

Year 2004

**VCE
Further Mathematics
Trial Examination 2**

Suggested Solutions

© Kilbaha Pty Ltd 2004



Kilbaha Multimedia Publishing ABN 47 065 111 373
PO Box 2227
Kew Vic 3101
Australia
Tel: 03 9817 5374
Fax: 03 9817 4334
chemas@chemas.com
www.chemas.com

IMPORTANT COPYRIGHT NOTICE

- This material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Pty Ltd.
 - The contents of this work are copyrighted. Unauthorised copying of any part of this work is illegal and detrimental to the interests of the author.
 - For authorised copying within Australia please check that your institution has a licence from Copyright Agency Limited. This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.
 - Teachers and students are reminded that for the purposes of school requirements and external assessments, students must submit work that is clearly their own.
 - Schools which purchase a licence to use this material may distribute this electronic file to the students at the school for their exclusive use. This distribution can be done either on an Intranet Server or on media for the use on stand-alone computers.
 - Schools which purchase a licence to use this material may distribute this printed file to the students at the school for their exclusive use.
-
- **The Word file (if supplied) is for use ONLY within the school**
 - **It may be modified to suit the school syllabus and for teaching purposes.**
 - **All modified versions of the file must carry this copyright notice**
 - **Commercial used of this material is expressly prohibited**

Question 1

a. The dependent variable is the number of workers' compensation claims. (1 mark)

b. Use your graphics calculator. Enter values using state, edit. Then use stat, calc.
4, L2, L1 if spending is in L2. $69.7 - 4.3 \times$ spending on safety procedures
(1 mark) (1 mark)

c.
 $0 = 69.7 - 4.3 \times x$
 $\Rightarrow 4.3x = 69.7$
 $\Rightarrow x = \frac{69.7}{4.3}$
 $\Rightarrow x = 16$ thousand dollars (1 mark)

Question 2

a.
 $\frac{4}{20} \times 100 = 20\%$ (1 mark)

b.
 $\frac{3}{20} \times 100 = 15\%$ (1 mark)

c.

Electrician		Plumber
9 8 7 6 6 5	0	3 7 8
6 4 2 1 0 0	1	2 4 6 7
8 8 7 4 0	2	0 2
0	3	0 2 4
8	4	1 3 8 9
2	5	3
	6	1 4
	7	
	8	9

(2 marks)

d. Middle number is half way between 3.0 and 3.2. Therefore, 3.1 hours (1 mark)

e. Use calculator or add the values in the Plumber's column and divide by 20.
3.315 hours (1 mark)

f. The median would be better because of the outlier of 8.9 (1 mark for median. 1 mark for outlier)

g. The distribution is positively skewed. (1 mark)

h. Median = 1.3 , Lower Quartile = 0.85, ($\frac{1}{2}$ mark) Upper Quartile = 2.75, ($\frac{1}{2}$ mark)
Interquartile range $2.75 - 0.85 = 1.9$ (1 mark)

Total = 15 marks

End of Core

Question 1

a. Arithmetic sequence $t_n = a + (n - 1)d$ $a = 12600$ $n = 4$ $d = 1400$ $\Rightarrow t_4 = 12600 + 3 \times 1400$ $\Rightarrow t_4 = 16,800$ (1 mark)	b. $t_{n+1} = t_n + 1400$ $t_1 = 12,600$ (1 mark)
c. $t_2 = 3 \times t_1 - 20000 = 3 \times 12600 - 20000 = 17800$ $t_3 = 3 \times 17800 - 20000 = 33,400$ (1 mark)	d. Geometric Sequence $a = 8000$ $n = 7$ $r = 1.03$ $t_7 = ar^6 = 8000(1.03)^6 = 9552$ people (1 mark)

Question 2

a. $\% \text{ water} = \frac{5}{8} \times 100 = 62.5\%$ (1 mark)	b. $\frac{3}{8} \times 5600 = 2,100$ litres (1 mark)
c. i. $1000 - 200 = 800$ mL (1 mark)	c. ii. $200 : 800 = 1 : 4$

Module 1 Number patterns and applications. Suggested Solutions

Question 3 a. $\text{Number of days} = \frac{8100}{50} = 162 \text{ days (1 mark)}$	b. Geometric Sequence $S_n = 10,000$ $a = 20$ $r = 2$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $\Rightarrow S_n = \frac{20(2^n - 1)}{2 - 1} = 10,000$ $\Rightarrow 2^n - 1 = 500 \quad (1 \text{ mark})$ $\Rightarrow 2^n = 501$ $\Rightarrow n \log_{10} 2 = \log_{10} 501$ $\Rightarrow n = \frac{\log_{10} 501}{\log_{10} 2} = 9 \text{ days (1 mark)}$
c. $\frac{8100}{x} = 9$ $\Rightarrow x = \frac{8100}{9} = 900 \text{ (1 mark)}$	d. Arithmetic Sequence $S_n = 7425$ $a = 55$ $d = 5$ $7425 = \frac{n}{2} [2 \times 55 + (n - 1)5]$ $\Rightarrow 7425 = \frac{n}{2} [110 + 5n - 5]$ $\Rightarrow 14850 = n[105 + 5n] \quad (1 \text{ mark})$ $\Rightarrow 5n^2 + 105n - 14850 = 0$ $\Rightarrow n^2 + 21n - 2970 = 0$ $\Rightarrow n = \frac{-21 \pm \sqrt{21^2 + 11880}}{2}$ $\Rightarrow n = \frac{-21 \pm 111}{2}$ $n > 0$ $\Rightarrow n = \frac{-21 + 111}{2} = \frac{90}{2} = 45 \text{ days (1 mark)}$

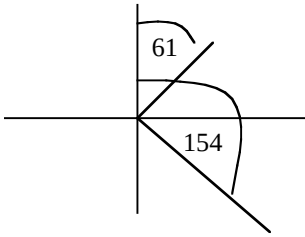
Total = 15 marks

End of Module 1

Module 2 Geometry and trigonometry. Suggested solutions.

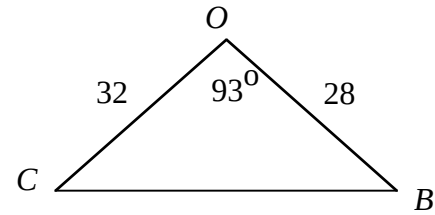
Question 1

a.



$$\angle BOC = 154 - 61 = 93^\circ \quad (1 \text{ mark})$$

b.



Area

$$= \frac{1}{2}bc \sin \theta$$

(1 mark)

$$= \frac{1}{2} \times 32 \times 28 \sin 93^\circ$$

$$= 447.386 \text{ m}^2$$

c.

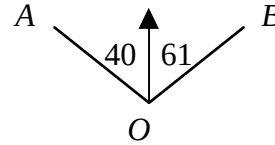
$$BC^2 = 28^2 + 32^2 - 2 \times 28 \times 32 \cos 93^\circ$$

$$BC^2 = 1808 + 93.786$$

$$BC = \sqrt{1901.786} = 43.61 \text{ m}$$

(1 mark)

d.



$$\angle AOB = 40 + 61 = 101^\circ \quad (1 \text{ mark})$$

e.

$$\frac{30}{\sin 101^\circ} = \frac{28}{\sin \angle BAO}$$

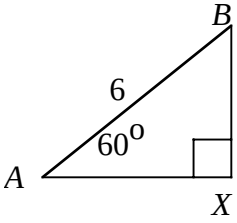
$$\Rightarrow \sin \angle BAO = \frac{28 \sin 101^\circ}{30} \quad (1 \text{ mark})$$

$$\Rightarrow \angle BAO = \sin^{-1} \left(\frac{28 \sin 101^\circ}{30} \right)$$

$$\Rightarrow \angle BAO = 66^\circ \quad (1 \text{ mark})$$

Module 2 Geometry and trigonometry. Suggested solutions.

Question 2

a. ABC is an equilateral triangle (all angles = 60°) Therefore, $AB = AC = 6$ m (1 mark)	b. $\tan 60^\circ = \frac{3}{x}$ (1 mark) $\Rightarrow x = \frac{3}{\tan 60^\circ} = 1.73$ m (1 mark)
c. Width of room = $6 - (2 \times 1.73) = 2.54$ m (1 mark)	d.  $\sin 60^\circ = \frac{BX}{6}$ (1 mark) $\Rightarrow BX = 6 \sin 60^\circ = 5.20$ (1 mark)
e. Area of ceiling = $2.54 \times 10 = 25.4$ m² (1 mark)	f. Perpendicular height of triangular end = $5.20 - 3 = 2.20$ (1 mark) Volume of space above ceiling = area of base $\times$ height = area of triangular end $\times 10$ = $\frac{1}{2} \times 2.54 \times 2.2 \times 10$ = 27.94 m³ (1 mark)

Total = 15 marks

End of Module 2

Module 3 Graphs and relations. Suggested solutions.

Question 1

a. $90 - 30 = 60$ minutes or 1 hour

(1 mark)

b.

80 km in 2 hours

$80 \div 2 = 40$ km/hr (1 mark)

Question 2

a.

$$C = 30x + 15y$$

(1 mark)

b.

(1) $x \leq 6$

(1 mark)

(2) $x + y \geq 7$

(1 mark)

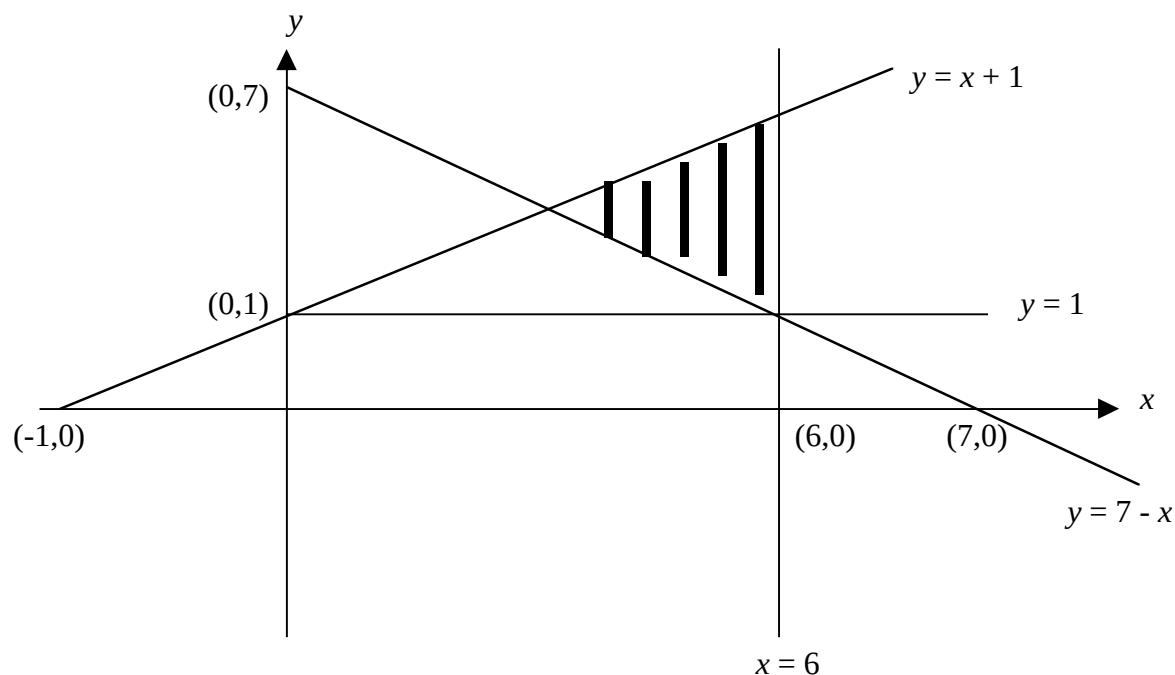
(3) $x \geq y - 1$ or $y \leq x + 1$

(1 mark)

(4) $y \geq 1$

(1 mark)

c.



The shaded region is the required region.

- $\frac{1}{2}$ mark for the line $y = 1$
- $\frac{1}{2}$ mark for the line $x = 6$
- 1 mark for the line $y = x + 1$
- 1 mark for the line $y = 7 - x$
- 1 mark for the correct area identified

Module 3 Graphs and relations. Suggested solutions.

Question 2 (continued) d. Intersection of $y = x + 1$ and $y = 7 - x$ $y - x = 1 \quad (1)$ $y + x = 7 \quad (2)$ $(1) + (2)$ $2y = 8$ $y = 4$ From (1), when $y = 4$, $x = 3$ $(3, 4)$ (1 mark) $\left\{ \begin{array}{l} \text{Intersection of } x = 6, y = 1 \quad (6, 1) \\ \text{Intersection of } x = 6, y = x + 1 \\ y = 6 + 1 = 7 \quad (6, 7) \end{array} \right\} \text{ (1 mark)}$	Cost at $(3, 4)$ $= 30 \times 3 + 15 \times 4 = 90 + 60 = \150 Cost at $(6, 1)$ $= 30 \times 6 + 15 \times 1 = 180 + 15 = \195 Cost at $(6, 7)$ $= 30 \times 6 + 15 \times 7 = 180 + 105 = \285 Minimum cost = \$150 (1 mark)
e. 3 art books and 4 science books (1 mark)	

Total = 15 marks

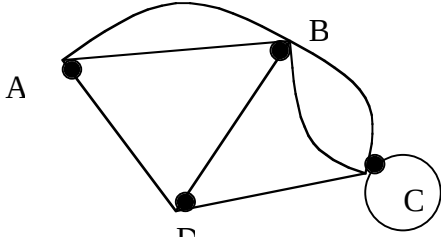
End of Module 3

Module 4 Business-related mathematics. Suggested solutions.

Question 1 a. $\frac{15}{100} \times 15,700 = \$2,355$ (1 mark)	b. $P = 15700 - 2355 = 13,345$ (1 mark) $I = \frac{PRT}{100}$ $I = \frac{13345 \times 12.5 \times 6}{100}$ $I = \$10,008.75$ (1 mark)
c. Amount to be repaid $= 13345 + 10008.75 = 23,353.75$ (1 mark) Number of repayments $= 6 \times 12 = 72$ Amount of each repayment $= 23353.75 \div 72 = \$324.36$ (1 mark)	d. Effective interest rate $= \frac{2n}{n+1} \times 12.5$ $= \frac{2 \times 72}{73} \times 12.5$ $= 24.7\%$ (1 mark)
Question 2 a. $\frac{14}{12} = 1.17\%$ (1 mark)	b. i. $R = 1 + \frac{1.16667}{100} = 1.0117$ (1 mark) b. ii. $n = 6 \times 12 = 72$ (1 mark)
c. $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$ $0 = 15700(1.0117)^{72} - \frac{Q(1.0117^{72} - 1)}{0.0117}$ (1 mark) $\Rightarrow \frac{Q(1.0117^{72} - 1)}{0.0117} = 15700(1.0117)^{72} = 36276.52279$ $\Rightarrow Q(1.0117^{72} - 1) = 424.4353$ $\Rightarrow Q = \$323.85$ (1 mark)	d. Total amount paid $= 323.85 \times 72 = 23,317.20$ Interest $= 23,317.20 - 15,700$ $= \$7,617.20$ (1 mark)
e. $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$ $A = 15700(1.0117)^{24} - \frac{323.85(1.0117^{24} - 1)}{0.0117}$ $\Rightarrow A = 20753.90 - 8913.68$ $\Rightarrow A = \$11,842.22$ (1 mark)	f. $0 = 11842.22(1.0117)^n - \frac{323.85(1.0117^n - 1)}{0.0117}$ $11842.22(1.0117)^n = 20110.77(1.0117^n)$ $1.58885 = 1.0117^n$ $\log_{10} 1.58885 = n \log_{10} 1.0117$ $n = 39.8$ $n = 40$ months (1 mark) 40 months = 3 years and 4 months Total time = 2 years + 3 years 4 months $= 5$ years and 4 months (1 mark)

End of Module 4

Total = 15 marks

Question 1 a.  (1 mark)	b. The network has no loops (1 mark)
c. It represents a directed graph. (1 mark)	
Question 2 a. - Immediate predecessor for F is B (1 mark) - Immediate predecessors for I are G and F (1 mark) - Immediate predecessors for J are H and I (1 mark) - Earliest starting time for F is the earliest time B finishes = 5 (1 mark) - Earliest starting time for I depends on when F and G finish. Earliest time for F to finish is $5 + 6 = 11$. Earliest time for G to finish is $12 + 2 = 14$. I cannot start until both F and G finish so earliest starting time for I = 14. (1 mark) - Earliest starting time for J depends on when H and I finish. Earliest time for H to finish = $10 + 7 = 17$. Earliest time for I to finish = $14 + 4 = 18$. Therefore, earliest starting time for J is 18. (1 mark) - Latest starting time for J = total time for the whole job which is 21 less 3 = 18. (1 mark) - Latest starting time for H = $18 - 7 = 11$ (1 mark) - To get the latest starting time for F, backtrack from the end along J, I and F = $21 - 3 - 4 - 6 = 8$ (1 mark)	
b. From the Table, for activity E, float time = latest start time – earliest start time = $8 - 5 = 3$ hours. (1 mark)	c. The critical path is the longest path which is C G I J (1 mark)
d. The length of the critical path = $12 + 2 + 4 + 3 = 21$ hours (1 mark)	

End of Module 5

Total = 15 marks

End of suggested solutions
2004 Further Mathematics Trial Examination 2

KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9817 5374
FAX: (03) 9817 4334
chemas@chemas.com
www.chemas.com