



VCE
2006 Mathematical Methods CAS
Trial Examination 2

Suggested Solutions

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PURPOSE OF THIS TRIAL EXAMINATION

This Mathematics Methods CAS Trial Examination is designed to assess

- understanding and communication of mathematical ideas
- interpretation, analysis and solution of routine problems
- interpretation, analysis and solution of non-routine problems

Assessment is by multiple-choice questions and extended answer questions involving multi-stage solutions of increasing complexity.

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Question 1 D Sketch the graphs on the calculator. There are three solutions at $x = -0.77$, 2 and 4.	Question 2 C $g\{f(x)\} = \log_e(x^2 - 9)$ $x^2 - 9 > 0$ $\Rightarrow x^2 > 9$ $\Rightarrow \pm x > 3$ $\Rightarrow x < -3$ or $x > 3$ Domain $(-\infty, -3) \cup (3, \infty)$												
Question 3 E The inverse only exists for a one-to-one function. The graph is one-to-one for $-\frac{\pi}{2} \leq 3x \leq \frac{\pi}{2} \Rightarrow -\frac{\pi}{6} \leq x \leq \frac{\pi}{6}$	Question 4 A <table><tr><td>x^2</td><td>1</td><td>4</td><td>9</td><td>16</td><td>25</td></tr><tr><td>$\Pr(X = x)$</td><td>k^2</td><td>$4k^2$</td><td>$9k^2$</td><td>$16k^2$</td><td>$25k^2$</td></tr></table> $\sum \Pr(X = x) = 1$ $\Rightarrow 55k^2 = 1$ $\Rightarrow k^2 = \frac{1}{55}$ $\Rightarrow k = \pm \frac{1}{\sqrt{55}} \times \frac{\sqrt{55}}{\sqrt{55}} = \pm \frac{\sqrt{55}}{55}$	x^2	1	4	9	16	25	$\Pr(X = x)$	k^2	$4k^2$	$9k^2$	$16k^2$	$25k^2$
x^2	1	4	9	16	25								
$\Pr(X = x)$	k^2	$4k^2$	$9k^2$	$16k^2$	$25k^2$								
Question 5 E (1) $-3x + p^2y = -6$ (2) $x - 4y = 2p$ $3 \times (2) + (1) \quad (p^2 - 12)y = 6p - 6$ $y = \frac{6(p - 1)}{p^2 - 12}$ There is a unique solution provided that $p^2 - 12 \neq 0 \quad p \neq \pm 2\sqrt{3} \quad \text{and } p \in R \text{ so}$ $p \in R \setminus \{-2\sqrt{3}, 2\sqrt{3}\}$	Question 6 A $f(x) = a(x + 1)^2(x - 3)$ When $x = 0$, $f(x) = 6$ $\Rightarrow 6 = a \times 1 \times -3$ $\Rightarrow a = -2$ $f(x) = -2(x + 1)^2(x - 3)$ $\Rightarrow f(x) = 2(x + 1)^2(3 - x)$												



Question 7 B only B works If $f(x) = \log_e(2x)$ $f(2x^2) = \log_e(2(2x^2))$ $= \log_e(4x^2)$ $= \log_e(2x)^2$ $= 2\log_e(2x) = 2f(x)$	Question 8 C $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dr} = 4\pi r^2$ $\frac{dr}{dt} = 0.1$ $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} = 4\pi r^2 \times 0.1 = 0.4\pi r^2$ When $r = 7$, $\frac{dV}{dt} = 0.4\pi \times 49 = 61.6\text{cm}^3 \text{sec}^{-1}$
Question 9 B Use a graphics calculator to sketch the graphs and find that the points of intersection are $x = 1.57$ and $x = 2.2$ Use the calculator to find the area between the curve and the X axis for $y = 6x - 11$ from $x = 1.57$ to $x = 2.2$ Area = 1.188 Use the calculator to find the area between the curve and the X axis for $y = x$ from $x = 1.57$ to $x = 2.2$ Area = 0.576 Area between these graphs $= 1.188 - 0.576 = 0.612$	Question 10 C $\lim_{\delta x \rightarrow 0} \sum_{i=1}^n \sin(x) \delta x$ $= \int_0^{\frac{\pi}{4}} \sin(x) dx$ $= \left[-\cos(x) \right]_0^{\frac{\pi}{4}}$ $= -\cos\left(\frac{\pi}{4}\right) + \cos(0)$ $= 1 - \frac{\sqrt{2}}{2}$ $= \frac{2 - \sqrt{2}}{2}$



Question 11 E When $t = 0$, $A = A_0 = 200$ $A = 200e^{-kt}$ When $t = 20$, $A = 100$ $0.5 = e^{-20k}$ $\log_e 0.5 = -20k$ $k = 0.035$ $A = 200e^{-0.035t}$ $\frac{dA}{dt} = 200 \times (-0.035)e^{-0.035t}$ When $t = 30$, $\frac{dA}{dt} = -2.45$ The isotope is decaying at 2.45 g/day.	Question 12 E The area in answer B can be checked as incorrect by using a graphics calculator. $\text{gradient} = \frac{dy}{dx} = xe^x + e^x \times 1$ $= e^x(x+1) = 0$ when $x = -1$ When $x < -1$, $\frac{dy}{dx} < 0$ Therefore, negative gradient when $x < -1$
Question 13 A A is false, all the other are true, since $E(X) = np$ $VAR(X) = np(1-p)$ and $0 < p < 1$ so $E(X) > VAR(X)$ $\frac{VAR(X)}{E(X)} = 1-p$ D is true $\Pr(X=0) = (1-p)^n$ E is true $\Pr(X=1) = np(1-p)^{n-1}$	Question 14 B $\sqrt{f(x)}$ only exists when $f(x) \geq 0$. That is, when y on the original graph is above or on the X axis.



Question 15 B

The gradient is positive everywhere except when $x \leq -1$ and when $x = 0$. $x = -1$ is a minimum as the graph goes from a negative to a positive gradient. $x = 0$ is a stationary point of inflexion as the graph goes from a positive to a positive gradient.

Question 16 D

$$\frac{dy}{dx} = \frac{2}{2x-1} + 4 = \frac{2+8x-4}{2x-1} = \frac{8x-2}{2x-1}$$

$$\text{When } x = 1, \frac{dy}{dx} = 6$$

$$\text{So, gradient of normal} = -\frac{1}{6}$$

$$\text{Equation of normal is } y = -\frac{1}{6}x + c$$

$$\text{When } x = 1 \text{ on curve } y = \log_e 1 + 4 = 4$$

So point (1, 4) lies on the normal.

$$\text{Therefore, } 4 = -\frac{1}{6} + c \Rightarrow \frac{25}{6} = c$$

$$\Rightarrow y = -\frac{1}{6}x + \frac{25}{6} \Rightarrow 6y + x - 25 = 0$$



Question 17 E

from the matrix equations

$$x' = 2y + 1$$

$$y' = -x + 3$$

$$\text{therefore } y = \frac{x' - 1}{2}$$

$$\text{and } x = 3 - y'$$

the equation

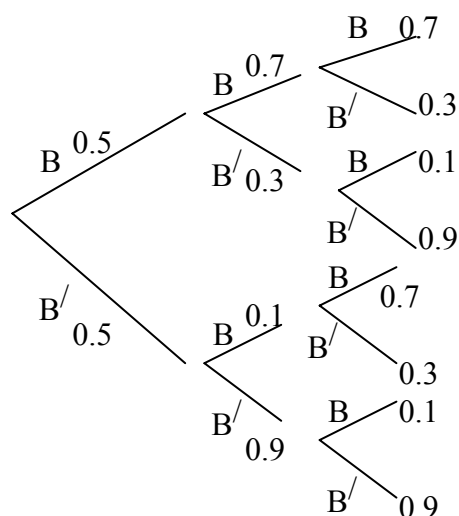
$$y = \sqrt{x} \text{ becomes}$$

$$\frac{x' - 1}{2} = \sqrt{3 - y'} \text{ drop the dashes}$$

$$x - 1 = 2\sqrt{3 - y}$$

$$x = 2\sqrt{3 - y} + 1$$

Question 18 B



$$B B B = 0.5 \times 0.7 \times 0.7 = 0.245$$

$$B B' B = 0.5 \times 0.3 \times 0.1 = 0.015$$

$$B' B B = 0.5 \times 0.1 \times 0.7 = 0.035$$

$$B' B' B = 0.5 \times 0.9 \times 0.1 = 0.045$$

$$\text{Total} = 0.34$$

Question 19 C

mean or average value

$$\bar{y} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$\bar{y} = \frac{1}{2-0} \int_0^2 (30 \cos(\pi x) + 40) dx$$

$$\bar{y} = \frac{1}{2} \left[40x + \frac{30}{\pi} \sin(\pi x) \right]_0^2$$

$$\bar{y} = \frac{1}{2} \left(80 + \frac{30}{\pi} \sin(2\pi) - \left(0 + \frac{30}{\pi} \sin(0) \right) \right)$$

$$\bar{y} = 40$$

Question 20 E

$$\Pr(A) = \frac{1}{6}$$

$$\Pr(B) = \frac{1}{2}$$

$$\Pr(A \cap B) = \frac{3}{36} = \frac{1}{12}$$

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B) = \frac{1}{12}$$

Therefore, independent events.

$$\Pr(A \cup B)$$

$$= \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$= \frac{1}{6} + \frac{1}{2} - \frac{1}{12} = \frac{7}{12}$$

Therefore, not mutually exclusive.



Question 21 A

Use a calculator. Enter data in the **list** menu. Use **binompdf** to get probabilities. Then use **graph** to get A.

Question 22 D

$\Pr(B | GG)$

$$\begin{aligned} &= \frac{\Pr(B \cap GG)}{\Pr(B \cap GG) + \Pr(Y \cap GG)} \\ &= \frac{\frac{3}{6} \times \frac{2}{5}}{\frac{3}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{1}{5}} \\ &= 0.75 \end{aligned}$$

Suggested Solutions Part II

Question 1

a.

$$f(x) = \frac{ax}{bx^2 + c}$$

$$f'(x) = \frac{a(c - bx^2)}{(bx^2 + c)^2} \quad (1 \text{ mark})$$

$$\text{when } x = 1, \quad f(1) = \frac{1}{2} = \frac{a}{b+c} \quad \text{and} \quad f'(1) = 0$$

$$\text{so } b = c \quad \text{and} \quad \frac{1}{2} = \frac{a}{2b} \quad \text{so } a = b = c$$

(2 marks)

$$f'(x) = 0 \quad \text{when } x = \pm 1$$

$$f(-1) = -\frac{1}{2}$$

$$\text{maximum} \left(1, \frac{1}{2} \right)$$

$$\text{minimum} \left(-1, -\frac{1}{2} \right) \quad (1 \text{ mark})$$

max and min are independent of a , b and c .

b.

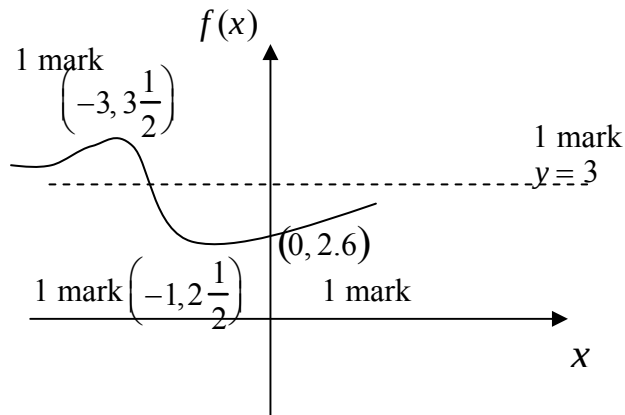
- Graph is reflected in the X axis.
- Graph is translated 2 units to the left parallel to the X axis.
- Graph is translated 3 units up parallel to the Y axis.

(1 mark for each)



Question 1 (continued)

c.



d.

$(-1, 1)$ (1 mark)

e.

$$y = \frac{x}{1+x^2}$$

Interchange x and y .

$$x = \frac{y}{1+y^2}$$

$$x(1+y^2) = y \quad (1 \text{ mark})$$

$$x + xy^2 - y = 0$$

$$xy^2 - y + x = 0$$

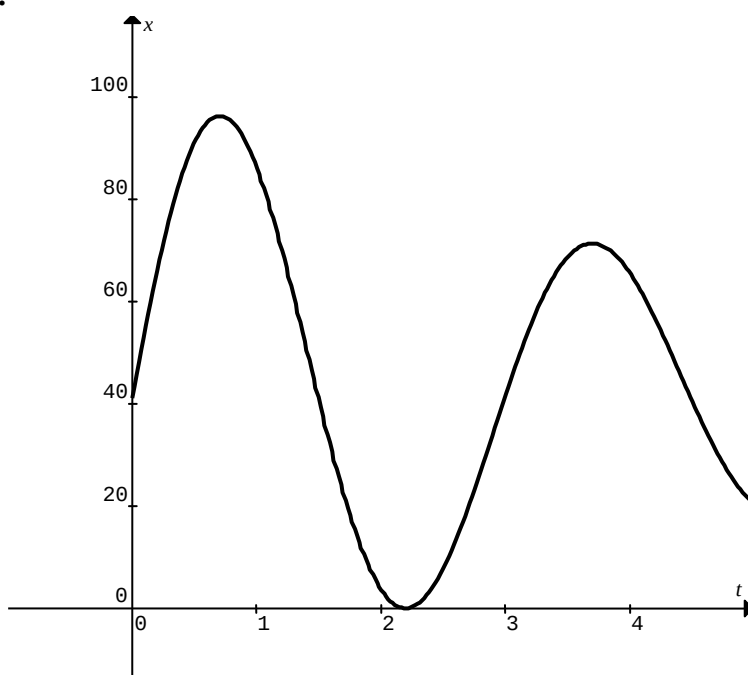
$$y = \frac{+1 \pm \sqrt{1-4x^2}}{2x}$$

$$\text{But } y > 0 \therefore y = \frac{+1 + \sqrt{1-4x^2}}{2x} \quad (1 \text{ mark})$$

$$\text{Domain } \left(0, \frac{1}{2}\right) \quad (1 \text{ mark})$$



Question 2

a. $x(0) = 41 \text{ cm}$ (1 mark)	c. since $64e^{-0.2t} \neq 0$ $-0.2 \sin\left(\frac{2\pi t}{3}\right) + \frac{2\pi}{3} \cos\left(\frac{2\pi t}{3}\right) = 0$ $\tan\left(\frac{2\pi t}{3}\right) = \frac{10\pi}{3}$ (1 mark) $\frac{2\pi t}{3} = \tan^{-1}\left(\frac{10\pi}{3}\right), \tan^{-1}\left(\frac{10\pi}{3}\right) + \pi$ $t = \frac{3}{2\pi} \tan^{-1}\left(\frac{10\pi}{3}\right), \frac{3}{2\pi} \left(\tan^{-1}\left(\frac{10\pi}{3}\right) + \pi\right)$ $t = 0.7045, 2.2045$ (1 mark) $x(0.7045) = 96.3367$ (1 mark) $x(2.2045) = 0.0056$ (1 mark)
b. $x = 64e^{-0.2t} \sin\left(\frac{2\pi t}{3}\right) + 41$ $\frac{dx}{dt} = 64e^{-0.2t} \left(-0.2 \sin\left(\frac{2\pi t}{3}\right) + \frac{2\pi}{3} \cos\left(\frac{2\pi t}{3}\right)\right)$ $a = -\frac{1}{5}$ and $b = \frac{2\pi}{3}$ (2 marks)	
d.  For the graph with correct intercept and max min and decaying amplitude (2 marks) The amplitude of the motion decreases exponentially, but the local maxima and minima are always equally separated. (1 mark)	



Question 2 (continued)

e. solving graphically

$$x = 64e^{-0.2t} \sin\left(\frac{2\pi t}{3}\right) + 41 = 75$$

gives $t_1 = 0.2852$ and $t_2 = 1.1503$ (1 mark)

the difference in times 0.865 sec (1 mark)

f. $\frac{dx}{dt}\bigg|_{t=1.5} = 64e^{-0.3} \left(-0.2 \sin(\pi) + \frac{2\pi}{3} \cos(\pi) \right)$

$\frac{dx}{dt}\bigg|_{t=\frac{3}{2}} = -\frac{128\pi}{3e^{0.3}}$ (1 mark)

g. $x\left(\frac{3}{2}\right) = 41$ $x(1) = 32\sqrt{3}e^{-0.2} + 41$

$$\frac{x\left(\frac{3}{2}\right) - x(1)}{\frac{3}{2} - 1} = -64\sqrt{3}e^{-\frac{1}{5}} \quad (1 \text{ mark})$$



Question 3

a. $y = a(x-b)^2 + c \quad (b=0)$ $y = ax^2 + c$ When $x=0, y=1$ so $y = ax^2 + 1$ (1 mark) When $x=-3, y=3.25$ so $3.25 = 9a + 1$ $\Rightarrow 2.25 = 9a$ $\Rightarrow \frac{2.25}{9} = a$ $\Rightarrow a = 0.25$ (1 mark)	b. Diameter = distance from -10 to $+10 = 20$ (1 mark)
c. Express x in terms of y. $y = \frac{x^2}{4} + 1$ $\Rightarrow y - 1 = \frac{x^2}{4}$ $\Rightarrow x^2 = 4(y - 1)$ $\Rightarrow x = \pm\sqrt{4(y-1)}$ (1 mark)	d. $\int_a^b 4\pi(y-1)dy$ (1 mark) a and b are the y values. $a = 1$ (1 mark) b is the value of y when $x = 10$ (1 mark) When $x = 10, y = \frac{1}{4} \times 100 + 1 = 26$ $\Rightarrow b = 26$ $V = 4\pi \left[\frac{y^2}{2} - y \right]_1^{26} = 3927$ cubic units (1 mark)
e. $r = 3 \Rightarrow x = 3$ When $x = 3, y = \frac{1}{4} \times 9 + 1 = \frac{13}{4}$ $V = 4\pi \int_1^{13/4} (y-1)dy$ (1 mark) $V = 4\pi \left[\frac{y^2}{2} - y \right]_1^{13/4}$ $V = 4\pi \left[\frac{169}{32} - \frac{13}{4} - \frac{1}{2} + 1 \right]$ $V = 4\pi \times \frac{81}{32} = \frac{81\pi}{8}$ cubic units (1 mark)	f. $\begin{bmatrix} \frac{3}{7} & \frac{5}{7} \end{bmatrix}^4 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1341}{2401} \\ \frac{1060}{2401} \end{bmatrix}$ (2 marks) $\text{Pr}(\text{apple on Friday}) = \frac{1341}{2401}$ (1 mark)



Question 3 (continued)

g. use CAS to consider large powers $\begin{bmatrix} \frac{3}{7} & \frac{5}{7} \\ \frac{4}{7} & \frac{2}{7} \end{bmatrix}^{100} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{9} \\ \frac{4}{9} \end{bmatrix} \quad (2 \text{ marks})$ have apples on 5 out of every 9 days in the long run, or exactly 55.5% 55.5555 repeating of the time (1 mark)	
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Question 4

a. i. $E(X) = \sum x \Pr(X = x)$ $= 0 + 0.01 + 0.1 + 0.6 + 1.6 + 1.5$ $= 3.81$ (1 mark)	a. ii. $\sigma^2 = E(X^2) - \mu^2$ $E(X^2) = 0.01 + 0.2 + 1.8 + 6.4 + 7.5$ $= 15.91$ (1 mark) $\sigma^2 = 15.91 - (3.81)^2$ $\sigma = \sqrt{15.91 - (3.81)^2} = 1.18$ (1 mark)																				
b. i. $1 - 0.85 = 0.15$ (1 mark)	b. ii. Probability(will not flower in 2008) <table><tr><td>2006</td><td>2007</td><td>2008</td><td></td></tr><tr><td>F</td><td>F</td><td>F/</td><td>$= 0.8 \times 0.8 \times 0.2$</td></tr><tr><td>F</td><td>F/</td><td>F/</td><td>$= 0.8 \times 0.2 \times 0.85$</td></tr><tr><td>F/</td><td>F/</td><td>F/</td><td>$= 0.2 \times 0.15 \times 0.2$</td></tr><tr><td>F/</td><td>F</td><td>F/</td><td>$= 0.2 \times 0.85 \times 0.85$</td></tr></table> (1 mark) $= 0.128 + 0.136 + 0.006 + 0.1445$ $= 0.4145$ (1 mark)	2006	2007	2008		F	F	F/	$= 0.8 \times 0.8 \times 0.2$	F	F/	F/	$= 0.8 \times 0.2 \times 0.85$	F/	F/	F/	$= 0.2 \times 0.15 \times 0.2$	F/	F	F/	$= 0.2 \times 0.85 \times 0.85$
2006	2007	2008																			
F	F	F/	$= 0.8 \times 0.8 \times 0.2$																		
F	F/	F/	$= 0.8 \times 0.2 \times 0.85$																		
F/	F/	F/	$= 0.2 \times 0.15 \times 0.2$																		
F/	F	F/	$= 0.2 \times 0.85 \times 0.85$																		
c. i. $a \int_0^1 (t^2 - t^3) dt = 1$ $\Rightarrow a \left[\frac{t^3}{3} - \frac{t^4}{4} \right]_0^1 = 1$ $\Rightarrow a \left(\frac{1}{3} - \frac{1}{4} \right) = 1$ $\Rightarrow \frac{a}{12} = 1 \Rightarrow a = 12$ (1 mark)	c. ii. $12 \int_0^{0.25} (t^2 - t^3) dt$ $= 12 \left[\frac{t^3}{3} - \frac{t^4}{4} \right]_0^{0.25}$ $= 0.05$ (1 mark)																				



Question 4 (continued)

c. iii.

$$\Pr(X \leq 3) | \Pr(X \leq 6) = \frac{\Pr(X \leq 3) \cap \Pr(X \leq 6)}{\Pr(X \leq 6)}$$

$$= \frac{\Pr(X \leq 3)}{\Pr(X \leq 6)}$$

$$\Pr(X \leq 6) = 12 \left[\frac{t^3}{3} - \frac{t^4}{4} \right]_0^{0.5} = 0.3125 \quad (1 \text{ mark})$$

$$\Pr(X \leq 3) | \Pr(X \leq 6) = \frac{0.05078125}{0.3125}$$
$$= 0.1625 \quad (1 \text{ mark})$$



Question 4 (continued)

d. i.

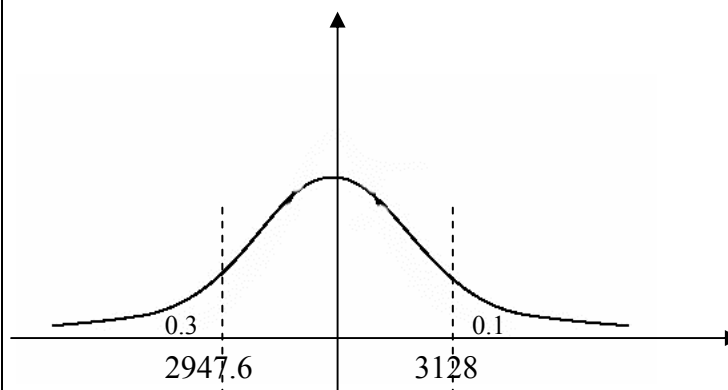
Binomial $n = 10$, $p = 0.3$

$\text{binompdf}(10, 0.3, 1) = 0.1493$

Probability = 0.1493

(1 mark)

d. ii.



$$\Pr(X < 3128) = \Pr(Z < a) = 0.9$$

$$a = 1.2816 \quad (1 \text{ mark})$$

$$\Pr(X < 2947.6) = \Pr(Z < -b) = 0.3$$

$$\Pr(Z > b) = 0.3 \Rightarrow \Pr(Z < b) = 0.7$$

$$b = 0.5244$$

$$-b = -0.5244 \quad (1 \text{ mark})$$

$$Z = \frac{x - \mu}{\sigma}$$

$$1.2816 = \frac{3128 - \mu}{\sigma} \dots (1)$$

$$-0.5244 = \frac{2947.6 - \mu}{\sigma} \dots (2)$$

$$\mu + 1.2816\sigma = 3128 \dots (1a)$$

$$\mu - 0.5244\sigma = 2947.6 \dots (2a) \quad (1 \text{ mark})$$

$$(1a) - (2a) \text{ gives } 1.806\sigma = 180.4$$

$$\sigma = 99.98 = 100 \text{ g to the nearest g} \quad (1 \text{ mark})$$

Substituting in (1a) gives

$$\mu = 3128 - 128.018 = 2999.98 = 3000 \text{ g to the nearest g} \quad (1 \text{ mark})$$

END OF SUGGESTED SOLUTIONS
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