

2007

VCE Further Mathematics Trial Examination 2

Suggested Solutions

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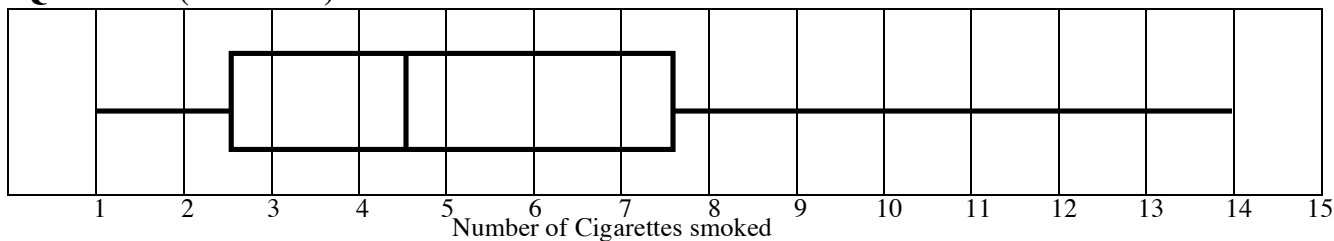
Question 1 a.

	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Total
Smoke	4	16	21	36	39	50	166
Do not Smoke	71	59	54	39	36	25	284

$\frac{1}{2}$ mark for each correct value

b. $\frac{93}{150} \times 100 = 62\%$ (1 mark)	c. Smoking/non smoking is a categorical variable (1 mark)
Question 2 a. On graphics calculator enter data for number of cigarettes smoked for 12 children in Stat Edit in L_1. Go to Stat calc 1 Var Stats L_1 Enter and read off the value for the median $Med=4.5$ $\left(\frac{1}{2} \text{ mark}\right)$	b. Using the same screen on the calculator read off Q_1, the lower quartile = 2.5 $\left(\frac{1}{2} \text{ mark}\right)$
c. Using the same screen on the calculator read off Q_3, the upper quartile = 7.5 $\left(\frac{1}{2} \text{ mark}\right)$	d. Interquartile range = $Q_3 - Q_1 = 7.5 - 2.5 = 5$ $\left(\frac{1}{2} \text{ mark}\right)$
e. $1.5 \times 5 = 7.5$ $Q_1 - 7.5 < 0$ $Q_3 + 7.5 = 15$ No data lies inside the range $0 < x < 15$ $\therefore$ no outliers (1 mark)	

Question 1 (continued) f.



(1 mark)

g. 75%

(1 mark)

Question 3

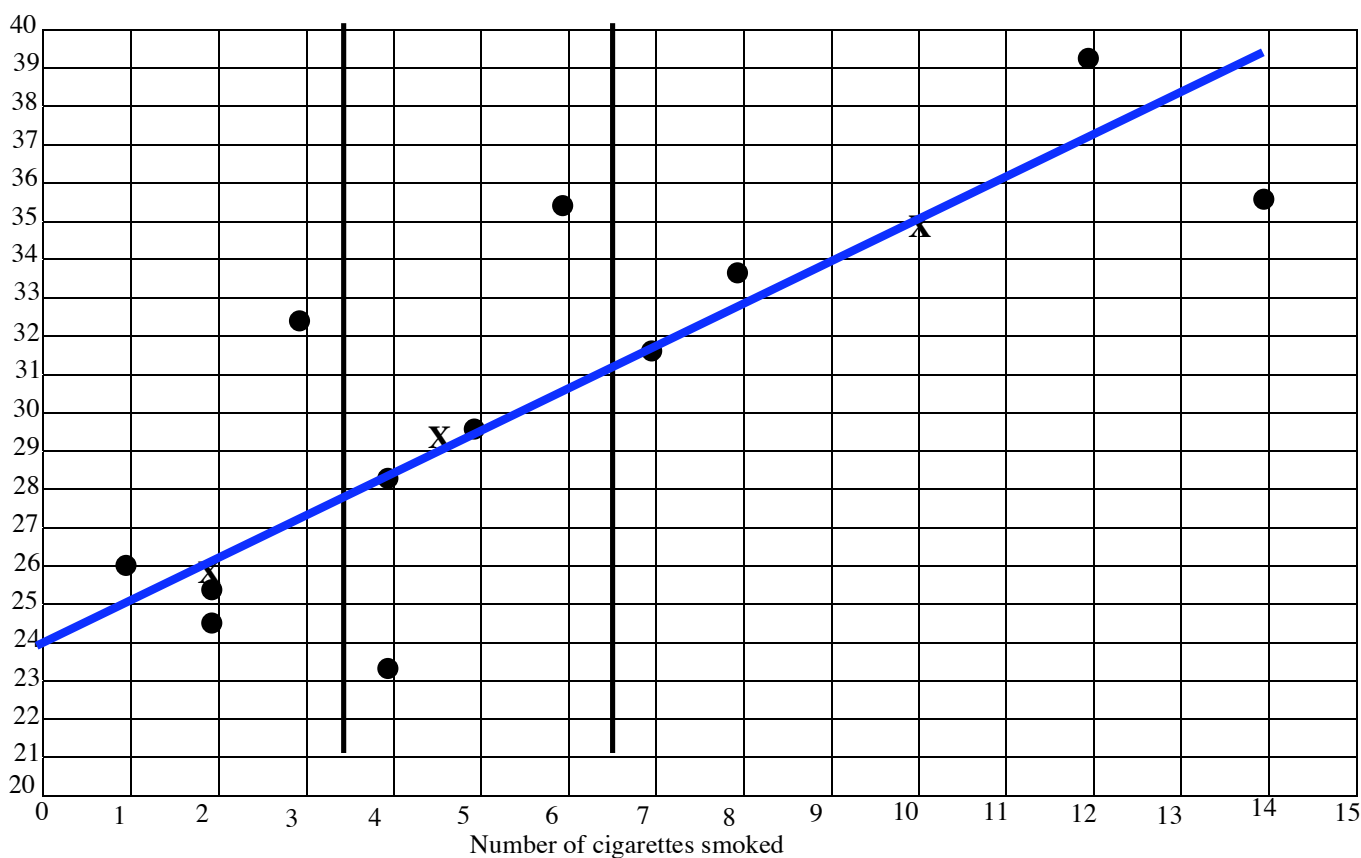
a. On graphics calculator enter data for the time taken to run 200 m for 12 children in Stat Edit in L_2 . Go to Stat calc 1 Var Stats L_2 Enter and read off the value for the mean, 30.5 and the standard deviation, 5.0

(1 mark)

b. On graphics calculator go to 2nd Dist normal cdf Enter -1, 2.5) enter. this gives $0.835 = 83.5\%$

(1 mark)

c.



(1 mark)

Question 3 (continued)

d.

$$\text{gradient} = \frac{y_u - y_l}{x_u - x_l}$$

$$x_l = 2$$

$$y_l = \frac{25.4 + 26}{2} = 25.7$$

$$x_u = \frac{8 + 12}{2} = 10$$

$$y_u = \frac{33.9 + 35.6}{2} = 34.75 \quad (1 \text{ mark})$$

$$\text{gradient} = \frac{34.75 - 25.7}{10 - 2}$$

$$= 1.13 \text{ to two decimal places. } (1 \text{ mark})$$

e.

Put ruler on line joining points (2, 25.7) and (10, 34.75). Move ruler one third of the way towards (4.5, 29.05).

This is

$$x_m = \frac{4 + 5}{2} = 4.5$$

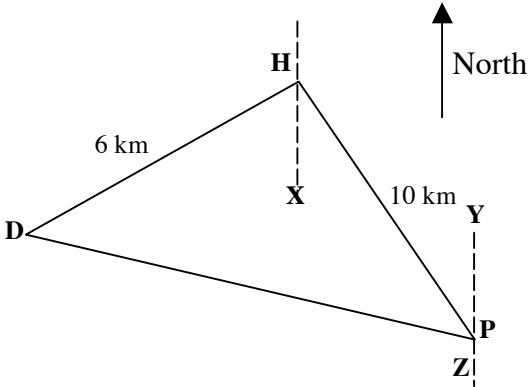
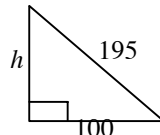
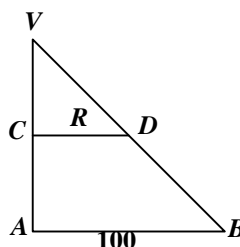
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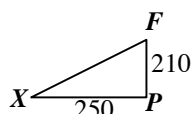
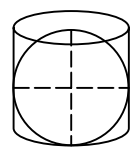
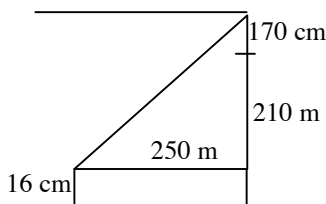
$$y_m = \frac{28.3 + 29.8}{2} = 29.05$$

(1 mark)

Question 1 a. Customers increase by 30 each year. $2004 = 140 + 30 = 170$ $2005 = 170 + 30 = 200$ (1 mark)	b. Arithmetic sequence $t_n = a + (n - 1)d$ $t_{11} = 80 + 10 \times 30$ $t_{11} = 80 + 300$ $t_{11} = 380$ or use graphics calculator in sequence mode. y = enter nMin = 1 $m_n = 80 + (n - 1)30$ $m(nMin) = 80$ Press 2nd table Go to $n = 11$, to find $m(n) = 380$ (1 mark)
c. Use sequence mode y = nMin = 1 $\mu_n = 80 + (n - 1)30$ $\mu(nMin) = 80$ Press 2nd table Go to $\mu_n = 500$, to find $n = 15$ Year = 2015 (1 mark)	d. From the table in (c) Number of customers in 2009 = 320 Number of customers in 2019 = 620 $S_n = \frac{n}{2}(a + l) = \frac{11}{2}(320 + 620) = 5170$ (1 mark)
Question 2 a. $4\% = 0.04$ $3.00 + 0.04 \times 3.00 = \\3.12 (1 mark)	e. $N = 800 - 40d$ (1 mark)
Question 2 b. This is a geometric sequence with $r = 1.04$ $a = 3.00$ $C_n = 3.00(1.04)^{n-1}$ (1 mark)	b. This is a geometric sequence with $r = 1.04$ $a = 3.00$ $C_n = 3.00(1.04)^{n-1}$ (1 mark)

Question 2 c. $t_{10} = 3(1.04)^9$ $t_4 = 3(1.04)^3$ $t_{10} - t_4 = 3(1.04)^9 - 3(1.04)^3 = 0.8953$ = 90 cents or use graphics calculator sequence mode with equation $m(n) = 3(1.04)^n$ This gives $m(n) = 4.27$ when $n = 10$ and $m(n) = 3.37$ when $n = 4$ $4.27 - 3.37 = 0.90 = 90$ cents. (1 mark)	d. Total cost if she sells one rose per year $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_6 = \frac{3(1.04^{10} - 1)}{(1.04 - 1)} = 36.01832137$ (1 mark) total cost for 8000 per year $= 8000 \times 36.01832137 = \$288,146.57$ (1 mark)
Question 3 a. $a = 1 + \frac{20}{100} = 1.2$ (1 mark)	b. A minus sign is necessary because overheads decrease the profit. - 10,000 (1 mark)
c. Use graphics calculator in sequence mode. y = enter $nMin = 1$ $\mu(n) = 1.2\mu(n - 1) - 10000$ $\mu(nMin) = 65000$ Press 2nd table Go to $n = 8$, to find $\mu(n) = 103748$ Profit = \$103,748 (1 mark)	d. Equation is now $\mu_n = 1.2\mu(n - 1) - 30000$ (1 mark) Use graphics calculator in sequence mode. y = enter $nMin = 1$ $\mu(n) = 1.2\mu(n - 1) - 30000$ $\mu(nMin) = 103748$ Press 2nd table and look for the first negative value in $\mu(n)$. 8 years later she will make her first loss. This is the year 2016. (1 mark)

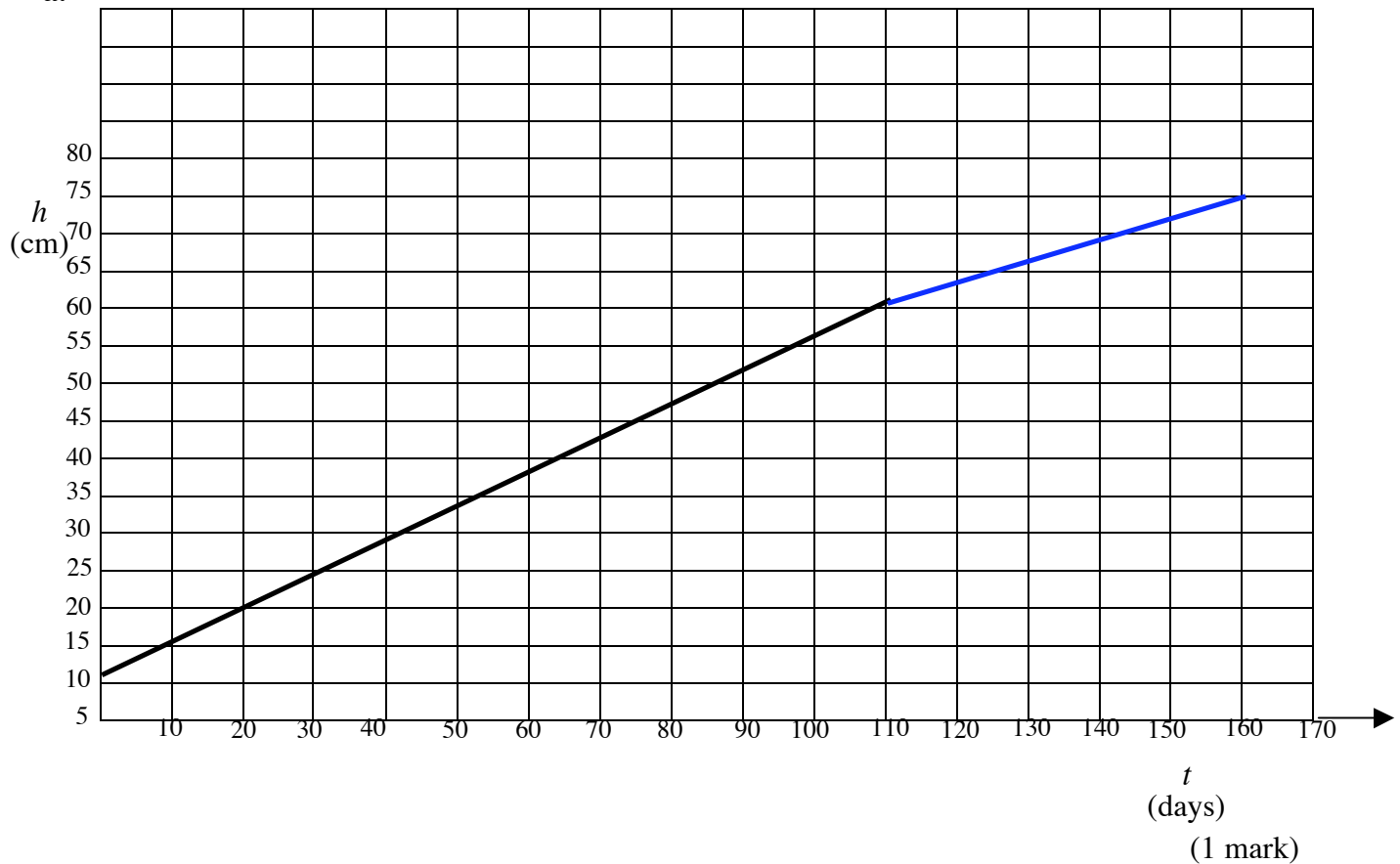
Question 1 a.  $\angle DHX = 40^\circ$ $\angle XHP = \angle HPY = 30^\circ$ (alternate angles) $\therefore \angle DHP = 40^\circ + 30^\circ = 70^\circ$ (1 mark)	b. Using cosine rule $DP^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \cos 70^\circ$ $DP = \sqrt{6^2 + 10^2 - 2 \times 6 \times 10 \cos 70^\circ}$ $DP = 9.74 \text{ km}$ (1 mark) c. $\frac{\sin \angle HPD}{6} = \frac{\sin 70^\circ}{9.74461845}$ $\sin \angle HPD = \frac{6 \sin 70^\circ}{9.74461845}$ $\angle HPD = \sin^{-1} \left(\frac{6 \sin 70^\circ}{9.74461845} \right) = 35^\circ \text{ (1 mark)}$ $\angle DPZ = 360 - (30 + 35) = 295^\circ T \text{ (1 mark)}$
d. $\text{Area} = \frac{1}{2} bc \sin A$ $\text{Area} = \frac{1}{2} \times 6 \times 10 \sin 70^\circ = 28.19 \text{ km}^2$ (1 mark)	Question 2 a.  $h^2 = 195^2 - 100^2$ $h = \sqrt{195^2 - 100^2}$ $h = 167.4 \text{ m to one decimal place}$ (1 mark)
b. Triangles VCD and VAB are similar AAA $\therefore \frac{R}{100} = \frac{3}{4}$ $\therefore R = 75 \text{ m.}$ (1 mark)	c.  (1 mark)

Question 2 (continued) c. Original volume = $\frac{1}{3}\pi r^2 h$ Original volume = $\frac{1}{3}\pi \times 100^2 \times 167.4$ = 1753008.701 Volume removed = $\frac{1}{3}\pi \times 75^2 \times 125.55$ = 739550.5456 (1 mark) Volume of coal remaining = 1753008.701 - 739550.5456 = 1013458.2 m³ (1 mark)	Question 3 a.  $XF^2 = 210^2 + 250^2$ $XF = \sqrt{210^2 + 250^2}$ = 326.5 m (1 mark)
b. Radius of sphere = 8 V of sphere = $\frac{4}{3}\pi r^3$ = $\frac{4}{3}\pi 8^3 = 2144.66 \text{ cm}^3$ (1 mark)  R of cylinder = 8 V of cylinder = $\pi r^2 h = \pi \times 8^2 \times 16 = 3216.99$ Space = $3216.99 - 2144.66 = 1072 \text{ cm}^3$ to the nearest cm³ (1 mark)	c.  (1 mark) Question 3 d. $\tan \theta = \frac{210 + 1.7 - 0.16}{250} = \frac{211.54}{250}$ (1 mark) $\theta = \tan^{-1}\left(\frac{211.54}{250}\right) = 40^\circ 14'$ (1 mark)

Question 1 a. 30 cm. (1 mark)	b. One complete cycle = 4 secs. (1 mark)
c. 30 cm to the right of the rest position. (1 mark)	

Question 2

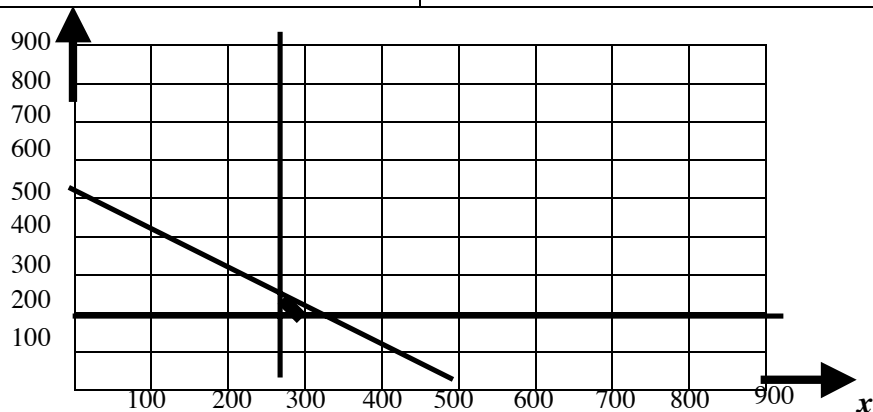
a.



b. Plant grows 50 cm in 110 days. $\frac{50}{110} = 0.45 \text{ cm/day}$ (1 mark)	c. $h = 0.45t + 10$ (1 mark)
d. Value of a is h value at end of first period of growth = 60 (1 mark)	e. $h = 0.1t + 60$ $t = 160$ $h = 0.1 \times 160 + 60 = 76 \text{ cm.}$ Join (110,60) and (160,76) on graph. (1 mark)

Question 3 a. $y \geq 180$ (1 mark)	b. $x + y \leq 500$ (1 mark)
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c.



($\frac{1}{2}$ mark for horizontal line) , ($\frac{1}{2}$ mark for vertical line) , ($\frac{1}{2}$ mark for line $x + y = 500$)

($\frac{1}{2}$ mark for shading)

d. $P = 45x + 60y$ (1 mark)	e. Points of intersection on graph are (260,180), (260,240) and (320,180) At (260,180) $P = 45 \times 260 + 60 \times 180 = 22,500$ At (260,240) $P = 45 \times 260 + 60 \times 240 = 26,100$ At (320,180) $P = 45 \times 320 + 60 \times 180 = 25,200$ $\therefore$ 260 kg. of day cream and 240 kg. of night cream. (1 mark)
f. From (e), maximum profit is \$26,100 (1 mark)	

Question 1 a. \$14,0247 (1 mark)	b. $140247 + 17335 - 12036.55 + 8642$ $= \\$154187.45$ (1 mark)
c. Minimum monthly balance for December $= 140247$ Interest for December = $140247 \times \frac{8}{12 \times 100} \times 1 = 934.98$ (1 mark) Minimum monthly balance for January $= 142150.9$ Interest for January = $142150.9 \times \frac{8}{12 \times 100} \times 1 = 947.67$ Minimum monthly balance for February $= 142150.9$ Interest for February = $142150.9 \times \frac{8}{12 \times 100} \times 1 = 947.67$ Total interest $= 934.98 + 2 \times 947.67 = \\2829.52 (1 mark)	d. $112\% = 160677$ $12\% = \frac{160677 \times 12}{112} = \\17215.39 (1 mark)
Question 2 a. Increase $= 38150 - 35000 = 3150$ $\% \text{ increase} = \frac{3150}{35000} \times 100 = 9\%$ (1 mark)	b. $35000(1.09)^4 = \\$49,405$ (1 mark)
c. Enter formula $y = 35000(1.09)^x$ in graphics calculator. Go to table to where $x = 21, y = 213808$ (1 mark) So in 21st year, where 2004 is year 0. $\therefore 2025$ (1 mark)	d. $35000 + \frac{35000 \times 16 \times 5}{100} = \\$63,000$ (1 mark)

Question 2 (continued) e. Enter $y_2 = 35000 + \frac{35000 \times 16 \times x}{100}$ in the graphics calculator. (1 mark) Keep the y_1 equation from (c) in the calculator. Go to table. When $x = 14, y_1 > y_2 \therefore 2018$ (1 mark)	
Question 3 a. i. \$2000 (1 mark)	a. ii. $1 + \frac{9.8}{12 \times 100}$ $= 1.008$ (1 mark)
b. Use graphics calculator Press Apps Finance Enter TVM Solver Enter $N =$ $I = 9.8$ $PV = 200000$ $PMT = -2000$ $FV = 0$ $P / Y = 12$ $C / Y = 12$ Put cursor on FV and press alpha solve $N = 204.22$ months = 17 years. (1 mark)	c. Amount repaid $= 204.2172486 \times 2000 = 408434.4972$ Interest = $408434.4972 - 200000$ $= \\$208,434.50$ (1 mark)

Question 1 a. $A = 2, B = 3, C = 3, D = 3, E = 1$ $2 + 3 + 3 + 3 + 1 = 12$ (1 mark)					b. D. (1 mark) It is the only town that can be reached from A, B, C and E (1 mark)				
c. A, C and D (1 mark)					d. To A B C D E A $\begin{bmatrix} 0 & 1 & 1 & 2 & 0 \end{bmatrix}$ B $\begin{bmatrix} 1 & 0 & 1 & 2 & 0 \end{bmatrix}$ From C $\begin{bmatrix} 1 & 1 & 0 & 2 & 0 \end{bmatrix}$ D $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ E $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ e.g. D can be reached from A in 2 ways, ABD or ABCD (1 mark) for getting any two rows correct (1 mark) for getting remaining rows correct.				
Question 2 a.	J	K	L	M	Step1 – Min. in rows	J	K	L	M
R	6	7	5	11	R	$6-5=1$	$7-5=2$	$5-5=0$	$11-5=6$
WS	10	5	3	12	WS	$10-3=7$	$5-3=2$	$3-3=0$	$12-3=9$
CP	11	3	4	13	CP	$11-3=8$	$3-3=0$	$4-3=1$	$13-3=10$
WP	8	4	6	10	WP	$8-4=4$	$4-4=0$	$6-4=2$	$10-4=6$
Step 2 – Minimum in columns					(1 mark)				
	J	K	L	M		J	K	L	M
R	$1-1=0$	$2-0=2$	$0-0=0$	$6-6=0$	R	0	2	0	0
WS	$7-1=6$	$2-0=2$	$0-0=0$	$9-6=3$	WS	6	2	0	3
CP	$8-1=7$	$0-0=0$	$1-1=0$	$10-6=4$	CP	7	0	0	4
WP	$4-1=3$	$0-0=0$	$2-0=2$	$6-6=0$	WP	3	0	2	0

(1 mark)

Question 2 a. (continued) Allocating jobs. $R \longrightarrow J$ $CP \longrightarrow K$ $WS \longrightarrow L$ $WP \longrightarrow M$ (1 mark)		b. Minimum time = $6 + 3 + 3 + 10 = 22$ hours (1 mark)
Question 3 a.		
Activity	Immediate Predecessor	b. The critical path is the longest path. ACDI (1 mark)
<i>A</i>	-----	
<i>B</i>	<i>A</i>	
<i>C</i>	<i>A</i>	
<i>D</i>	<i>B</i>	
<i>E</i>	<i>C</i>	
<i>F</i>	<i>E</i>	
<i>G</i>	<i>A, B</i>	
<i>H</i>	-----	
<i>I</i>	<i>D, F, G, H</i>	
(1 mark)		
c. $5 + 8 + 12 + 8 = 33$ hours. (1 mark)		d. This will alter the critical path to ABEFI Time taken = $5 + 6 + 4 + 14 + 8 = 37$ hours. This means it will take four hours longer to finish the project. (1 mark)
e. No effect, as F is not on the critical path. (1 mark)		

Question 1 a. $M = \begin{matrix} & R & W & C \\ \begin{matrix} X \\ Y \end{matrix} & \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \end{matrix}$ (1 mark)	b. 2 rows and 3 columns so 2×3 (1 mark)
c. $N = \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$ (1 mark)	d. $MN = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} = \begin{bmatrix} aP + bQ + cR \\ dP + eQ + fR \end{bmatrix}$ (1 mark)
e. $dP + eQ + fR$ represents the selling price of wines for company Y (1 mark)	
Question 2 a. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{7}{31} & \frac{2}{31} & \frac{8}{31} \\ \frac{8}{31} & \frac{11}{31} & -\frac{18}{31} \\ \frac{9}{31} & -\frac{7}{31} & \frac{3}{31} \end{bmatrix} \begin{bmatrix} 36.1 \\ 30.65 \\ 32.45 \end{bmatrix} = \begin{bmatrix} 2.20 \\ 30.65 \\ 32.45 \end{bmatrix}$ Use graphics calculator to get inverse matrix of matrix $\begin{bmatrix} 3 & 2 & 4 \\ 6 & 3 & 2 \\ 5 & 1 & 3 \end{bmatrix}$ which is $\begin{bmatrix} -\frac{7}{31} & \frac{2}{31} & \frac{8}{31} \\ \frac{8}{31} & \frac{11}{31} & -\frac{18}{31} \\ \frac{9}{31} & -\frac{7}{31} & \frac{3}{31} \end{bmatrix}$ Then use graphics calculator to do the multiplication to get the answer. 1 mark for inverse matrix 1 mark for giving answers as fractions 1 mark for correct solution (3 marks)	b. The cost of a tin of dog food is \$2.20 (1 mark)

Question 3 a. $0.05 = 5 \%$ (1 mark)	b. Next Purchase $A \quad B \quad C$ $T = \text{Current Purchase}$$\begin{bmatrix} A & 0.9 & 0.1 & 0.2 \\ B & 0.05 & 0.8 & 0.1 \\ C & 0.05 & 0.1 & 0.7 \end{bmatrix}$ (1 mark)
c. $\begin{bmatrix} 0.3 \\ 0.6 \\ 0.1 \end{bmatrix}$ (1 mark)	d. i. $\begin{bmatrix} 0.9 & 0.1 & 0.2 \\ 0.05 & 0.8 & 0.1 \\ 0.05 & 0.1 & 0.7 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.6 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.35 \\ 0.505 \\ 0.145 \end{bmatrix}$ (1 mark)
d. ii. The percentage of customers expected at Charisma pharmacy next week is 14.5%. (1 mark)	e. Find $T \wedge 40 S_0$ and $T \wedge 50 S_0$ These both give $K = \begin{bmatrix} 0.588 \\ 0.235 \\ 0.176 \end{bmatrix}$ To enter $T \wedge 40 K_0$ into calculator press 2nd matrix A $\wedge$ 40 2nd matrix B enter So expect 17.6% of customers in the Charisma pharmacy in the long term. (1 mark)

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