

Year 2004

**VCE
Mathematical Methods
Trial Examination 2**

Suggested Solutions

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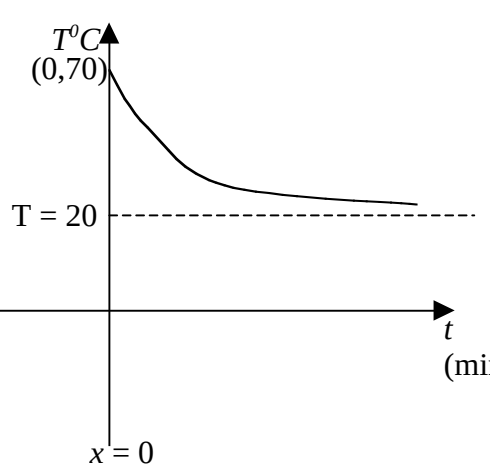


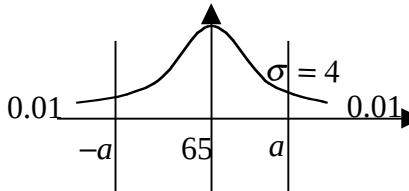
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Question 1

a. $t = 0$ $T = 50e^0 + 20 = 50 + 20 = 70^\circ C$ (1 mark)	b. $45 = 50e^{-3k} + 20$ (1 mark) $25 = 50e^{-3k}$ $0.5 = e^{-3k}$ $-3k = \log_e(0.5)$ $k = 0.231$ (1 mark)
c. $35 = 50e^{-0.231t} + 20$ $15 = 50e^{-0.231t}$ $0.3 = e^{-0.231t}$ $\log_e(0.3) = -0.231t$ $t = 5.2 \text{ min.}$ (1 mark)	d. 1 mark for shape 1 mark for y intercept 1 mark for equation of asymptote. 
e. From (c) when $T = 35, t = 5.2$ $\frac{dT}{dt} = -50 \times 0.231e^{-0.231t}$ (1 mark) When $t = 5.2$ $\frac{dT}{dt} = -11.55e^{-0.231 \times 5.2} = -3.47$ Temperature is decreasing at a rate of $3.47^\circ C / \text{min}$ (1 mark)	

f.(i) There is a turning point at (3,0) $\frac{dT}{dt} = 2a(3t - b) \times 3 = 6a(3t - b) = 0$ for T.P. When $t = 3$, $6a(9 - b) = 0$ $\Rightarrow 9 - b = 0$ since $a > 0$ $\Rightarrow b = 9$ (1 mark)	f.(ii) When $t = 4$, $T = 5$ $5 = a(12 - 9)^2$ $9a = 5$ $a = \frac{5}{9}$ (1 mark)
f.(iii) $\frac{5}{9}(3t - 9)^2 = 50e^{-0.231t} + 20$ In graphics calculator enter $y_1 = \frac{5}{9}(3t - 9)^2 - (50e^{-0.231t} + 20)$ Go to table and look for the answer closest to zero. This is 5.6 to one decimal place. (1 mark)	
Question 2 a. $\Pr(X > 75) = \Pr(Z > 2.5)$ $Z = \frac{75 - 65}{4}$ $Z = 2.5$ (1 mark) $\Pr(Z > 2.5) = 1 - \Pr(Z < 2.5)$ $= 1 - 0.9938 = 0.0062$ (1 mark)	b.  $\Pr(Z < -a) = \Pr(Z > a) = 1 - \Pr(Z < a) = 0.01$ $\Rightarrow \Pr(Z < a) = 0.99$ $a = 2.326$ $-a = -2.326$ (1 mark) $-2.326 = \frac{x - 65}{4}$ $x = 65 - 9.304 = 55.7$ (1 mark)

c. Total probability = 1 Probability acceptable = $1 - [0.01 + 0.0062] = 0.9838$ (1 mark)	d.(i)	Acceptable	Not Acceptable
	Y	4	−6
	Pr(Y=y)	0.9838	0.0162
	• (1 mark for each value)		
d.(ii) $E(X) = \sum x \Pr(X = x)$ $= (4 \times 0.9838) - (6 \times 0.0162) = 3.838$ per trip (1 mark) Expected value for 80 trips = 80×3.838 $= \$307$ (1 mark)	e. $\Pr(55.7 < X < 75) / X > 65) 0.5$ $= \frac{\Pr(55.7 < X < 75 / X > 65) \cap X > 65}{\Pr(X > 65)}$ (1 mark) $= \frac{\Pr(65 < X < 75)}{0.5}$ $= \frac{\Pr(X < 75) - \Pr(X < 65)}{0.5}$ $= \frac{0.9938 - 0.5}{0.5}$ $= \frac{0.4938}{0.5} = 0.9876$ (1 mark)		
f. Binomial: Number of independent trials and only two outcomes. $\Pr(X \geq 1) = 1 - \Pr(X = 0)$ (1 mark) $= 1 - \binom{20}{0} (0.0062)^0 (0.9938)^{20}$ $= 1 - (0.9938)^{20} = 0.117$ (1 mark)	g. $\Pr(X \geq 1) = 1 - \Pr(X = 0) = 0.285$ $\Pr(X = 0) = 0.715$ (1 mark) $(0.9938)^n = 0.715$ $n \log_e(0.9938) = \log_e(0.715)$ $n = \frac{\log_e(0.715)}{\log_e(0.9938)} = 54$ (1 mark)		

Question 3

a.

$$y = \frac{1}{2}e^{-x}(\sin(x) - \cos(x))$$

$$\frac{dy}{dx} = \frac{1}{2}e^{-x}(\cos(x) + \sin(x)) +$$

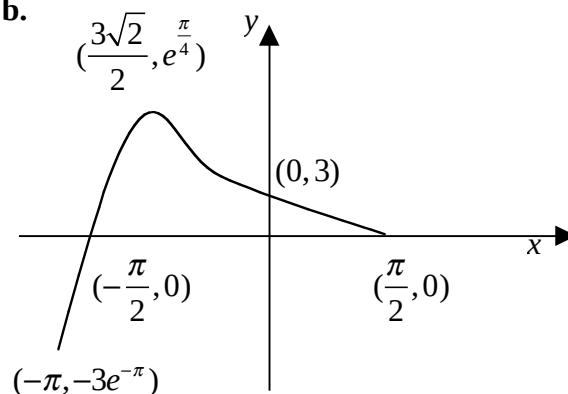
$$(\sin(x) - \cos(x))\left(-\frac{1}{2}e^{-x}\right) \quad (1 \text{ mark})$$

$$= \frac{1}{2}e^{-x}[\cos(x) + \sin(x) - \sin(x) + \cos(x)]$$

$$= \frac{1}{2}e^{-x} \times 2 \cos(x)$$

$$= e^{-x} \cos(x) \quad (1 \text{ mark})$$

b.



X intercept when $y = 0$

$$3e^{-x} \cos(x) = 0$$

$$\text{But } 3e^{-x} \neq 0$$

$$\therefore \cos(x) = 0 \Rightarrow x = -\frac{\pi}{2}, \frac{\pi}{2} \quad (1 \text{ mark})$$

Y intercept when $x = 0$

$$y = 3e^0 \cos(0) = 3 \quad (1 \text{ mark})$$

Turning point when $f'(x) = 0$

$$3e^{-x}(-\sin(x)) + \cos(x)(-3e^{-x}) = 0 \quad (1 \text{ mark})$$

$$-3e^{-x}[\sin(x) + \cos(x)] = 0$$

$$\Rightarrow \sin(x) = -\cos(x)$$

$$\Rightarrow \tan(x) = -1 \quad (1 \text{ mark})$$

$$\Rightarrow x = -\frac{\pi}{4} \quad (1 \text{ mark})$$

$$\text{When } x = -\frac{\pi}{4}, y = 3e^{\frac{\pi}{4}} \cos(-\frac{\pi}{4})$$

$$y = \frac{3}{\sqrt{2}}e^{\frac{\pi}{4}} = \frac{3\sqrt{2}}{2}e^{\frac{\pi}{4}} \quad (1 \text{ mark})$$

$$\text{End point when } x = -\pi, y = 3e^{-\pi} \cos(-\pi) = -3e^{-\pi} \quad (1 \text{ mark})$$

(1 mark for shape of graph)

Question 3 (continued)

c.

$$A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} 3e^{-x} \cos(x) dx$$

$$= 3 \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} e^{-x} \cos(x) dx$$

$$= 3 \left[\frac{1}{2} (\sin(x) - \cos(x)) e^{-x} \right]_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \quad (1 \text{ mark})$$

$$= \frac{3}{2} \left[\left(\sin\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right) \right) \times e^{-\frac{\pi}{2}} \right.$$

$$\left. - \left(\sin\left(-\frac{\pi}{4}\right) - \cos\left(-\frac{\pi}{4}\right) \right) \times e^{\frac{\pi}{4}} \right] \quad (1 \text{ mark})$$

$$= \frac{3}{2} \left[(1 - 0) e^{-\frac{\pi}{2}} - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right) e^{\frac{\pi}{4}} \right]$$

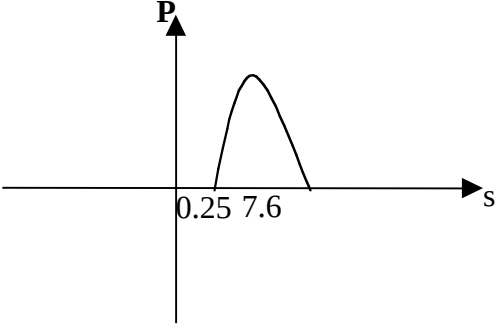
$$= \frac{3}{2} \left[e^{-\frac{\pi}{2}} + \frac{2}{\sqrt{2}} e^{\frac{\pi}{4}} \right]$$

$$= \frac{3}{2} \left[e^{-\frac{\pi}{2}} + \sqrt{2} e^{\frac{\pi}{4}} \right] \quad (1 \text{ mark})$$

Question 4

a. $20,000 = a + 5b$ (1) $10,000 = a + 10b$ (2) $(1)-(2) \rightarrow 10,000 = -5b$ $\Rightarrow b = -2,000$ (1 mark) Substituting in (1) $20,000 = a - 10,000$ $a = 30,000$ (1 mark)	b. $C = 1,500 + 0.2(a + bs)$ (1 mark) $C = 1,500 + 0.2(30,000 - 2000s)$ $C = 1,500 + 6,000 - 400s$ (1 mark) $C = 7,500 - 400s$
c. $P = \text{Selling Price} - \text{Cost Price}$ $P = s(30,000 - 2,000s) - (7,500 - 400s)$ (1 mark) $P = 30,000s - 2,000s^2 - 7,500 + 400s$ $P = 30,400s - 2,000s^2 - 7,500$ (1 mark)	d. $\frac{dP}{ds} = 30,400 - 4,000s = 0$ for TP $\Rightarrow 4,000s = 30,400$ $s = \frac{30,400}{4,000}$ $s = \\$7.60$ (1 mark) Since curve of $P = 30,400s - 2,000s^2 - 7,500$ is concave down, then maximum profit when $s = \\$7.60$ (1 mark)

Question 4 (continued)

e. $P_{\max} = 30,400 \times 7.6 - 2,000(7.6)^2 - 7,500$ $= 231,040 - 115,520 - 7,500$ $= \$108,020 \quad (1 \text{ mark})$	f. To make this profit, $s = \\$7.60$ $\therefore n = 30,000 - 2,000 \times 7.6$ $\therefore n = 14,800 \quad (1 \text{ mark})$
g. $P = 0$ $\therefore 30,400s - 2,000s^2 - 7,500 = 0$ $\Rightarrow -20s^2 + 304s - 75 = 0$ $s = \frac{-304 \pm \sqrt{304^2 - 6000}}{-40}$ $s = 25 \text{ cents} \quad (1 \text{ mark})$	h.  $n = a + bs$ $12,000 = 30,000 - 2,000s$ $2,000s = 18,000$ $s = 9 \quad (1 \text{ mark})$ When $s = 9$ $P = 30,400 \times 9 - 2,000 \times 81 - 7,500$ $P = \$104,100 \quad (1 \text{ mark})$

END OF SUGGESTED SOLUTIONS
2004 Mathematical Methods Trial Examination 2

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