



VCE
2006 Mathematical Methods
Trial Examination 1

Suggested Solutions

© Kilbaha Multimedia Publishing 2006



Kilbaha Multimedia Publishing ABN 47 065 111 373
PO Box 2227
Kew Vic 3101
Australia
Tel: 03 9817 5374
Fax: 03 9817 4334
chemas@chemas.com
www.chemas.com



PURPOSE OF THIS TRIAL EXAMINATION

This Mathematics Methods Trial Examination is designed to assess

- knowledge of mathematical concepts
- skills in carrying out mathematical algorithms
- ability to apply concepts and skills in standard ways

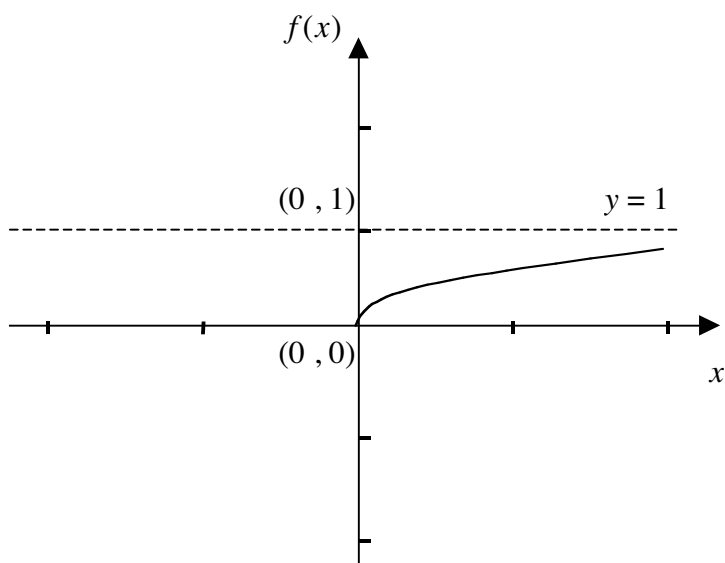
IMPORTANT COPYRIGHT NOTICE

- This material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Multimedia Publishing.
- The contents of this work are copyrighted. Unauthorised copying of any part of this work is illegal and detrimental to the interests of the author.
- For authorised copying within Australia please check that your institution has a licence from Copyright Agency Limited. This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.
- Teachers and students are reminded that for the purposes of school requirements and external assessments, students must submit work that is clearly their own.
- Schools which purchase a licence to use this material may distribute an electronic file to the students at the school for their exclusive use. This distribution can be done either on an Intranet Server or on media for the use on stand-alone computers.
- Schools which purchase a licence to use this material may distribute a printed copy to the students at the school for their exclusive use.
- The Word file of this material (if supplied) is for use **only** within the school
- The Word file may be modified to suit the school syllabus and for teaching purposes.
- All modified versions of the Word file must carry this copyright notice.
- Commercial use of any content in this Trial Examination is expressly prohibited.



Question 1

a.



(1 mark) for point (0 , 0).
(1 mark) for the equation of the asymptote.
(1 mark) for the shape of the graph.

b.

Range is $[0, 1)$ (1 mark)

Question 2

a.

At the point of intersection:

$$e^x = 6e^{-x} - 1$$

Multiply both sides by e^x

$$\Rightarrow e^{2x} = 6 - e^x$$

$$\Rightarrow e^{2x} + e^{-x} - 6 = 0$$

Let $y = e^x$

$$y^2 + y - 6 = 0$$

$$(y + 3)(y - 2) = 0$$

$$\Rightarrow y = -3 \text{ or } y = 2 \quad (1 \text{ mark})$$

$$\Rightarrow e^x = -3 \text{ or } e^x = 2$$

But $e^x > 0 \therefore e^x = 2$ only (1 mark)

$$y = 2, x = \log_e 2$$

$$(\log_e 2, 2) \quad (1 \text{ mark})$$



Question 3

a. $f'(x) = \frac{1}{x^2 + 5} \times 2x \quad (1 \text{ mark})$ $f'(x) = \frac{2x}{x^2 + 5} \quad (1 \text{ mark})$	b. $f(x) = \int (1 - 6 \sin 3x) dx$ $f(x) = x + 2 \cos 3x + c \quad (1 \text{ mark})$ $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + 0 + c = \frac{\pi}{3}$ $\Rightarrow c = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ $f(x) = x + 2 \cos 3x + \frac{\pi}{6} \quad (1 \text{ mark})$
---	---

Question 4

a. $x = \frac{1}{3y - 1}$ $\Rightarrow 3y - 1 = \frac{1}{x} \quad (1 \text{ mark})$ $\Rightarrow 3y = \frac{1}{x} + 1 = \frac{1 + x}{x}$ $\Rightarrow y = \frac{1 + x}{3x} \quad (1 \text{ mark})$	b. Domain is $(0, \infty)$ (1 mark) Range is $\left(\frac{1}{3}, \infty\right)$ (1 mark)
--	--

Question 5

$2 \sin 3\left(\theta + \frac{\pi}{4}\right) - 1 = 0, \quad 0 \leq \theta \leq \frac{\pi}{2}$ $\Rightarrow 2 \sin 3\left(\theta + \frac{\pi}{4}\right) = 1, \quad \frac{3\pi}{4} \leq 3\left(\theta + \frac{\pi}{4}\right) \leq \frac{9\pi}{4} \quad (1 \text{ mark})$ $\Rightarrow \sin 3\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2}$ $\Rightarrow 3\left(\theta + \frac{\pi}{4}\right) = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$ $\Rightarrow 3\left(\theta + \frac{\pi}{4}\right) = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ $\Rightarrow \theta + \frac{\pi}{4} = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18} \quad (1 \text{ mark})$ $\Rightarrow \theta = \frac{\pi}{18} - \frac{\pi}{4}, \frac{5\pi}{18} - \frac{\pi}{4}, \frac{13\pi}{18} - \frac{\pi}{4} \Rightarrow \theta = \frac{-7\pi}{36}, \frac{\pi}{36}, \frac{17\pi}{36}$ But $0 \leq \theta \leq \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{36}, \frac{17\pi}{36} \quad (1 \text{ mark})$	
---	--



Question 6

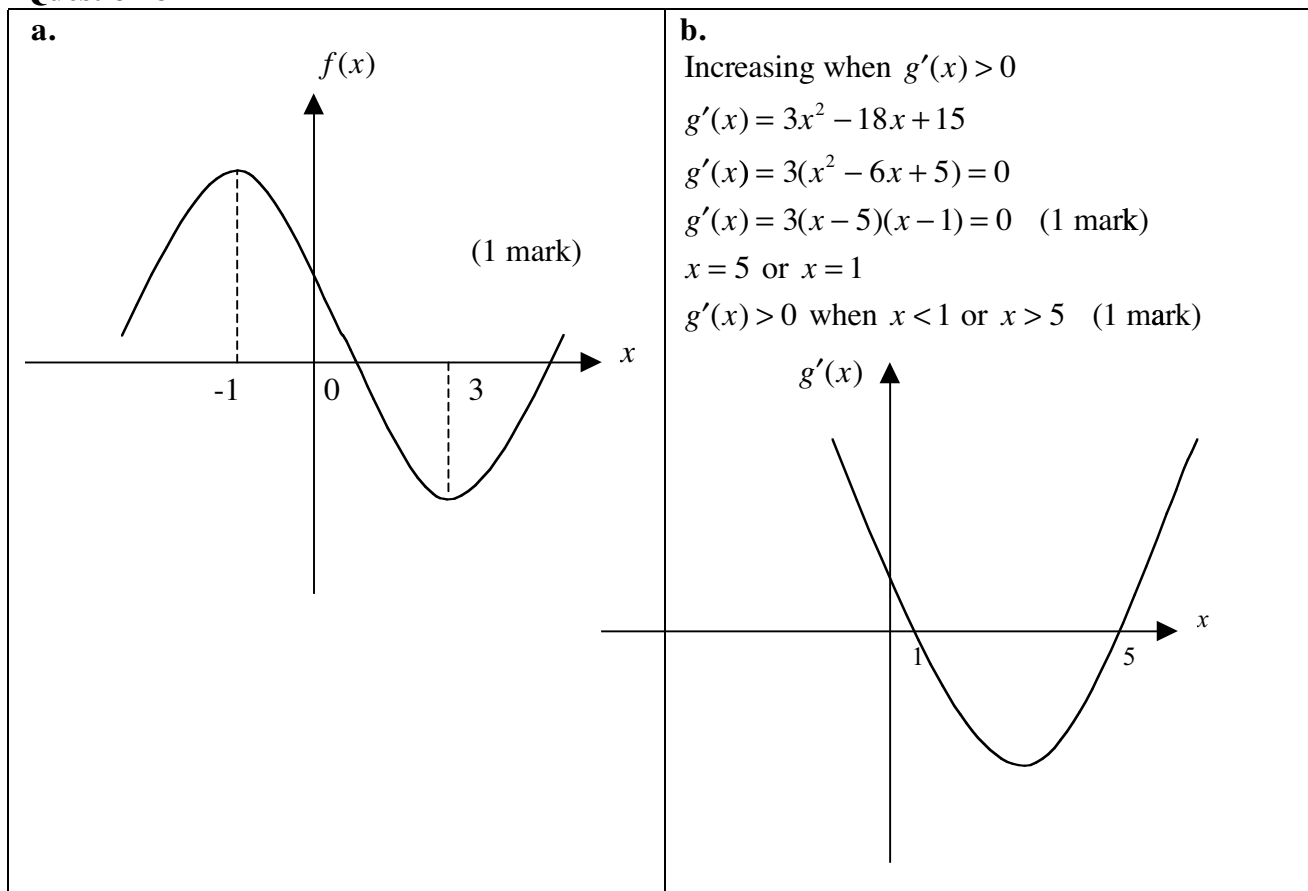
a. $\int_0^a 4x^3 dx = x^4 \Big _0^a = \frac{1}{4} \quad (1 \text{ mark})$ $a^4 = \frac{1}{4}$ $\Rightarrow a^2 = \pm \frac{1}{2}$ $\Rightarrow a = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$ But $a > 0$ $\Rightarrow a = \frac{\sqrt{2}}{2} \quad (1 \text{ mark})$	b. $\int_{1/2}^{3/4} 4x^3 dx = x^4 \Big _{1/2}^{3/4} \quad (1 \text{ mark})$ $= \frac{81}{256} - \frac{1}{16}$ $= \frac{81}{256} - \frac{16}{256}$ $= \frac{65}{256} \quad (1 \text{ mark})$
---	--

Question 7

a. $\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ $\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B)$ $= 0.3 + 0.6 - 0.8$ $= 0.1 \quad (1 \text{ mark})$ $\Pr(A B) = \frac{0.1}{0.6} = \frac{1}{6} \quad (1 \text{ mark})$	b. $\Pr(X < 12 X > 8) = \frac{\Pr(X < 12 \cap X > 8)}{\Pr(X > 8)}$ $= \frac{\Pr(8 < X < 12)}{\Pr(X > 8)}$ $Z = \frac{12 - 8}{2} = 2$ 95% lie within 2 standard deviations of mean 5% lie outside 2 standard deviations of mean (1 mark) 2.5% are greater than $Z = 2$ $\therefore \frac{\Pr(8 < X < 12)}{\Pr(X > 8)} = \frac{0.5 - 0.025}{0.5} \quad (1 \text{ mark})$ $= \frac{0.475}{0.5}$ $= 0.475 \times 2$ $= 0.95 \quad (1 \text{ mark})$
---	---



Question 8



Question 9

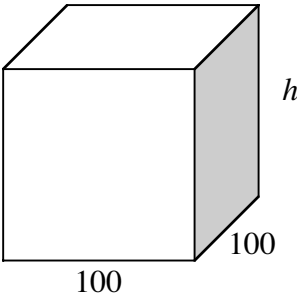
a. $\sigma^2 = np(1 - p)$ $2.25 = 9p(1 - p)$ $\Rightarrow \frac{2.25}{9} = p(1 - p)$ $\Rightarrow 0.25 = p - p^2 \quad (1 \text{ mark})$ $\Rightarrow p^2 - p + 0.25 = 0$ $\Rightarrow 4p^2 - 4p + 1 = 0$ $\Rightarrow (2p - 1)(2p - 1) = 0$ $\Rightarrow 2p - 1 = 0$ $\Rightarrow p = \frac{1}{2} \quad (1 \text{ mark})$	b. $\Pr(X \geq 1) = 1 - \Pr(X = 0)$ $= 1 - \binom{9}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^9$ $= 1 - \left(\frac{1}{2}\right)^9$ $= 1 - \frac{1}{512} = \frac{511}{512} \quad (1 \text{ mark})$
--	--



Question 10

a. $\frac{dy}{dx} = 3e^{3x} \cos 3x + 3e^{3x} \sin 3x \quad (1 \text{ mark})$	b. $\int (3 \sin 3x(e^{3x} - 1) + 3e^{3x} \cos 3x) dx$ $= \int (3 \sin 3x e^{3x} - 3 \sin 3x + 3e^{3x} \cos 3x) dx$ $= \int (3e^{3x} \cos 3x + 3e^{3x} \sin 3x) dx - \int 3 \sin 3x dx \quad (1 \text{ mark})$ $= e^{3x} \sin 3x + \cos 3x + c \text{ where } c \text{ is a constant} \quad (1 \text{ mark})$
---	---

Question 11

a.  100 $\frac{dV}{dt} = -50 \text{ ml sec}^{-1} = -50 \text{ cm}^3 \text{ sec}^{-1} \quad (1 \text{ mark})$ $\frac{dh}{dt} = \frac{dh}{dV} \frac{dV}{dt} \text{ where } h = \text{depth of water.}$ $V = 100 \times 100 \times h = 10000h$ $\frac{dV}{dh} = 10000 \quad (1 \text{ mark})$ $\frac{dh}{dt} = \frac{1}{10000} \times \frac{-50}{1} = -0.005$ Height of water is falling at $0.005 \text{ cm sec}^{-1}$ (1 mark)	b. $\frac{60}{0.005} = \frac{60000}{5} = 12000 \text{ seconds}$ $12000 \text{ seconds} = \frac{12000}{60} \text{ minutes} = 200 \text{ minutes}$ $200 \text{ minutes} = \frac{200}{60} = 3\frac{2}{3} \text{ hours} \quad (1 \text{ mark})$
---	---

END OF SUGGESTED SOLUTIONS
2006 Mathematical Methods Trial Examination 1

KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9817 5374
FAX: (03) 9817 4334
chemas@chemas.com
www.chemas.com