



Year 2006

VCE

Mathematical Methods CAS

Trial Examination 2



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9817 5374
FAX: (03) 9817 4334
chemas@chemas.com
www.chemas.com

© Kilbaha Multimedia Publishing 2006
ABN 47 065 111 373



PURPOSE OF THIS TRIAL EXAMINATION

This Mathematics Methods CAS Trial Examination is designed to assess

- understanding and communication of mathematical ideas
- interpretation, analysis and solution of routine problems
- interpretation, analysis and solution of non-routine problems

Assessment is by multiple-choice questions and extended answer questions involving multi-stage solutions of increasing complexity.

IMPORTANT COPYRIGHT NOTICE

- This material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Multimedia Publishing.
- The contents of this work are copyrighted. Unauthorised copying of any part of this work is illegal and detrimental to the interests of the author.
- For authorised copying within Australia please check that your institution has a licence from Copyright Agency Limited. This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.
- Teachers and students are reminded that for the purposes of school requirements and external assessments, students must submit work that is clearly their own.
- Schools which purchase a licence to use this material may distribute an electronic file to the students at the school for their exclusive use. This distribution can be done either on an Intranet Server or on media for the use on stand-alone computers.
- Schools which purchase a licence to use this material may distribute a printed copy to the students at the school for their exclusive use.
- The Word file of this material (if supplied) is for use **only** within the school
- The Word file may be modified to suit the school syllabus and for teaching purposes.
- All modified versions of the Word file must carry this copyright notice.
- Commercial use of any content in this Trial Examination is expressly prohibited.



VICTORIAN CERTIFICATE OF EDUCATION 2006

	STUDENT NUMBER		Letter
Figures			
Words			

MATHEMATICAL METHODS CAS

Trial Written Examination 2

Reading time: 15 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
Total			80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one approved graphics calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator, one bound reference.

Materials supplied

- Question and answer book of 22 pages, with a detachable sheet of miscellaneous formulas.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct **and** sign your name in the space provided to verify this.
- All written responses must be in English

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

© KILBAHA MULTIMEDIA PUBLISHING 2006



MATHEMATICAL METHODS CAS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.



Mathematical Methods and Mathematical Methods CAS

Formulas

Mensuration

area of a trapezium $\frac{1}{2}(a+b)h$

curved surface of a cylinder $2\pi rh$

volume of a cylinder $\pi r^2 h$

volume of a cone $\frac{1}{3}\pi r^2 h$

volume of a pyramid $\frac{1}{3}Ah$

volume of a sphere $\frac{4}{3}\pi r^3$

area of a triangle $\frac{1}{2}bc \sin A$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(x^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum xp(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$



continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} xf(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$
------------	------------------------------------	---	---

END OF FORMULA SHEET
VCE MATHEMATICAL METHODS CAS 2006
Trial Written Examination 2
ANSWER SHEET

NAME: _____

STUDENT _____
NUMBER

SIGNATURE _____

Instructions

- Write your name in the space provided above.
- Write your student number in the space provided above. Sign your name.
- Use a **PENCIL** for **ALL** entries.
If you make a mistake, **ERASE** it - **DO NOT** cross it out.
- Marks will **NOT** be deducted for incorrect answers.
- **NO MARK** will be given if more than **ONE** answer is completed for any question.
- All answers must be completed like **THIS** example.

A	B	C	D	E
---	----------	---	---	---

1	A	B	C	D	E	12	A	B	C	D	E
2	A	B	C	D	E	13	A	B	C	D	E
3	A	B	C	D	E	14	A	B	C	D	E
4	A	B	C	D	E	15	A	B	C	D	E
5	A	B	C	D	E	16	A	B	C	D	E
6	A	B	C	D	E	17	A	B	C	D	E
7	A	B	C	D	E	18	A	B	C	D	E
8	A	B	C	D	E	19	A	B	C	D	E
9	A	B	C	D	E	20	A	B	C	D	E
10	A	B	C	D	E	21	A	B	C	D	E
11	A	B	C	D	E	22	A	B	C	D	E

Please DO NOT fold, bend or staple this form

**Instructions**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The number of solutions of the equation $2^x = x^2$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

Question 2

If $f(x) = x^2 - 9$ and $g(x) = \log_e x$, then $g\{f(x)\}$ equals

- A. $\log_e(x^2 - 9), x \in [-3, 3]$
- B. $\log_e(x^2 - 9), x \in (-3, 3)$
- C. $\log_e(x^2 - 9), x \in (-\infty, -3) \cup (3, \infty)$
- D. $[\log_e x]^2 - 9, x \in (0, \infty)$
- E. $2\log_e x - 9, x \in (0, \infty)$

**Question 3**

If $f(x) = \sin(3x)$, then a restricted domain for which $f(x)$ has an inverse could be

- A. $\left[0, \frac{2\pi}{3}\right]$
- B. $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$
- C. $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$
- D. $\left[0, \frac{\pi}{3}\right]$
- E. $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

Question 4

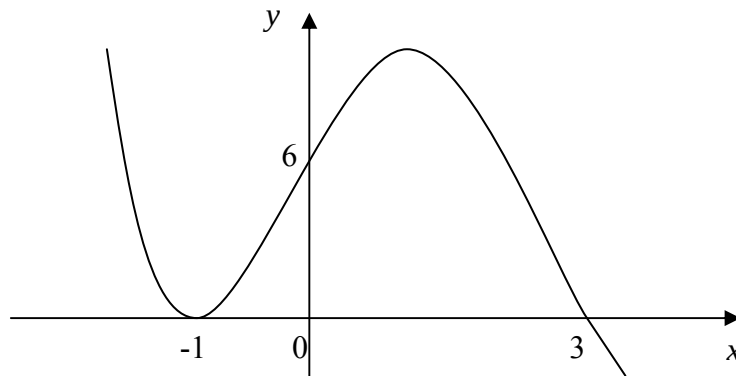
A discrete random variable X is defined only for values of $x = 1, 2, 3, 4$, and 5 , with probabilities given by $\Pr(X = x) = (kx)^2$ where k is a constant. The value of k is

- A. $\pm \frac{\sqrt{55}}{55}$
- B. $\frac{\sqrt{55}}{55}$
- C. $\pm \frac{\sqrt{15}}{15}$
- D. $\frac{\sqrt{15}}{15}$
- E. $\frac{1}{15}$

**Question 5**

The simultaneous equations $-3x + p^2y = -6$ and $x - 4y = 2$ have a unique solution for

- A. $p \in \mathbb{R} \setminus \{1\}$
- B. $p = \pm 2\sqrt{3}$
- C. $p \in \mathbb{R} \setminus (-2, 2)$
- D. $p \in \mathbb{R} \setminus (-2\sqrt{3}, 2\sqrt{3})$
- E. $p \in \mathbb{R} \setminus \{-2\sqrt{3}, 2\sqrt{3}\}$

Question 6

The graph of the function shown above could be

- A. $y = 2(x+1)^2(3-x)$
- B. $y = -2(x+1)^2(3-x)$
- C. $y = -2(x-1)^2(x+3)$
- D. $y = -2(x+1)(x-3)^2$
- E. $y = 2(x+1)(x-3)^2$

**Question 7**

If $f(2x^2) = 2f(x)$ for $x > 0$ then $f(x)$ could be

- A. $\log_e(x)$
- B. $\log_e(2x)$
- C. $\log_e\left(\frac{x}{2}\right)$
- D. e^{2x}
- E. e^x

Question 8

The volume of a spherical balloon of radius r cm is $\frac{4}{3}\pi r^3$. The balloon is being inflated at a rate of 0.1 cm sec^{-1} . The rate at which the volume is increasing when the radius is 7 cm, is closest to

- A. $6.2 \text{ cm}^3 \text{ sec}^{-1}$
- B. $14.4 \text{ cm}^3 \text{ sec}^{-1}$
- C. $61.6 \text{ cm}^3 \text{ sec}^{-1}$
- D. $143.7 \text{ cm}^3 \text{ sec}^{-1}$
- E. $615 \text{ cm}^3 \text{ sec}^{-1}$

**Question 9**

The area of the region bounded by the graphs of $y = |6x - 11|$ and $y = |x|$ is closest to

- A. 0.576
- B. 0.612
- C. 1.188
- D. 6.601
- E. 11.11

Question 10

The interval $\left[0, \frac{\pi}{4}\right]$ is divided into n equal subintervals by the points, $x_0, x_1, x_2, \dots, x_{n-1}, x_n$

where $0 = x_0 < x_1 < x_2 < \dots < x_{n-1} < x_n = \frac{\pi}{4}$. Let $\delta x_i = x_i - x_{i-1}$ for $i = 1, 2, \dots, n$.

Then $\lim_{\delta x \rightarrow 0} \sum_{i=1}^n \sin(x) \delta x$ is equal to

- A. $\int_{\frac{\pi}{4}}^0 \sin(x) dx$
- B. $\int_0^{\frac{\pi}{4}} \cos(x) dx$
- C. $\frac{2 - \sqrt{2}}{2}$
- D. $-\frac{\sqrt{2}}{2}$
- E. $\frac{\sqrt{2}}{2}$

**Question 11**

A radioactive isotope decays according to the equation $A = A_0 e^{-kt}$ where A is the amount of the isotope remaining after t days. Initially, there is 200 g of the isotope which has a half life of 20 days. The rate at which the isotope is decaying when $t = 30$ days, is

- A. 0.099 g/day.
- B. -0.996 g/day.
- C. -0.885 g/day.
- D. 1.856 g/day.
- E. 2.45 g/day.

Question 12

Given the function $f : R \rightarrow R$ where $f(x) = xe^x$, which one of the following statements is true?

- A. The gradient is positive for all values of x .
- B. The area between the X axis and this graph and the ordinates $x = 0$ and $x = 2$ is $2e^x$.
- C. The graph has a stationary point of inflexion when $x = 1$.
- D. The gradient is positive when $x \geq -1$.
- E. The gradient is negative when $x < -1$.

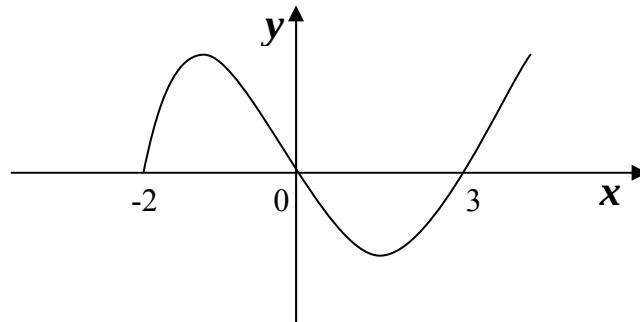
Question 13

If X denotes the number of successes in n independent trials, each with a probability of p , then X has a binomial distribution. Which of the following statements is FALSE?

- A. $\Pr(X > 1) = 1 - (1 - p)^n$
- B. $E(X) > \text{VAR}(X)$
- C. $\frac{\text{VAR}(X)}{E(X)} = 1 - p$
- D. $\Pr(X = 0) = (1 - p)^n$
- E. $\Pr(X = 1) = np(1 - p)^{n-1}$

**Question 14**

The graph of $y = f(x)$ is shown below.

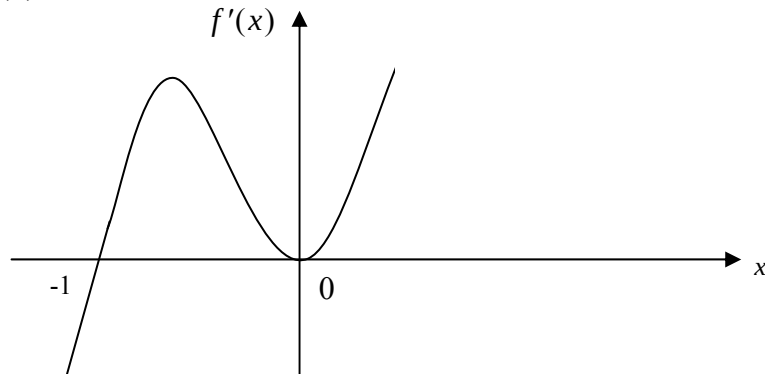


Which one of the following could be the graph of $y = \sqrt{f(x)}$?

A. 	B.
C. 	D.
E. 	

**Question 15**

The graph of $y = f'(x)$ is shown below



Which one of the following could be the graph of $y = f(x)$?

A. Graph A shows a function $f(x)$ that is a parabola opening upwards. It has a local minimum at $x = -1$ and passes through the origin $(0, 0)$. The function is increasing for $x > 0$ and decreasing for $x < -1$.	B. Graph B shows a function $f(x)$ that is a cubic-like curve. It has a local minimum at $x = -1$ and passes through the origin $(0, 0)$. The function is increasing for $x > 0$ and decreasing for $x < -1$.
C. Graph C shows a function $f(x)$ that is a cubic-like curve. It has a local maximum at $x = -1$ and passes through the origin $(0, 0)$. The function is decreasing for $x > 0$ and increasing for $x < -1$.	D. Graph D shows a function $f(x)$ that is a cubic-like curve. It has a local maximum at $x = -1$ and passes through the origin $(0, 0)$. The function is increasing for $x > 0$ and decreasing for $x < -1$.
E. Graph E shows a function $f(x)$ that is a cubic-like curve. It has a local minimum at $x = -1$ and passes through the origin $(0, 0)$. The function is decreasing for $x > 0$ and increasing for $x < -1$.	

**Question 16**

The equation of the normal to the curve $f(x) = \log_e(2x-1) + 4x$ at the point where $x = 1$, is

- A. $y = 6x - 2$
- B. $y = \frac{1}{6}x - 2$
- C. $6y + x - 2 = 0$
- D. $6y + x - 25 = 0$
- E. $6y - x - 23 = 0$

Question 17

The linear transformation $T : R^2 \rightarrow R^2$ is defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

The equation of the image of the curve $y = \sqrt{x}$ under T is given by

- A. $y = \sqrt{\frac{x-1}{2}} - 3$
- B. $2x = \sqrt{3-y} + 1$
- C. $x = 3 - \sqrt{2y+1}$
- D. $y = 3 - \sqrt{2x+1}$
- E. $x = 2\sqrt{3-y} + 1$

**Question 18**

When John gets a birdie on a hole in golf, the probability he will get a birdie on the next hole is 0.7. When he does not get a birdie on a particular hole, the probability that he will not get a birdie on the next hole is 0.9. If the probability that John will get a birdie on the first hole is 0.5, then the probability that he will get a birdie on the third hole is

- A. 0.12
- B. 0.34
- C. 0.63
- D. 0.67
- E. 0.68

Question 19

The mean value of the function $f(x) = 30 \cos(\pi x) + 40$ over $0 \leq x \leq 2$ is closest to

- A. 80
- B. 70
- C. 40
- D. 69.94
- E. 139.88

**Question 20**

Tran rolls a six sided die twice.

A is the event “getting a six on the first roll”.

B is the event “getting an even number on the second roll”.

Which one of the following statements is true?

A. A and B are mutually exclusive events.

B. $\Pr(A \cup B) = \frac{5}{12}$

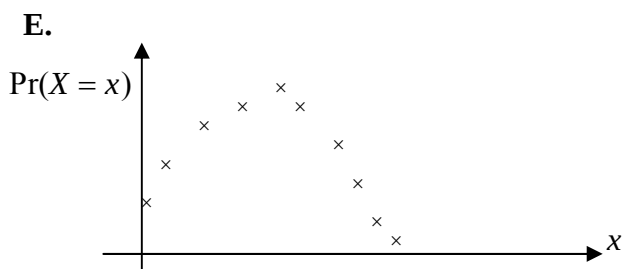
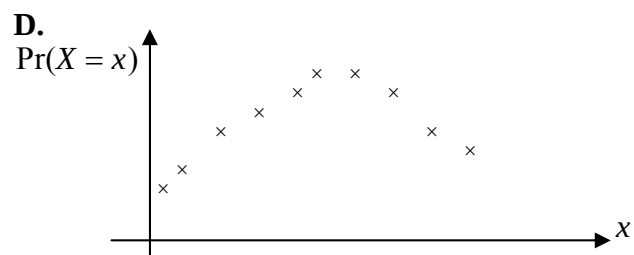
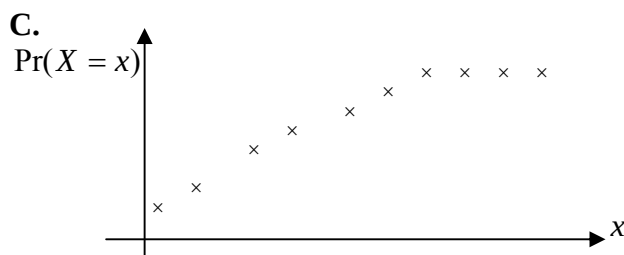
C. $\Pr(A \cap B) = 0$

D. $\Pr(A) \times \Pr(B) = \frac{1}{36}$

E. A and B are independent events.

Question 21

Which one of the following graphs best represents the shape of a distribution where there are 10 independent trials, each with a probability of 0.3 of a success.





Question 22

There are 2 boxes each containing red and green balls of the same size. The yellow box has 4 red and 2 green balls. The black box has 3 red and 3 green balls. A box is chosen at random and 2 balls are chosen from this box, without replacement, and found to be both green. The probability that it was the black box that was chosen is closest to

- A. 0.42
- B. 0.53
- C. 0.67
- D. 0.75
- E. 0.80

**END OF PART I
MULTIPLE CHOICE QUESTIONS**

**Question 1**

- a. If the function $f(x) = \frac{ax}{bx^2 + c}$ has a turning point at $\left(1, \frac{1}{2}\right)$ then show that $a = b = c$ and find the coordinates of all the turning points.

4 marks

For the remainder of this question let $a = b = c = 1$ and consider the function $f(x) = \frac{x}{1+x^2}$.

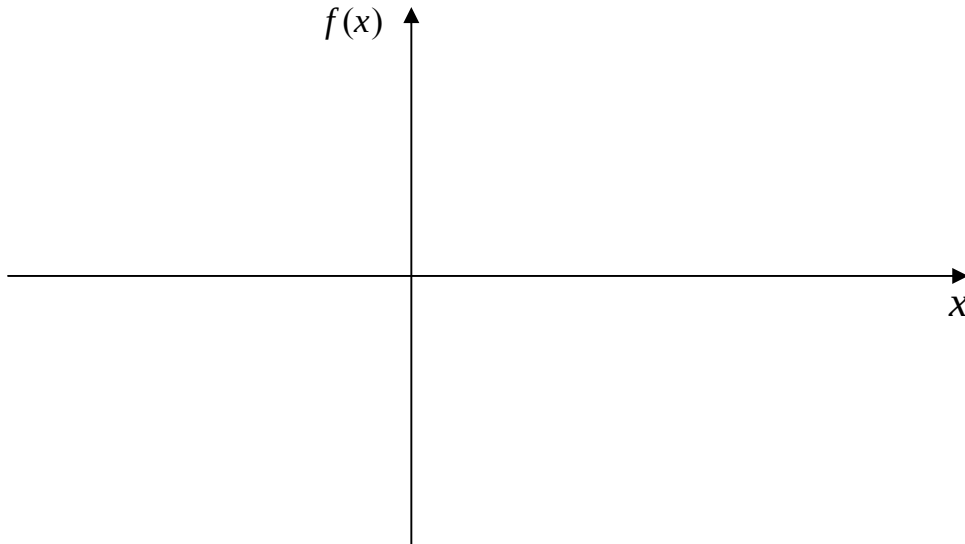
- b. State the transformations that have occurred in changing $f(x)$ to $-f(x+2)+3$

3 marks



Question 1 (continued)

- c. On the axes below, sketch the graph of $-f(x+2)+3$, showing all turning points, intercepts with the axes and the equations of any asymptotes.



4 marks

- d. State the maximal domain of $f(x)$ such that $f(x)$ has a positive gradient.

1 mark

- e. Find $f^{-1}(x)$ if $f(x)$ has a domain $(0, \infty)$ and give the domain of $f^{-1}(x)$.

2 marks

Total = 14 marks

**Question 2**

A mass is suspended from the ceiling on the end of a spring. It is pulled down and then released. The mass oscillates up and down. The length of the spring x cm at a time t seconds, where $t \geq 0$ is given by the equation:

$$x = 64e^{-0.2t} \sin\left(\frac{2\pi t}{3}\right) + 41$$

- a. What is the initial length of the spring?

1 mark

- b. Given that $\frac{dx}{dt} = 64e^{-0.2t} \left(a \sin\left(\frac{2\pi t}{3}\right) + b \cos\left(\frac{2\pi t}{3}\right) \right)$ find the exact values of a and b .

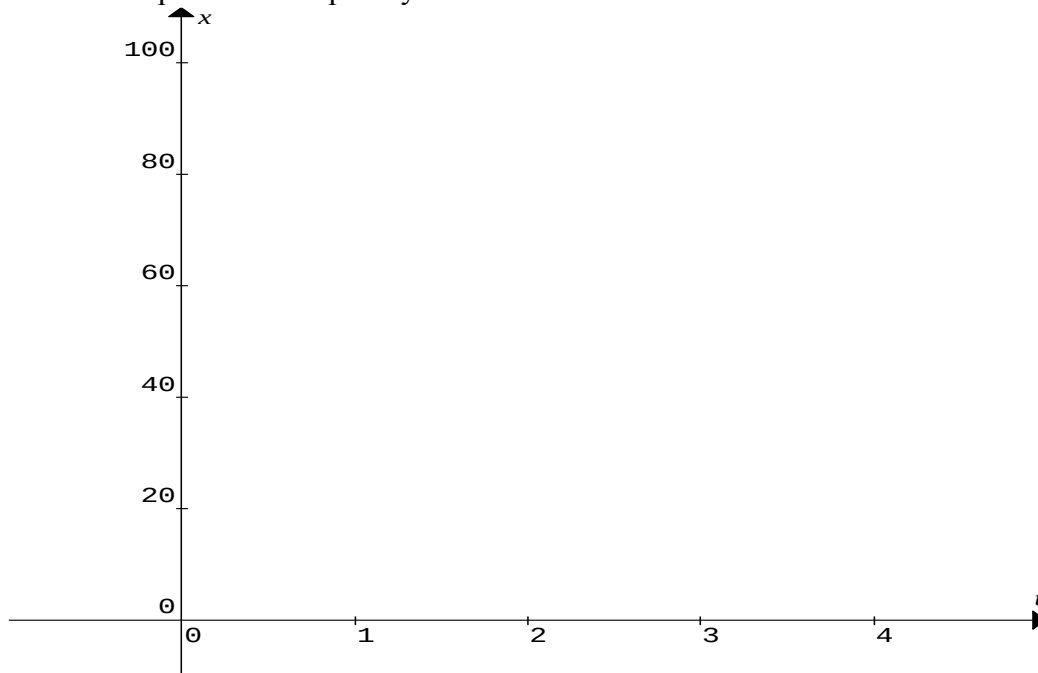
2 marks

- c. Hence find the times when the length of the spring is a maximum and minimum, and find the maximum and minimum lengths of the spring, giving all answers correct to four decimal places.

4 marks

**Question 2 (continued)**

- d. Sketch the graph of x versus t , on the axes below.
Is the motion periodic? Explain your answer.



3 marks

- e. For how long is the length of the spring greater than 75 cm.
Give your answer correct to three decimal places.

2 marks

- f. What is the exact rate of change of the length of the spring when $t = 1.5$ seconds?

1 mark

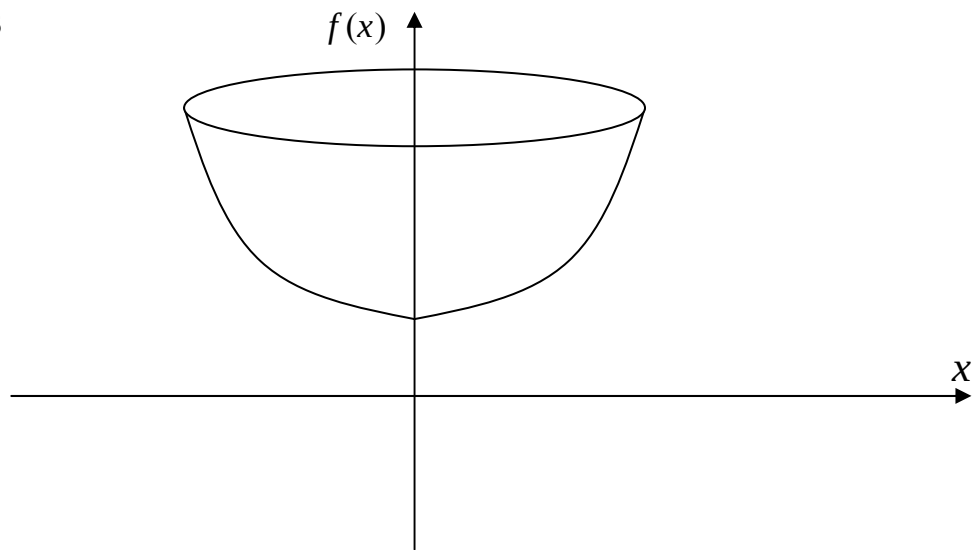
- g. What is the average rate of change of the length of the spring from $t = 1$ to $t = 1.5$ seconds?
Give an exact answer.

1 mark

Total = 14 marks



Question 3



The curved part of a bowl shown on the axes above can be modelled by the equation of a parabola that contains the points $(0, 1)$ and $(-3, 3.25)$, $-10 \leq x \leq 10$

- a. Find the equation of the parabola.

2 marks

- b. Find the diameter of the circular top of the bowl.

1 mark

- c. Express x in terms of y .

1 mark



Question 3 (continued)

- d. The volume of the bowl can be found using the formula $V = \int_a^b \pi x^2 dy$.

Find the volume of the bowl and give your answer to the nearest cubic unit.

2 marks

- e. Water is poured into the bowl so that the radius of the top of the water level is 3 units.
Give the **exact** value of the volume of water in the container.

2 marks

**Question 3 (continued)**

The bowl is now filled with two types of fruits, apples and bananas. Each day a person decides to have one piece of fruit, either an apple or a banana. The fruit chosen depends only on the selection of the fruit chosen on the previous day. If they choose an apple on one day, then the probability that they choose an apple on the next day is $\frac{3}{7}$. The transition matrix for the probability that the person

having either an apple or a banana given the piece of fruit from the previous day is
$$\begin{bmatrix} \frac{3}{7} & \frac{5}{7} \\ \frac{4}{7} & \frac{2}{7} \end{bmatrix}$$

- f. Suppose that the person had an apple on Monday, what is the exact probability of having an apple on the Friday of the same week.

3 marks

- g. In the long term, what is the exact percentage of days on which apples are chosen?

3 marks

Total = 14 marks



Question 4

- a. A retailer sells tulip bulbs in packets of 5. The probability that the bulbs in the packet will flower has the probability distribution given in the table below.

x	0	1	2	3	4	5
$\Pr(X = x)$	0.04	0.01	0.05	0.2	0.4	0.3

- i. How many bulbs would you expect to flower from each packet?
Give your answer to 2 decimal places

1 mark

- ii. What is the standard deviation of this distribution. Give your answer to 2 decimal places.

2 marks

- b. From past experience, the retailer knows that if a tulip bulb flowers one year, it has an 80% chance of flowering the next year, and if a tulip bulb does **not** flower one year, it has an 85% chance of **not** flowering the next year.

- i. What is the probability that a tulip bulb that did **not** flower in 2005 will flower in 2006?

1 mark

- ii. What is the probability that a tulip bulb that flowered in 2005 will **not** flower in 2008?
Give your answer to 4 decimal places.

2 marks

**Question 4 (continued)**

- c. The time, t , in years, taken for a bulb to flower after it is planted is described by the probability density function:

$$f(t) = \begin{cases} at^2(1-t) & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- i. Find the value of a .

1 mark

- ii. What is the probability that a bulb will flower within 3 months of planting?
Give your answer to 2 decimal places.

1 mark

- iii. Given that a particular tulip flowered within 6 months of planting, what is the probability that it flowered within 3 months? Give your answer to 4 decimal places.

2 marks

Question 4 (continued)

- d. A nursery wholesaler sells tulip bulbs in boxes of 1000. It is known that the weight of the boxes is normally distributed and that 10% of the boxes weigh more than 3128 g while 30% of the boxes weigh less than 2947.6 g.
- i. A nursery retailer buys 10 boxes of tulip bulbs from the wholesaler. What is the probability that not more than one of the boxes will have a weight less than 2947.6 g?
Give your answer to 4 decimal places.

1 mark

- ii. What is the mean weight and the standard deviation of the wholesaler's boxes. Give your answers to the nearest g.

5 marks

Total = 16 marks

END OF QUESTION AND ANSWER BOOK

KILBAHA MULTIMEDIA PUBLISHING PO BOX 2227 KEW VIC 3101 AUSTRALIA	TEL: (03) 9817 5374 FAX: (03) 9817 4334 chemas@chemas.com www.chemas.com
---	--