

2013 VCE Further Mathematics Trial Examination 1 Suggested Answers



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Core: Data analysis

Question 1 C Number of cars exceeding the speed limit $= 8 + 6 + 5 + 2 = 21$	Question 2 D Number of males who are between 16 and 31 $= 284$ Total number of males $= 280 + 284 + 290 + 105 = 959$ Percentage $= \frac{284}{959} \times 100 = 29.6\%$
Question 3 D 139 grams is the lower quartile. 25% are less than the lower quartile so $100 - 25 = 75\%$ are greater than the lower quartile.	Question 4 E IQR $= 169 - 139 = 30$ $1.5 \times 30 = 45$ $169 + 45 = 214$ $139 - 45 = 94$ Any Seville orange with a weight greater than 214 or less than 94 is an outlier result. Hence, E is the only true option.
Question 5 E English $Z = \frac{74 - 60}{7} = 2$ Maths $Z = \frac{69 - 55}{5} = 2.8$ History $Z = \frac{78 - 66}{6} = 2$ French $Z = \frac{77 - 59}{9} = 2$ Chemistry $Z = \frac{65 - 50}{5} = 3$ Since Chemistry has the biggest Z value, Susan did best in Chemistry relative to the rest of the class.	Question 6 B P has quite strong positive correlation and S has quite strong negative correlation, so neither of these would have a correlation between -0.1 and 0.1. Both Q and R have very low correlation. Hence, answer B.
Question 7 D Blood pressure is the dependent variable. $s_y = 18.66$ $s_x = 14.7$ $r = 0.8382$ gradient $= \frac{rs_y}{s_x} = \frac{0.8382 \times 18.66}{14.7} = 1.06$	

Core: Data analysis

Question 8 D SI for Spring = $4 - (1.88 + 0.74 + 0.52) = 0.86$ $0.86 \times 2,400,000 = \\$2,064,000$,	Question 9 A 720 is 2 standard deviations below the mean. 2.5% are less than 2 standard deviations below the mean. 1920 is 3 standard deviations above the mean. 0.15% of light globes are greater than 3 standard deviations above the mean. Hence, percentage greater than 3 standard deviations above the mean or less than 2 standard deviations below the mean = $0.15 + 2.5 = 2.65\%$
Question 10 B The smoothed graph will start in February and end in November, so not D or E. The mean of the first 3 points of the given graph = $(1 + 3 + 4) \div 3 = 2.7$, so not A or C. Hence, B	Question 11 C Smoothing is used on a time series plot to eliminate some variation so that seasonality and trend are more easily identified.
Question 12 C The independent variable is time. This is the x value and has to be squared in the transformation. Hence, not A, B or D. The gradient is 3, which is the coefficient of time^2. Hence, C.	Question 13 D There are 10 points, so put 3 points on each end section and 4 points in the middle. The gradient is obtained from the point in the lower third and the point in the upper third. Median point for lower third is (2,2000) Median point for upper third is (9,5000) $\text{Gradient} = \frac{5000 - 2000}{9 - 2} = \frac{3000}{7} = 428.6$

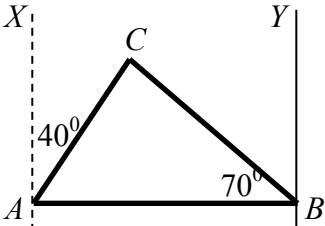
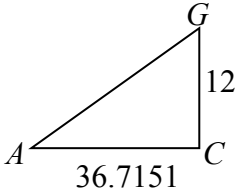
Module 1 Number patterns

Question 1 C When $n = 1$ $t_2 = t_1 + 9 = 6 + 9 = 15$ $t_3 = t_2 + 9 = 15 + 9 = 24$	Question 2 C This is an arithmetic sequence with $a = 7, d = 2$ $t_n = 7 + (n - 1)2 = 1001$ $5 + 2n = 1001$ $2n = 996$ $n = 498$ There are 498 terms
Question 3 B This is the sum of an arithmetic sequence with $a = 4, d = 7$ Number of terms = $\frac{438 - 4}{7} + 1 = 63$ $S_n = \frac{n}{2}(a + l)$ $S_n = \frac{63}{2}(4 + 438) = 13923$	Question 4 D 3,6,12..... This is a geometric sequence with $a = 3, r = 2$ $t_n = ar^{n-1}$ $t_{10} = 3 \times 2^9 = 1536$
Question 5 D $\frac{x}{32} = \frac{\left(11 + \frac{13}{25}\right)}{x}$ Use Cas calculator to get $x = -19\frac{1}{5}$ or $19\frac{1}{5}$	Question 6 E $t_1 = 4$ $t_2 = 3 \times 4$ $t_3 = 3 \times 3 \times 4 = 3^2 \times 4$ $t_n = 3^{n-1} \times 4$

Module 1 Number patterns

Question 7 A $ar = \frac{8}{9}$ $ar^4 = 3$ Use Cas calculator to get $a = \frac{16}{27}, r = 1.5$ $S_n = \frac{a(r^n - 1)}{r - 1} = \frac{\frac{16}{27}(1.5^{15} - 1)}{1.5 - 1} = 517.8$	Question 8 E 30, 22.5, 16.875..... Downward motion of ball = S_∞ with $a = 30, r = \frac{3}{4}$ Upward motion of ball = S_∞ with $a = 22.5, r = \frac{3}{4}$ Total distance travelled by ball $= \frac{30}{1 - \frac{3}{4}} + \frac{22.5}{1 - \frac{3}{4}} = 210 \text{ m}$
Question 9 A Let original population be N Half this population is $0.5 \times N$ Number of elephants after n years = $(1 - 0.03)^n N$ $0.97^n N = 0.5N$ $0.97^n = 0.5$ Use Cas calculator $n = 22.76$	

Module 2 Geometry and trigonometry

Question 1 B $\angle BCE = 180 - 74 = 106$ (angles in a straight line add to 180°) $\angle BAE + 82 + 106 + 90 = 360$ (angles in a quadrilateral add to 360°) $\angle BAE = 360 - 278 = 82^\circ$	Question 2 B  $\angle XAC = 40^\circ$ (since C is $040^\circ T$ from A) $\angle ABC = 340^\circ - (\angle YBZ + \angle ZBA)$ $\angle YBC = 340^\circ - (180 + 90)$ $\angle ABC = 70^\circ$ $\angle BAC = 90 - 40 = 50^\circ$ $\angle ACB = 180 - (50 + 70) = 60^\circ$ (angles in triangle add to 180°)
Question 3 C Surface area of a face = $3 \times 3 = 9$ There are 4 cubes. 3 of the cubes have 5 faces each and one of the cubes has 3 faces that make up the surface area. Total number of faces = $3 \times 5 + 3 = 18$ Surface area of shape = $18 \times 9 = 162 \text{ cm}^2$	Question 4 A B is 30 m high and C is 10 m high so E is incorrect. We do not have enough information to know how far A and C are from B, so D is incorrect. Because we do not know the horizontal distances, we cannot work out the average slope, so not C. When contour lines are close together, the slope is greatest, so the slope increases as we go from A to B.
Question 5 E Area of triangle $= \frac{1}{2} \times 12 \times 14 \times \sin 22^\circ = 31.467$ Area of rectangle = $10 \times 16 = 160$ Area of shaded region = $160 - 31.467 = 128.53 \text{ cm}^2$	Question 6 E $AC = \sqrt{32^2 + 18^2} = 36.7151$  $\tan \angle AGC = \frac{36.7151}{12}$ $\angle AGC = \tan^{-1} \left(\frac{36.7151}{12} \right) = 71.9^\circ$

Module 2 Geometry and trigonometry

Question 7 A

Angles in a regular pentagon =
 $\frac{3 \times 180}{5} = 108^\circ$

In triangle EDC , $ED = DC = 8$

$\angle EDC = 108$

$$EC = \sqrt{(8^2 + 8^2 - 2 \times 8 \times 8 \cos 108^\circ)}$$

$$EC = 12.9$$

Question 8 D

Similar figures

$$\text{Length } 20 : 50 = 2 : 5$$

$$\text{Volume } 2^3 : 5^3 = 8 : 125$$

960 : Volume of cone

$$\text{Volume of cone} = \frac{960 \times 125}{8} = 15000$$

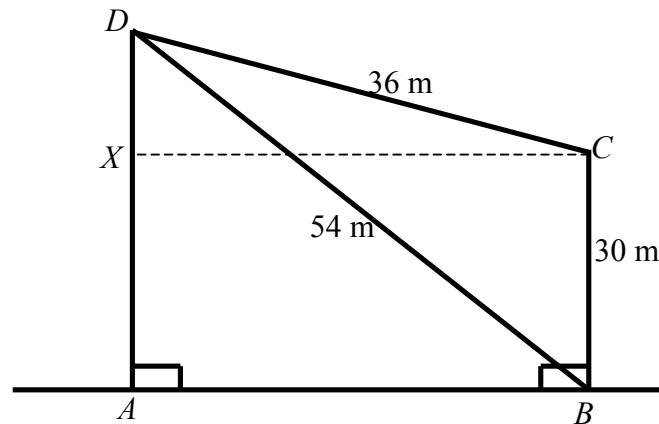
$$\text{Volume of cone} = \frac{1}{3} \pi R^2 H = 15000$$

$$\text{Volume of cone} = \frac{1}{3} \pi R^2 \times 50 = 15000$$

$$R = 16.9$$

$$\text{Diameter} = 2R = 2 \times 16.9 = 33.9 \text{ cm}$$

Question 9 D



In triangle DCB ,

$$54^2 = 36^2 + 30^2 - 2 \times 36 \times 30 \cos \angle DCB$$

$$\angle DCB = 109.47^\circ$$

$$\angle DCB = \angle DCX + \angle XCB$$

$$109.47^\circ = \angle DCX + 90^\circ$$

$$\angle DCX = 19.47^\circ$$

$$\cos 19.47^\circ = \frac{XC}{36}$$

$$XC = 33.9$$

$$AB = XC = 33.9 \text{ m}$$

Module 3 Graphs and relations

Question 1 C $y = 3 - 2x$ When $x = -5, y = 3 + 10 = 13$, so not A. When $x = 5, y = 3 - 10 = -7$, so not E. When $x = 1, y = 3 - 2 = 1$, so not D. When $x = -1, y = 3 + 2 = 5$, so C, not B.	Question 2 D $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 0}{1 - 7} = -1.7$
Question 3 B Let cost of pencil be \$x. Let cost of biro be \$y. $2x + 3y = 18$ $3x + 2y = 17$ Cas calculator gives $x = 3, y = 4$ $4x + y = 4 \times 3 + 4 = \\16	Question 4 E B has an x value of 6 and a y value of 0, (6, 0) D has an x value of 0 and a y value of 3, (0, 3)

Module 3 Graphs and relations

Question 5 E Rate of change for $AB = \frac{10}{5} = 2$ litres/min Rate of change for $BC = 0$ Rate of change for $CD = -\frac{10}{5} = -2$ litres/min Rate of change for $DE = \frac{15}{10} = 1.5$ litres/min Rate of change for $EF = -\frac{15}{5} = -3$ litres/min In the section EF, the volume of water is decreasing by 3 litres per minute, which is the greatest change.	Question 6 A Equation of line is $y = mx + c$ where y is volume and x is time. From the solutions to question 5, $m = -2$ $y = -2x + c$ When $x = 20$, $y = 0$, so $0 = -40 + c$ $c = 40$ Volume = $40 - 2 \times \text{time}$.
Question 7 D When $\frac{1}{x} = 4$, $x = \frac{1}{4}$, $y = 7$ If $y = \frac{7}{x} = 7 \times \frac{1}{x} = 7 \times 4 = 28$ so not A. If $y = \frac{4x}{7} = \frac{4 \times \frac{1}{4}}{7} = \frac{1}{7}$ so not B. If $y = \frac{x}{28} = \frac{1}{4} \times \frac{1}{28} = \frac{1}{112}$ so not C. If $y = \frac{28}{x} = 28 \times \frac{1}{x} = 28 \times 4 = 112$ so not E. If $y = 28x = 28 \times \frac{1}{4} = 7$ so D.	Question 8 A Let them meet after t hours. Jane is $30t$ from her house, and Joe is $160 - 50t$ from Jane's house. $30t = 160 - 50t$ $t = 2$ When $t = 2$, they will be $30 \times 2 = 60$ km from Jane's house.
Question 9 C x chairs, y tables, z desks $x = 2z$ $z = \frac{x}{2}$ $x \leq 8y$ so not A $2x + 3y + z \leq 200$ $2x + 3y + \frac{x}{2} \leq 200 \Rightarrow \frac{5x}{2} + 3y \leq 200$, so not B $6x + 2y + 4z \leq 80 \Rightarrow 6x + 2y + 2x \leq 80 \Rightarrow 8x + 2y \leq 80 \Rightarrow 4x + y \leq 40$ so not E but C. $3x + 5y + 2z \leq 220 \Rightarrow 3x + 5y + x \leq 220 \Rightarrow 4x + 5y \leq 220$, so not D	

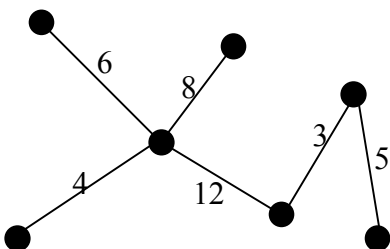
Module 4 Business-related mathematics

Question 1 C $BV = 800(1 - 0.2)^3 = \\$409.60$	Question 2 A $\text{Final value} = 2760 \left(1 + \frac{5}{100 \times 365} \right)^{15} = \\2766
Question 3 E Total amount earned in week = $32 \times 8 \times 5$ $= \\$1280$ Super = 9% of 1280 = $\frac{9 \times 1280}{100} = 115.20$ Hex = 3% of 1280 = $\frac{3 \times 1280}{100} = 38.40$ Remainder of earnings $= 1280 - (115.20 + 38.40) = 1126.40$ Tax = 35% of 1126.40 = $\frac{35 \times 1126.40}{100}$ $= 394.24$ Take home pay = $1126.40 - 394.24$ $= \\$732.16$	Question 4 D Amount repaid = $300 \times 3 \times 12 = \\10800 Interest = $10800 - 8000 = 2800$ $I = \frac{PRT}{100}$ $2800 = \frac{8000 \times R \times 3}{100}$ $R = 11.7\%$
Question 5 C Total cost of chairs = $940 \times 2 = 1880$ Amount owing after deposit = $1880 - 500 = 1380$ Interest = $\frac{1380 \times 8 \times 3}{100} = 331.20$ Amount repaid = $1380 + 331.20 = 1711.20$ Amount repaid per month = $1711.20 \div 36$ $= \\$47.53$	Question 6 E Taking any point on the graph, say, (4000,7000) Final value = Amount invested + interest $7000 = 4000 + \frac{4000 \times R \times 5}{100}$ $R = 15\%$
Question 7 B In 2 years car travels $16080 + 25300 = 41380$ km. Depreciation = $41380 \times \frac{24.5}{100} = \\10138.10 Value of car = $35000 - 10138.10 = \\$24861.90$	

Module 4 Business-related mathematics

Question 8 A Use TVM solver $N = 36$ $I = 10$ $PV = 25000$ $PMT = -1260$ $P/Y = 12$ $C/Y = 12$ This gives $FV = 18940.54851$ $I = 7$ $PV = 18940.54851$ $PMT = -1260$ $FV = 0$ $P/Y = 12$ $C/Y = 12$ This gives $N = 15.8$ months Total time over which repayments are made $= 3 + 1.3$ years $= 4.3$ years	Question 9 C $3161 = 1840 \left(1 + \frac{r}{100} \right)^8$ $r = 7$ $9200 = 1840 \left(1 + \frac{7}{100} \right)^n$ $n = 24$
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Module 5 Networks and decision mathematics

Question 1 D The sum of the degrees is $1 + 3 + 3 + 2 + 4 + 1 + 2 + 2 = 18$	Question 2 D S cannot be reached from T.
Question 3 B For an Eulerian circuit to exist, all the vertices must be of even degree. Only the second of the given shapes have all vertices of even degree.	Question 4 C 2 people study English 2 people study Maths 2 people study Chemistry 3 people study Art 2 people study French Hence, A is true. Ben is the only one who studies Art and French, so B is true. Ann and Eddie study English, Ann and Ben study Maths. Hence, Ann studies both English and Maths, but Eddie and Ben study English or Maths but not both. This is two students, so C is not true. Dan only studies one subject which is the least number studied, so D is true. Cassie studies 2 subjects and ben studies 3, so E is true.
Question 5 C  $3 + 5 + 12 + 4 + 6 + 8 = 38$	Question 6 A Capacity = $2 + 4 = 6$. The 5 and the 7 have already been cut off by the 2 cut.

Module 5 Networks and decision mathematics

Question 7 E

The minimum time for E to start is
 $8 + 9 + 7 = 24$ hours. E takes 4 hours to complete, so $24 + 4 = 28$.
 The minimum time for J to start is
 $28 + 5 + 3 = 36$ hours. J takes another 2 hours, so project takes $36 + 2 = 38$ hours.

Question 8 B

J can start at $38 - 2 = 36$ hours.
 E can start at $36 - 3 - 5 - 4 = 24$ hours.
 C can start 15 hours before this at $24 - 15 = 9$ hours.

Question 9 E

Warehouse					
Factory		<i>P</i>	<i>Q</i>	<i>R</i>	<i>S</i>
	<i>V</i>	30	26	25	27
	<i>W</i>	22	28	18	12
	<i>X</i>	42	30	30	35
	<i>Y</i>	15	36	32	38

Subtract the minimum number in each row from every element in that row

Warehouse					
Factory		<i>P</i>	<i>Q</i>	<i>R</i>	<i>S</i>
	<i>V</i>	30-25	26-25	25-25	27-25
	<i>W</i>	22-12	28-12	18-12	12-12
	<i>X</i>	42-30	30-30	30-30	35-30
	<i>Y</i>	15-15	36-15	32-15	38-15

Warehouse					
Factory		<i>P</i>	<i>Q</i>	<i>R</i>	<i>S</i>
	<i>V</i>	5	1	0	2
	<i>W</i>	10	16	6	0
	<i>X</i>	12	0	0	5
	<i>Y</i>	0	21	17	23

Assign warehouses to factories:

Y → *P*

W → *S*

V → *R*

X → *Q*

Module 6 Matrices

Question 1 B $\begin{bmatrix} -18 & -3 & -12 \\ -24 & 6 & -15 \\ -12 & 0 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 1 & -2 \\ -1 & 3 & 4 \\ 0 & -5 & 2 \end{bmatrix}$ $= \begin{bmatrix} -21 & -2 & -14 \\ -25 & 9 & -11 \\ -12 & -5 & 5 \end{bmatrix}$	Question 2 E $P^3 = P \times P \times P$ $= 2 \times 2$ matrix $\times 2 \times 2$ matrix $\times 2 \times 2$ matrix $= 2 \times 2$ matrix $\times 2 \times 2$ matrix $= 2 \times 2$ matrix, so A can be found. $Q^2 = Q \times Q$ $= 3 \times 3$ matrix $\times 3 \times 3$ matrix $= 3 \times 3$ matrix, so B can be found. $RQ = 2 \times 3$ matrix $\times 3 \times 3$ matrix which equals a 2×3 matrix, so C can be found. $T^3 = T \times T \times T$ $= 3 \times 3$ matrix $\times 3 \times 3$ matrix $\times 3 \times 3$ matrix $= 3 \times 3$ matrix $\times 3 \times 3$ matrix $= 3 \times 3$ matrix, so D can be found. $TR = 3 \times 3$ matrix $\times 2 \times 3$ matrix. Since the number of columns of T does not equal the number of rows of R, this cannot be done.
Question 3 D $\begin{bmatrix} 4 & 6 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 7 & 2 \\ 1 & 4 \end{bmatrix} =$ $\begin{bmatrix} 4 \times 7 + 6 \times 1 & 4 \times 2 + 6 \times 4 \\ -2 \times 7 + 3 \times 1 & -2 \times 2 + 3 \times 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $d = -2 \times 2 + 3 \times 4$	Question 4 B $\begin{bmatrix} 0 & 1 & 1 \\ 3 & 0 & -1 \\ 4 & -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 9 \end{bmatrix}$ Put into calculator $A = \begin{bmatrix} 0 & 1 & 1 \\ 3 & 0 & -1 \\ 4 & -2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 1 \\ 9 \end{bmatrix}$ Evaluate $A^{-1} \times B$ to get $\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ so $y = -3$
Question 5 C 40% is going from X to Y, so 60% is going from X to X. Hence, the element in the first column, first row is 0.6 and the element in the first column second row is 0.4.	Question 6 D The number of roads connecting any 2 towns is either 0 or 1. Hence, not A. There is no road connecting A to E so not B. There is a road connecting A to C so not C. There is no road connecting B to B so not E.

Module 6 Matrices

Question 7 C

$$\begin{bmatrix} 8 & 10 & 6 \\ 5 & 2 & 1 \\ 12 & 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 50 \\ 13 \\ 30 \end{bmatrix}$$

this is 50 units of carbohydrate, 13 units of fat and 30 units of protein

Question 8 E

Let a profit on a gidget, a widget and a midget be a, b and c respectively.

$$\begin{bmatrix} 4 & 3 & 5 \\ 8 & 2 & 6 \\ 7 & 9 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 158000 \\ 220000 \\ 205000 \end{bmatrix}$$

Put into calculator $A = \begin{bmatrix} 4 & 3 & 5 \\ 8 & 2 & 6 \\ 7 & 9 & 4 \end{bmatrix}$

$$B = \begin{bmatrix} 158000 \\ 220000 \\ 205000 \end{bmatrix}$$

Evaluate $A^{-1} \times B$ to get $\begin{bmatrix} 12000 \\ 5000 \\ 19000 \end{bmatrix}$

So the profit on one midget is \$19000

Question 9 A

$$\begin{bmatrix} p + 4q + r \\ 2p - 3q \\ p \end{bmatrix} = \begin{bmatrix} b \\ 2a \\ a \end{bmatrix} \Rightarrow p = a$$

$$2p - 3q = 2a \Rightarrow 2a - 3q = 2a \Rightarrow q = 0$$

$$p + 4q + r = b \Rightarrow a + 0 + r = b \Rightarrow r = b - a$$

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