Use calculator to get Week 11 = \$88 saved + \$189 from granny. $88 + 189 = \\$277$, which is not enough for the cricket bat. Week 12 = \$103.20 saved + \$381 from granny which is sufficient to buy the cricket bat. It would take 12 weeks to save. (1 mark)
e. Use calculator In week 9 Mitch saves \$11.60 and Granny gives \$24. Week 9. (1 mark)	

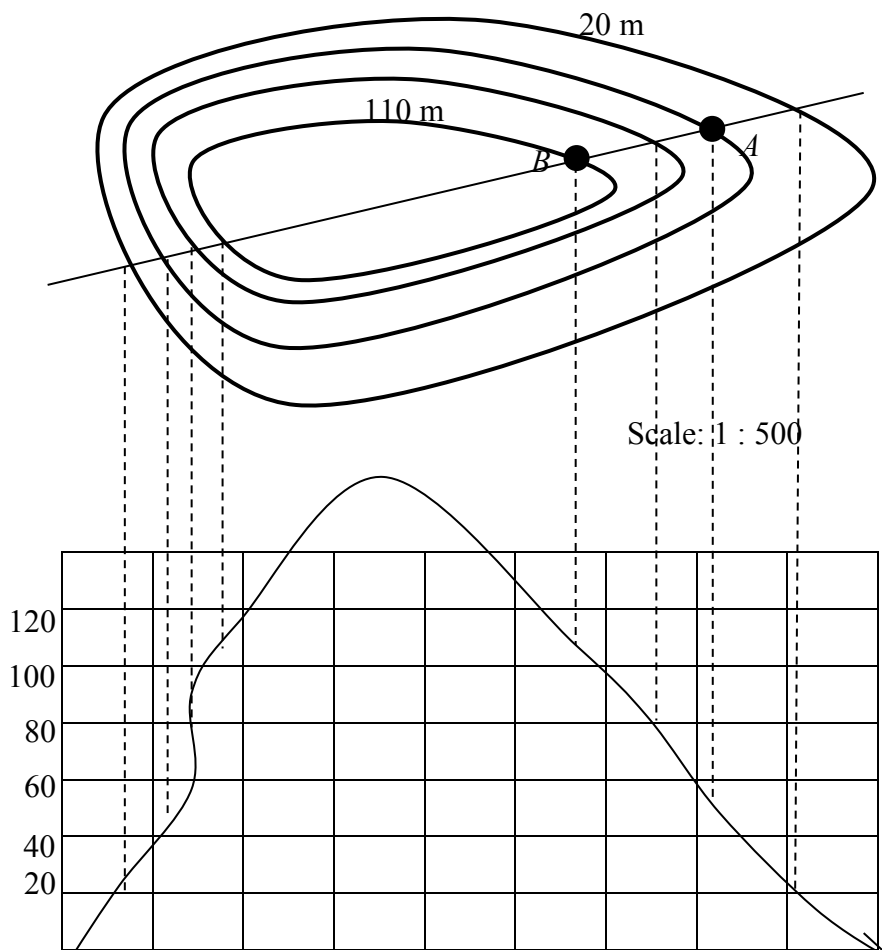
Module 2: Geometry and trigonometry

Question 1

a.
There are 4 contour lines of evenly spaced heights between 20 m and 110 m.
 $110 - 20 = 90$
 $90 \div 3 = 30$
 A is $20 + 30 = 50$ m above sea level.

(1 mark)

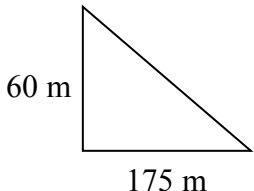
b.



(1 mark)

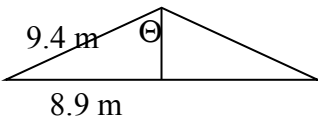
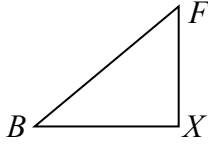
Module 2: Geometry and trigonometry

Question 1 (continued)

c. 1 cm on map = 500 cm in reality. 35 cm on map = $500 \times 35 = 17500$ cm in reality = 175 m. (1 mark)	d.  (1 mark) Gradient = $\frac{\text{rise}}{\text{run}} = \frac{60}{175} = 0.343$
e. $\tan \theta = 0.343$ $\theta = \tan^{-1}(0.343)$ $\theta = 19^\circ$ (1 mark)	

Module 2: Geometry and trigonometry

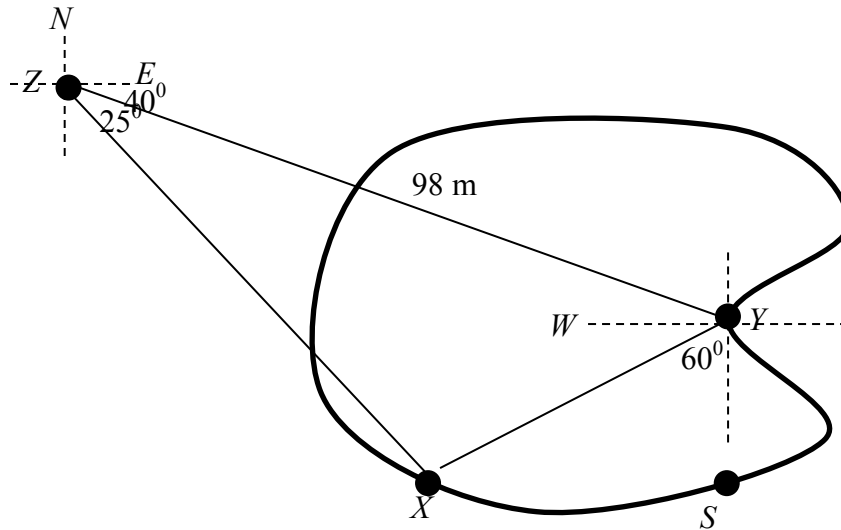
Question 2

a. $HG = 25.8 - 3 - 17.8 = 5 \text{ m}$ (1 mark)	b. $\tan 62^\circ = \frac{BJ}{3}$ $BJ = 5.64 \text{ m}$ (1 mark)
c.  $\sin \Theta = \frac{8.9}{9.4}$ $\Theta = \sin^{-1}\left(\frac{8.9}{9.4}\right) = 71.228$ $2\Theta = 142.5^\circ = \angle CDE$ (1 mark)	d.  $\tan 68^\circ = \frac{FX}{5}$ $FX = 12.375$ $FX = 12.375 - 5.64 = 6.735$ $XB = 17.8$ $FB = \sqrt{17.8^2 + 6.735^2} = 19 \text{ m.}$ (1 mark)
e. Height of triangle $CDE = \sqrt{9.4^2 - 8.9^2} = 3.02$ Area = Area of $\triangle ABJ$ + Area of $\triangle FGH$ + Area of $\triangle CDE$ + Area of $\square CEHJ$ $\text{Area} = \frac{3 \times 5.64}{2} + \frac{5 \times 12.375}{2} + \frac{1}{2} \times 9.4 \times 9.4 \times \sin 142.5^\circ + 17.8 \times (18.6 - 3.02)$ $\text{Area} = 8.46 + 30.94 + 26.9 + 277.32 = 344 \text{ m}^2$ (1 mark)	

Module 2: Geometry and trigonometry

Question 3

a.



Mark Z somewhere to the left of X and above Y.

(1 mark)

b.

$$\begin{aligned}\angle EZY &= 130 - 90 = 40^\circ \\ \angle NZX &= 155^\circ \\ \angle XZY &= 155 - 90 - 40 = 25^\circ\end{aligned}$$

(1 mark)

c.

$$\begin{aligned}\angle SYX &= 240 - 180 = 60^\circ \\ \angle WYX &= 90 - 60 = 30^\circ \\ \angle WYZ &= 40^\circ \text{ (alternate angles are equal)} \\ \angle ZYX &= \angle WYZ + \angle WYX = 40 + 30 = 70^\circ\end{aligned}$$

(1 mark)

Module 2: Geometry and trigonometry

Question 3 (continued)

d.

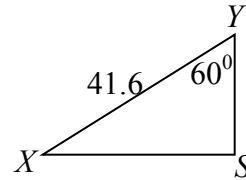
$$\angle ZXY = 180 - (70 + 25) = 85$$

$$\frac{XY}{\sin 25^\circ} = \frac{98}{\sin 85^\circ}$$

$$XY = 41.6 \text{ m}$$

(1 mark)

b.



$$\sin 60^\circ = \frac{XS}{41.6}$$

$$XS = 36$$

2000 m in 60 min

$$36 \text{ m in } \frac{60 \times 36}{2000} = 1.08 \text{ min.}$$

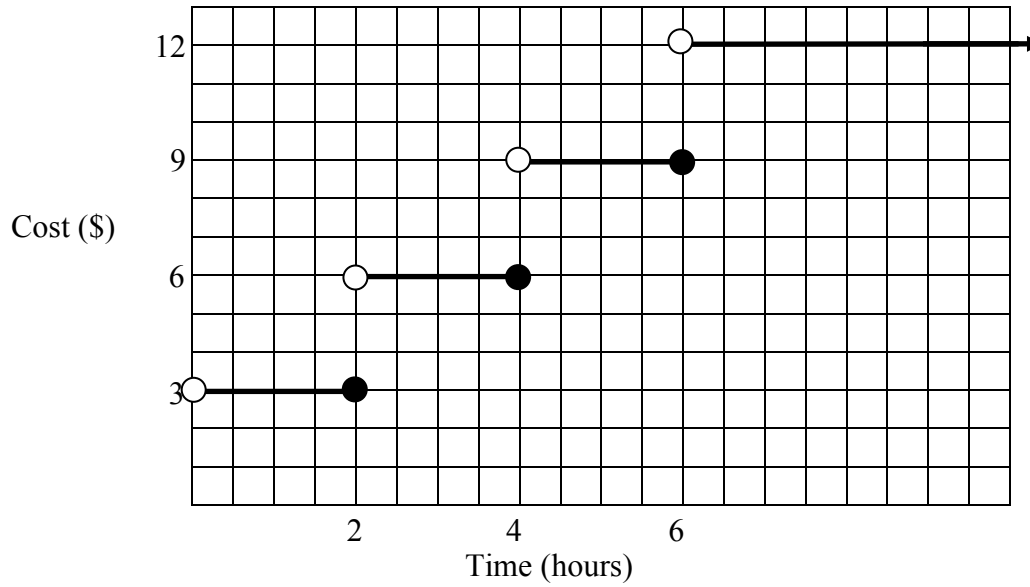
1 minute to the nearest minute

(1 mark)

Module 3: Graphs and relations

Question 1

a.



(1 mark)

b. \$3 (1 mark)	c. \$12 - \$3 = \$9 (1 mark)
d. $\begin{cases} C = 3, 0 < t \leq 2 \\ C = 6, 2 < t \leq 4 \\ C = 9, 4 < t \leq 6 \\ C = 12, t > 6 \end{cases}$ (2 marks)	

Module 3: Graphs and relations

Question 2

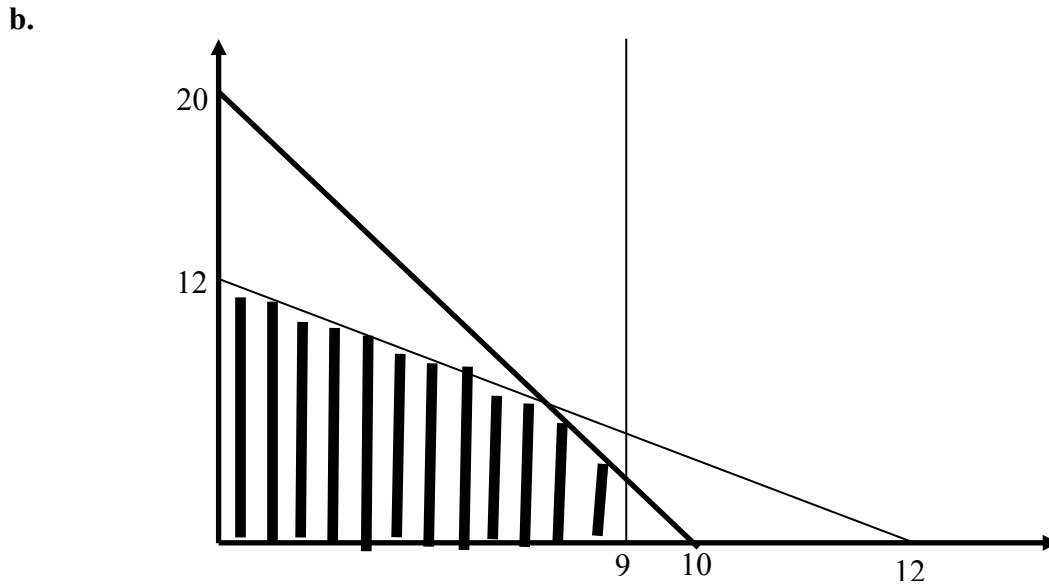
a. From 6:30 till 6:45 = 15 minutes. (1 mark)	b. 50 km (1 mark)
c. $\frac{50 - 30}{\frac{1}{2}} = 40 \text{ km/hr}$ (1 mark)	
d. Distance petrol station is from home = $50 - 45 = 5$ Speed from car park to petrol station = $\frac{5 - 50}{1\frac{1}{2}} = 30 \text{ km/hr homewards}$ (1 mark) Speed from petrol station to home = $2 \times 30 = 60 \text{ km/hr homewards}$ $60 = \frac{5}{x}$ $x = \frac{1}{12} \text{ hr} = 5 \text{ minutes}$ time arrives home = $6:45 + 0:05 = 6:50 \text{ pm}$ (1 mark)	

Module 3: Graphs and relations

Question 3

a.
 $30x + 15y \leq 300$
 $\Rightarrow 2x + y \leq 20$

(1 mark)



(1 mark)

c.

(1 mark)

d.
 $P = 3000x + 2500y$
 Solving $x + y = 12$ and $2x + y = 20$ gives the point (8, 4)
 Solving $x = 9$ and $x + y = 12$ gives the point (9, 3)
 Other corner points are (0, 12) and (9, 0).
 Profit at (0, 12) = \$30000
 Profit at (8, 4) = \$34000
 Profit at (9, 3) = \$34500
 Profit at (9, 0) = \$27000
 Maximum profit = \$34500

(1 mark)

e.
 If profit \$30000, tax = 28% of \$30000 = \$8400
 After tax profit = \$30000 – \$8400 = **\$21600**
 If profit \$34000, tax = 28% of \$30000 + 30% of \$2000 + 45% of \$2000 = \$8400 + \$600 + \$900 = \$9900.
 After tax profit = \$34000 – \$9900 = **\$24100**
 If profit \$34500, tax = 28% of \$30000 + 30% of \$2000 + 45% of \$2500 = \$8400 + \$600 + \$1125 = \$10125.
 After tax profit = \$34500 – \$10125 = **\$24375**
 If profit \$27000, tax = 28% of \$27000 = \$7560
 After tax profit = \$27000 – \$7560 = **\$19440**
 The best after tax profit occurs when Ali makes a total profit of \$34500. This occurs when he makes 9 Alpha cars and 3 Beta cars in a week.

(1 mark)

Module 4: Business-related mathematics

Question 1

a. $100\% = 32$ $1\% = 32 \div 100$ $120\% = 32 \div 100 \times 120 = \\38.40 (1 mark)	b. $120\% = 180$ $1\% = 180 \div 120$ $100\% = 180 \div 120 \times 100 = \\150 (1 mark)
c. Normal selling price: $90\% = 270$ $1\% = 270 \div 90$ $100\% = 270 \div 90 \times 100 = \\300 Buying Price: $120\% = 300$ $1\% = 300 \div 120$ $100\% = 300 \div 120 \times 100 = \\250 (1 mark)	d. Buying price = \$500 Selling Price: $100\% = 500$ $1\% = 500 \div 100$ $120\% = 500 \div 100 \times 120 = \\600 Sale Price: $100\% = 600$ $1\% = 600 \div 100$ $90\% = 600 \div 100 \times 90 = \\540 Selling Price to Glenda: $100\% = 540$ $1\% = 540 \div 100$ $95\% = 540 \div 100 \times 95 = \\513 Profit = $513 - 500 = \\$13$ (1 mark)

Module 4: Business-related mathematics

Question 2

a. i. $1850 - x = 1630$ $x = \\$220$ (1 mark)	a. ii. Balance = $3310 - 440 = \\$2870$ (1 mark)
b.(i) The minimum balance for March is \$1630. This is the balance in the account on 1st and 2nd of March. (1 mark)	b. (ii) Feb interest = $1400 \times \frac{5.6}{12 \times 100} = \\6.53 March interest = $1630 \times \frac{5.6}{12 \times 100} = \\7.61 April interest = $1770 \times \frac{5.6}{12 \times 100} = \\8.26 Total interest = $\\$6.53 + \\$7.61 + \\$8.26$ = \$22.40 to nearest 10 cents. (1 mark)

Question 3

a. i. Use TVM solver N = 120 I = 6.3 PV = 350000 - 80000 FV = 0 P/Y = 12 C/Y = 12 This gives PMT = \$3038.39 (1 mark)	a.ii Total amount paid = $3038.39 \times 120 = 364606.80$ Interest = $364606.80 - 270000 = \\$94606.80$ (1 mark)
b. i Use TVM solver N = 24 I = 6.3 PV = 270000 PMT = -3038.39 P/Y = 12 C/Y = 12 This gives FV = \$228657	b. i. (continued) I = 6.3 PV = 228657 PMT = -3838.39 FV = 0 P/Y = 12 C/Y = 12 This gives N = 71.6 months. Hence 72 more months. (1 mark)

Module 4: Business-related mathematics

Question 3 (continued)

b. ii.

Total amount paid

$$= 24 \times 3038.39 + 71.6 \times 3838.39$$

$$= 347750.084$$

$$\text{Interest} = 347750.084 - 270000 = \$77750.09$$

Difference in interest paid

$$= 94606.80 - 77750.09 = \$16857$$

(1 mark)

Question 4

a. i.

$$Q = \frac{PR}{100}$$

$$3000 \times 12 = \frac{P \times 5.8}{100}$$

$$P = \$620690$$

(1 mark)

a. ii.

None of the \$620690 will be used up as Tran will only be using the interest.

(1 mark)

b.

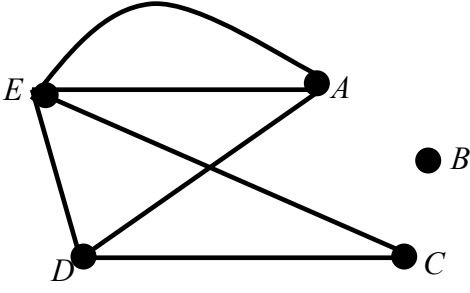
$$620690 = P \left(1 + \frac{7.5}{100} \right)^{30}$$

$$P = \$70896$$

(1 mark)

Module 5: Networks and decision mathematics

Question 1

a. No town is connected to itself by a road. (1 mark)	b. B is not connected to any other town by a road. (1 mark)
c.  $\frac{1}{2}$ mark for each road and vertex (2 marks)	
d. The road could be built between either B and A or B and D. Obviously B has to be connected to the rest of the road system. To run each road once is an Euler path. This means there can only be 2 odd vertices. B will be one of the odd vertices. A and D are odd at the moment, so by connecting the road from B to either one of these will leave just 2 odd vertices. (1 mark)	

Module 5: Networks and decision mathematics

Question 2

a.

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
Jack	0	11	10	0	2
Kate	4	11	0	10	4
Len	0	2	7	2	6
Maya	4	7	9	0	6
Nina	0	7	5	4	0

(1 mark)

b.

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
Jack	0	11	10	0	2
Kate	4	11	0	10	4
Len	0	11	7	2	6
Maya	4	2	9	0	6
Nina	0	7	5	4	0

4 lines.

(1 mark)

c.

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
Jack	0	$11 - 2 = 9$	10	0	2
Kate	4	$11 - 2 = 9$	0	10	4
Len	0	$2 - 2 = 0$	7	2	6
Maya	4	$7 - 2 = 5$	9	0	6
Nina	0	$7 - 2 = 5$	5	4	0

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
Jack	0	9	10	0	2
Kate	4	9	0	10	4
Len	0	0	7	2	6
Maya	4	5	9	0	6
Nina	0	5	5	4	0

(1 mark)

Module 5: Networks and decision mathematics

Question 2 (continued)

d.

Town	Guide
<i>A</i>	Jack
<i>B</i>	Len
<i>C</i>	Kate
<i>D</i>	Maya
<i>E</i>	Nina

Kate must do *C* and Maya must do *D*.

This leaves Jack to do *A* and Len to do *B* and Nina to do *E*.

(1 mark)

e.

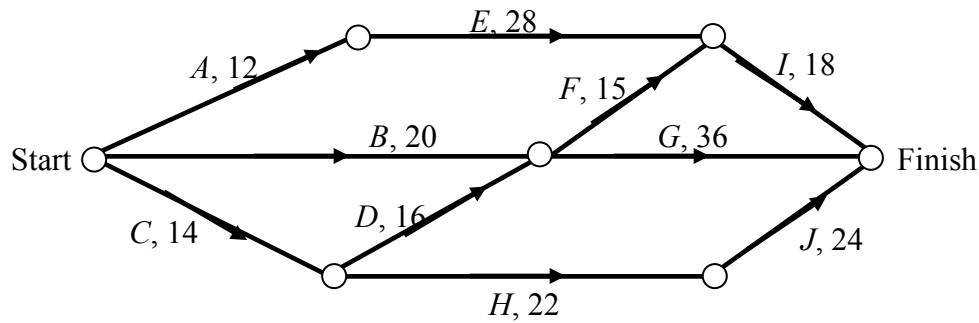
Total time = $7 + 10 + 8 + 10 + 9 = 44$ minutes.

(1 mark)

Module 5: Networks and decision mathematics

Question 3

a.



(1 mark)

b. The critical path is the longest path. $C + D + G = 14 + 16 + 36 = 66$ Critical path is C - D - G (1 mark)	c. $14 + 16 + 36 = 66$ weeks. (1 mark)
d. I cannot start until C, D and F are finished. This will be $14 + 16 + 15 = 45$ weeks. (1 mark)	e. Earliest completion time for E = 40 weeks Latest completion time for E = $66 - 18 = 48$ weeks. Slack time = $48 - 40 = 8$ weeks. (1 mark)

Module 6: Matrices

Question 1

a. There is no unique solution if $\det \begin{bmatrix} 3 & -2 \\ 6 & p \end{bmatrix} = 0$ $3p + 12 = 0$ $3p = -12$ (1 mark)	b. There is no unique solution but there are an infinite number of solutions because the 2 lines are identical. Hence every point on the line would be a solution. (1 mark)
Question 2 a. This is a 3×4 matrix (1 mark)	b. The number of days for the artist to draw the outline of a portrait. (1 mark)
c. $B = \begin{bmatrix} 50 \\ 45 \\ 60 \\ 32 \end{bmatrix} \begin{matrix} \text{Landscape} \\ \text{Portrait} \\ \text{Still Life} \\ \text{Naive} \end{matrix}$ (1 mark)	d. $\begin{bmatrix} 0.2 & 0.5 & 0.3 & 0.4 \\ 2 & 4 & 5 & 0.9 \\ 12 & 8 & 16 & 20 \end{bmatrix} \begin{bmatrix} 50 \\ 45 \\ 60 \\ 32 \end{bmatrix}$ $= \begin{bmatrix} 63.3 \\ 608.8 \\ 2560 \end{bmatrix}$ (1 mark)
e. \$2560 represents the total cost of the colour component of the four types of painting. (1 mark)	

Module 6: Matrices

Question 3

a. $100 - (70 + 20) = 10\%$ (1 mark)	b. <i>Last Week</i> <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td style="text-align: center;"><i>Landscape</i></td> <td style="text-align: center;"><i>Portrait</i></td> <td style="text-align: center;"><i>Still Life</i></td> </tr> <tr> <td style="text-align: right;"><i>This Week</i></td> <td style="text-align: center;"><i>Landscape</i></td> <td style="text-align: center;"><i>Portrait</i></td> <td style="text-align: center;"><i>Still Life</i></td> </tr> </table> $\begin{bmatrix} 0.9 & 0.1 & 0.2 \\ 0.05 & 0.8 & 0.1 \\ 0.05 & 0.1 & 0.7 \end{bmatrix}$ (1 mark)		<i>Landscape</i>	<i>Portrait</i>	<i>Still Life</i>	<i>This Week</i>	<i>Landscape</i>	<i>Portrait</i>	<i>Still Life</i>
	<i>Landscape</i>	<i>Portrait</i>	<i>Still Life</i>						
<i>This Week</i>	<i>Landscape</i>	<i>Portrait</i>	<i>Still Life</i>						

c.

$\frac{1}{2}$ mark each.

(2 marks)

Module 6: Matrices

Question 3 (continued)

d. $S_0 = \begin{bmatrix} 300 \\ 400 \\ 300 \end{bmatrix}$ (1 mark)	e. 90% of 300 + 80% of 400 + 70 % of 300 = 800 students. (1 mark)
f. $T^3 S_0$ $= \begin{bmatrix} 0.9 & 0.1 & 0.2 \\ 0.05 & 0.8 & 0.1 \\ 0.05 & 0.1 & 0.7 \end{bmatrix}^3 \begin{bmatrix} 300 \\ 400 \\ 300 \end{bmatrix} = \begin{bmatrix} 462 \\ 315 \\ 223 \end{bmatrix}$ Expect 315 students will paint a portrait in the fourth week. (1 mark)	g. $T^{30} S_0$ $= \begin{bmatrix} 0.9 & 0.1 & 0.2 \\ 0.05 & 0.8 & 0.1 \\ 0.05 & 0.1 & 0.7 \end{bmatrix}^{30} \begin{bmatrix} 300 \\ 400 \\ 300 \end{bmatrix} = \begin{bmatrix} 588 \\ 235 \\ 177 \end{bmatrix}$ $T^{50} S_0$ $= \begin{bmatrix} 0.9 & 0.1 & 0.2 \\ 0.05 & 0.8 & 0.1 \\ 0.05 & 0.1 & 0.7 \end{bmatrix}^{50} \begin{bmatrix} 300 \\ 400 \\ 300 \end{bmatrix} = \begin{bmatrix} 588 \\ 235 \\ 177 \end{bmatrix}$ 588 to paint a landscape each week. (1 mark)

End of Suggested Solutions 2013 Further Mathematics VCE Trial Examination 2

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