

Year 2004

**VCE
Further Mathematics
Trial Examination 1**

Suggested Solutions

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Question 1 D Use your graphics calculator. Press stat → edit and type in the values in the given table. Points scored in L1 and number of students in L2. Press stat → calc → 1 : 1 – Var Stats → Enter Type L1, L2 ENTER Read off the mean as $\bar{x} = 6$	Question 2 E Use the same screen from the calculator. The sample standard deviation $S_x = 2.25$ $\bar{x} \pm S_x$ $= 6 \pm 2.25$ $= 3.75 \rightarrow 8.25$ the number of scores that lie within 3.75 and 8.25 is the number of students who got a score of 4, 5, 6, 7 or 8. That is, $3 + 7 + 7 + 6 + 5 = 28$
Question 3 D From the same screen on the calculator, the upper quartile is $Q_3 = 8$	Question 4 D This is bivariate data where time is the dependent variable. The relationship between the two variables is most easily seen in a scatter plot.
Question 5 C 95% of scores lie within ± 2 standard deviations of the mean. That is, between $45 \pm 2 \times 13$. That is, from 19 to 71. 5% of results lie outside this range. 2.5% less than 19 and 2.5% greater than 71. Therefore, 2.5% lie above 71.	Question 6 D $r^2 = 0.49$ $\Rightarrow r = \pm\sqrt{0.49} = \pm 0.7$ But from the graph we can see that the correlation is negative. Therefore, $r = -0.7$
Question 7 E This box plot is negatively skewed because the median is up towards the right hand end of the box plot. 25% of the results lie within each whisker. The percentage of results in the box = 50% and the percentage of results in both whiskers together = 50%. 12 is the upper quartile so $25\% + 50\% = 75\%$ of results are less than this value.	Question 8 D Use your graphics calculator and enter the data using stat – edit. Then use stat – calc – linear regression. L2, L1 if you entered number of units sold (y) in list 2. Press ENTER. This gives $y = 15.66 + 0.56x$

Module 1 Number patterns and applications. Suggested solutions.

Question 9 A Using the same screen, read $r = 0.51$	Question 10 E The given graph is $y = \frac{1}{x}$ If we plot y against $\frac{1}{x}$, we will get a straight line.
Question 11 A Taking the values for 2000, 2001 and 2002 gives 23, 21 and 38. The median of 21, 23, 38 is 23 Therefore, sales = $23 \times 100,000 = \\$2,300,000$	Question 12 A The sum of the seasonal indices = 4. Therefore, seasonal index for winter = $4 - (1.3 + 1.5 + 0.7) = 0.5$
	Question 13 D Seasonally adjusted value = $\frac{1.2}{0.7}$ = \$1.7 million

Module 1 Number patterns and applications. Suggested solutions.

Question 1 B

Arithmetic sequence

$$a = -9$$

$$d = 4$$

$$n = 20$$

$$t_n = a + (n-1)d$$

$$t_n = -9 + (20-1) \times 4$$

$$t_n = -9 + 19 \times 4$$

$$t_n = 67$$

Question 2 B

$$S_{\infty} \text{ of geometric sequence} = \frac{a}{1-r}$$

$$a = 12$$

$$r = \frac{1}{3}$$

$$S_{\infty} = \frac{12}{1 - \frac{1}{3}} = \frac{12}{\frac{2}{3}} = 18$$

Question 3 D

Geometric sequence

$$a = 100$$

$$r = 0.8$$

$$n = 12$$

$$t_n = ar^{n-1}$$

$$t_{12} = 100 \times 0.8^{11}$$

$$t_{12} = 8.6$$

Question 4 E

A straight line graph represents an arithmetic sequence. Because the graph is decreasing, the common difference is negative.

Question 5 E

$$ar^2 = 27 \quad (1)$$

$$ar^5 = 8 \quad (2)$$

$$(2) \div (1) \rightarrow r^3 = \frac{8}{27}$$

$$\Rightarrow r = \sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$$

Substituting in (1)

$$\Rightarrow a \times \frac{4}{9} = 27$$

$$\Rightarrow a = 27 \div \frac{4}{9} = 60 \frac{3}{4}$$

Question 6 D

Bananas Oranges Apples

$$\begin{array}{ccc} 5 & : & 4 \\ & & 3 \end{array} \quad : \quad 2$$

Make number of oranges = 12 for both ratios.

$$\begin{array}{ccc} 5 \times 3 & : & 4 \times 3 \\ & & 3 \times 4 \end{array} \quad : \quad 2 \times 4$$

$$15 \quad : \quad 12 \quad : \quad 8$$

so the ratio of Bananas to Apples is 15 : 8

Module 1 Number patterns and applications. Suggested solutions.

Question 7 C

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_5 = \frac{5}{2}[2a + 4d] = 75$$

$$\Rightarrow 5a + 10d = 75 \quad (1)$$

$$S_{10} = \frac{10}{2}[2a + 9d] = 175$$

$$\Rightarrow 10a + 45d = 175 \quad (2)$$

$$(1) \times 2 \rightarrow 10a + 20d = 150 \quad (3)$$

$$(2) - (3) \rightarrow 25d = 25$$

$$\Rightarrow d = 1$$

Substituting in (1)

$$5a = 65$$

$$\Rightarrow a = 13$$

$$S_{15} = \frac{15}{2}[2 \times 13 + 14 \times 1]$$

$$\Rightarrow S_{15} = \frac{15}{2} \times 40 = 300$$

Question 8 C

This is a geometric sequence with

$$a = 2, r = -3$$

$$t_n = ar^{n-1} = 13122$$

$$\Rightarrow 2 \times (-3)^{n-1} = 13122$$

$$\Rightarrow (-3)^{n-1} = 6561$$

Use $y =$ on the graphics calculator

and enter $(-3) \wedge X$

Press second table and read the value of X that gives $y = 6561$

This is $X = 8$

$$\Rightarrow n - 1 = 8$$

$$\Rightarrow n = 9$$

Question 9 D

Geometric sequence

$$a = 6$$

$$r = 1 - 0.05 = 0.95$$

$$t_n = ar^{n-1}$$

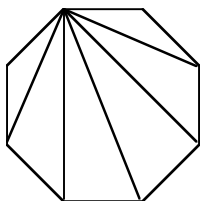
$$t_6 = 6 \times (0.95)^5$$

$$t_6 = 4.6$$

Module 2 Geometry and trigonometry. Suggested solutions.

Question 1 D

The sum of the angles in each triangle = 180°



Therefore, the sum of the angles in 6 triangles

$$= 6 \times 180 = 1080^\circ$$

In the regular octagon, there are 8 equal angles. Therefore, the size of each angle

$$= \frac{1080}{8} = 135^\circ$$

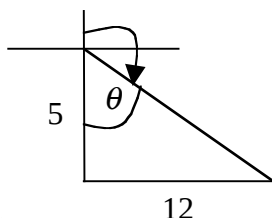
Question 2 E

$$\text{Volume of cylinder A} = \pi r^2 h$$

$$\text{Volume of cylinder B} = \pi(3r)^2 \times 2h = 18\pi r^2 h$$

$$\text{Ratio } V_A : V_B = \pi r^2 h : 18\pi r^2 h$$

$$\Rightarrow \text{Ratio is } 1 : 18$$

Question 3 B

$$\tan \theta = \frac{12}{5}$$

$$\theta = \tan^{-1}\left(\frac{12}{5}\right) = 67.38^\circ$$

$$\text{True bearing} = 90^\circ + (90 - 67.38)^\circ$$

$$= 112.62^\circ$$

which is closest to 113°

Question 4 C

Remaining angle of triangle

$$= 180 - (80 + 38)$$

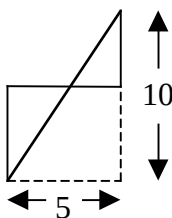
$$= 62^\circ$$

Using Sine Rule

$$\frac{x}{\sin 62^\circ} = \frac{12}{\sin 38^\circ}$$

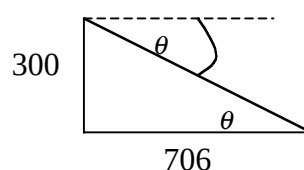
$$\Rightarrow x = \frac{12 \sin 62^\circ}{\sin 38^\circ}$$

$$\Rightarrow x = 17.2$$

Question 5 B

$$x^2 = 5^2 + 10^2 = 125$$

$$x = \sqrt{125} = 11.18$$

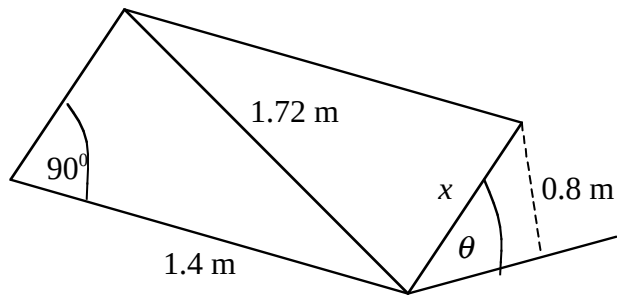
Question 6 A

θ is the angle of depression.

$$\tan \theta = \frac{300}{706}$$

$$\theta = \tan^{-1}\left(\frac{300}{706}\right) = 23^\circ$$

Module 2 Geometry and trigonometry. Suggested solutions.

Question 7 C

Using Pythagoras' Theorem

$$x = \sqrt{1.72^2 - 1.4^2} = 0.999$$

$$\sin \theta = \frac{0.8}{0.999}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{0.8}{0.999}\right)$$

$$\Rightarrow \theta = 53^\circ$$

Question 8 B

Using the Cosine Rule

$$2^2 = 5^2 + 4^2 - 2 \times 5 \times 4 \cos \theta$$

$$4 = 25 + 16 - 40 \cos \theta$$

$$\Rightarrow 40 \cos \theta = 41 - 4$$

$$\Rightarrow 40 \cos \theta = 37$$

$$\Rightarrow \cos \theta = \frac{37}{40}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{37}{40}\right)$$

$$\Rightarrow \theta = 22.3^\circ$$

Question 9 C

Triangles ABE and CDE are similar (AAA)
Therefore, their sides are in the same ratio.

Let $DE = x$ cm

Therefore, $AE = 25 - x$

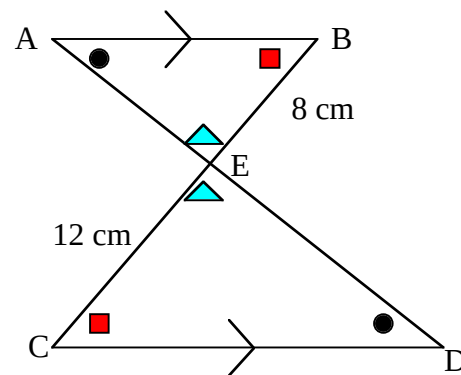
$$\Rightarrow \frac{8}{12} = \frac{25 - x}{x}$$

$$\Rightarrow 8x = 12(25 - x)$$

$$\Rightarrow 8x = 300 - 12x$$

$$\Rightarrow 20x = 300$$

$$\Rightarrow x = \frac{300}{20} = 15$$



Module 3 Graphs and relations. Suggested solutions.

Question 1 E Highest value of the graph is 600. This is the greatest distance from his house. Therefore, the shop is 600 m from his house.	Question 2 D It takes 18 minutes to reach the shop. He returns from the shop in half this time. That is, he takes 9 minutes to return home. He leaves the shop after 20 minutes. So time to reach home = $20 + 9 = 29$ minutes.
Question 3 A He travelled 100 m in 6 minutes. Therefore, speed = $\frac{100}{6} = 16\frac{2}{3}$ m/min	Question 4 B The break even point is where the two graphs intersect.
Question 5 C $60 < 8x + 4 \leq 120$ $\Rightarrow 60 - 4 < 8x + 4 - 4 \leq 120 - 4$ $\Rightarrow 56 < 8x \leq 116$ $\Rightarrow \frac{56}{8} < \frac{8x}{8} \leq \frac{116}{8}$ $\Rightarrow 7 < x \leq 14.5$	Question 6 A A straight line graph has the form $y = mx + c$ c is the y intercept = 1 so $y = mx + 1$ m is the gradient of the line joining the points $(0,1)$ and $(-\frac{1}{2}, 0)$ $m = \frac{1 - 0}{0 - -\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$ $\Rightarrow y = 2x + 1$

Question 7 A $x \geq 0$ $y \geq 0 \therefore \text{not E}$ $x + 2y \leq 8$ $2y \leq -x + 8$ $y \leq -\frac{1}{2}x + 4$ This line joins the points (0,4) and (8,0) Less than this line. Therefore, not C or D. $2x - y \leq 6$ $-y \leq -2x + 6$ $y \geq 2x - 6$ This line joins the points (0,-6) and (3,0) Greater than this line. Therefore, A	Question 8 D $x + 2y = 8 \quad (1)$ $2x - y = 6 \quad (2)$ $(1) \times 2$ $\Rightarrow 2x + 4y = 16 \quad (1a)$ $2x - y = 6 \quad (2)$ $(1a) - (2)$ $\Rightarrow 5y = 10$ $\Rightarrow y = 2$
Question 9 A x is at least double y $x \geq 2y$ z is no greater than one third x $z \leq \frac{1}{3}x$ If $x \geq 2y$ then $2y \leq x$ so $y \leq \frac{1}{2}x$ If $z \leq \frac{1}{3}x$ then $\frac{1}{3}x \geq z$ so $x \geq 3z$ Therefore, A is not true.	

Module 4 Business-related mathematics. Suggested solutions.

Question 1 E $I = \frac{PRT}{100}$ $I = \frac{5000 \times 10 \times 4}{100} = \2000 Value of investment = 5000 + 2000 = \$7000	Question 2 D Depreciation = 2400 $\text{Fraction depreciation} = \frac{2400}{10500}$ $\% \text{ depreciation} = \frac{2400}{10500} \times 100 = 22.86\%$
Question 3 C $A = 23460 \left(1 + \frac{0.075}{2} \right)^4$ $\Rightarrow A = \$27,181.94$	Question 4 C Amount depreciation $= 20,000 \times \frac{20}{100} \times 5$ $= \$20,000$ Value of car $= 30,000 - 20,000$ $= \$10,000$
Question 5 C Effective interest rate $= \frac{2n}{n+1} \times \text{flat rate}$ $= \frac{2 \times 36}{36+1} \times 12$ $= 23.35\%$	Question 6 D $18,463 = P \left(1 + \frac{0.1}{4} \right)^{20}$ $\Rightarrow P = \frac{18,463}{\left(1 + \frac{0.1}{4} \right)^{20}}$ $\Rightarrow P = \$11,267.43$

Module 4 Business-related mathematics. Suggested solutions.**Question 7 E**

Minimum amount for February is the amount in the account from 1 Feb to 19 Feb 19 inclusive.
This equals the amount in the account after 24 Jan = \$15,631.46

Question 8 A

Minimum amount in the account in December = \$14,247.19

$$\text{Interest} = \frac{0.05}{12} \times 14247.19 = \$59.36$$

Question 9 B

$$\text{Cash deposit} = \frac{10}{100} \times 3500 = \$350$$

$$\text{Interest paid on balance of } 3500 - 350 = 3150$$

$$\text{Interest} = 3150 \times \frac{8.5}{100} \times 3 = 803.25$$

$$\text{Amount to be paid back} = 3150 + 803.25 = 3953.25$$

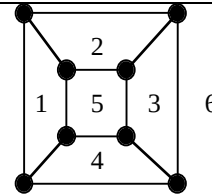
$$\text{Number of repayments} = 3 \times 12 = 36$$

$$\text{Value of each repayment} = 3953.25 \div 36 = \$109.82$$

Module 5 Networks and decision mathematics. Suggested solutions.

Question 1 D

A Hamiltonian Path passes through each vertex once only, starting and finishing at different vertices. This cannot be done with diagram D.

Question 2 C

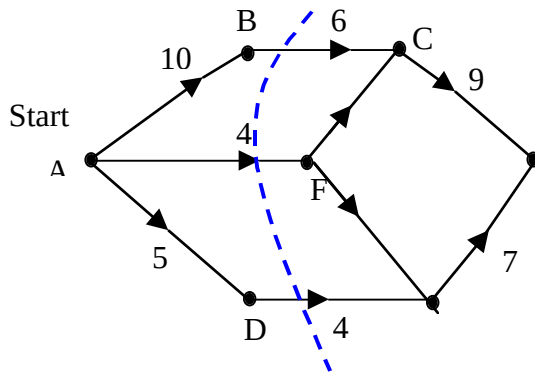
Number of faces = 6

Question 3 B

$$2 + 2 + 6 + 3 = 13$$

Question 4 E

For an Eulerian circuit to exist, all vertices must be of even degree. Since D and A are of odd degree, adding an edge joining A to D would make both these vertices even.

Question 5 B

Maximum flow = minimum cut

$$\text{Minimum cut} = 6 + 4 + 4 = 14$$

Therefore, maximum flow = 14

Question 6 D

DE can hold 4 and the maximum amount of 4 can flow along here when maximum flow is occurring.

Module 5 Networks and decision mathematics. Suggested solutions.

Question 7 C

From A to A = 0

From A to B = 2

From A to C = 0

So first row is 0 2 0 therefore, not B

From B to A = 1

From B to B = 0

From B to C = 1

So second row is 1 0 1 therefore, C

Question 8 E

Circle the smallest value in each row

5 4 3 69 10 6 910 13 8 76 9 10 10

Subtract this circled value from each value in its row.

2 1 0 3

3 4 0 3

3 6 1 00 3 4 4

Circle the smallest value in each column.

Subtract this value from each value in its column.

2 0 0 3

3 3 0 3

3 5 1 00 2 4 4

Take a row and column where only one zero occurs. (See the lines drawn above)

⇒ Jake makes the Racing Bike

⇒ Ida makes the Super Athlete Bike

⇒ Greg makes the Mountain bike

Therefore, Helen makes the Non-geared Bike.

Therefore, E

Question 9 C

Earliest time G can start is when C and F are both finished. C finishes in 8 hours. F finishes in $4 + 3 + 2 = 9$ hours. Therefore G must wait to start until F finishes. So, the earliest G can start is 9 hours from the beginning.

End of suggested solutions**2004 Further Mathematics Trial Examination 1**

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