



2006

VCE

Further Mathematics

Trial Examination 2



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PURPOSE OF THIS TRIAL EXAMINATION

This Further Mathematics Trial Examination is designed to assess

- understanding and communication of mathematical ideas
- interpretation, analysis and solution of routine problems
- interpretation, analysis and solution of non-routine problems

Assessment is by extended answer questions involving multi-stage solutions of increasing complexity.

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**STUDENT NUMBER****Letter**

Figures									
Words									

VICTORIAN CERTIFICATE OF EDUCATION 2006

FURTHER MATHEMATICS

Trial Written Examination 2 (Analysis task)

Reading time: 15 minutes

Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core

Number of questions	Number of questions to be answered
2	2

Modules

Number of modules	Number of modules to be answered
6	3

Directions to students

Materials

Question and answer book of 24 pages.

Working space is provided throughout the book.

There is a detachable sheet of miscellaneous formula supplied.

You may bring to the examination one bound reference that may be annotated. The reference may be typed or handwritten. The reference may be a textbook.

One approved graphics calculator or CAS and, if desired, one scientific calculator may be used.

You should have at least one pencil and an eraser.

The task

Detach the formula sheet from the book during reading time.

Please ensure that your **student number** is written in the space provided on the front cover of this book.

Answer **all** questions in the core and in each of the three chosen modules.

The marks allotted to each part of each question are indicated at the end of the part.

The core is worth 15 marks. Each module is worth 15 marks. There is a total of 60 marks available for the examination.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π , e , surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

All written responses should be in English.



FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

**Further Mathematics Formulas****Core: Data analysis**

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \dots = \frac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc \sin A$

Heron's formula: $A = \sqrt{s(s - a)(s - b)(s - c)}$ where $s = \frac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base $\times$ height

volume of a pyramid: $\frac{1}{3}$ area of base $\times$ height



Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{\text{Pr}T}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times \text{flat rate}$

annuities: $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$, where $R = 1 + \frac{r}{100}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET

**Specific Instructions**

This task paper consists a core and five modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

	Page
Core: Data analysis	2
Module	
Module 1: Number patterns and applications	6
Module 2: Geometry and trigonometry	9
Module 3: Graphs and relations	13
Module 4: Business-related mathematics	15
Module 5: Networks and decision mathematics	18
Module 6: Matrices	21



Question 1

At the supermarket, two brands of butter, *Rising Start* and *Local* can be bought in 500 g tubs. A consumer affairs employee checks the weights of 20 tubs of each brand from a particular supermarket and draws up the following Table.

<i>Rising Start Butter</i> Weight (g)	<i>Local Butter</i> Weight (g)
495.7	493.8
495.9	496.4
491.1	500.2
501.1	499.1
499.9	494.5
498.3	495.7
500.2	507.9
500.1	502.7
498.8	499.6
497.4	497.2
498.3	495.4
497.2	503.0
499.6	498.5
500.0	498.2
496.5	496.3
498.4	500.4
496.8	497.7
498.7	497.2
497.3	501.3
499.8	498.1

- a. What is the outlier in each set of data?

Rising Star Butter outlier

1 mark

Local Butter outlier

1 mark



Question 1 (continued)

- b. Complete the following back to back stem and leaf for the given data.

Rising Start Butter

Local butter

491	
492	
493	8
494	5
495	4 , 7
496	3 , 4
497	2 , 2 , 7
498	1 , 2 , 5
499	1 , 6
500	2 , 4
501	3
502	7
503	0
504	
505	
506	
507	9

2 marks

- c. Which two words out of **categorical**, **numerical**, **continuous**, **discrete** would you use to describe the data in b. ?

1 mark

- d. Find the mean and standard deviation for the twenty weights of the *Rising Start* data. Give your answers to 1 decimal place.

Mean

1 mark

Standard deviation

1 mark



Question 1 (continued)

- e. This sample mean and standard deviation for *Rising Start Butter* are similar to the population mean and standard deviation. If the population has a normal distribution, what weight would the butter have to be so that 97.5% of the tubs of butter weigh more than this value?

1 mark

- f. The mean and standard deviation for the normal population of *Local Butter* are 494.2 g and 3.0 g respectively. You buy a tub of each type of butter and find that both tubs weigh 500.2 g. Compare the chances of obtaining this exact weight for *Local Butter* and *Rising Start Butter*.

3 marks

Question 2

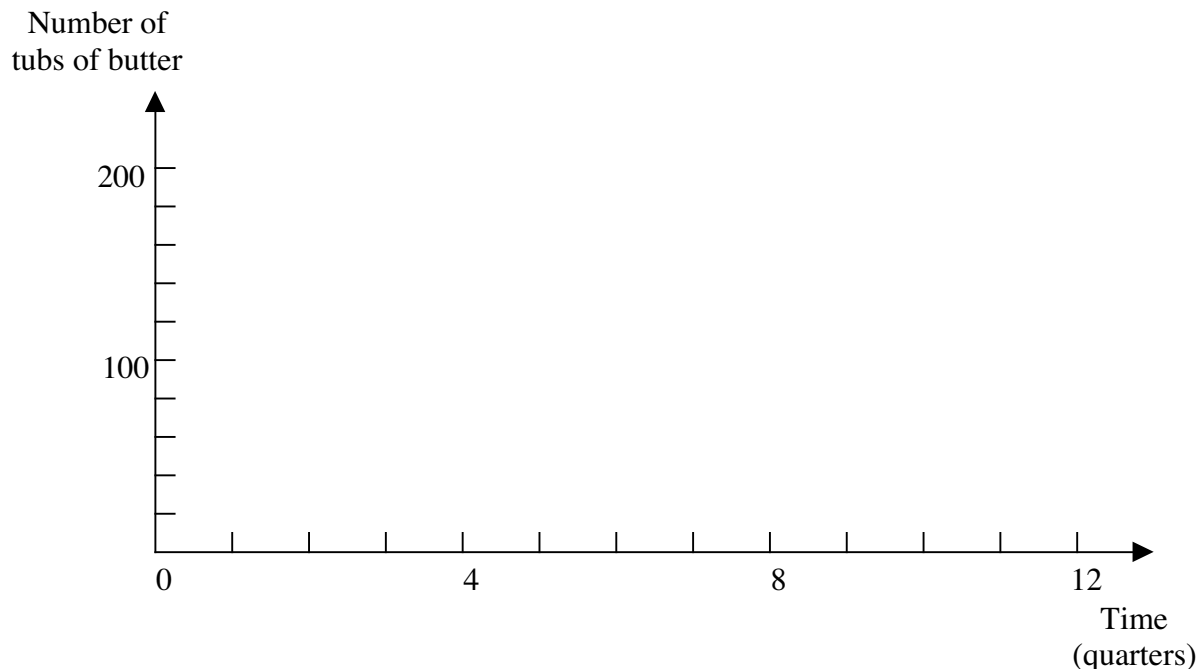
The following Table shows the number of tubs of butter sold by the local milk bar over four quarters of a year for three years.

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2004	160	100	80	60
2005	180	120	90	70
2006	200	150	100	80



Question 2 (continued)

- a. Plot the time series on the graph below.



1 mark

- b. The seasonal index for each of the quarters for the years 2005 and 2006 have been calculated and listed in the Table below. Complete the Table for the year 2004.

Seasonal Indices

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2004				
2005	1.6	1.0	0.8	0.6
2006	1.5	1.1	0.8	0.6

1 mark

- c. What is the three year quarterly average for the fourth quarter and what does it mean?

2 marks

Total = 15 marks

End of Core: Data analysis



If you choose this module, all questions are to be answered.

Question 1

Vincenzo Corelli began building units at the beginning of 1960. In this year he built three different blocks of units called Apollo, Radon and Zeus with two, four and six units in each block respectively. Each year until the end of 1979, he increased the number of units he built the year before by four and each block of units he built contained an extra two units. Hence, in 1961, he built seven blocks of units containing 8, 10, 12, 14, 16, 18 and 20 units.

- a.** How many units did he build in 1961?

1 mark

- b.** How many blocks of units did he build during 1979?

1 mark

- c.** How many individual units were built in 1962?

2 marks



Question 1 (continued)

- d. In which year would the number of blocks of units built that year exceed 70?

2 marks

Question 2

Vincenzo's daughter, Antonia, took over her father's business in January 1980. In her first year of business, she built 80 new apartment blocks. Each year after that the number of new apartment blocks increased by 2.5% of the number built during the previous year.

- a. i. Write a difference equation that would generate the terms of this sequence.

1 mark

- ii. How many apartment blocks did she build in 1990?
Give your answer to the nearest whole number.

1 mark

- b. The number of individual apartments Antonia built increased each year such that $t_n = t_{n-1} + t_{n-2}$. If she built 300 apartments in 1980 and 637 apartments in 1982, how many apartments did she build in 1981?

1 mark



Question 2 (continued)

- c. i.** In 2000, Antonia built 5000 new apartments and only sold 1000 of them. Each year after that, she decreased the number of apartments she built to 85% of the unsold apartments that had been built in the previous year. If she sold 1000 apartments in each year from 2000, write a difference equation to show the number of unsold apartments.

1 mark

- ii.** How many apartments built in 2002 were left unsold?

1 mark

- iii.** What would be the first year that the number of sales for that year was less than the number of apartments Antonia built that year?

1 mark

Question 3

Mark and Jane buy an apartment at the beginning of 2003. After paying a deposit, they need a further \$325,000 which they borrow from a bank. The following Table shows the amount that they owe to the bank at the end of each year.

Year	2003	2004	2005
Amount owing (\$)	317,350	309,654	301,912

- a.** Is this sequence geometric? Give a reason for your answer.

1 mark

- b.** The difference equation that generates the sequence for the amount owed is $A_{n+1} = bA_n - 9600$, $A_0 = 325,000$. Calculate the value of b .
 Give your answer to 3 decimal places.

1 mark

- c.** How much money would Mark and Jane owe at the end of 2006?
 Give your answer to the nearest dollar

1 mark

Total = 15 marks

End of Module 1: Number patterns and applications

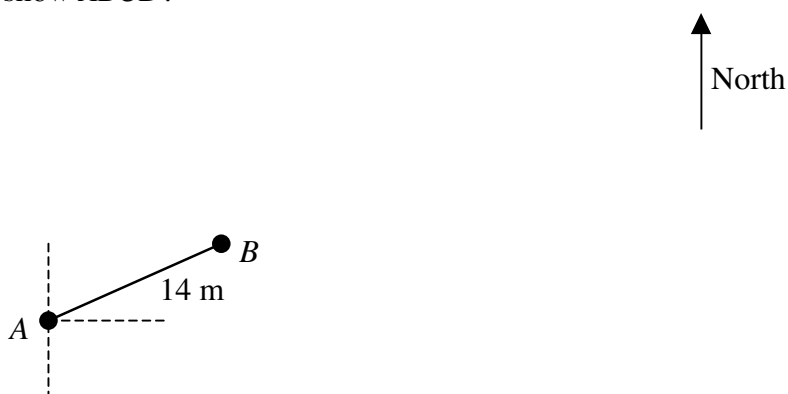


If you choose this module, all questions are to be answered.

Question 1

A landscape designer sets out to build a children's playground in a park. She sets out from a point A in the park and walks 14 m on a bearing 055° T. Here, she places a peg at B . From here she walks 10 m on a bearing 300° T to a point C . From C she walks 14 m on a bearing 235° T to point D . The points $ABCD$ map out the area of the children's playground.

- a. Complete the diagram below to show $ABCD$.



2 marks

- b. Find angle ABC ?

1 mark

- c. Find angle BCD ?

1 mark

- d. What is the shape of the playground $ABCD$? Give a reason for your answer.

2 marks



Question 1 (continued)

- e. How far north of A is B ? Give your answer to 2 decimal places.

1 mark

- f. What is the true bearing of A from D ?

1 mark

Question 2

A hollow, vertical cylindrical climbing tower, 4 m high with a diameter of 80 cm , is to be placed at point C in the playground. The tower is made from a sheet of steel and is to have steps made from a metal alloy added to the outside of the cylinder.

- a. How many square metres of steel are needed to make the cylinder?
Give your answer to 2 decimal places.

2 marks



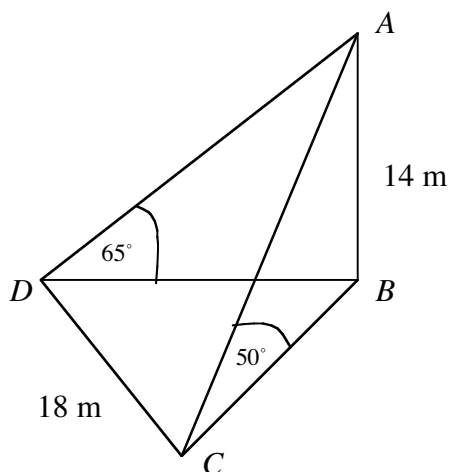
Question 2 (continued)

- b. A bird sits on top of the tower and looks at a 60 cm tall child standing at point B. Find the angle of depression from the top of the tower to the child's head. Give your answer to the nearest minute.

2 marks

Question 3

A solid sculpture in the shape of a tetrahedron, $ABCD$, is to be built just at the entrance to the children's playground. $\angle ADB = 65^\circ$, $\angle ACB = 50^\circ$, $AB = 14$ m and $DC = 18$ m. AB is perpendicular to BD . BDC sits on the horizontal ground.



- a. Find AC . Give your answer to 2 decimal places.

1 mark



Question 3 (continued)

- b.** Find $\angle DAC$ to the nearest degree.

2 marks

Total = 15 marks

End of Module 2: Geometry and trigonometry



If you choose this module, all questions are to be answered.

Question 1

A company manufactures both LCD and Plasma TV sets. Each LCD TV requires 6 hours of labour and 2 hours of processing. Each Plasma TV requires 2 hours of labour and 4 hours of processing. The total number of labour hours available is 18 and the total number of processing hours available is 16. If x is the number of LCD TVs produced each day and y is the number of Plasma TVs produced each day, then an inequality involving the number of labour hours is $6x + 2y \leq 18$.

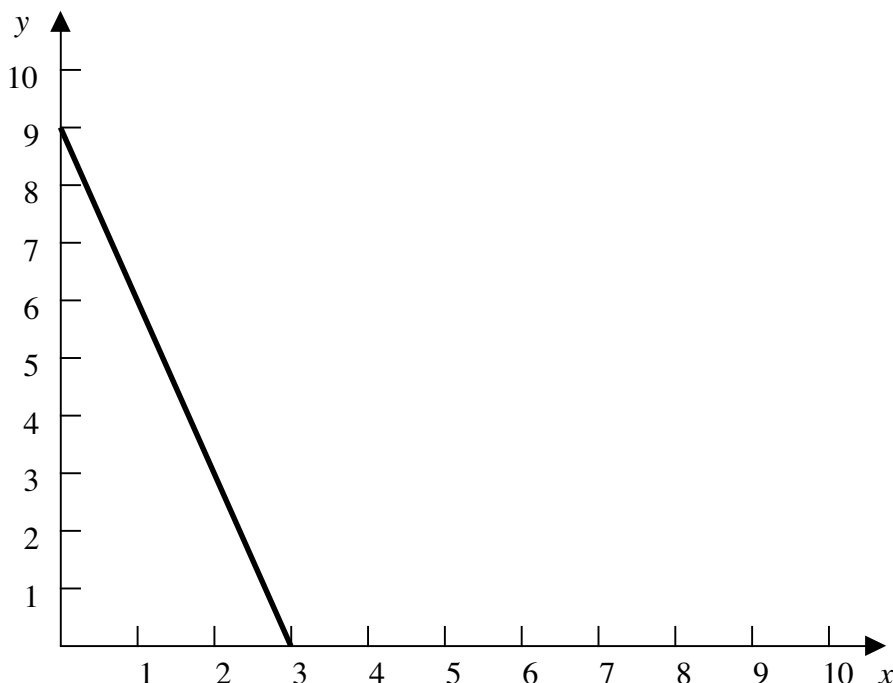
- a. Write an inequality involving the number of processing hours.

1 mark

- b. Write another **two inequalities** to represent the manufacture of these two types of TV screens.

1 mark

The graph below shows the boundary line $6x + 2y = 18$.



- c. On this graph, show the boundary line represented by you answer in a. Show the intercepts on this line with the axes.

2 marks



- d.** Find the point of intersection of the boundary lines that have a gradient less than zero.

- e.** If the maximum number of Plasma TVs produced each day is 2, write an inequation to show this constraint? 3 marks

- f.** Mark the boundary line for this constraint on your graph. 1 mark

- g.** Shade the region on the graph that satisfies all of the constraints. 1 mark

- h.** Find the co-ordinates of all the intersection points of the boundary of the shaded region. 1 mark

- i.** If the profit on each LCD and Plasma TV is \$300 and \$800 respectively, set up an equation for the total daily profit. 2 marks

- j.** What is the maximum profit that can be made each day? 1 mark

- i.** How many of each type of TV should be manufactured each day to ensure this maximum profit? 1 mark

1 mark
Total = 15 marks

End of Module 3: Graphs and relations



If you choose this module, all questions are to be answered.

Question 1

- a.** Jason inherits \$180,000 from his grandmother. He decides to use 70% of this money as a deposit on a house and to borrow the rest of the money from a bank. Calculate the amount of his deposit.

1 mark

- b.** The house he buys costs \$355,000. How much will he need to borrow from the bank if the government gives him a first home owner's grant of \$10,000?

1 mark

- c.** The bank charges Jason an interest rate of 6.00% per annum compounded monthly. Jason wishes to repay the bank in full in 18 years time. What will his monthly repayment have to be to achieve this goal?

1 mark

- d.** Jason actually repays \$2000 per month for the first 4 years and then reduces his repayments to \$1300 per month for the next 2 years while he saves for a holiday. How much does he owe the bank at the end of 6 years?

2 marks



Question 1 (continued)

- e. At the end of this 6 year period, Jason decides to let out one of his rooms and charge his tenant \$360 per calendar month. He now returns to his repayment of \$2000 per month plus the rent he gets from his tenant. How many more years will it take for him to repay his loan? Give your answer to 1 decimal place.

1 mark

- f. How much interest would Jason have paid to the bank from the time he originally took out the loan?

2 marks

- g. Jason's friend Lucy said it would have been better for him to have paid \$1000 per fortnight for every year of the loan instead of paying various amounts per month over the period of the loan. She said that when you do this, the interest is compounded fortnightly and the time needed to repay the loan is reduced. Is Lucy correct? How much interest would have been saved or how much extra interest would have been paid using Lucy's method?

3 marks



Question 2

- a.** Lucy wishes to invest some money in an account paying a flat 5.2% interest per annum so that she can have \$30,000 in 5 year's time to buy a car. How much does she need to invest now to achieve her aim? Give your answer to the nearest dollar.

2 marks

- b.** Lucy eventually buys a \$50,000 car and her accountant tells her that the car will need to be written off when it has done 100,000 km. At this time, its scrap value will be \$5000.

- i.** What is the value of the depreciation per kilometre?

1 mark

- ii.** What is the book value of the car when it has done 40,000 km?

1 mark

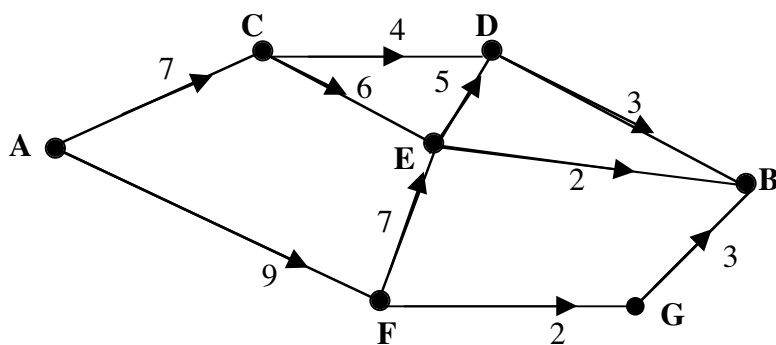
Total = 15 marks

End of Module 4: Business-related mathematics



If you choose this module, all questions are to be answered.

Question 1



The directed graph above shows the number of seats on a bus going from A to B .

- a.** What is the degree of the vertex E ?

1 mark

- b. i.** On the diagram, mark a cut to show the maximum flow.

1 mark

- ii.** What is the maximum number of people who can travel from A to B on this bus?

1 mark

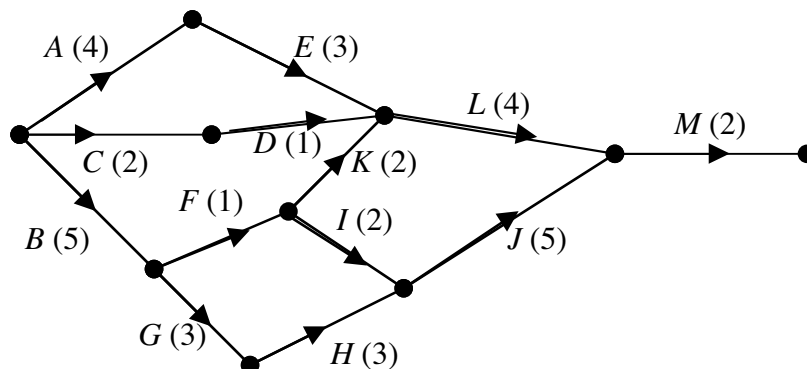
- c.** If 10 extra seats became available from E to B , how many people would now be able to travel from A to B ?

1 mark



Question 2

The network diagram below shows the activities required to build an extra room on a house and the time, in days, to complete each activity.



a. Using this information, complete the following Table.

Activity	Immediate Predecessor	Earliest starting time (days)
<i>A</i>	-----	0
<i>B</i>	-----	0
<i>C</i>	-----	0
<i>D</i>	<i>C</i>	2
<i>E</i>	<i>A</i>	4
<i>F</i>	<i>B</i>	5
<i>G</i>	<i>B</i>	5
<i>H</i>	<i>G</i>	8
<i>I</i>	<i>F</i>	6
<i>J</i>	<i>H, I</i>	
<i>K</i>		6
<i>L</i>		

2 + 2 = 4 marks

b. i. What is the shortest time in which the room can be built?

1 mark

ii. What is the critical path?

1 mark



Question 2 (continued)

- c. i.** What is the latest start time for activity *L* ?

1 mark

- ii.** If activity *K* had to be delayed, what would be the latest start time for this activity without delaying the whole project?

1 mark

- d.** For some of the activities required to build the room, the site manager can employ extra workers. This can speed up the operation but it costs money. The following is a Table showing the amount of time saving and the costs incurred.

Activity	Time reduction (days)	Cost per day for reducing each activity (\$)
<i>A</i>	2	150
<i>B</i>	3	400
<i>C</i>	1	100
<i>G</i>	1	100
<i>H</i>	2	300
<i>L</i>	1	350
<i>J</i>	2	150

- i.** What is the new minimum time required to build the room?

1 mark

- ii.** How much extra will it cost the builder to complete the room in this time?

2 marks

Total = 15 marks

End of Module 5: Networks and decision mathematics



If you choose this module, all questions are to be answered.

Question 1

In the country of *Chrysanthia*, health card numbers carry a fifteen digit code for each person. When using the health card at a doctor's surgery, the number on the health card is entered as a 3×5 matrix.

- a. Write the 3×5 matrix for a health card containing the numbers

1 4 7 6 1 2 2 7 9 7 3 6 5 5 5

1 mark

- b. This matrix, Y , is then coded for security reasons. To do this, Y is multiplied by a 3×3 matrix, B . **Tran**, who wishes to work for the coding company, says that he would calculate YB to obtain the coded matrix. Explain why **Tran** is wrong.

1 mark

- c. Susan, who works for the coding company, enters the coding matrix each day.

Today, the coding matrix is $\begin{bmatrix} 1 & 3 & -2 \\ 7 & 5 & 4 \\ 2 & -3 & 1 \end{bmatrix}$. Find the coded matrix BY .

1 mark



Question 1 (continued)

- d. Write down the element $a_{3,2}$ in the coded matrix.

1 mark

- e. Dan decodes the coded matrix. To do this he has a matrix key, K , so that he can enter the details of the person's actual health card number. Find the matrix key that would unlock the coded matrix, BY . Find each element of K to 3 decimal places.

2 marks

- f. Susan also enters a coding matrix each day for credit cards.
The credit cards are made up of 16 digits which are entered into a 2×8 matrix.

Explain why the matrix $\begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix}$ could **not** be used as a coding matrix.

2 marks

Question 2

In a certain town, people buy their bread from *Colds* or *Saveway*. Research shows that 60% of people who buy their bread from *Colds* today will buy their bread from *Colds* tomorrow, while 40% of people who buy their bread from *Colds* today will buy their bread from *Saveway* tomorrow. On the other hand, 25% of people who buy their bread from *Saveway* today will buy their bread from *Colds* tomorrow.

- a. What percentage of people who buy their bread from *Saveway* today will buy their bread from *Saveway* tomorrow?

1 mark



Question 2 (continued)

- b. Complete the transition matrix for the information in a.
Give the elements of the matrix as decimals.

$$T = \begin{bmatrix} & \\ & \end{bmatrix}$$

1 mark

- c. On one Monday, out of 7000 people, 4800 buy their bread from *Colds* and the remainder buy their bread from *Saveway*. What is the initial state matrix, S_0 , for this Monday?

1 mark

- d. Find S_1 .

1 mark



Question 2 (continued)

- e. How many people will buy their bread from *Saveway* on Tuesday, the day after the original Monday?

1 mark

- f. How many people will buy their bread from *Colds* on the Tuesday of the following week?
Give your answer to the nearest whole number.

1 mark

- g. What is the percentage change in *Colds*' sales of bread from Monday to the Tuesday of the following week? Give your answer to 1 decimal place.

1 mark

Total = 15 marks

End of Module 6: Matrices

END OF QUESTION AND ANSWER BOOK
2006 Further Mathematics Trial Examination 2

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