

Year 2004

VCE

Specialist Mathematics

Trial Examination 1

Suggested Solutions

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Question 1

$$y = \frac{x^3 + b^3}{x^2} = \frac{x^3}{x^2} + \frac{b^3}{x^2} = x + \frac{b^3}{x^2}$$

The denominator is zero when $x = 0$, so $x = 0$ is a vertical asymptote.

As $x \rightarrow \pm \infty$ $y \rightarrow x$, so $y = x$ is an oblique asymptote.

Both $x = 0$ and $y = x$ are asymptotes.

Answer: D.

Question 2

The ellipse has centre $(-3, 1)$ semi-major axis is 3, semi-minor is 1, the general form is

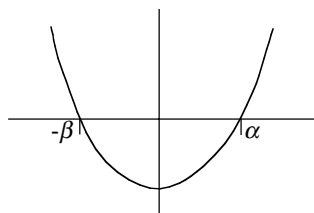
$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

with $h = -3$ $k = 1$ $a = 3$ and $b = 1$ so the equation is $\frac{(x + 3)^2}{9} + \frac{(y - 1)^2}{1} = 1$.

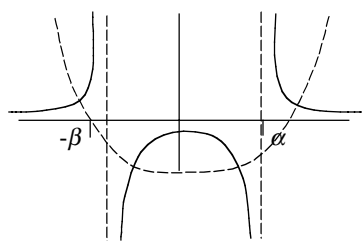
Answer: E.

Question 3

Given $f(x) = a(x - \alpha)(x + \beta)$ where $a > 0$ this graph crosses the x -axis at $x = \alpha$ and $x = -\beta$.



the graph of $\frac{1}{f(x)}$



has vertical asymptotes at $x = \alpha$ and $x = -\beta$. So alternatives A. and B. are incorrect.

$$f(x) = a(x^2 + (\beta - \alpha)x + \alpha\beta)$$

$$f'(x) = a(2x + (\beta - \alpha)) = 0 \text{ when}$$

$$2x + (\beta - \alpha) = 0$$

$$2x = (\alpha - \beta)$$

$$x = \frac{\alpha - \beta}{2}$$

So $\frac{1}{f(x)}$ has a local maximum at $x = \frac{\alpha - \beta}{2}$.

Answer: C.

Question 4

The range of $y = \cos^{-1}x$ is $[0, \pi]$

The range of $f(x) = a + b \cos^{-1}(cx)$ is dilated by b and translated by a ,
its range is $[a, a + b\pi]$

(the value of c has no effect on the range).

Answer: E.

Question 5

Given $|a| = 7$ $|b| = 3$ $a \cdot b = 2$, now

$$\begin{aligned} |a - b|^2 &= (a - b) \cdot (a - b) \\ &= a \cdot a - b \cdot a - a \cdot b + b \cdot b \\ &= |a|^2 - 2a \cdot b + |b|^2 \\ &= 49 - 4 + 9 \end{aligned}$$

$$|a - b|^2 = 54$$

so $|a - b| = \sqrt{54}$

Answer: C.

Question 6

Given $P(2, -8, 10)$ $Q(x, 4, -5)$ then

$$\overrightarrow{OP} = 2\mathbf{i} - 8\mathbf{j} + 10\mathbf{k} \quad \overrightarrow{OQ} = x\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (x - 2)\mathbf{i} + 12\mathbf{j} - 15\mathbf{k}$$

Checking each alternative:

A When $x = 2$ $\overrightarrow{PQ} = 12\mathbf{j} - 15\mathbf{k}$ so it is parallel to the YZ plane.

Alternative A is correct.

B When $x = -1$, $\overrightarrow{OP} = -2\overrightarrow{OQ}$, so $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are linearly dependent

Alternative B is incorrect.

C When $x = -3$, $|\overrightarrow{OQ}| = \sqrt{9 + 16 + 25} = \sqrt{50} = 5\sqrt{2}$

D When $x = 3$, $|\overrightarrow{OQ}| = \sqrt{9 + 16 + 25} = \sqrt{50} = 5\sqrt{2}$. So alternatives C and D are correct.

E If $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$,

$$\overrightarrow{OP} \cdot \overrightarrow{OQ} = 0$$

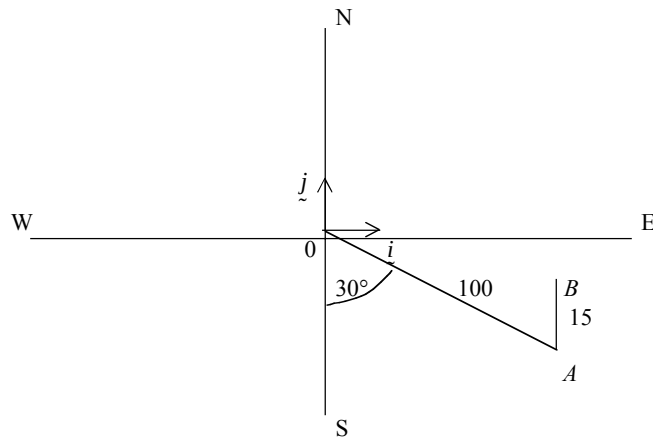
$$\therefore 2x - 32 - 50 = 0$$

So alternative E is correct.

$$\therefore 2x = 82 \Rightarrow x = 41$$

Answer: B.

Question 7



$$\vec{OA} = 100 \sin 30^\circ \vec{i} - 100 \cos 30^\circ \vec{j} = 50\vec{i} - 50\sqrt{3}\vec{j}$$

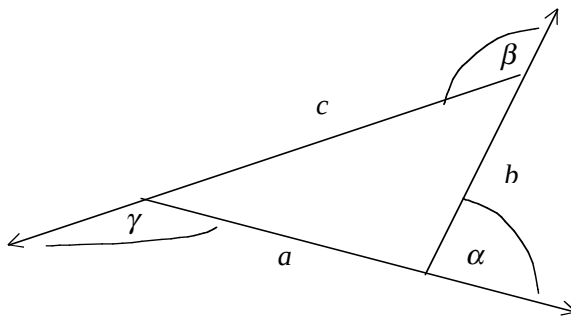
$$\vec{AB} = 15\vec{k}$$

$$\therefore \vec{OB} = \vec{OA} + \vec{AB} = 50\vec{i} - 50\sqrt{3}\vec{j} + 15\vec{k}$$

Answer: B.

Question 8

If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ form the sides of a triangle.



The angles given are the exterior angles.

So the interior angles are $\pi - \alpha$, $\pi - \beta$ and $\pi - \gamma$

$$\Rightarrow \pi - \alpha + \pi - \beta + \pi - \gamma = \pi$$

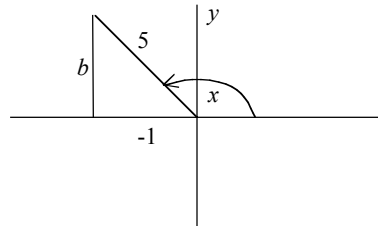
$$\Rightarrow 3\pi - \pi = \alpha + \beta + \gamma$$

$$\Rightarrow \alpha + \beta + \gamma = 2\pi$$

Answer: E.

Question 9

Given $\sec(x) = -5$ so $\sec(x) = \frac{1}{\cos(x)} = -5$ so $\cos(x) = -\frac{1}{5}$, but $\frac{\pi}{2} < x < \pi$ so x is the 2nd quadrant.



by Pythagoras' theorem: $1^2 + b^2 = 5^2$

$$b^2 = 25 - 1 = 24$$

$$b = \sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$$

$$\text{Now } \operatorname{cosec}(x) = \frac{1}{\sin(x)} = \frac{1}{\frac{2\sqrt{6}}{5}} = \frac{5}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{12}$$

since $\sin(x)$ is positive in the 2nd quadrant.

Answer: E.

Question 10

Let $\underline{a} = 2\underline{i} - 3\underline{j} + 4\underline{k}$, the scalar resolute of $\underline{a}$ in the direction of $\underline{b}$ is $\underline{a} \cdot \hat{\underline{b}} = 2\sqrt{6}$.

The vector resolute of $\underline{a}$ perpendicular to $\underline{b}$ is $\underline{a} - (\underline{a} \cdot \hat{\underline{b}})\hat{\underline{b}} = \underline{j} + 2\underline{k}$.

$$2\underline{i} - 3\underline{j} + 4\underline{k} - 2\sqrt{6} \hat{\underline{b}} = \underline{j} + 2\underline{k}$$

$$2\sqrt{6} \hat{\underline{b}} = (2\underline{i} - 3\underline{j} + 4\underline{k}) - (\underline{j} + 2\underline{k})$$

$$2\sqrt{6} \hat{\underline{b}} = 2\underline{i} - 4\underline{j} + 2\underline{k}$$

$$\text{so } \hat{\underline{b}} = \frac{1}{\sqrt{6}}(\underline{i} - 2\underline{j} + \underline{k})$$

Answer: C.

Question 11

$P(z) = z^3 + az^2 + bz + c$ given that $P(\alpha - i\beta) = 0$ and a, b and c are real then

- A $P(\alpha + i\beta) = 0$ is correct by the conjugate root theorem.
B $P(z)$ has three roots (1 pair of complex conjugates and one root). This is correct.
C This is correct.
D Now $u = \alpha + i\beta$ $v = \alpha - i\beta$

$$u + v = 2\alpha \quad uv = \alpha^2 - i^2\beta^2 = \alpha^2 + \beta^2 \text{ so}$$

$$z^2 - (u + v)z + uv = 0$$

$$z^2 - (\text{sum of the roots})z + (\text{product of the roots}) = 0$$

$$\text{so} \quad z^2 - 2\alpha z + (\alpha^2 + \beta^2) = 0 \text{ is a factor of } P(z).$$

Option D is INCORRECT.

- E is correct since

$$z^3 + az^2 + bz + c = 0$$

$$= (z^2 - 2\alpha z + (\alpha^2 + \beta^2))(z + d) = 0$$

$$\text{so } d(\alpha^2 + \beta^2) = c$$

$$d = \frac{c}{\alpha^2 + \beta^2} \text{ and } P(-d) = 0$$

Answer: D.

Question 12

The shaded region is the intersection of the inside of a circle with centre at the origin and radius 2, with the region above the line $y = -x$.

The circle is $z\bar{z} \leq r^2 = 4$ (where r is the radius).

since if $z = x + iy$ $\bar{z} = x - iy$ so

$$z\bar{z} = x^2 - i^2 y^2 = x^2 + y^2 = r^2$$

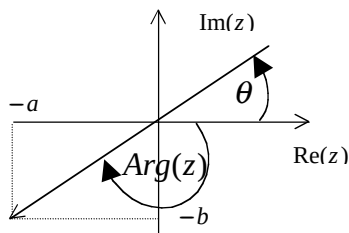
The line is $y \geq -x$, if $z = x + iy$

$$\operatorname{Re}(z) = x \text{ and } \operatorname{Im}(z) = y$$

$$\text{so } \operatorname{Im}(y) + \operatorname{Re}(z) \geq 0$$

Answer: A.

Question 13



$$\text{Now } z = -a - bi$$

a and b are real positive

$$-\pi < \operatorname{Arg}(z) \leq \pi$$

$$\tan \theta = \frac{b}{a} > 0 \quad \theta = \operatorname{Tan}^{-1}\left(\frac{b}{a}\right)$$

$$\operatorname{Arg}(z) = -\pi + \theta$$

$$= -\pi + \operatorname{Tan}^{-1}\left(\frac{b}{a}\right)$$

Answer: D.

Question 14

Solving $z^3 + a^3 i = 0$ since $i^2 = -1$

$$z^3 = -a^3 i = a^3 i^3, \text{ one answer is:}$$

$$z = ai \text{ where } a > 0$$

There are three answers one of which is D are all equally spaced around the circle. The roots are D, H and L

Answer: B.

Question 15

$$\vec{r}(t) = \cos^2(t)\vec{i} + \cos(2t)\vec{j}$$

$$x = \cos^2(t)$$

$$y = \cos(2t) = 2\cos^2(t) - 1$$

$y = 2x - 1$, this is a straight line.

Answer: A.

Question 16

$$y = \cos^{-1}\left(\frac{5x}{4}\right) = \cos^{-1}\left(\frac{u}{4}\right) \text{ where } u = 5x \text{ Chain Rule}$$

$$\frac{dy}{du} = \frac{-1}{\sqrt{16 - u^2}} \quad \frac{du}{dx} = 5$$

$$\frac{dy}{dx} = \frac{-5}{\sqrt{16 - 25x^2}}, \text{ now at } x = 0 \quad m_T = \left. \frac{dy}{dx} \right|_{x=0} = f'(0) = \frac{-5}{4}$$

at $x = 0$ $y = \cos^{-1}(0) = \frac{\pi}{2}$ so $P\left(0, \frac{\pi}{2}\right)$ and the gradient of the normal is $\frac{4}{5}$,

so the equation of the normal is: $y - \frac{\pi}{2} = \frac{4}{5}(x - 0)$

$$\text{or } y = \frac{4x}{5} + \frac{\pi}{2}$$

Answer: A.

Question 17

$$y = \tan^{-1}\left(\frac{3}{4x}\right) = \tan^{-1} u \text{ where } u = \frac{3}{4x} = \frac{3}{4} x^{-1}$$

$$\frac{dy}{du} = \frac{1}{1+u^2} \quad \frac{du}{dx} = -\frac{3}{4} x^{-2} = \frac{-3}{4x^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} \text{ (chain rule)}$$

$$= -\frac{3}{4x^2} \left(\frac{1}{1 + \frac{9}{16x^2}} \right)$$

$$= -\frac{3}{4x^2} \left(\frac{1}{\frac{16x^2 + 9}{16x^2}} \right)$$

$$= -\frac{3}{4x^2} \left(\frac{16x^2}{16x^2 + 9} \right)$$

$$= \frac{-12}{9 + 16x^2}$$

Answer: A.

Question 18

$$\int_0^{\frac{\pi}{4}} \cos^3(2x) \sin(2x) dx$$

$$\text{let } u = \cos(2x) \quad \frac{du}{dx} = -2 \sin(2x)$$

changing terminals, when

$$x = \frac{\pi}{4} \quad u = \cos\left(\frac{\pi}{2}\right) = 0 \quad \text{and when } x = 0 \quad u = \cos(0) = 1$$

$$= \int_1^0 u^3 \times -\frac{1}{2} \frac{du}{dx} \cdot dx = -\frac{1}{2} \int_1^0 u^3 du = \frac{1}{2} \int_0^1 u^3 du$$

Answer: E.

Question 19

$$y = (A + Bx)e^{-2x} \text{ satisfies } \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$$

$$\frac{dy}{dx} = Be^{-2x} - 2(A + Bx)e^{-2x}$$

$$= ((B - 2A) - 2Bx)e^{-2x}$$

$$\frac{d^2y}{dx^2} = -2((B - 2A) - 2Bx)e^{-2x} - 2Be^{-2x}$$

$$= ((4A - 4B) + 4Bx)e^{-2x}$$

$$\text{so } \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y$$

$$= (4A - 4B + 4Bx + 4B - 8A - 8Bx + 4A + 4Bx)e^{-2x} = 0$$

If $B = 0$ and $A = 3$ or $A = 5$ so alternatives A. and B. both satisfy the differential equation.

If $A = 0$ and $B = 2$ or if $A = 4$ and $B = 2$ so alternatives C. and D. both satisfy the differential equation.

Alternative E. **does not** satisfy the differential equation.

Answer: E.

Question 20

$y_1 = \frac{\cos(2x)}{2x-1}$ using a graphic calculator, in the Radians mode we find the graph crosses the x -axis at $x \approx 2.36$, the area is:

$$\int_2^{2.36} \frac{\cos(2x)}{2x-1} dx + \int_{2.36}^2 \frac{\cos(2x)}{2x-1} dx$$

$$= A_1 + A_2$$

but $A_1 = -0.0376$ and $A_2 = 0.0796$ the shaded area is:

$$A = |A_1| + A_2$$

$$= 0.0376 + 0.0796$$

$$= 0.1173$$

Answer: D.

Question 21

Rearranging to $\frac{dy}{dx} = \frac{\cos(2x)}{2x-1}$ with the calculator in the radians mode, using a PRGM Euler

$$Y_1 = \frac{\cos(2x)}{2x-1} \quad x_0 = 0 \quad y_0 = -1 \quad h = 0.2$$

$$y(0.4) = -1.507$$

Answer: D.

Question 22

$$\frac{d}{dx} (x \log_e (4+x)) = \log_e (4+x) + \frac{x}{4+x}$$

so it follows that:

$$\int \left(\log_e (4+x) + \frac{2x}{4+x} \right) dx = x \log_e (4+x)$$

$$\begin{aligned} \int \log_e (4+x) dx &= x \log_e (4+x) - \int \frac{x}{4+x} dx \\ &= x \log_e (4+x) - \int \left(\frac{x+4-4}{4+x} \right) dx \\ &= x \log_e (4+x) - \int \left(1 - \frac{4}{4+x} \right) dx \\ &= x \log_e (4+x) - x + 4 \log_e (4+x) \\ &= (x+4) \log_e (x+4) - x \end{aligned}$$

Answer: C.

Question 23

Since for

$$x < 0 \quad \frac{dy}{dx} < 0 \quad \text{and} \quad 0 < x < 1 \quad \frac{dy}{dx} > 0$$

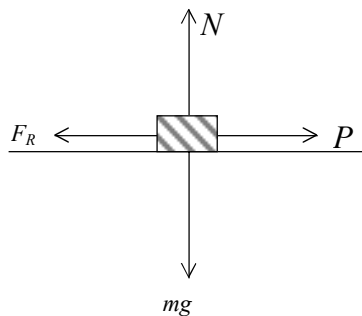
while at $x = 0 \quad \frac{dy}{dx} = 0$ the graph has a local minimum at $x = 0$

$$\text{and at } x = 4 \quad \frac{d^2y}{dx^2} = 0$$

The graph has a stationary point of inflexion at $x = 4$

Answer: B.

Question 24



$$m = 12 \text{ kg} \quad P = 11.65 \text{ N}$$

$$\mu = 0.1$$

$$N = mg$$

$$N = 12 \times 9.8$$

$$= 117.6 \text{ N}$$

$$\text{Now } \mu N = 0.1 \times 117.6$$

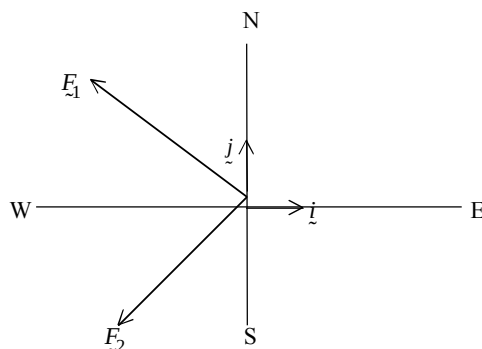
$$= 11.76 \text{ N}$$

$$\text{since } \mu N = 11.76 > P = 11.65$$

The suitcase remains at rest.

Answer: E.

Question 25



$$\underline{F}_1 = -3\hat{i} + 3\hat{j} \quad |\underline{F}_1| = 3\sqrt{2}$$

$$\underline{F}_2 = -4\hat{i} - 4\hat{j} \quad |\underline{F}_2| = 4\sqrt{2}$$

$$\underline{R} = \underline{F}_1 + \underline{F}_2 = -7\hat{i} - \hat{j} = 2\underline{a}$$

so $\underline{a} = -\frac{1}{2}(7\hat{i} - \hat{j})$

$$|\underline{a}| = \frac{1}{2}\sqrt{49+1} = \frac{\sqrt{50}}{2} = \frac{5\sqrt{2}}{2}$$

Answer: B.

Question 26

Resolving vertically gives: $T_1 \sin \beta + T_2 \sin \alpha - mg = 0$
so alternative B. is correct.

Resolving horizontally gives: $T_1 \cos \beta - T_2 \cos \alpha = 0$

so $T_1 \cos \beta = T_2 \cos \alpha$

$$\frac{T_1}{T_2} = \frac{\cos \alpha}{\cos \beta} = \frac{\frac{L_2}{\alpha}}{\frac{L_1}{\alpha}} = \frac{L_2}{L_1} \text{ so alternatives A. and C. are correct.}$$

The vector equation $\underline{T}_1 + \underline{T}_2 + m\underline{g} = \underline{0}$ is true.

Alternative D. is correct. Alternative E. is INCORRECT.

The shorter string carries more tension.

Answer: E.

Question 27

Let $y_1 = 4e^{-\frac{x}{2}}$ and $y_2 = 3 \sin\left(\frac{x}{2}\right)$ the volume is

$$V = \pi \int_0^b (y_1^2 - y_2^2) dx = \pi \int_0^b \left(16e^{-x} - 9 \sin^2\left(\frac{x}{2}\right) \right) dx$$

Answer: A.

Question 28

Since $v > 0$ for $0 \leq t < c$ and $v = 0$ for $t = c$

$v < 0$ for $t > c$ Alternative B. is correct.

Answer: B.

Question 29

Given the initial conditions $t = 0$ $x = 0$ $v = v_0$

Now $\frac{dv}{dx} = 1$ so $v \frac{dv}{dx} = a = v$ Alternative A. is correct.

Since $\frac{dv}{dx} = 1$ integrating wrt x

$$v = \int 1 dx = x + c \text{ but } v = v_0 \text{ when } x = 0$$

so $c = v_0$ and $v = x + v_0$ Alternative B. is correct.

Since $a = \frac{dv}{dt} = v$ this has the solution $v = v_0 e^t$ Alternative C. is correct.

Alternative D. is the **constant** acceleration formula $v^2 = v_0^2 + 2vx$ is FALSE.

Alternative E. is correct since $v = \frac{dx}{dt} = v_0 e^t$ integrating wrt t

$$x = \int v_0 e^t dt = v_0 e^t + c$$

but when $t = 0$ $x = 0$ so $0 = v_0 + c$ so $c = -v_0$

$$\text{and } x = v_0 (e^t - 1)$$

Answer: D.

Question 30

Resolving perpendicular to the incline gives

$$(1) \quad N - mg \cos \theta = 0$$

resolving downwards parallel to the plane gives

$$(2) \quad mg \sin \theta - \mu N = ma$$

from (1) $N = mg \cos \theta$ into (2)

$$mg \sin \theta - \mu mg \cos \theta = ma$$

$$a = g(\sin \theta - \mu \cos \theta) \quad \text{Alternative A. and B. are correct.}$$

The acceleration is constant, using $u = 0 \quad t = T \quad s = D$

$$v^2 = u^2 + 2as \text{ gives}$$

$$v^2 = 0 + 2g(\sin \theta - \mu \cos \theta) D$$

$$\text{so} \quad v = \sqrt{2gD(\sin \theta - \mu \cos \theta)} \quad \text{Alternative C. is correct.}$$

$$\text{Using: } s = \left(\frac{u+v}{2} \right) t \text{ gives}$$

$$D = \left(\frac{0+V}{2} \right) T = \frac{VT}{2}$$

$$\text{so} \quad T = \frac{2D}{V}$$

Alternative D. is correct.

Alternative E. is false, terminal or limiting velocity occurs in a situation when falling vertically with air resistance proportional to the velocity.

Answer: E.

Question 1

$$\text{Let } u = \sqrt{x} = x^{\frac{1}{2}} \quad \frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \quad \text{so } 2 du = \frac{1}{\sqrt{x}} dx$$

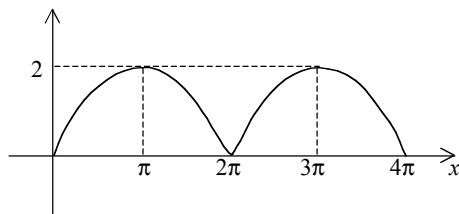
$$\text{terminals } x = 4 \quad u = 2$$

$$x = 0 \quad u = 0$$

$$\begin{aligned} \int_0^4 \frac{\cos \sqrt{x}}{\sqrt{x}} dx &= 2 \int_0^2 \cos(u) du \\ &= 2 \sin u \Big|_0^2 = 2 \sin(2) - 2 \sin(0) \\ &= 2 \sin(2) \end{aligned}$$

Question 2

i. $x = 2t - \sin(2t) \quad y = 1 - \cos(2t) \quad 0 \leq t \leq 2\pi$



ii. $\vec{r}(t) = (2t - \sin(2t))\vec{i} + (1 - \cos(2t))\vec{j} \quad 0 \leq t \leq 2\pi$

$$\dot{\vec{r}}(t) = (2 - 2 \cos(2t))\vec{i} + 2 \sin(2t)\vec{j}$$

$$|\dot{\vec{r}}(t)| = \sqrt{(2 - 2 \cos 2t)^2 + (2 \sin 2t)^2}$$

$$= \sqrt{4 - 8 \cos(2t) + 4 \cos^2(2t) + 4 \sin^2(2t)}$$

$$= \sqrt{8 - 8 \cos(2t)}$$

$$= \sqrt{8(1 - \cos(2t))}$$

$$\text{since } \cos(2A) = 1 - 2\sin^2(A)$$

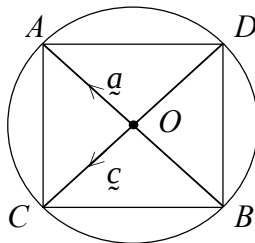
$$= \sqrt{8(2 \sin^2(t))}$$

$$\text{and } 2\sin^2(A) = 1 - \cos(2A)$$

$$= 4 \sin t$$

$$\text{so } c = 4$$

Question 3



Now $\overrightarrow{OA} = \overrightarrow{BO} = \underline{q}$ and $\overrightarrow{OC} = \overrightarrow{DO} = \underline{c}$

$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = \underline{c} - \underline{q}$$

$$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD} = -\underline{q} - \underline{c}$$

$$\overrightarrow{CB} = \overrightarrow{CO} + \overrightarrow{OB} = -\underline{q} - \underline{c} \Rightarrow \overrightarrow{AD} = \overrightarrow{CB}$$

consider

$$\begin{aligned} \overrightarrow{AC} \cdot \overrightarrow{CB} &= (\underline{c} - \underline{q}) \cdot (-\underline{q} - \underline{c}) \\ &= -\underline{c} \cdot \underline{q} + \underline{q} \cdot \underline{q} - \underline{c} \cdot \underline{c} + \underline{q} \cdot \underline{c} \\ &= |\underline{q}|^2 - |\underline{c}|^2 \\ &= 0 \text{ since } |\underline{q}| = |\underline{c}| \text{ (both are radii of the circle)} \end{aligned}$$

so $\overrightarrow{AC}$ is perpendicular to $\overrightarrow{CB}$ and $\overrightarrow{AD} = \overrightarrow{CB}$

$ABCD$ is rectangle

Question 4

i. $u = 2\sqrt{2}(1-i) \quad \text{Arg}(u) = -\frac{\pi}{4}$

so $u = 4 \text{cis}\left(-\frac{\pi}{4}\right)$

And $\arg(u^8) = 8 \times -\frac{\pi}{4} = -2\pi$

But $\text{Arg}(u^8) = -2\pi + 2\pi = 0$

ii. $u^2 = \left(4 \text{cis}\left(-\frac{\pi}{4}\right)\right)^2$

$= 16 \text{cis}\left(-\frac{\pi}{2}\right)$

$= -16i$

iii. $z^3 + 16iz = 0$

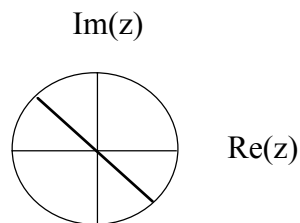
$z(z^2 + 16i) = 0$

$z = 0$ or $z^2 = -16i$ hence from i. and ii.

$z = 2\sqrt{2}(1-i) = 4 \text{cis}\left(-\frac{\pi}{4}\right) = z_1$

$z = 2\sqrt{2}(-1+i) = 4 \text{cis}\left(\frac{3\pi}{4}\right) = z_2$

note that $z_2 = -z_1$ these are **not** conjugates.



Question 5

a. $\vec{a} = -2\vec{i} + y\vec{j} + 5\vec{k}$

$$|\vec{a}| = \sqrt{4 + y^2 + 25} = 6$$

$$\sqrt{29 + y^2} = 6$$

$$29 + y^2 = 36$$

$$y^2 = 7$$

$$y = \pm \sqrt{7} \quad \text{both } \pm \text{ are both acceptable answers.}$$

b. $\cos \beta = -\frac{\sqrt{7}}{6} = \frac{y}{|\vec{a}|} = \frac{y}{\sqrt{29 + y^2}}$

$$6y = -\sqrt{7} \left(\sqrt{29 + y^2} \right)$$

$$36y^2 = 7(29 + y^2)$$

$$= 7 \times 29 + 7 \times y^2$$

$$29y^2 = 7 \times 29$$

$$y = \pm \sqrt{7} \quad \text{but}$$

$$y = -\sqrt{7} \quad \text{since } y < 0 \quad \text{it is an obtuse angle, there is only one answer.}$$

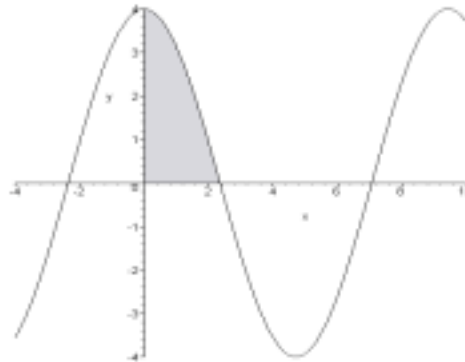
Question 6

to find the first x intercept

$$\cos\left(\frac{2x}{3}\right) = 0$$

$$\frac{2x}{3} = \frac{\pi}{2}$$

$$x = \frac{3\pi}{4}$$



Now the volume $V_x = \pi \int_a^b y^2 dx$

$$= \pi \int_0^{\frac{3\pi}{4}} 16 \cos^2\left(\frac{2x}{3}\right) dx$$

$$= 8\pi \int_0^{\frac{3\pi}{4}} \left(1 + \cos\left(\frac{4x}{3}\right)\right) dx$$

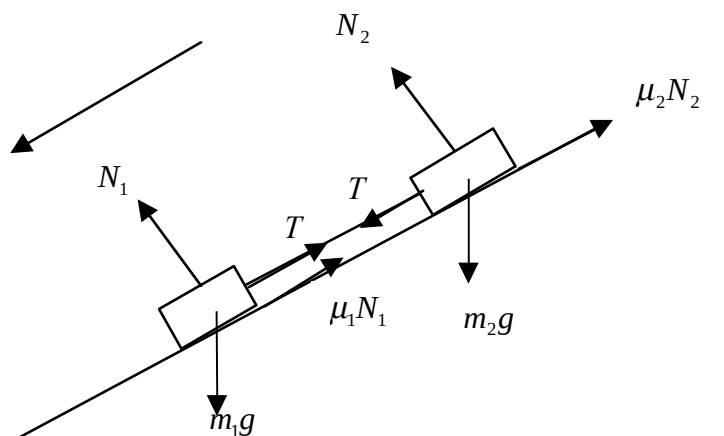
$$= 8\pi \left[x + \frac{3}{4} \sin\left(\frac{4x}{3}\right) \right]_0^{\frac{3\pi}{4}}$$

$$= 8\pi \left[\frac{3\pi}{4} + \frac{3}{4} \sin(\pi) - \left(0 + \frac{3}{4} \sin 0 \right) \right]$$

$$= 6\pi^2$$

Question 7

i.



- ii. isolating the particles and resolving around each individually,
resolving perpendicular to the plane around the mass m_1

$$N_1 - m_1 g \cos \alpha = 0 \quad \text{so that (1)} \quad N_1 = m_1 g \cos \alpha$$

resolving perpendicular to the plane around the mass m_2

$$N_2 - m_2 g \cos \alpha = 0 \quad \text{so that (2)} \quad N_2 = m_2 g \cos \alpha$$

resolving parallel and up the plane around the mass m_1 gives

$$T + \mu_1 N_1 - m_1 g \sin \alpha = 0 \quad \text{using (1) gives (3)} \quad T = m_1 g (\sin \alpha - \mu_1 \cos \alpha)$$

resolving parallel and up the plane around the mass m_2 gives

$$\mu_2 N_2 - T - m_2 g \sin \alpha = 0 \quad \text{using (2) gives (4)} \quad T = m_2 g (\mu_2 \cos \alpha - \sin \alpha)$$

Equating (3) and (4)

$$m_1 (\sin \alpha - \mu_1 \cos \alpha) = m_2 (\mu_2 \cos \alpha - \sin \alpha)$$

$$(m_1 + m_2) \sin \alpha = (m_1 \mu_1 + m_2 \mu_2) \cos \alpha$$

$$\tan \alpha = \frac{m_1 \mu_1 + m_2 \mu_2}{m_1 + m_2} \quad \text{shown.}$$

END OF SUGGESTED SOLUTIONS
2004 Specialist Mathematics Trial Examination 1

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