

2007

VCE Further Mathematics Trial Examination 1

Suggested Solutions

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Question 1 A $\frac{61}{73 + 61 + 146} \times \frac{100}{1} = \frac{61}{280} \times \frac{100}{1} = 21.786\%$	Question 2 D Having or not having a water tank or having one on order is categorical data. Scatter plots and histograms are used when both variables are numerical. For a stem and leaf you need two categories, but there are three categories here. You cannot find the median, lower quartile, etc. so cannot draw a box plot. Bar graphs are used for categorical data.
Question 3 C A long whisker to the left indicates the box plot is negatively skewed. The interquartile range = $80 - 60 = 20$	Question 4 C 99.7% lie within 3 standard deviations of the mean, so 0.3% lie outside this range. Half of $0.3\% = 0.15\%$. this means 0.15% lie above 3 standard deviations above the mean. $43 + 3 \times 15 = 88$. So the lowest mark for an A is 88
Question 5 A The standard deviation remains at 5 g., because the spread is unaltered. The mean weight of each bag is increased by 2g to 452g.	Question 6 B $r^2 = 0.1296$ $r = \pm\sqrt{0.1296}$ $r = \pm 0.36$ But the gradient of the line is negative, (-0.73) so the correlation coefficient must be negative $\therefore r = -0.36$
Question 7 B Draw 2 vertical lines so that there are 3 points to the left of the first vertical line, 4 points in the middle, and 3 points to the right of the last vertical line. $x_l = 2, y_l = 2$ $x_u = 10, y_u = 18$ gradient = $\frac{18 - 2}{10 - 2} = \frac{16}{8} = 2$	Question 8 E The gradient is approximately -28. This means that for every dollar increase in fares, the number of passengers decreased by 28.
Question 9 D For y versus x the graph would look like A or C, not B. However, the graph would contain the point (1,2). Hence, not A, B, or C. For y versus log x the graph would look like D or E but would contain the point (1,5). Hence, D	

Question 10 B Enter data in graphics calculator in L_1 and L_2 using Stat Edit Press Stat Calc Linear Regression Line L_1, L_2 This gives $r^2 = 0.305$	Question 11 D To use seasonal indices, there must be a secular trend and the variation must be seasonal.
Question 12 D Quarterly average = $\frac{240 + 226 + 210 + 120}{4} = 199$ Seasonal Index for 3rd quarter = $\frac{210}{199} = 1.055$	Question 13 A The median of the first three points is 600 so the point (1985,600) should lie on the graph, hence not C, D or E. The median of points 3,4, and 5 is 450, so the third point on the smoothed graph should be (1995,450). Hence A

Question 1 D Each term in the sequence is found by adding 6 to the term before it. Hence it is an arithmetic sequence with a common difference of 6	Question 2 B This is a geometric sequence with a common ratio of $-\frac{1}{2}$ $a = 8, n = 12$ $t_n = ar^{n-1}$ $t_n = 8\left(-\frac{1}{2}\right)^{11} = -\frac{1}{256}$
Question 3 C $t_7 = 3 \times t_6 + 4$ $8017 = 3 \times t_6 + 4$ $8013 = 3 \times t_6$ $2671 = t_6$ $t_6 = 3 \times t_5 + 4$ $2671 = 3 \times t_5 + 4$ $2667 = 3 \times t_5$ $889 = t_5$	Question 4 C $t_4 = t_{1+3} = t_{1+2} + t_{1+1} + t_1 = t_3 + t_2 + t_1 = 1 + 1 + 1 = 3$ $t_5 = t_{2+3} = t_{2+2} + t_{2+1} + t_2 = t_4 + t_3 + t_2 = 3 + 1 + 1 = 5$
Question 5 C $4 + 6 + 8 + \dots$ Sum of Arithmetic Sequence $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{10} = \frac{10}{2}[8 + 9 \times 2] = 5 \times 26 = 130\text{m}$	Question 6 E Value at beginning of first year = 32,000 Value at beginning of second year = $32,000(0.85)$ Value at beginning of third year = $32,000(0.85)^2$ Following the pattern, Value at beginning of fifth year = $32,000(0.85)^4$ $= \\$16704.20$ This is closest to \$16704

Question 7 D

Arithmetic Sequence.

$$a = 85$$

$$d = -7$$

$$t_n = a + (n - 1)d = 85 + (n - 1)(-7)$$

Use sequence mode on graphics calculator and
press $y =$

$$n(\min) = 1$$

$$\mu(n) = 85 + (n - 1)(-7)$$

$$\mu(n \text{ min}) = 85$$

Press second table and scroll down to get - 48

This corresponds to $n = 20$

Question 8 A

$$\frac{72}{x} = \frac{x}{8}$$

$$x^2 = 576$$

$$x = \pm\sqrt{576} = \pm 24$$

$$\text{Common ratio} = \frac{\pm 24}{8} = \pm 3$$

Question 9 D

$$S_{\infty} = \frac{a}{1 - r} = \frac{2}{1 - \frac{2}{5}} = 3.3333$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{2(1 - 0.4^n)}{0.6}$$

Enter this equation in sequence mode in the graphics calculator

$$n(\min) = 1$$

$$\mu(n \text{ min}) = 2$$

Using the table scroll down

$$n = 5 \quad \mu(n) = 3.2992$$

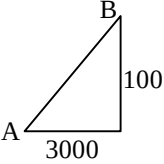
$$n = 6 \quad \mu(n) = 3.3197$$

$$n = 7 \quad \mu(n) = 3.3279$$

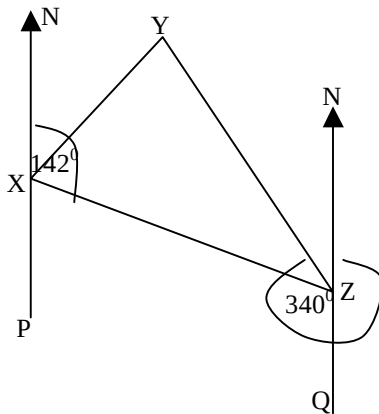
$$3.3333 - 3.2992 = 0.0341 \text{ which is too big}$$

$$3.3333 - 3.3197 = 0.0136 \text{ which is too big}$$

$$3.3333 - 3.3279 = 0.0054 \text{ which is less than } 0.01$$

Question 1 E $b^2 = a^2 + c^2 - 2ac \cos 50^\circ$ $b^2 = 27^2 + 41^2 - 2 \times 27 \times 41 \times \cos 50^\circ$ $b = \sqrt{27^2 + 41^2 - 2 \times 27 \times 41 \times \cos 50^\circ}$ $b = 31.4$ AC is closest to 31cm	Question 2 D $\text{Area} = \frac{1}{2}ac \sin b$ $\text{Area} = \frac{1}{2} \times 4 \times 7 \times \sin 64^\circ$ $\text{Area} = 12.58 \text{ cm}^2$
Question 3 E In this right angled isosceles triangle, AB = AC, so $\angle BAC = \theta$. The sum of the angles of a triangle = 180° $\therefore \theta = 45^\circ$ $\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$	Question 4 A  $\text{gradient} = \frac{100}{3000} = 0.033$ which is closest to 0.03
Question 5 C In $\triangle ADB$, $AD^2 = 20^2 - 16^2$ $AD = \sqrt{20^2 - 16^2} = 12$ In $\triangle BDC$, $DC^2 = 34^2 - 16^2$ $DC = \sqrt{34^2 - 16^2} = 30$ $AC = AD + DC = 12 + 30 = 42 \text{ cm}$	Question 6 E Let the length of the original cube = x Then the original volume = x^3 Length of larger cube = $4x$ Volume of larger cube = $(4x)^3 = 64x^3$ $\therefore$ volume has increased by a factor of 64

Question 7 A



$$\angle NXZ = \angle XZQ \text{ (alternate angles)} = 142^\circ$$

$$\angle NZQ = 180^\circ$$

$$\angle XZY = 340 - (180 + 142) = 18^\circ$$

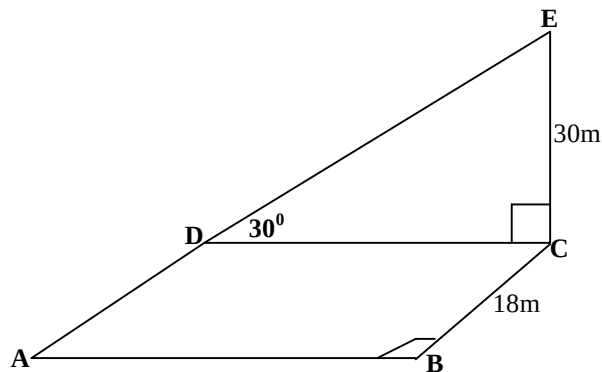
Question 8 C

$\Delta s ABO$ and CDO are similar (AAA)

$$\therefore \frac{x}{5.2} = \frac{2.5}{10}$$

$$x = \frac{2.5 \times 5.2}{10} = 1.3 \text{ m} = 130 \text{ cm}$$

Question 9 A



$$\tan 30^\circ = \frac{30}{DC}$$

$$DC = \frac{30}{\tan 30^\circ} = 51.9615 = AB$$

$$AC^2 = AB^2 + 18^2$$

$$AC^2 = 51.9615^2 + 18^2$$

$$AC = \sqrt{51.9615^2 + 18^2}$$

$$AC = 54.99$$

$$\tan \theta = \frac{30}{AC} = \frac{30}{54.99}$$

$$\theta = \tan^{-1} \left(\frac{30}{54.99} \right) = 28.6^\circ \text{ which is closest to } 29^\circ$$

Question 1 A They both meet where the lines meet. Read the distance from the distance axis. This is 15 km.	Question 2 E Jake travels $25 + 25 = 50$ km Jenny travels $5 + 15 = 20$ km. So Jake travels 30 km further than Jenny
Question 3 E $28 = a \times 2^3$ $28 = a \times 8$ $a = \frac{28}{8} = 3.5$	Question 4 D $2 \times 3 - 5 \times 2 + b = 0$ $6 - 10 + b = 0$ $b = 10 - 6 = 4$
Question 5 B The charge per hour is the gradient of the line $= \frac{90 - 30}{3 - 0} = \frac{60}{3} = 20$ The fixed cost is the y intercept = 30	Question 6 D The shaded region is above the line or equal to the line $y = x - 3$ $\therefore y \geq x - 3$ $\therefore$ not C or E The vertical line is $x = 5$ and the required region is greater than or equal to this line $\therefore x \geq 5$, so not B The horizontal line is $y = 4$ and the required region is less than or equal to this line so $y \leq 4$ $\therefore$ D

Question 7 B $3y = -6x + 2$ $y = -2x + \frac{2}{3}$ gradient of this line is -2 $\therefore$ gradient of parallel line is -2 $y = -2x + c$ when $x = 4$, $y = 10$ $10 = -2 \times 4 + c$ $10 = -8 + c$ $c = 18$ $\therefore y = -2x + 18$ $y + 2x - 18 = 0$	Question 8 E If you multiplied equation 1 by 2 and equation 2 by 3 you would have $-6b$ in each equation. The b could be eliminated by subtraction. Question 9 A Eqn of revenue $R = mx + c$ Points on revenue line are (0,0) and (200,600) $m = \frac{600 - 0}{200 - 0} = 3$ $y = 3x + c$ c is the y intercept = 0 $R = 3x$ Eqn of cost $C = mx + c$ Points on cost line are (0,400) and (200,600) $m = \frac{600 - 400}{200 - 0} = \frac{200}{200} = 1$ $c = 400$ $C = x + 400$ When $x = 150$, $C = 150 + 400 = 550$ When $x = 150$, $R = 3 \times 150 = 450$ Therefore, loss = 100
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Question 1 C $R = \frac{100I}{PT} = \frac{100 \times 945}{4800 \times 3} = 6.5625$ This is closest to 6.6%	Question 2 D Amount paid = $7000 + 24 \times 1330 = \\$38,920$ Interest = Amount paid – Cost Interest = $38920 - 36000 = \\$2,920$
Question 3 B Sale price = 85% of 1300 = 1105 Price paid by customer = 88% of 1105 = \$972.40	Question 4 C Use TVM solver under finance on a graphics calculator. $N = 48$ $I = 10$ $Pv = -3000$ $PMT = 0$ $FV =$ $P / Y = 4$ $C / Y = 4$ This gives $FV = 9814.47$ Interest = Final value – Amount invested Interest = $9814.47 - 3000 = 6814.47$ This is closest to \$6814
Question 5 C On 9 Jan. have $7648 + 2004 = 9652$ On 29 Jan have 7632 Amt. taken out of account = $9652 - 7632$ = \$2020	
Question 6 B Interest will be paid for the months of Dec., Jan. and Feb. Dec. min monthly balance = 9243 Jan. min monthly balance = 7632 Feb. min monthly balance = 7532 Interest for Dec. = $9243 \times \frac{1}{100} \times 1 = 92.43$ Interest for Jan. = $7632 \times \frac{1}{100} \times 1 = 76.32$ Interest for Feb. = $7532 \times \frac{1}{100} \times 1 = 75.32$ Total Interest = $92.43 + 76.32 + 75.32 = \\244.07	Question 7 E $(0.92)^n \times 2000 = 800$ Enter $y = (0.92)^n \times 2000$ in graphics calculator Then 2nd Table When $x = 11$ $y = 799.27$ So 11 years.

Question 8 E

Compound interest rate increases more rapidly with time. This means the gradient of the graph increases with time. The only graph that does this is E

Question 9 C

Use TVM solver under finance on a graphics calculator.

$$N = 48$$

$$I = 9.2$$

$$PV = 20000$$

$$PMT = -300$$

$$FV =$$

$$P / Y = 12$$

$$C / Y = 12$$

This gives $FV = 11,395.90$

Paid back lump sum of 2000 leaves

9395.90 owing

$$N =$$

$$I = 9.2$$

$$PV = 9395.90$$

$$PMT = -400$$

$$FV = 0$$

$$P / Y = 12$$

$$C / Y = 12$$

This gives $N = 25.8$ months which is closest to 2 years.

Question 1 A A complete graph has each vertex linked to every other vertex	Question 2 E For an Euler circuit to exist, each vertex must be of even degree.
Question 3 E A to A is 0 B to B is 0 C to C is 1 D to D is 0 Hence, the diagonal is 0,0,1,0	Question 4 C A tree is a connected graph with no loops, circuits or multiple edges. A, B, D, and E have loops, multiple edges or circuits.
Question 5 E Degree of A = 3 Degree of B = 4 Degree of C = 5 Degree of D = 3 Degree of E = 5 Degree of F = 4 Need a line joining any two of A, C, D or E, so that two of the vertices only remain odd. From given choices it has to be C to D	Question 6 D Cameron is the parent of Donald and Anthea, not of Chris

Question 7 B

F is a prerequisite for D so not C
C is a prerequisite for D so not A or D
D is a prerequisite for E so not E

Question 8 B

Ben has dominance over Carrie who in turn has dominance over David.

Question 9 B

B,C,D,G lie on the critical path because their float time is zero. Hence, A is not true.
If all the crash times are implemented, only those on the critical path will reduce the overall time of the project. Hence, C is not true. E does not lie on the critical path, so D is not true. The total time for the project is the length of the critical path so E is not true. E does not lie on the critical path, so it does not affect the overall time of the project.

Question 1 A $\det A = (4 \times -3) - (-2 \times -1) = -12 - 2 = -14$	Question 2 D $2A = \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$ $2A - B = \begin{bmatrix} 2a - e & 2b - f \\ 2c - g & 2d + h \end{bmatrix}$
Question 3 B AB does not exist because the number of columns of A does not equal the number of rows of B. BA is a 4×3 matrix. Order of BA is number of rows of B $\times$ number of columns of A	Question 4 C A has no solutions because the lines are parallel. B has one, unique, solution. (2,4) D has a unique solution because $\det D = 3 - 2 = 1$ E has a unique solution because $\det E = -12 - 12 = -24$ C has many solutions because the two lines are the same.
Question 5 A This can be done on the graphics calculator or $A - B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$ $C^2 = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 6 & 7 \end{bmatrix}$ $(A - B)C^2 = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 7 & 4 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 20 & 15 \\ 26 & 22 \end{bmatrix}$	Question 6 E From Y to X is 0.25 so the first row second column is 0.25 so not B or C or D From X to Z is 0.05 so not A Hence, answer E

Question 7 A S_0 is the initial number of people choosing milk and dark chocolate. $S_0 = \begin{bmatrix} 2180 \\ 1260 \end{bmatrix}$	Question 8 B $T = \begin{bmatrix} 0.72 & 0.35 \\ 0.28 & 0.65 \end{bmatrix}$ $T^5 S_0 = \begin{bmatrix} 0.72 & 0.35 \\ 0.28 & 0.65 \end{bmatrix}^{\wedge 5} \times \begin{bmatrix} 2180 \\ 1260 \end{bmatrix} = \begin{bmatrix} 1913 \\ 1527 \end{bmatrix},$ using graphics calculator Number choosing dark chocolate = 1527																																								
Question 9 D <table><tr><td></td><td>D</td><td>N</td><td>C</td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>X</td><td>50</td><td>60</td><td>200</td><td>$\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$</td><td>=</td><td>$\begin{bmatrix} 100 + 60 + 600 \\ 40 + 100 + 90 \\ 160 + 40 + 210 \end{bmatrix}$</td><td>=</td><td>$\begin{bmatrix} 760 \\ 230 \\ 410 \end{bmatrix}$</td><td></td></tr><tr><td>Y</td><td>20</td><td>100</td><td>30</td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>Z</td><td>80</td><td>40</td><td>70</td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>			D	N	C							X	50	60	200	$\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$	=	$\begin{bmatrix} 100 + 60 + 600 \\ 40 + 100 + 90 \\ 160 + 40 + 210 \end{bmatrix}$	=	$\begin{bmatrix} 760 \\ 230 \\ 410 \end{bmatrix}$		Y	20	100	30							Z	80	40	70						
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End of suggested solutions
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