



2006

VCE Further Mathematics Trial Examination 1

Suggested Solutions

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Question 1 B The line in the box represents the median. This is equal to 38 years.	Question 2 D 28 years is the lower quartile for Japanese ages. Therefore, 25 % are younger than 28 years and 75 % are older than 28 years.
Question 3 B % of part time workers who are women $= \frac{45}{45 + 37} \times 100 = 54.9\% = 55\%$	Question 4 C Median = 68 Lower quartile = 58 Upper quartile = $\frac{76 + 82}{2} = \frac{158}{2} = 79$ Interquartile range = $79 - 58 = 21$
Question 5 E r^2 can equal 1. This is the maximum value for r^2. If $r^2 = 1$, then $r = +1$ or -1. If $r = -1$, then the more one studies, the lower the results in tests. So not A, B or D. If $r^2 = 1$, then there is a strong correlation between the two variables and so reliable predictions can be made. Therefore, not C.	Question 6 D $D = \text{decay}$. $S = \text{sweets}$. $D = 0.22S + 1.4$ $35 = 0.22S + 1.4$ $S = \frac{35 - 1.4}{0.22} = 152.7 = 153$ (153 is the nearest whole number) 150 is the closest answer.
Question 7 E You could only use an x^2 or a y^2 transformation to linearise this shape.	Question 8 B Split points into 3 , 4 , 3 Median of 3 points to the left = (3 , 4) Median of 3 points to the right = (9 , 7) $m = \frac{7 - 4}{9 - 3} = \frac{3}{6} = 0.5$
Question 9 A $b = \frac{rS_y}{S_x} = -0.06$ $20.25 = -0.06 \times 194.58 + a$ $a = 31.9 \approx 32$	Question 10 B $7 - (0.5 + 0.6 + 1 + 1.4 + 1.5 + 1.6)$ $= 0.4$
Question 11 D Take the average of June to September and put the value between July and August. $\frac{0.4 + 0.3 + 0.5 + 0.7}{4} = 0.475$ Take the average of July to October and put the value between August and September $\frac{0.3 + 0.5 + 0.7 + 0.8}{4} = 0.575$ Get the average of these two values $\frac{0.475 + 0.575}{2} = 0.525$	Question 12 B $56 + 2 \times 6 = 68$ g Greater than 2 standard deviations from the mean = $\frac{5}{2} = 2.5\%$. 2.5 % of 1000 = 25 Question 13 C You would need to standardise each score using $z = \frac{x - \bar{x}}{s_x}$. This has the effect of moving each subject's marks onto a standard normal curve with mean = 0 and standard deviation = 1.



Question 1 C $S_{\infty} = \frac{a}{1-r} \quad a = 8, r = \frac{1}{2}$ $S_{\infty} = \frac{8}{1 - \frac{1}{2}}$ $S_{\infty} = \frac{8}{\frac{1}{2}} = 16$	Question 2 E $t_n = a + (n-1)d$ $a = -9$ $t_4 = 3 \text{ when } n = 4$ $3 = -9 + 3d$ $12 = 3d$ $4 = d$ $\therefore -9, -5, -1, 3$ So second and third terms are respectively -5 and -1
Question 3 D $t_1 = -3$ $t_2 = -6 + 3 = -3$ $t_3 = -6 + 3 = -3$ $t_4 = -6 + 3 = -3$ $\therefore -3, -3, -3, -3$	Question 4 B Let $n = 11$ $t_{12} = t_{11} + t_{10}$ $144 = 89 + t_{10}$ $t_{10} = 55$ Let $n = 10$ $t_{11} = t_{10} + t_9$ $89 = 55 + t_9$ $t_9 = 34$
Question 5 D Use a graphics calculator. Use sequence mode. $a = 3$ $d = 4$ $S_n = \frac{n}{2}[6 + 4n - 4]$ $S_n = \frac{n}{2}[2 + 4n]$ $S_n = n + 2n^2$ • Enter this expression in the calculator. • Use 2nd Table to get sequence. • Look for value of n that gives the first value greater than 320	Question 6 C $A = 50,000(1 - 0.15)^7 = \$16,029$



Question 7 D

Sequence is 9 , 11 , 13 , ...

Arithmetic

$$a = 9$$

$$d = 2$$

$$t_n = a + (n - 1)d$$

$$t_{20} = 9 + 19 \times 2$$

$$t_{20} = 9 + 38 = 47$$

Question 8 C

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{7(2^n - 1)}{2 - 1} = 7(2^n - 1)$$

This is a geometric sequence with
 $a = 7$ and $r = 2$.

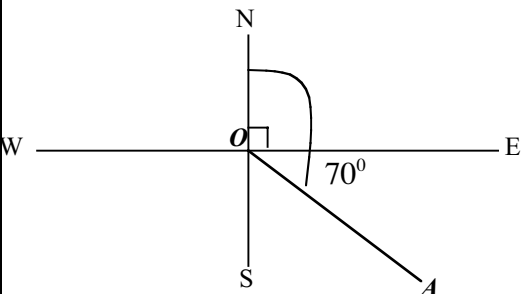
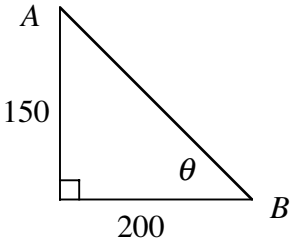
$$t_n = ar^{n-1}$$

$$t_3 = 7 \times 2^2 = 7 \times 4 = 28$$

Question 9 A

Since $a < 0$ and $0 < r < 1$, the sum of the geometric sequence, $S_n = \frac{a(r^n - 1)}{r - 1}$, must always be negative and becoming more negative.



Question 1 D $90^\circ + 70^\circ = 160^\circ$ T 	Question 2 B $\sin \theta = \frac{47}{68}$ $\theta = \sin^{-1}\left(\frac{47}{68}\right) = 43^\circ 43'$
Question 3 E $\frac{6.2}{\sin 43^\circ} = \frac{7.8}{\sin \angle BAC}$ $\sin \angle BAC = \frac{7.8 \times \sin 43^\circ}{6.2} = 0.86$	Question 4 B $DC^2 = 65^2 - 30^2$ $DC = \sqrt{65^2 - 30^2}$ $DC = 57.66$ $AD = \frac{57.66}{2} = 28.83$ $AC = AD + DC$ $AC = 28.83 + 57.66$ $AC = 86.49$
Question 5 A  $\tan \theta = \frac{150}{200}$ $\theta = \tan^{-1}\left(\frac{150}{200}\right) = 36^\circ 52'$	Question 6 C Volume of cone = $\frac{1}{3}\pi r^2 h$ $\Rightarrow 9047.8 = \frac{1}{3}\pi r^2 \times 60$ $\Rightarrow r^2 = 144$ $\Rightarrow r = 12$ $\Rightarrow \text{Diameter} = 2 \times 12 = 24 \text{ cm}$



Question 7 A

Triangles ADE and ABC are similar (AAA).
Therefore, corresponding sides in the same ratio.

$$\frac{AB}{3} = \frac{24}{15}$$

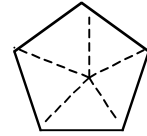
$$AB = \frac{24 \times 3}{15} = 4.8$$

Question 8 B

The pentagon can be divided into 5 congruent triangles. The apex angle of each triangle

$$= \frac{360}{5} = 72^\circ$$

$$\text{Base angle} = \frac{180 - 72}{2} = 54^\circ$$



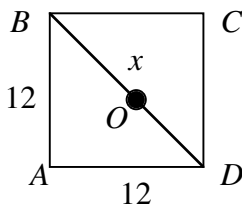
$$\text{Height of one triangle} = 2.5 \times \tan 54^\circ$$

$$\text{Area of one triangle} = \frac{5 \times 2.5 \times \tan 54^\circ}{2}$$

$$\text{Area of 5 triangles} = 5 \times \frac{5 \times 2.5 \times \tan 54^\circ}{2}$$

$$\text{Area of pentagon} = 43.01$$

Question 9 D

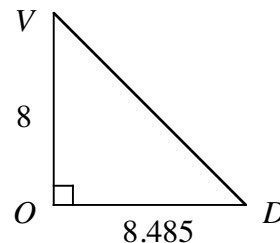


Length of diagonal base

$$x = \sqrt{12^2 + 12^2} = \sqrt{288} = 16.9706$$

Half the diagonal base

$$\frac{x}{2} = \frac{1}{2} \times 16.9706 = 8.485$$



$$VD = \sqrt{(8)^2 + (8.485)^2} = 11.66$$

4 straws of 11.66 and 4 straws of 12

$$= 4 \times 11.66 + 4 \times 12$$

$$= 94.64 \text{ (closest to 95 cm)}$$



Question 1 A In the first 5 minutes, water is flowing into the tank at 100 litres (L) in 5 minutes or 20 L/min. From 5 to 15 minutes, the increase is 50 litres, so 50 litres in 10 minutes is a rate of 5 L/min. From 35 to 40 minutes, the increase is 25 litres, so 25 litres in 5 minutes is a rate of 5 L/min. At other times, water is not flowing into the tank. Water is flowing into the tank most rapidly in the first 5 minutes.	Question 2 E $x - 3y = -10$ (1) $x - 5y = 6$ (2) Subtract (2) from (1) $\Rightarrow 2y = -16$ $\Rightarrow y = -8$ Substitute into equation (1) $\Rightarrow x + 24 = -10$ $\Rightarrow x = -34$
Question 3 C $2y = 4x - 7$ $\Rightarrow y = 2x - \frac{7}{2}$ $\Rightarrow m = 2$ $3y - 6x = 0$ $\Rightarrow 3y = 6x$ $\Rightarrow y = 2x$ $\Rightarrow m = 2$ Parallel lines have the same gradient.	Question 4 E When $x = 10$ and $y = 11$, $2 \times 11 - 10 - 12 = 22 - 10 - 12 = 0$ So, (10,11) lies on the line $2y - x - 12 = 0$
Question 5 E $y = \frac{k}{x^2}$ When $x = 5$ $y = \frac{k}{25}$ so $k = 3$ $\Rightarrow y = \frac{3}{x^2}$	Question 6 A $P = 2n \times 80 - 40,000$ $P = 160n - 40,000$



Question 7 E

When $t = 8$,

$$c = 70 + 40 \times 8 = 70 + 320 = 390$$

$$\Rightarrow c = dt + f = 30t + 390$$

$$\Rightarrow f = 390$$

Question 8 B

$$m = kr^3$$

$$\Rightarrow k = \frac{m}{r^3} = \frac{36.45}{(1.5)^3} = 10.8$$

$$\Rightarrow m = 10.8r^3$$

$$\Rightarrow r^3 = \frac{m}{10.8}$$

$$\Rightarrow r = \sqrt[3]{\frac{m}{10.8}} = \sqrt[3]{\frac{72.9}{10.8}} = 1.89$$

$$= 1.9 \text{ cm to 1 decimal place}$$

Question 9 C

Constraints are

$$5000x + 3000y \leq 50000$$

$$\Rightarrow 5x + 3y \leq 50$$

$$10000x + 4000y \geq 90000$$

$$\Rightarrow 5x + 2y \geq 45$$

$$8000x + 1000y \geq 50000$$

$$\Rightarrow 8x + y \geq 50$$



Question 1 C $V = 33000 - \frac{PRT}{100}$ $\Rightarrow V = 33000 - \frac{33000 \times 12 \times 6}{100}$ $\Rightarrow V = \$9240$	Question 2 D $A = PR^n$ $18000 = 9000 \left(1 + \frac{(6.3/12)}{100} \right)^n$ Enter into Math Solver $0 = P \left(1 + \frac{(R/12)}{100} \right)^{NT} - A$ Enter $P = 9000, R = 6.3, N = 12, A = 18000$ Solve for T $T = 11.03$ Closest to 11
Question 3 B Balance 13 Aug = $3420 - 685 = 2735$ Balance 17 Aug = $2735 + 1800 = 4535$ Balance 21 Aug = $4535 - 1300 = 3235$ Minimum monthly balance = 2735 $\text{Interest} = 2735 \times \frac{5.8}{1200} = \13.22 $\text{Balance} = 3235 + 13.22 = \3248.22	Question 4 C Original price $= 5.4 \times \frac{100}{104.2}$ $= \$5.18$
Question 5 E Amount borrowed = $3000 - 400 = 2600$ Total amount repaid = $36 \times 100 = 3600$ Interest = $3600 - 2600 = 1000$ $I = \frac{PRT}{100}$ $\Rightarrow 1000 = \frac{2600 \times R \times 3}{100}$ $\Rightarrow 1000 = 78R$ $\Rightarrow R = \frac{1000}{78} = 12.8\%$ Closest to 13%	Question 6 E Use TVM solver under finance on a graphics calculator. $I = 7.2$ $PV = -120000$ $PMT = 5000$ $FV = 0$ $P/Y = 12$ $C/Y = 12$ $N = 26 \text{ years}$



Question 7 D $P + I = 8700$ $I = (8700 - P)$ $(8700 - P) = \frac{P \times 5.2 \times 3}{100}$ $\Rightarrow 870000 - 100P = 15.6P$ $\Rightarrow 870000 = 115.6P$ $\Rightarrow P = \frac{870000}{115.6} = 7525.95$ $\Rightarrow P = \\$7526$ to the nearest dollar	Question 8 B $R = \frac{100I}{PT} = \frac{100 \times 1340}{4000 \times 5} = 6.7\%$ per annum Effective rate $= r \times \frac{2n}{n+1}$ where $n = 5 \times 12 = 60$ Effective rate $= 6.7 \times \frac{2 \times 60}{60+1} = 13.18\%$
Question 9 C $I = \frac{PRT}{100} = \frac{12000 \times 5.4T}{100}$ $\Rightarrow I = 648T$ But, $3240 = 648 \times 5$ Therefore, graph C.	

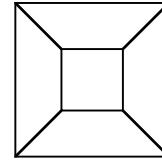


Question 1 B

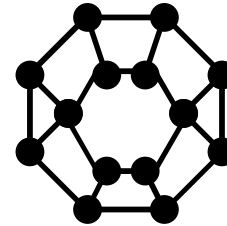
		TO			
		<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
FROM	<i>A</i>	0	1	2	1
	<i>B</i>	1	0	2	0
	<i>C</i>	2	2	0	1
	<i>D</i>	1	0	1	0

Question 2 E

This is a planar graph of a cube, so **not A**. This graph has 8 vertices and 12 edges, so **not B**.



This planar graph has 10 faces, so **not C**.



This is a planar graph with only one vertex, so **not D**.



Question 3 C

$$V + F - E = 2$$

$$\Rightarrow E = V + F - 2$$

$$\Rightarrow E = 9 + 5 - 2$$

$$\Rightarrow E = 12$$

Question 4 B

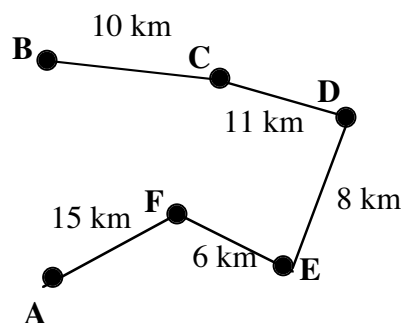
Teams are *A, B, C, D, E, F, G, H, I, J, K, L*.
A plays 11 games. *B* plays another 10 games.
C plays another 9 games. *D* plays another 8 games.
E plays another 7 games. *F* plays another 6 games.
G plays another 5 games. *H* plays another 4 games.
I plays another 3 games. *J* plays another 2 games.
K plays another 1 game. *L* plays no more games.

Total number of games
= $11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$
= 66 games.

Question 5 D

There is no hamiltonian path or circuit because it is not possible to pass through each vertex exactly once. There is no eulerian circuit because it is not possible to start and finish at the same point and pass through each edge. There is an eulerian path because if you start at *H*, it is possible to pass over each edge once and finish at *B*.

Question 6 C



$$15 + 6 + 8 + 11 + 10 = 50 \text{ km}$$



Question 7 C

J starts on day 9 and finishes at the end of day 16. Therefore, 16 days to complete.

Question 8 C

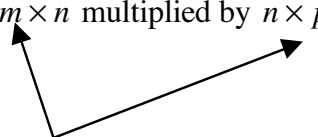
$$6 + 2 + 6 = 14$$

Question 9 A

$$\begin{bmatrix} 3 & 8 & \textcircled{2} & 4 \\ \textcircled{3} & 8 & 5 & 4 \\ 12 & 8 & \textcircled{6} & 10 \\ \textcircled{5} & 8 & 7 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 6 & 0 & 2 \\ 0 & 5 & 2 & 1 \\ 6 & \textcircled{2} & 0 & 4 \\ 0 & 3 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 0 & 2 \\ 0 & 3 & 2 & 1 \\ 6 & 0 & 0 & 4 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

So Brian does Algebra and Danielle does Calculus.



Question 1 A $2A = \begin{bmatrix} 2p & 2q \\ 2r & 2s \end{bmatrix}$ $2A - B = \begin{bmatrix} 2p + p & 2q - q \\ 2r - r & 2s - s \end{bmatrix}$ $\Rightarrow 2A - B = \begin{bmatrix} 3p & q \\ r & 3s \end{bmatrix}$	Question 2 C $\begin{bmatrix} 2 \times 1 + 3 \times 2 & 2 \times 3 + 3 \times -1 \\ 1 \times 1 + -4 \times 2 & 1 \times 3 + -4 \times -1 \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ -7 & 7 \end{bmatrix}$ or use 2nd Matrix on a graphics calculator.
Question 3 D $m \times n$ multiplied by $n \times p$  Order of resulting matrix is $m \times p$.	Question 4 D Q and U are both column matrices. No other given matrix is a column matrix.
Question 5 D Rewrite the equations. $3x + 2y = 4$ $5x - 2y = 12$ Therefore, $\begin{bmatrix} 3 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$	Question 6 D Use a graphics calculator. After entering matrix, go to 2nd-matrix-math-det. Answer is 8



Question 7 B

$$\Delta = 2 - (-3) = 2 + 3 = 5$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{2}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

$$\Rightarrow a_{1,2} = -\frac{3}{5}$$

Question 8 E

20% less means 80% of week 2 values =
 0.8×28 and 0.8×60 and 0.8×50 and 0.8×44

$$= \begin{bmatrix} 22.4 \\ 48 \\ 40 \\ 35.2 \end{bmatrix}$$

Question 9 C

$$S_1 = TS_0 = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix} \times \begin{bmatrix} 520 \\ 380 \end{bmatrix} = \begin{bmatrix} 426 \\ 474 \end{bmatrix}$$

$$S_2 = TS_1 = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix} \times \begin{bmatrix} 426 \\ 474 \end{bmatrix} = \begin{bmatrix} 397.8 \\ 502.2 \end{bmatrix} = \begin{bmatrix} 398 \\ 502 \end{bmatrix}$$

End of suggested solutions
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