



Year 2006

VCE

Mathematical Methods

Trial Examination 2



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PURPOSE OF THIS TRIAL EXAMINATION

This Mathematics Methods Trial Examination is designed to assess

- understanding and communication of mathematical ideas
- interpretation, analysis and solution of routine problems
- interpretation, analysis and solution of non-routine problems

Assessment is by multiple-choice questions and extended answer questions involving multi-stage solutions of increasing complexity.

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VICTORIAN CERTIFICATE OF EDUCATION 2006

STUDENT NUMBER								Letter
Figures								
Words								

MATHEMATICAL METHODS

Trial Written Examination 2

Reading time: 15 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	4	4	58
Total			80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one approved graphics calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator, one bound reference.

Materials supplied

- Question and answer book of 22 pages, with a detachable sheet of miscellaneous formulas.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct **and** sign your name in the space provided to verify this.
- All written responses must be in English

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.



Mathematical Methods and Mathematical Methods CAS

Formulas

Mensuration

area of a trapesium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin A$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = \sec^2(ax)$	
product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	
mean: $\mu = E(X)$	variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(x^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum xp(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} xf(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

END OF FORMULA SHEET



VCE MATHEMATICAL METHODS 2006
Trial Written Examination 2
ANSWER SHEET

NAME: _____

**STUDENT
NUMBER** _____

SIGNATURE _____

Instructions

- Write your name in the space provided above.
- Write your student number in the space provided above. Sign your name.
- Use a **PENCIL** for **ALL** entries.
If you make a mistake, **ERASE** it - **DO NOT** cross it out.
- Marks will **NOT** be deducted for incorrect answers.
- **NO MARK** will be given if more than **ONE** answer is completed for any question.
- All answers must be completed like **THIS** example.

A	B	C	D	E
---	---	---	---	---

1	A	B	C	D	E	12	A	B	C	D	E
2	A	B	C	D	E	13	A	B	C	D	E
3	A	B	C	D	E	14	A	B	C	D	E
4	A	B	C	D	E	15	A	B	C	D	E
5	A	B	C	D	E	16	A	B	C	D	E
6	A	B	C	D	E	17	A	B	C	D	E
7	A	B	C	D	E	18	A	B	C	D	E
8	A	B	C	D	E	19	A	B	C	D	E
9	A	B	C	D	E	20	A	B	C	D	E
10	A	B	C	D	E	21	A	B	C	D	E
11	A	B	C	D	E	22	A	B	C	D	E

Please DO NOT fold, bend or staple this form

**Instructions**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The number of solutions of the equation $2^x = x^2$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

Question 2

If $f(x) = x^2 - 9$ and $g(x) = \log_e x$, then $g\{f(x)\}$ equals

- A. $\log_e(x^2 - 9), x \in [-3, 3]$
- B. $\log_e(x^2 - 9), x \in (-3, 3)$
- C. $\log_e(x^2 - 9), x \in (-\infty, -3) \cup (3, \infty)$
- D. $[\log_e x]^2 - 9, x \in (0, \infty)$
- E. $2 \log_e x - 9, x \in (0, \infty)$

**Question 3**

If $f(x) = \sin(3x)$, then a restricted domain for which $f(x)$ has an inverse could be

- A. $\left[0, \frac{2\pi}{3}\right]$
- B. $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$
- C. $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$
- D. $\left[0, \frac{\pi}{3}\right]$
- E. $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

Question 4

A discrete random variable X is defined only for values of $x = 1, 2, 3, 4$, and 5 , with probabilities given by $\Pr(X = x) = (kx)^2$ where k is a constant. The value of k is

- A. $\pm \frac{\sqrt{55}}{55}$
- B. $\frac{\sqrt{55}}{55}$
- C. $\pm \frac{\sqrt{15}}{15}$
- D. $\frac{\sqrt{15}}{15}$
- E. $\frac{1}{15}$

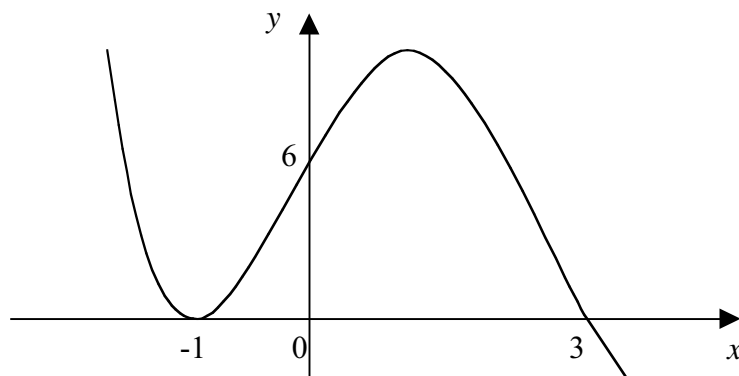


Question 5

The sum of the solutions of $2\sin^2\theta = 7\cos\theta - 2$, $0 \leq \theta \leq 2\pi$, is

- A. $\frac{\pi}{3}$
- B. $\frac{5\pi}{3}$
- C. π
- D. $\frac{11\pi}{6}$
- E. 2π

Question 6



The graph of the function shown above could be

- A. $y = 2(x+1)^2(3-x)$
- B. $y = -2(x+1)^2(3-x)$
- C. $y = -2(x-1)^2(x+3)$
- D. $y = -2(x+1)(x-3)^2$
- E. $y = 2(x+1)(x-3)^2$

**Question 7**

Given that $f(x) = 4 - \frac{1}{x-3}$, then $f^{-1}(x)$ equals

- A. $3 + \frac{1}{x-4}$
- B. $3 - \frac{1}{x-4}$
- C. $3 + \frac{1}{x+4}$
- D. $3 - \frac{1}{x+4}$
- E. $4 + \frac{1}{x-3}$

Question 8

The volume of a spherical balloon of radius r cm is $\frac{4}{3}\pi r^3$. The balloon is being inflated at a rate of 0.1 cm sec^{-1} . The rate at which the volume is increasing when the radius is 7 cm, is closest to

- A. $6.2 \text{ cm}^3 \text{ sec}^{-1}$
- B. $14.4 \text{ cm}^3 \text{ sec}^{-1}$
- C. $61.6 \text{ cm}^3 \text{ sec}^{-1}$
- D. $143.7 \text{ cm}^3 \text{ sec}^{-1}$
- E. $615 \text{ cm}^3 \text{ sec}^{-1}$

**Question 9**

The area of the region bounded by the graphs of $y = |6x - 11|$ and $y = |x|$ is closest to

- A. 0.576
- B. 0.612
- C. 1.188
- D. 6.601
- E. 11.11

Question 10

An antiderivative of $\frac{1}{\sqrt{3x+2}}$ could be

- A. $\frac{\log_e(\sqrt{3x+2})}{3}$
- B. $\frac{2\log_e(\sqrt{3x+2})}{3}$
- C. $\frac{2\sqrt{3x+2}}{3}$
- D. $\frac{\sqrt{3x+2}}{2}$
- E. $\frac{\sqrt{3x+2}}{6}$

**Question 11**

A radioactive isotope decays according to the equation $A = A_0 e^{-kt}$ where A is the amount of the isotope remaining after t days. Initially, there is 200 g of the isotope which has a half life of 20 days. The rate at which the isotope is decaying when $t = 30$ days, is

- A. 0.099 g/day.
- B. -0.996 g/day.
- C. -0.885 g/day.
- D. 1.856 g/day.
- E. 2.45 g/day.

Question 12

Given the function $f : R \rightarrow R$ where $f(x) = xe^x$, which one of the following statements is true?

- A. The gradient is positive for all values of x .
- B. The area between the X axis and this graph and the ordinates $x = 0$ and $x = 2$ is $2e^x$.
- C. The graph has a stationary point of inflexion when $x = 1$.
- D. The gradient is positive when $x \geq -1$.
- E. The gradient is negative when $x < -1$.

Question 13

A continuous random variable X has a probability density function

$$f(x) = \begin{cases} kx^2 & 0 \leq x < 9 \\ 0 & \text{elsewhere} \end{cases}$$

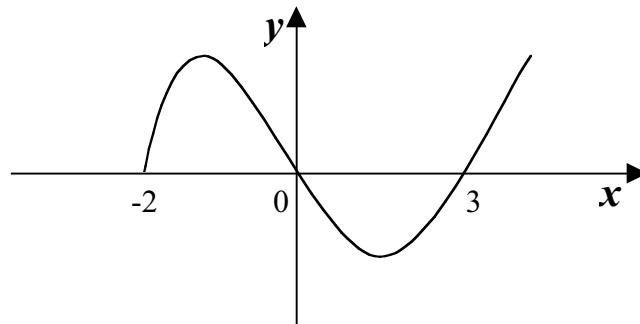
The value of k is closest to

- A. 0.004
- B. 0.006
- C. 0.02
- D. 0.04
- E. 0.06



Question 14

The graph of $y = f(x)$ is shown below.



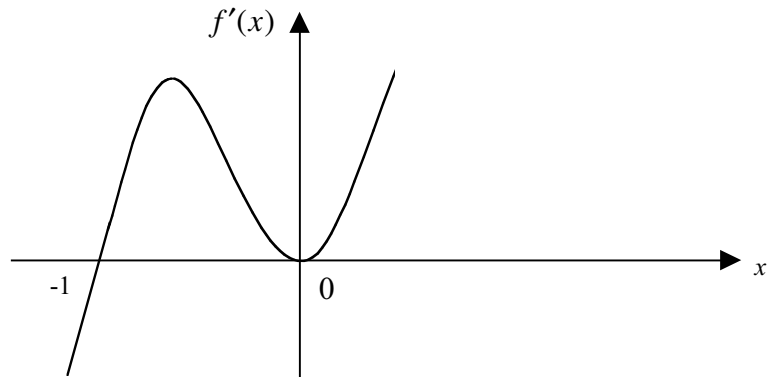
Which one of the following could be the graph of $y = \sqrt{f(x)}$?

A.	B.
C.	D.
E.	

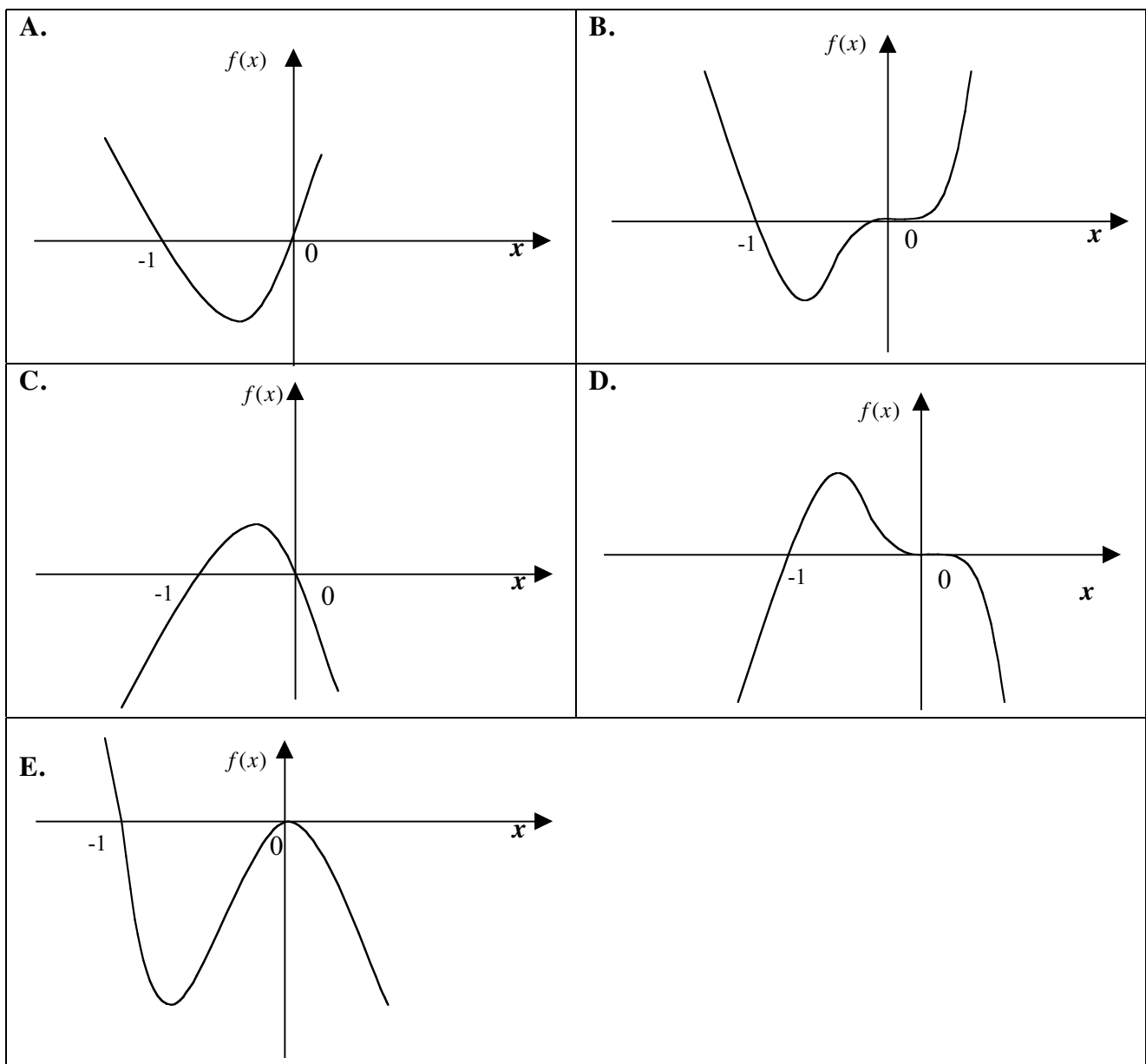


Question 15

The graph of $y = f'(x)$ is shown below



Which one of the following could be the graph of $y = f(x)$?





Question 16

The equation of the normal to the curve $f(x) = \log_e(2x - 1) + 4x$ at the point where $x = 1$, is

- A. $y = 6x - 2$
- B. $y = \frac{1}{6}x - 2$
- C. $6y + x - 2 = 0$
- D. $6y + x - 25 = 0$
- E. $6y - x - 23 = 0$

Question 17

The area between the X axis, the curve $f(x) = \frac{2}{12 - 5x}$ and the ordinates $x = 1$ and $x = 2$ can be given by

- A. $2[\log_e 7 - \log_e 2]$
- B. $\frac{1}{2}[\log_e 7 - \log_e 2]$
- C. $0.4 \log_e 7$
- D. $2.5 \log_e(3.5)$
- E. $0.4 \log_e(3.5)$

**Question 18**

When John gets a birdie on a hole in golf, the probability he will get a birdie on the next hole is 0.7. When he does not get a birdie on a particular hole, the probability that he will not get a birdie on the next hole is 0.9. If the probability that John will get a birdie on the first hole is 0.5, then the probability that he will get a birdie on the third hole is

- A. 0.12
- B. 0.34
- C. 0.63
- D. 0.67
- E. 0.68

Question 19

X is a continuous random variable such that the probability density function is given by

$$f(x) = \begin{cases} \frac{2}{\pi} & 0 \leq x \leq \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

The expected value of $\cos(x)$ is closest to

- A. 0.32
- B. 0.5
- C. 0.64
- D. 1
- E. 1.28

**Question 20**

Tran rolls a six sided die twice.

A is the event “getting a six on the first roll”.

B is the event “getting an even number on the second roll”.

Which one of the following statements is true?

A. A and B are mutually exclusive events.

B. $\Pr(A \cup B) = \frac{5}{12}$

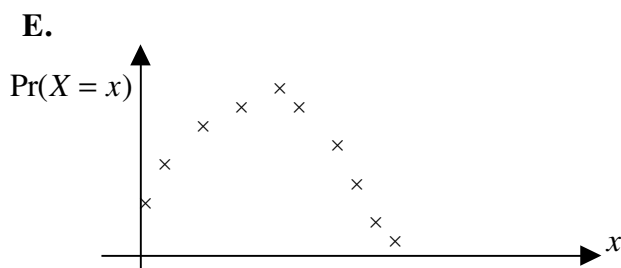
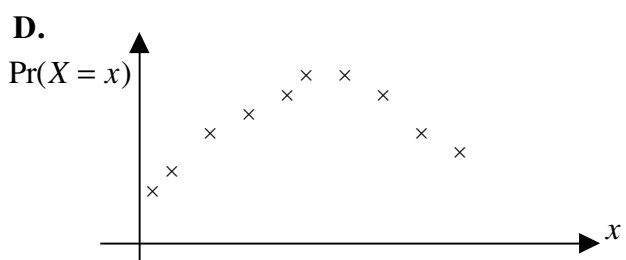
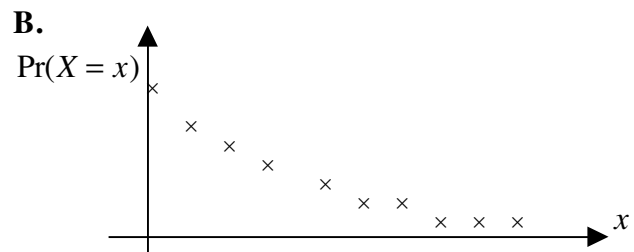
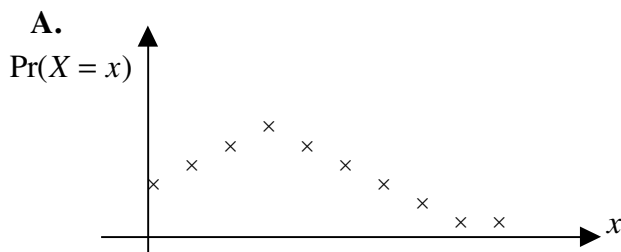
C. $\Pr(A \cap B) = 0$

D. $\Pr(A) \times \Pr(B) = \frac{1}{36}$

E. A and B are independent events.

Question 21

Which one of the following graphs best represents the shape of a distribution where there are 10 independent trials, each with a probability of 0.3 of a success.



**Question 22**

There are 2 boxes each containing red and green balls of the same size. The yellow box has 4 red and 2 green balls. The black box has 3 red and 3 green balls. A box is chosen at random and 2 balls are chosen from this box, without replacement, and found to be both green. The probability that it was the black box that was chosen is closest to

- A. 0.42
- B. 0.53
- C. 0.67
- D. 0.75
- E. 0.80

**END OF PART I
MULTIPLE CHOICE QUESTIONS**



Question 1

All parts of this question refer to the function $f(x) = \frac{x}{1+x^2}$.

- a.** Use calculus to find the turning points of the graph $f(x) = \frac{x}{1+x^2}$ and state the types of turning points.

3 marks

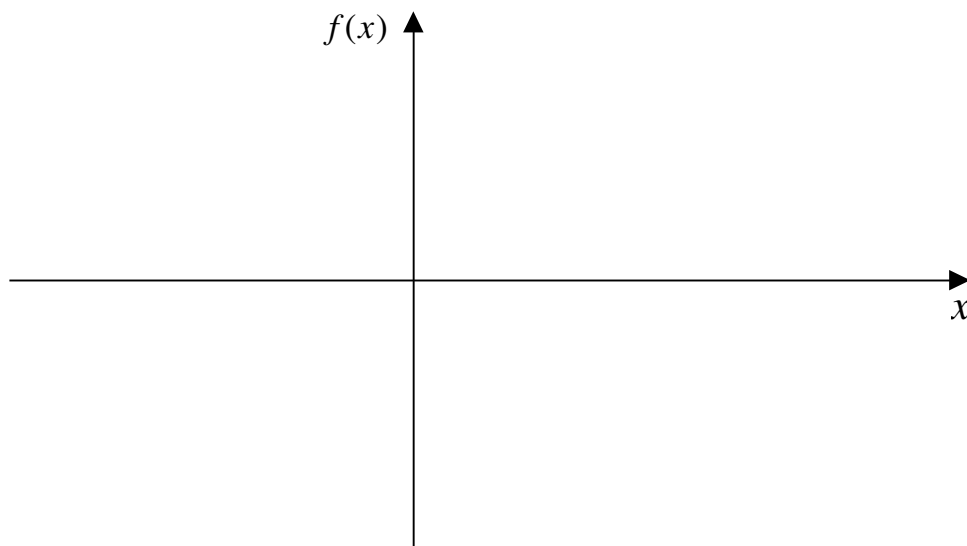
- b.** State the transformations that have occurred in changing $f(x)$ to $-f(x+2)+3$

3 marks

**Question 1 (continued)**

All parts of this question refer to the function $f(x) = \frac{x}{1+x^2}$.

- c. On the axes below, sketch the graph of $-f(x+2)+3$, showing all turning points, intercepts with the axes and the equations of any asymptotes.



4 marks

- d. State the maximal domain of $f(x)$ such that $f(x)$ has a positive gradient.

1 mark

- e. Find $f^{-1}(x)$ if $f(x)$ has a domain $(0, \infty)$ and give the domain of $f^{-1}(x)$.

2 marks

Total = 13 marks

**Question 2**

A mass is suspended from the ceiling on the end of a spring. It is pulled down and then released. The mass oscillates up and down. The length of the spring x cm at any time t seconds, is given by the equation:

$$x = a + b \sin\left(\frac{2\pi t}{3}\right) \text{ where } a > b$$

- a. What is the maximum length of the spring?

1 mark

- b. What is the minimum length of the spring?

1 mark

- c. If $a = 64$ and $b = 8$, find the first 4 times when the spring has a length of 68 cm. Give **exact** values for these answers.

3 marks

- d. How long does one complete oscillation take?

1 mark



Question 2 (continued)

e. For how long is the length of the spring greater than 60 cm in the first 2 seconds?

2 marks

f. What is the average rate of change of the length of the spring from $t = 0$ to $t = 0.25$ seconds?

2 marks

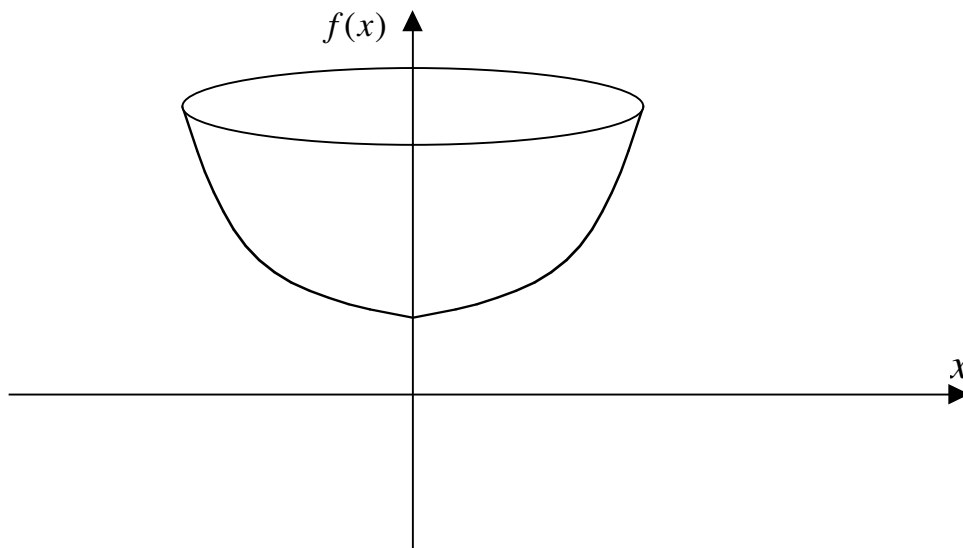
g. What is the **exact** value of the rate of change of the length of the spring when $t = 2.75$ seconds?

2 marks

Total = 12 marks



Question 3



The curved part of a bowl shown on the axes above can be modelled by the equation of a parabola that contains the points $(0, 1)$ and $(-3, 3.25)$, $-10 \leq x \leq 10$

- a. Find the equation of the parabola.

2 marks

- b. Find the diameter of the circular top of the bowl.

1 mark

- c. Express x in terms of y .

1 mark



Question 3 (continued)

- d. The volume of the bowl can be found using the formula $V = \int_a^b \pi x^2 dy$.

Find the volume of the bowl and give your answer to the nearest cubic unit.

4 marks

- e. Water is poured into the bowl so that the radius of the top of the water level is 3 units.
Give the **exact** value of the volume of water in the container.

2 marks

- f. Differentiate $\frac{1}{9}e^{3x}(3x+2)$.

2 marks



Question 3 (continued)

- g. The bowl is being filled with water so that the rate of change of height of water at any time t is $4te^{3t}$.
- i. Find the depth of water in the bowl when $t = 1$. Give your answer to 4 decimal places.

3 marks

- ii. Find the rate of change of volume when $t = 1$. Give your answer to 2 decimal places.

2 marks

Total = 17 marks



Question 4

- a. A retailer sells tulip bulbs in packets of 5. The probability that the bulbs in the packet will flower has the probability distribution given in the table below.

x	0	1	2	3	4	5
$\Pr(X = x)$	0.04	0.01	0.05	0.2	0.4	0.3

- i. How many bulbs would you expect to flower from each packet?
Give your answer to 2 decimal places

1 mark

- ii. What is the standard deviation of this distribution. Give your answer to 2 decimal places.

2 marks

- b. From past experience, the retailer knows that if a tulip bulb flowers one year, it has an 80% chance of flowering the next year, and if a tulip bulb does **not** flower one year, it has an 85% chance of **not** flowering the next year.

- i. What is the probability that a tulip bulb that did **not** flower in 2005 will flower in 2006?

1 mark

- ii. What is the probability that a tulip bulb that flowered in 2005 will **not** flower in 2008?
Give your answer to 4 decimal places.

2 marks



Question 4 (continued)

- c. The time, t , in years, taken for a bulb to flower after it is planted is described by the probability density function:

$$f(t) = \begin{cases} at^2(1-t) & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- i. Find the value of a .

1 mark

- ii. What is the probability that a bulb will flower within 3 months of planting?
Give your answer to 2 decimal places.

1 mark

- iii. Given that a particular tulip flowered within 6 months of planting, what is the probability that it flowered within 3 months? Give your answer to 4 decimal places.

2 marks

**Question 4 (continued)**

- d. A nursery wholesaler sells tulip bulbs in boxes of 1000. It is known that the weight of the boxes is normally distributed and that 10% of the boxes weigh more than 3128 g while 30% of the boxes weigh less than 2947.6 g.
- i. A nursery retailer buys 10 boxes of tulip bulbs from the wholesaler. What is the probability that not more than one of the boxes will have a weight less than 2947.6 g?
Give your answer to 4 decimal places.

1 mark

- ii. What is the mean weight and the standard deviation of the wholesaler's boxes.
Give your answers to the nearest g.

5 marks

Total = 16 marks

END OF QUESTION AND ANSWER BOOK

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