



2006

VCE Further Mathematics Trial Examination 2

Suggested Solutions

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2006 Further Mathematics Trial Examination 2
Core: Data analysis Suggested Solutions

Question 1

a. *Rising Star Butter* outlier = 491.1 g (1 mark) *Local Butter* outlier = 507.9 g (1 mark)

b.

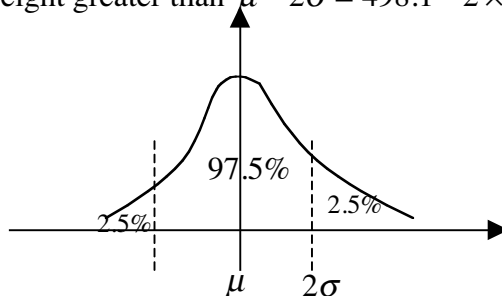
Rising Start Butter					Local butter			
8	7	4	3	1	491			
					492			
					493	8		
					494	5		
				9	7	495	4 , 7	
				8	5	496	3 , 4	
				4	3	2	497	2 , 2 , 7

(2 marks - $\frac{1}{2}$ mark for each error up to 2 marks)

c. continuous and numerical (1 mark)

d. Mean = 498.1 g (1 mark) Standard deviation = 2.3 g (1 mark)

e. 97.5% will have a weight greater than $\mu - 2\sigma = 498.1 - 2 \times 2.3 = 493.4$ g (1 mark)





Question 1 (continued)

f.

$$\text{Rising Star } Z = \frac{500.2 - 494.2}{3.0} = 2 \quad (1 \text{ mark})$$

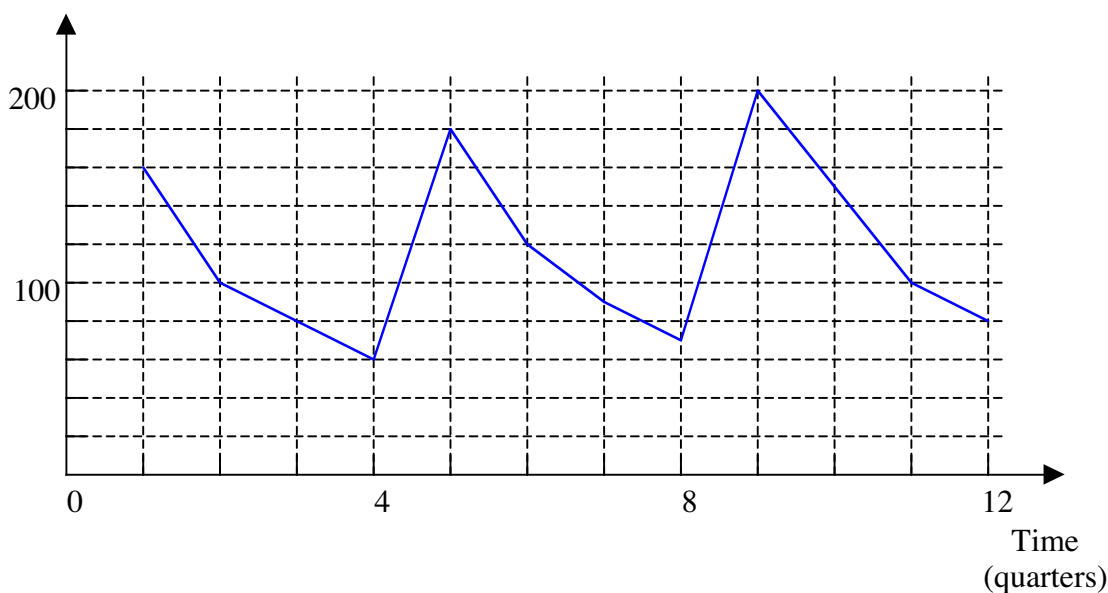
$$\text{Local } Z = \frac{500.2 - 498.1}{2.3} = 0.9 \quad (1 \text{ mark})$$

Since the weight for *Rising Star* is 2 standard deviations above the standardised mean and only 0.9 for *Local*, there would be less chance of getting a 500.2 tub of *Rising Star Butter*. (1 mark)

Question 2

a.

Number of
tubs of butter



(1 mark)

b.

Seasonal Indices

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2004	1.6	1.0	0.8	0.6
2005	1.6	1.0	0.8	0.6
2006	1.5	1.1	0.8	0.6

(1 mark)

c. 0.6 (1 mark).

This means that sales in the fourth quarter are 40% below average. (1 mark)

Total = 15 marks

End of Core: Data analysis



Question 1

a. Arithmetic sequence $a = 8, d = 2, n = 7$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_6 = \frac{7}{2}[16 + 6 \times 2] = 98 \quad (1 \text{ mark})$	b. Arithmetic sequence $a = 3, d = 4, n = 20$ $t_n = a + (n-1)d$ $t_{10} = 3 + 19 \times 4 = 79 \text{ blocks} \quad (1 \text{ mark})$
c. In 1962 he built 4 more blocks than in 1961. Therefore, $7 + 4 = 11$ blocks containing 22, 24, . . . units. (1 mark) Arithmetic sequence $a = 22, d = 2, n = 11$ $S_{11} = \frac{11}{2}[44 + 10 \times 2] = 352 \text{ units} \quad (1 \text{ mark})$	d. $a = 3, d = 4, t_n = 70$ $t_n = a + (n-1)d$ $70 = 3 + (n-1)4$ Use sequence mode of graphics calculator and enter $Y =$ $nMin = 1$ $\mu(nMin) = 3 + (n-1)4$ $\mu(nMin) = 3$ 2nd Table Scroll down the Table to find when $\mu(n)$ is first greater than 70. $n = 18$ (1 mark) So 18 years from the first year 1960 = 1977 (1 mark)

Question 2

a. i. $t_{n+1} = (1.025)t_n, t_1 = 80 \quad (1 \text{ mark})$	a. ii. This is a geometric sequence. $t_{11} = (1.025)^{10} \times 80 = 102 \quad (1 \text{ mark})$
b. $637 - 300 = 337 \quad (1 \text{ mark})$	c. i. $t_n = 0.85t_{n-1} - 1000, t_1 = 4000 \quad (1 \text{ mark})$
c. ii. 2001 : $0.85 \times 4000 - 1000 = 2400$ 2002 : $0.85 \times 2400 - 1000 = 1040 \quad (1 \text{ mark})$	c. iii. Use sequence mode of graphics calculator and enter $Y =$ $nMin = 1$ $\mu(n) = 0.85\mu(n-1) - 1000$ $\mu(nMin) = 4000$ 2nd Table $\Rightarrow n = 4$ $\Rightarrow \text{Year } 2003 \quad (1 \text{ mark})$



Question 3

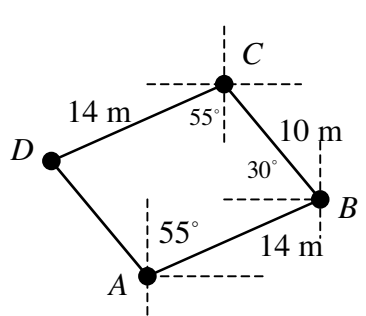
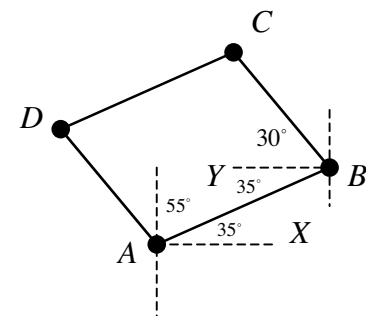
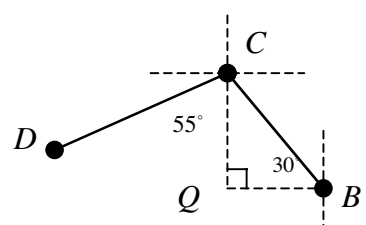
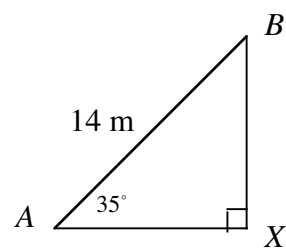
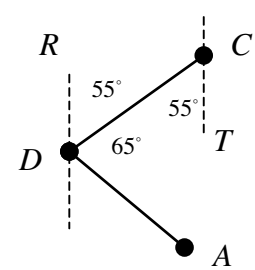
a. No because $\frac{309654}{317350} \neq \frac{301912}{309654}$ (1 mark)	b. $309654 = b \times 317350 - 9600$ $b \times 317350 = 319254$ $b = 1.006$ (1 mark)
c. $A_{n+1} = 1.006 \times 301912 - 9600 = \294123 $\Rightarrow A_{n+1} = \$294123$ (1 mark)	

Total = 15 marks

End of Module 1: Number patterns and applications



Question 1

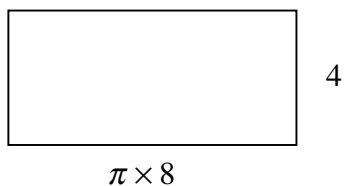
a.  (1 mark) for shape. (1 mark) for at least one angle shown.	b.  $\angle XAB = 35^\circ$ since $\angle XAY = 90^\circ$ $AX \parallel YB$ so $\angle ABY = 35^\circ$ So $\angle ABC = 35 + 30 = 65^\circ$ (1 mark)
c.  $\angle CQB = 90^\circ$ $\angle CBQ = 30^\circ$ $\therefore \angle QCB = 60^\circ$ So $\angle BCD = 60 + 55 = 115^\circ$ (1 mark)	d. It is a parallelogram (1 mark) because the opposite sides, AB and DC, are equal and $\angle ABC + \angle BCD = 180^\circ$ (1 mark)
e.  $\sin 35^\circ = \frac{BX}{14}$ $BX = 14 \times \sin 35^\circ = 8.03$ m B is 8.03 m north of A. (1 mark)	f.  Because $ABCD$ is a parallelogram, then opposite angles are equal. So $\angle ADC = 65^\circ$ $\angle RDC = \angle DCT = 55^\circ$ (alternate angles) True bearing of A from D is 120° T. (1 mark)



Question 2

a.

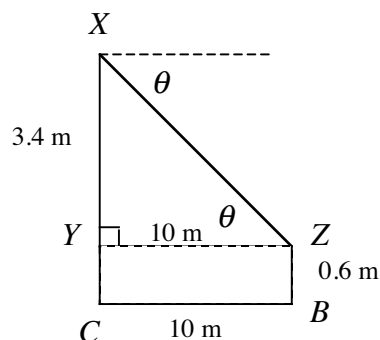
$$80 \text{ cm} = 0.8 \text{ m} \quad (1 \text{ mark})$$



Surface area =

$$4 \times \pi \times 0.8 = 10.05 \text{ m}^2 \quad (1 \text{ mark})$$

b.



$$XY = 4 - 0.6 = 3.4$$

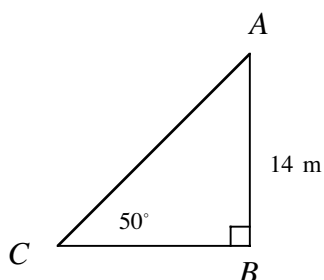
$$\tan \theta = \frac{3.4}{10}$$

$$\theta = \tan^{-1}\left(\frac{3.4}{10}\right) \quad (1 \text{ mark})$$

$$\theta = 18^\circ 47' \quad (1 \text{ mark})$$

Question 3

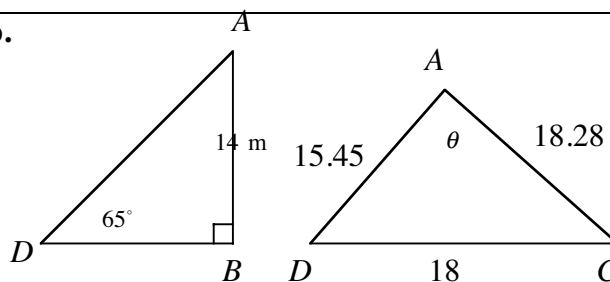
a.



$$\sin 50^\circ = \frac{14}{AC}$$

$$\Rightarrow AC = \frac{14}{\sin 50^\circ} = 18.28 \text{ m} \quad (1 \text{ mark})$$

b.



$$\sin 65^\circ = \frac{14}{AD}$$

$$\Rightarrow AD = \frac{14}{\sin 65^\circ} = 15.45 \text{ m} \quad (1 \text{ mark})$$

$$18^2 = 15.45^2 + 18.28^2 - 2 \times 15.45 \times 18.28 \cos \theta$$

$$\Rightarrow -248.8609 = -564.852 \cos \theta$$

$$\Rightarrow \cos \theta = 0.4406$$

$$\Rightarrow \theta = 64^\circ \quad (1 \text{ mark})$$

Total = 15 marks

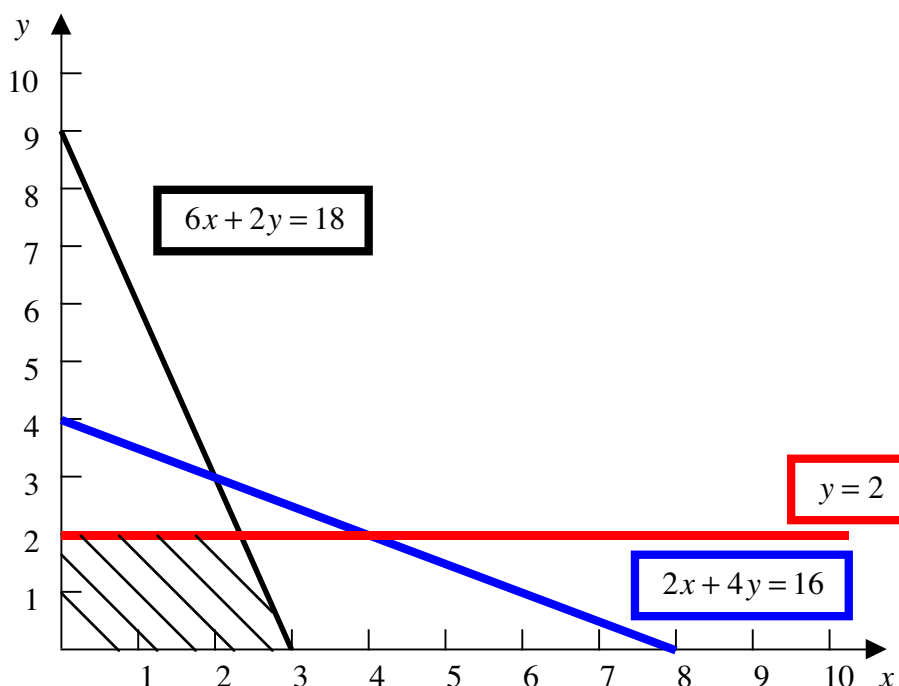
End of Module 2: Geometry and trigonometry



Question 1

a. $2x + 4y \leq 16$ (1 mark)	b. $x \geq 0$ and $y \geq 0$ (1 mark)
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- c.** The line $2x + 4y = 16$ can be found using a graphics calculator and then plotted, making sure to get the X and Y intercepts. (1 mark) (1 mark)
- f.** The boundary line for this constraint is $y = 2$. (1 mark)
- g.** The shaded region is shown. (1 mark)



d. $2y = -6x + 18$ $y = -3x + 9$ (1 mark) Enter this in a graphics calculator $y =$ $4y = -2x + 16$ $y = -\frac{1}{2}x + 4$ (1 mark) Enter this in a graphics calculator $y =$ Use graphics calculator “calc” to find that the point of intersection is $x = 2$, $y = 3$. Point of intersection is $(2, 3)$ (1 mark)	e. $y \leq 2$ (1 mark)
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Question 1 (continued)

h. When $y = 2$ $6x = 4 = 18$ $\Rightarrow 6x = 14$ $\Rightarrow x = 2\frac{1}{3}$ Intersection points $(0,0)$, $(0,2)$, $(3,0)$ (1 mark) $(2\frac{1}{3},2)$ (1 mark)	i. $P = 300x + 800y$ (1 mark)
j. At $(0,2)$, $P = \$1600$ At $(3,0)$, $P = \$900$ Cannot make $\frac{1}{3}$ of a TV set. So at $(2,2)$, $P = 600 + 1600$ Maximum profit = \$2200 (1 mark)	k. 2 LCD and 2 Plasma (1 mark)

Total = 15 marks

End of Module 3: Graphs and relations



Question 1

a. $\frac{70}{100} \times 180000 = \\126000 (1 mark)	b. $355000 - 126000 - 10000 = \\219000 (1 mark)
c. Using TVM solver under Finance Enter $N = 216$ (12×18) $I = 6$ $PV = -219000$ $FV = 0$ $P / Y = 12$ $C / Y = 12$ This gives a monthly repayment of \$1660.38 (1 mark)	d. Use TVM solver to find FV with $N = 48$ and $PMT = 2000$. Amount owing after 4 years is 170041.4618 (1 mark) This amount owing becomes the present value, PV, for the next 2 years and $N = 24$ now and $PMT = 1300$. All other values remain the same as you look for FV, the future value. This equals \$145708.65 (1 mark)
e. Use TVM solver to find N. $PV = -145708.6523$ $PMT = 2360$ $FV = 0$ Other values remain the same. $N = 74.022$ Number of years = $74.022 \div 12 = 6.2$ years (1 mark)	f. For first 4 years paid $= 2000 \times 48 = 96000$ Next 2 years paid $= 1300 \times 24 = 31200$ Next 6.2 years paid $2360 \times 6.2 \times 12$ $= 175584$ (1 mark) Total amount repaid = 302784 Amount borrowed = 219000 Interest paid = \$83784 (1 mark)
g. Use TVM solver with $PV = -219000$, $PMT = 1000$, $FV = 0$, $P / Y = C / Y = 26$ N is found to equal 305.41 repayments (1 mark) Amount repaid = $305.41 \times 1000 = 305410$ Interest = $305410 - 219000 = \\$86410$ (1 mark) More interest is paid by Lucy's method With Lucy's method, an extra $86410 - 83784 = \\$2626$ would have been paid in interest (1 mark)	



Question 2

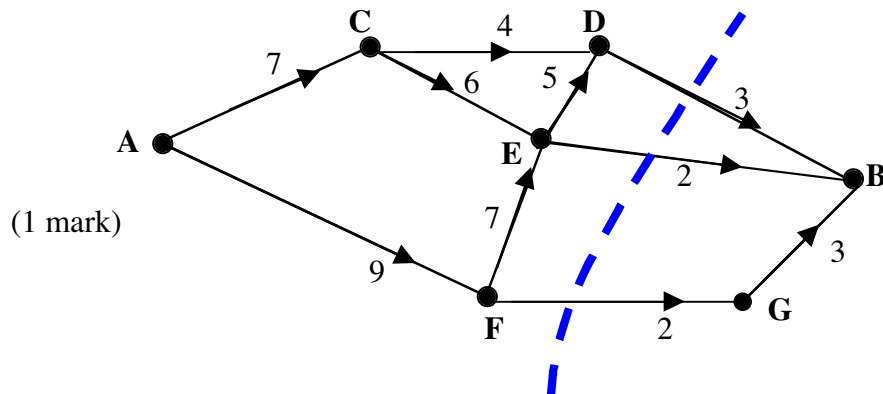
a. $I = \frac{PRT}{100}$ $A = P + I$ $A = P + \frac{PRT}{100} \quad (1 \text{ mark})$ $30000 = P + \frac{P \times 5.2 \times 5}{100}$ $\Rightarrow 30000 = P + 0.26P$ $\Rightarrow 30000 = 1.26P$ $\Rightarrow P = \$23809.52$ $P = \$23810 \quad (1 \text{ mark})$	b. i $\text{Total depreciation} = 50000 - 5000 = 45000$ $\text{Depreciation/km} = \frac{45000}{100000} = 45 \text{ cents/km}$ (1 mark) b. ii $BV = 50000 - 0.45 \times 40000 = \32000 (1 mark)
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Total = 15 marks

End of Module 4: Business-related mathematics



2006 Further Mathematics Trial Examination 2
Module 5: Networks and decision mathematics. Suggested solutions.

Question 1**a.**Degree of vertex E is 4. (1 mark)**b. i.****b. ii.** $3 + 2 + 2 = 7$ (1 mark)**c.** $7 + 9 = 16$ (1 mark)**Question 2****a.**

Activity	Immediate Predecessor	Earliest starting time (days)
A	-----	0
B	-----	0
C	-----	0
D	C	2
E	A	4
F	B	5
G	B	5
H	G	8
I	F	6
J	H, I	11
K	F	6
L	D, E, K	8

(1 + 1 + 1 + 1 = 4 marks)



Question 2 (continued)

- b. i.** 18 days (1 mark)
- b. ii.** B, G, H, J, M (1 mark)
- c. i.** Day 12 (1 mark)
- c. ii.** Latest start time for K is day 10.
Earliest start time for K is day 6.
Therefore, float time = $10 - 6 = 4$ days. (1 mark)
- d. i.** Critical path activities that can be reduced are B, G, H, J .
Therefore, time saved = $3 + 1 + 2 + 2 = 8$ days.
Therefore, minimum time = $18 - 8 = 10$ days (1 mark)
- d. ii.** The earliest L can now start is day 5 so there is no value in crashing C . (1 mark)
All other activities can be crashed by the time in the Table, so
Cost = $2 \times 150 + 3 \times 400 + 1 \times 100 + 2 \times 300 + 1 \times 350 + 2 \times 150$
= $300 + 1200 + 100 + 600 + 350 + 300$
= \$2850 (1 mark)

Total = 15 marks

End of Module 5: Networks and decision mathematics



Question 1

a. $Y = \begin{bmatrix} 1 & 4 & 7 & 6 & 1 \\ 2 & 2 & 7 & 9 & 7 \\ 3 & 6 & 5 & 5 & 5 \end{bmatrix} \quad (1 \text{ mark})$	b. He would be multiplying a 3×5 matrix by a 3×3 matrix. To multiply matrices, the number of columns in the first matrix must equal the number of rows in the second matrix. Hence, YB cannot be calculated. (1 mark)
c. $BY = \begin{bmatrix} 1 & 3 & -2 \\ 7 & 5 & 4 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 7 & 6 & 1 \\ 2 & 2 & 7 & 9 & 7 \\ 3 & 6 & 5 & 5 & 5 \end{bmatrix}$ Enter B as $[A]$ in a graphics calculator and Y as $[B]$. Then on home page find $[A][B]$. $[A][B] = \begin{bmatrix} 1 & -2 & 18 & 23 & 12 \\ 29 & 62 & 104 & 107 & 62 \\ -1 & 8 & -2 & -10 & -14 \end{bmatrix} \quad (1 \text{ mark})$	d. 8 (1 mark)
e. Decoder key, K is $B^{-1} = [A]^{-1}$ $K = \begin{bmatrix} 0.207 & 0.037 & 0.268 \\ 0.122 & 0.061 & -0.220 \\ -0.378 & 0.110 & -0.195 \end{bmatrix}$ (1 mark) for matrix. (1 mark) for correct rounding.	f. $\begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix} \text{ has determinant} = 0 \quad (1 \text{ mark})$ Therefore, it has no inverse. So it could not be decoded if it were used. (1 mark)



Question 2

a. $100 - 25 = 75\%$ (1 mark)	b. $T = \begin{matrix} c & s \\ \begin{bmatrix} 0.6 & 0.25 \\ 0.4 & 0.75 \end{bmatrix} \end{matrix}$ (1 mark)
c. $7000 - 4800 = 2200$ $S_0 = \begin{bmatrix} 4800 \\ 2200 \end{bmatrix}$ (1 mark)	d. $S_1 = TS_0$ $S_1 = \begin{bmatrix} 0.6 & 0.25 \\ 0.4 & 0.75 \end{bmatrix} \begin{bmatrix} 4800 \\ 2200 \end{bmatrix} = \begin{bmatrix} 3430 \\ 3570 \end{bmatrix}$ (1 mark)
e. 3570 (1 mark)	f. $S_8 = T^8 S_0 = \begin{bmatrix} 2693 \\ 4307 \end{bmatrix}$ 2693 (1 mark)
g. $\% \text{ change} = \frac{4800 - 2693}{4800} \times 100 = 43.9\%$ (1 mark)	

Total = 15 marks

End of Module 6: Matrices

End of suggested solutions
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