

Year 2012
VCE
Mathematical Methods
CAS
Solutions
Trial Examination 1



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Question 1

a. Let $y = \sqrt{16 - x^3} = (16 - x^3)^{\frac{1}{2}} = u^{\frac{1}{2}}$ where $u = 16 - x^3$

$$\frac{dy}{du} = \frac{1}{2} u^{-\frac{1}{2}} \quad \frac{du}{dx} = -3x^2 \quad \text{using the Chain rule} \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = -3x^2 \times \frac{1}{2} u^{-\frac{1}{2}} = -\frac{3x^2}{2\sqrt{u}}$$

$$\frac{dy}{dx} = \frac{-3x^2}{2\sqrt{16 - x^3}} \quad \text{A1}$$

b. $\int \frac{x^2}{\sqrt{16 - x^3}} dx = -\frac{2}{3} \sqrt{16 - x^3} \quad \text{A1}$

Question 2

$$4 \cos\left(\frac{\pi x}{3}\right) - 2 = 0 \Rightarrow \cos\left(\frac{\pi x}{3}\right) = \frac{1}{2}$$

$$\frac{\pi x}{3} = 2n\pi \pm \cos^{-1}\left(\frac{1}{2}\right) = 2n\pi \pm \frac{\pi}{3} \quad \text{M1}$$

$$x = 6n \pm 1 \quad \text{where } n \in \mathbb{Z} \quad \text{A1}$$

Question 3

$$f(x) = \log_e(x+3) \quad g(x) = 5 + 2x - x^2$$

$$f(g(x)) = f(5 + 2x - x^2)$$

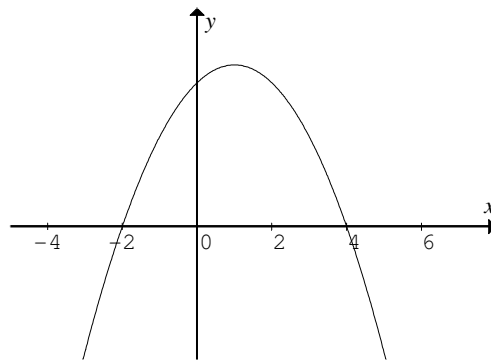
$$f(g(x)) = \log_e(5 + 2x - x^2 + 3)$$

$$f(g(x)) = \log_e(8 + 2x - x^2) \quad \text{A1}$$

$$f(g(x)) = \log_e(-(x^2 - 2x - 8)) = \log_e(-(x-4)(x+2))$$

we require $y = -(x-4)(x+2) > 0$ from the graph

$$-2 < x < 4 = (-2, 4) \quad \text{A1}$$



Question 4

$$x' = 2x + 3y - 4 \quad \text{and} \quad y' = -y + 2 \quad \text{A1}$$

$$\Rightarrow y = 2 - y' \quad \text{M1}$$

$$2x = x' - 3y + 4 = x' - 3(2 - y') + 4 = x' + 3y' - 2$$

$$\text{so } 4x + y = 3 \text{ becomes}$$

$$2(x' + 3y' - 2) + 2 - y' = 3 \text{ drop the dashes}$$

$$2x + 5y = 5 \quad \text{so } a = 2 \quad b = 5 \quad k = 5 \quad \text{A1}$$

Question 5

$$\text{i.} \quad q = \frac{10p}{p-10}$$

$$\frac{dq}{dp} = \frac{10(p-10) - 1(10p)}{(p-10)^2} \quad \text{using the quotient rule} \quad \text{M1}$$

$$\frac{dq}{dp} = \frac{-100}{(p-10)^2}$$

$$\text{ii.} \quad p = 15 \quad \Delta p = 0.1 \quad \Delta q = ?$$

$$\frac{dq}{dp} \approx \frac{\Delta q}{\Delta p} \quad \text{M1}$$

$$\Rightarrow \Delta q \approx \left. \frac{dq}{dp} \right|_{p=15} \Delta p$$

$$\Delta q = \frac{-100}{5^2} \times 0.1 = -0.4 \text{ cm} \quad \text{or} \quad 0.4 \text{ cm decrease} \quad \text{A1}$$

$$\text{iii.} \quad \frac{dp}{dt} = 0.25 \text{ cm/sec} \quad \text{and} \quad \frac{dq}{dt} = \frac{dq}{dp} \cdot \frac{dp}{dt} \quad \text{M1}$$

$$\frac{dq}{dt} = \frac{-100}{(p-10)^2} \times 0.25 \quad \text{when } p = 15$$

$$\left. \frac{dq}{dt} \right|_{p=15} = \frac{-100}{5^2} \times 0.25 = -1 \text{ cm/sec} \quad \text{or} \quad 1.0 \text{ cm/sec decrease} \quad \text{A1}$$

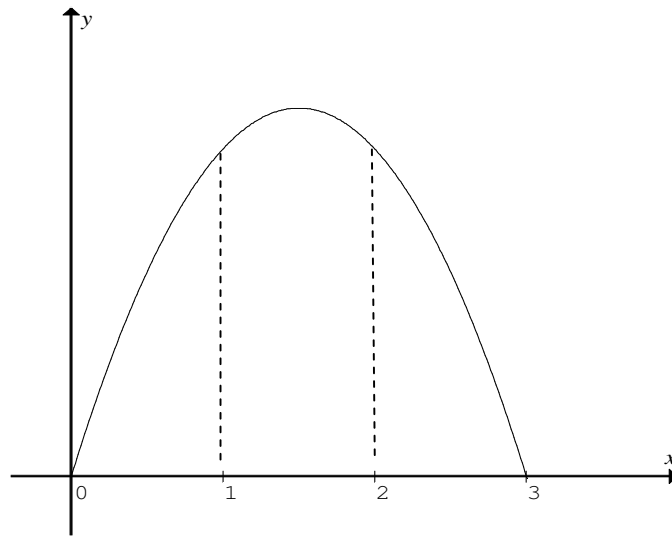
Question 6

i. $k \int_0^3 x(3-x) dx = k \int_0^3 (3x - x^2) dx = 1$

$$k \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^3 = k \left(\frac{27}{2} - 9 - 0 \right) = \frac{9k}{2} = 1$$

M1

$$k = \frac{2}{9}$$



ii. Now $\Pr(2 < X < 3) = \Pr(0 < X < 1)$ by symmetry

$$\Pr(0 < X < 1) = \frac{2}{9} \int_0^1 (3x - x^2) dx$$

$$= \frac{2}{9} \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{2}{9} \left(\frac{3}{2} - \frac{1}{3} - 0 \right)$$

M1

$$= \frac{7}{27}$$

$$\Pr(X > 2 | X > 1) = \frac{\Pr(X > 2)}{\Pr(X > 1)} = \frac{\Pr(2 < X < 3)}{1 - \Pr(X < 1)} = \frac{\Pr(0 < X < 1)}{1 - \Pr(X < 1)}$$

$$\Pr(X > 2 | X > 1) = \frac{\frac{7}{27}}{1 - \frac{7}{27}} = \frac{\frac{7}{27}}{\frac{20}{27}}$$

M1

$$\Pr(X > 2 | X > 1) = \frac{7}{20}$$

A1

Question 7

- i. $\sum \Pr(X = x) = \log_8(k+1) + \log_8(2k-4) = 1$
 $\log_8((k+1)(2k-4)) = 1$
 $(k+1)(2k-4) = 8$ M1
 $2k^2 - 2k - 4 = 8$
 $2k^2 - 2k - 12 = 0$
 $2(k^2 - k - 6) = 0$ M1
 $2(k-3)(k+2) = 0$
 $k = 3$ or $k = -2$ but $k > 2$
 $k = 3$ only A1
- ii. $E(X) = \log_8(k+1) + 2\log_8(2k-4)$ with $k = 3$
 $E(X) = \log_8(4) + 2\log_8(2) = \log_8(4) + \log_8(4) = \log_8(16) = m$ M1
 $8^m = 16 \quad (2^3)^m = 2^4 \Rightarrow 3m = 4$
 $m = E(X) = \frac{4}{3}$ A1

Question 8

- $y = f(x) = a(x-h)^3 + k$
 since there is a stationary point at $x = 2 \Rightarrow h = 2$
 crosses the x -axis at $x = 4 \quad f(4) = 0 \Rightarrow 8a + k = 0$ so $k = -8a$ A1
 $y = f(x) = a(x-2)^3 - 8a = a((x-2)^3 - 8)$
 $A = a \int_0^4 ((x-2)^3 - 8) dx = 64$ M1
 $A = a \left[\frac{1}{4}(x-2)^4 - 8x \right]_0^4 = a \left(\left(\frac{1}{4}(2^4) - 32 \right) - \left(\frac{1}{4}(-2)^4 - 0 \right) \right) = -32a = 64$ M1
 $a = -2 \quad k = 16 \quad h = 2$ A1

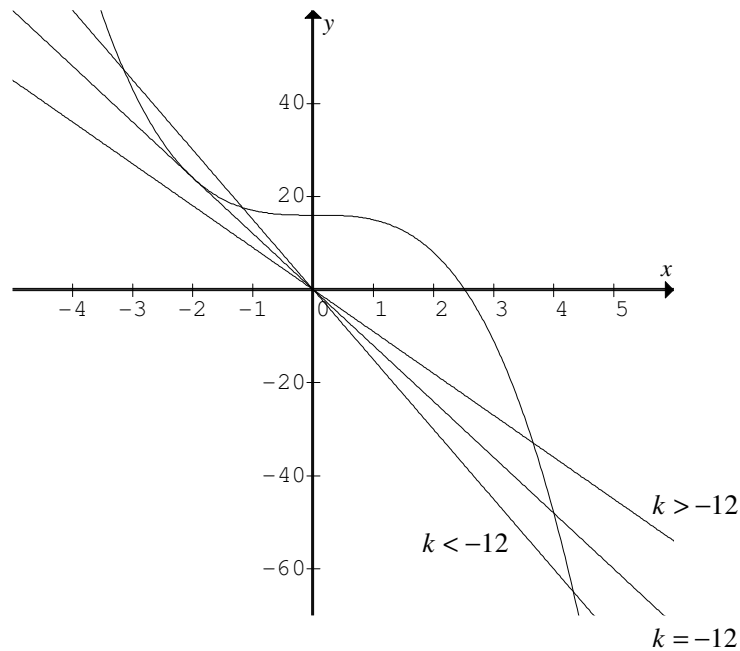
Question 9

$$y = 16 - x^3 \text{ at the point } (a, f(a)) = (a, 16 - a^3) \quad \frac{dy}{dx} = -3x^2 \quad m_T = -3a^2$$

$$-3a^2 = \frac{16 - a^3 - 0}{a - 0} \quad \Rightarrow 16 - a^3 = -3a^3 \quad \text{M1}$$

$$\text{at the point of contact } -2a^3 = 16 \quad a^3 = -8 \quad \text{so } a = -2 \quad \text{A1}$$

$$\text{The tangent is } y = -12x \quad \text{A1}$$



i. one solution $k > -12$

ii. two solutions $k = -12$

iii. three solutions $k < -12$

A2

Question 10

$$v(t) = \frac{24}{\sqrt{4t+9}}$$

$$s = \int_0^4 \frac{24}{\sqrt{4t+9}} dt$$

$$s = \frac{24}{4} \times 2 \left[\sqrt{4t+9} \right]_0^4 \quad \text{A1}$$

$$s = 12(\sqrt{25} - \sqrt{9})$$

$$s = 24 \text{ metres.} \quad \text{A1}$$

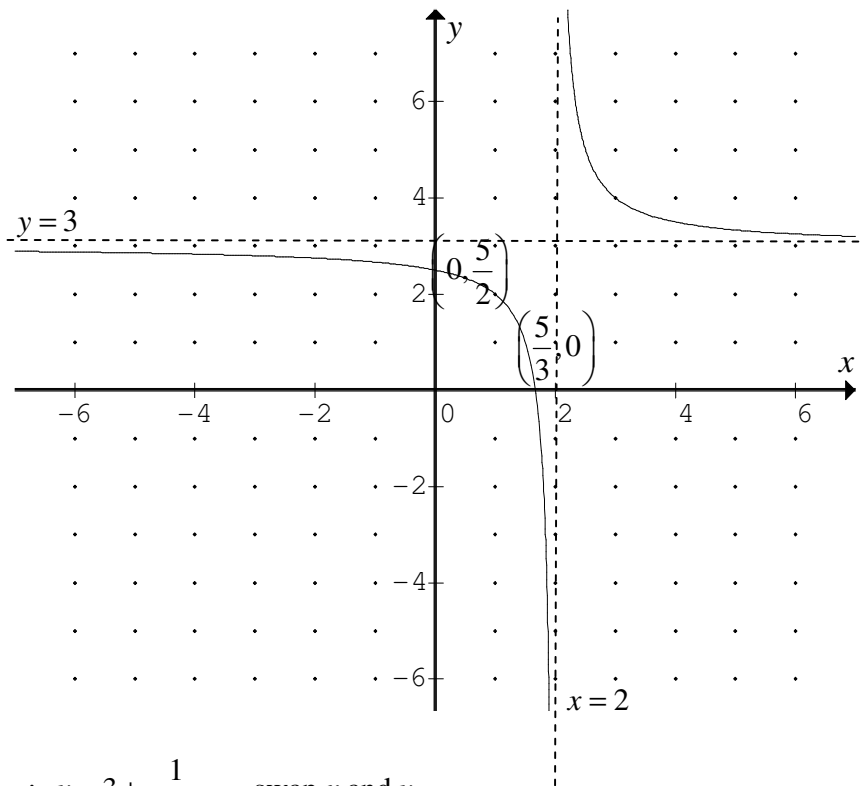
Question 11

i. $f(x) = \frac{3x-5}{x-2} = \frac{3(x-2)+1}{x-2} = 3 + \frac{1}{x-2}$ A1

ii. crosses the x-axis $y=0 \Rightarrow 3x-5=0 \Rightarrow x=\frac{5}{3} \left(\frac{5}{3}, 0\right)$

crosses the y-axis $x=0 \Rightarrow y=\frac{5}{2} \left(0, \frac{5}{2}\right)$ A1

the line $x=2$ is a vertical asymptote and
the line $y=3$ is a horizontal asymptote. A1



iii. $f : y = 3 + \frac{1}{x-2}$ swap x and y
 $f^{-1} : x = 3 + \frac{1}{y-2} \Rightarrow \frac{1}{y-2} = x-3$
 $y-2 = \frac{1}{x-3} \quad y = f^{-1}(x) = 2 + \frac{1}{x-3}$ A1

must state the domain of the function, $\text{dom } f^{-1} = \text{ran } f = \mathbb{R} \setminus \{3\}$

$f^{-1} : \mathbb{R} \setminus \{3\} \rightarrow \mathbb{R} \quad f^{-1}(x) = 2 + \frac{1}{x-3}$ A1

END OF SUGGESTED SOLUTIONS