

**VCE  
Mathematical Methods  
Trial Examination 1**

# **Suggested Solutions**

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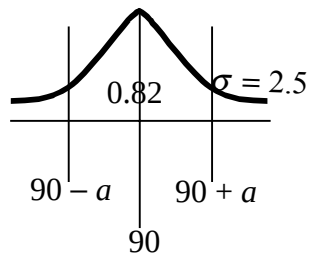
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|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>Question 1 C</b></p> $  \begin{array}{r}  \phantom{x-5} \overline{) 2x+2} \\  \underline{2x-10} \phantom{0} \\  +12 \phantom{0} \\  \hline  = 2 + \frac{12}{x-5}  \end{array}  $                                                                                                                                                                                                                                                                                                                                                                                                        | <p><b>Question 2 B</b></p> $  \begin{aligned}  P(-1) &= 1 - 1 + a - b + 30 = 0 \\  \Rightarrow a - b &= -30 \quad (1) \\  P(2) &= 16 + 8 + 4a + 2b + 30 = 0 \\  \Rightarrow 4a + 2b &= -54 \quad (2) \\  (1) \times 4 \rightarrow 4a - 4b &= -120 \quad (1a) \\  (2) - (1a) \rightarrow 6b &= 66 \\  b &= 11 \\  \text{Substituting } b = 11 \text{ in (1)} \\  a - 11 &= -30 \\  a &= -19  \end{aligned}  $     |
| <p><b>Question 3 B</b></p> <p>To have an inverse that is a function <math>f(x)</math> must be a one-to-one function. In the domains given, the only function that is one-to-one is <math>f(x) = x^3 - x</math></p>                                                                                                                                                                                                                                                                                                                                                                            | <p><b>Question 4 D</b></p> <p>The basic graph of <math>\log_e(x)</math> which passes through the point <math>(1,0)</math> and has an asymptote <math>x = 0</math> is translated <math>a</math> units to the right and <math>b</math> units up, to pass through the point <math>(a+1, b)</math>. The asymptote is translated <math>a</math> units to the right to become the vertical line <math>x = a</math></p> |
| <p><b>Question 5 E</b></p> $  \frac{\sqrt{3} \sin\left(\frac{3\pi}{2}x\right)}{\cos\left(\frac{3\pi}{2}x\right)} = -1 \quad 0 \leq x \leq 2  $ $  \sqrt{3} \tan\left(\frac{3\pi}{2}x\right) = -1 \quad 0 \leq \frac{3\pi}{2}x \leq 3\pi  $ $  \tan\left(\frac{3\pi}{2}x\right) = -\frac{1}{\sqrt{3}}  $ $  \frac{3\pi}{2}x = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}  $ $  3\pi x = \frac{5\pi}{3}, \frac{11\pi}{3}, \frac{17\pi}{3}  $ $  x = \frac{5}{9}, \frac{11}{9}, \frac{17}{9}  $ $  \text{Sum} = \frac{5}{9} + \frac{11}{9} + \frac{17}{9} = \frac{33}{9} = \frac{11}{3}  $ | <p><b>Question 6 A</b></p> <p>Domain of <math>f(x)</math> is <math>R^+ \cup \{0\}</math></p> <p>Range of <math>f(x)</math> is <math>[1, \infty)</math></p> <p>Domain of <math>f^{-1}(x)</math> = range of <math>f(x)</math></p> <p><math>\therefore</math> Domain of <math>f^{-1}(x)</math> is <math>x \geq 1</math></p>                                                                                         |

|                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                     |
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| <p><b>Question 7 E</b><br/>This graph is translated 2 units to the right and 3 units up,<br/>so maximum at <math>(-1 + 2, 16 + 3)</math> i.e. (1,19)<br/>minimum at <math>(3 + 2, -16 + 3)</math> i.e. (5, -13)</p>                                                                                                                                                                                    | <p><b>Question 8 C</b><br/>X intercept when <math>y = 0</math><br/><math>3\log_3(x + 2) - 1 = 0</math><br/><math>3\log_3(x + 2) = 1</math><br/><math>\log_3(x + 2) = \frac{1}{3}</math><br/><math>x + 2 = 3^{\frac{1}{3}} = \sqrt[3]{3}</math><br/><math>x = \sqrt[3]{3} - 2</math></p>                                                             |
| <p><b>Question 9 C</b><br/><math>\frac{dy}{dx} = -6\cos(2x)</math><br/>This cos graph has a maximum of 6 and a minimum of -6</p>                                                                                                                                                                                                                                                                       | <p><b>Question 10 A</b><br/>Product rule<br/><math>f'(x) = x^4(-3e^{-3x}) + e^{-3x} \times 4x^3</math><br/><math>= x^3e^{-3x}(4 - 3x)</math></p>                                                                                                                                                                                                    |
| <p><b>Question 11 E</b><br/><math>\frac{dy}{dx} = \frac{(x^3 + 2)4x - 2x^2(3x^2)}{(x^3 + 2)^2} = 0</math> (Quotient rule)<br/><math>\frac{4x^4 + 8x - 6x^4}{(x^3 + 2)^2} = 0</math><br/><math>\frac{8x - 2x^4}{(x^3 + 2)^2} = 0</math><br/><math>8x - 2x^4 = 0</math><br/><math>2x(4 - x^3) = 0</math><br/><math>x(4 - x^3) = 0</math></p>                                                             | <p><b>Question 12 C</b><br/>Chain rule<br/><math>\frac{dV}{dx} = \frac{dV}{dy} \frac{dy}{dx}</math><br/><math>\frac{dV}{dy} = 5(2y - 3)^4 \times 2 = 10(2y - 3)^4</math><br/><math>\frac{dy}{dx} = 2(2x - 3) \times 2 = 4(2x - 3)</math><br/><math>\frac{dV}{dx} = 10(2y - 3)^4 \times 4(2x - 3)</math><br/><math>= 40(2y - 3)^4(2x - 3)</math></p> |
| <p><b>Question 13 E</b><br/><math>f(x) = \int (3 - x)(5 - 2x)dx</math><br/><math>= \int (15 - 6x - 5x + 2x^2)dx</math><br/><math>= \int (15 - 11x + 2x^2)dx</math><br/><math>= 15x - \frac{11x^2}{2} + \frac{2x^3}{3} + c</math><br/><math>66 = 90 - 198 + 144 + c</math><br/><math>66 - 36 = c</math><br/><math>c = 30</math><br/><math>f(x) = \frac{2x^3}{3} - \frac{11x^2}{2} + 15x + 30</math></p> | <p><b>Question 14 A</b><br/>Area <math>= \int_0^4 (4ax - ax^2)dx</math><br/><math>= [2ax^2 - \frac{ax^3}{3}]_0^4</math><br/><math>= (32a - \frac{64a}{3}) - 0</math><br/><math>= \frac{96a}{3} - \frac{64a}{3}</math><br/><math>= \frac{32a}{3} = 10\frac{2}{3}a</math></p>                                                                         |

|                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                              |
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| <p><b>Question 15 E</b></p> $A = \int_{-1}^2 [f(x) - g(x)] dx$                                                                                                                                                                                                                                           | <p><b>Question 16 A</b></p> $A = \int \frac{2x^2}{x} dx - \int \frac{1}{x} dx + \int \frac{1}{x-2} dx$ $= x^2 - \log_e(x) + \log_e(x-2)$ $= x^2 + \log_e\left(\frac{x-2}{x}\right)$                                                                                                                                                          |
| <p><b>Question 17 B</b></p> $A = \frac{1}{2}[(f(0) + f(1)) + (f(1) + f(2)) + (f(2) + f(3)) + (f(3) + f(4))]$ $= \frac{1}{2} f(0) + f(1) + f(2) + f(3) + \frac{1}{2} f(4)$ $= \frac{1}{2} + 3^1 + 3^2 + 3^3 + \frac{1}{2} \times 3^4$ $= \frac{1}{2} + 3 + 9 + 27 + \frac{81}{2}$ $= 80$                  | <p><b>Question 18 E</b></p> $2y = x - 6$ $\Rightarrow y = \frac{1}{2}x - 3$ $m = \frac{1}{2}$ <p>gradient of perpendicular line <math>\times m = -1</math></p> <p>gradient of perpendicular line <math>= -2</math></p> $y = -ax - b$ $\therefore a = 2$ $y = -2x - b$ <p>When <math>x = 4, y = -1</math></p> $-1 = -8 - b$ $-b = 7$ $b = -7$ |
| <p><b>Question 19 E</b></p> $f(x+h) \approx f(x) + hf'(x)$ $f(x+0.2) \approx f(x) + 0.2 \times f'(x)$ $f(x) = SA = 6x^2$ $f'(x) = 12x$ $\Delta SA = f(x+0.2) - f(x) \approx 0.2 \times f'(x)$ $= 0.2 \times 12x$ <p>When <math>x = 10</math></p> $\Delta SA = 0.2 \times 12 \times 10 = 24 \text{ cm}^2$ | <p><b>Question 20 A</b></p> $x^2 - 2x + 3 = Ax^2 + 2A + Bx^2 + Bx + Cx + C$ $x^2 - 2x + 3 = (A+B)x^2 + (B+C)x + (2A+C)$ <p>Equating coefficients</p> $A+B=1 \quad (1)$ $B+C=-2 \quad (2)$ $2A+C=3 \quad (3)$ $(2)-(1) \rightarrow C-A=-3$ $\therefore -2A+2C=-6 \quad (4)$ $(4)+(3) \rightarrow 3C=-3$ $\Rightarrow C=-1$                    |

|                                                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
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| <p><b>Question 21 C</b><br/>The domain is <math>R</math> and the range is <math>[2, \infty)</math>.<br/>This is the graph of <math>y = x^3</math> translated 3 units to the right and 2 units up, so local minimum is <math>(3, 2)</math>. The dilation is by a factor of 4 parallel to the <math>y</math> axis.</p>                                             | <p><b>Question 22 A</b><br/>Vertical asymptote when <math>2x - 4 = 0</math><br/><math>\Rightarrow x = 2</math><br/>Graph of <math>y = \frac{5}{2x - 4}</math> is moved 3 units up by <math>+3</math> so the horizontal asymptote is <math>y = 3</math></p>                                                                                                                                                                                                                                               |
| <p><b>Question 23 E</b><br/><math display="block">\text{Pr} = \frac{17}{17 + 23} = \frac{17}{40}</math></p>                                                                                                                                                                                                                                                      | <p><b>Question 24 A</b><br/><br/>For A, when <math>x = 0</math>, <math>\text{Pr} = \frac{1}{4}</math><br/><br/>When <math>x = 1</math>, <math>\text{Pr} = \frac{3}{4}</math><br/><br/><math display="block">\sum \text{Pr} = \frac{1}{4} + \frac{3}{4} = 1</math><br/><math>\therefore</math> this is a probability function.</p>                                                                                                                                                                        |
| <p><b>Question 25 B</b><br/>Hypergeometric<br/><math display="block">\sigma = \sqrt{n \frac{D}{N} \left(1 - \frac{D}{N}\right) \left(\frac{N - n}{N - 1}\right)}</math><br/><math>D = 8, N = 18, n = 5</math><br/><math display="block">\sigma = \sqrt{5 \times \frac{8}{18} \left(1 - \frac{8}{18}\right) \left(\frac{18 - 5}{18 - 1}\right)} = 0.97</math></p> | <p><b>Question 26 A</b><br/>Binomial with <math>n = 10, p = \frac{1}{6}, x = 9</math> or <math>10</math><br/><math display="block">\text{Pr}(X = 9) + \text{Pr}(x = 10)</math><br/><math display="block">= \binom{10}{9} \left(\frac{1}{6}\right)^9 \left(\frac{5}{6}\right)^1 + \binom{10}{10} \left(\frac{1}{6}\right)^{10} \left(\frac{5}{6}\right)^0</math><br/><math display="block">= \binom{10}{9} \left(\frac{1}{6}\right)^9 \left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^{10}</math></p> |

Question 27 E



$$\Pr(90 - a < X < 90 + a) = 0.82$$

$$\Pr(X < (90 + a)) - \Pr(X < (90 - a)) = 0.82$$

$$\Pr(X < (90 + a)) - \Pr(X > (90 + a)) = 0.82$$

$$\Pr(X < (90 + a)) - (1 - \Pr(X < (90 + a))) = 0.82$$

$$2 \Pr(X < (90 + a)) - 1 = 0.82$$

$$2 \Pr(X < (90 + a)) = 1.82$$

$$\Pr(X < (90 + a)) = 0.91$$

$$Z = \frac{x - \mu}{\sigma} = \frac{90 + a - 90}{2.5} = \frac{a}{2.5}$$

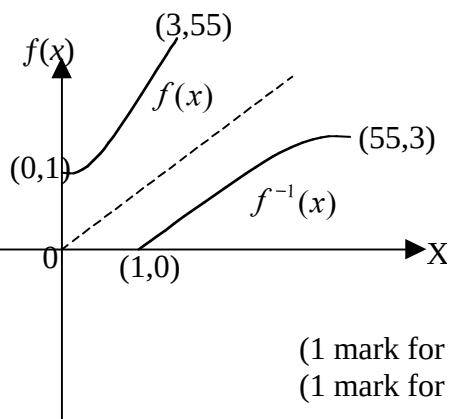
$$\Pr(Z < \frac{a}{2.5}) = 0.91$$

$$\frac{a}{2.5} = 1.341$$

$$a = 2.5 \times 1.341 = 3.3525$$

**Question 1**

a. & c.



b. Range of  $f(x)$  is  $[1,55]$  (1 mark)

**Question 2**

a.

$$\begin{aligned} g(x) &= 2\left[x^2 - 6x + \frac{31}{2}\right] \\ &= 2\left[x^2 - 6x + 9 - 9 + \frac{31}{2}\right] \quad (1 \text{ mark}) \\ &= 2\left[(x-3)^2 - \frac{18}{2} + \frac{31}{2}\right] \\ &= 2\left[(x-3)^2 + \frac{13}{2}\right] \\ &= 2(x-3)^2 + 13 \quad (1 \text{ mark}) \end{aligned}$$

b.

$f(x)$  is translated 3 units to the right parallel to the  $X$  axis (1 mark)  
 $f(x)$  is translated 13 units up parallel to the  $Y$  axis (1 mark)  
 $f(x)$  is dilated by a factor of 2 parallel to the  $Y$  axis (1 mark)

**Question 3**

a.

When  $x = 0, y = 0$  (0,0) (1 mark)  
 When  $x = -2, y = 4e^{-2}$  (-2,  $4e^{-2}$ ) (1 mark)

b.

$\frac{dy}{dx} = x^2 e^x + e^x \times 2x = 0$  for tangents that are horizontal. (1 mark)

$\therefore xe^x(x+2) = 0$   
 $\Rightarrow x = 0$  or  $x = -2, e^x > 0$  for all  $x$  (1 mark)

**Question 4**

a.

$$\begin{aligned} \sin(2\pi x)(2\cos(2\pi x) + \sqrt{3}) &= 0 \quad (1 \text{ mark}) \\ \sin(2\pi x) &= 0 \text{ or } 2\cos(2\pi x) = -\sqrt{3} \quad 0 \leq x \leq 2 \\ \sin(2\pi x) &= 0 \text{ or } \cos(2\pi x) = -\frac{\sqrt{3}}{2} \quad 0 \leq 2\pi x \leq 4\pi \\ 2\pi x &= 0, \pi, 2\pi, 3\pi, 4\pi \text{ or } 2\pi x = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, \\ &3\pi - \frac{\pi}{6}, 3\pi + \frac{\pi}{6} \quad (1 \text{ mark}) \\ 2x &= 0, 1, 2, 3, 4 \text{ or } 2x = \frac{5}{6}, \frac{7}{6}, \frac{17}{6}, \frac{19}{6} \\ x &= 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{12}, \frac{7}{12}, \frac{17}{12}, \frac{19}{12} \quad (1 \text{ mark}) \end{aligned}$$

**Question 5**

$$\log_a(x^2) + \log_a(5) = \log_a(a) + \log_a(8x - 3a)$$

$$\log_a(5x^2) = \log_a(a(8x - 3a)) \quad (1 \text{ mark})$$

$$5x^2 = 8ax - 3a^2$$

$$5x^2 - 8ax + 3a^2 = 0 \quad (1 \text{ mark})$$

$$x = \frac{8a \pm \sqrt{64a^2 - 60a^2}}{10}$$

$$x = \frac{8a \pm \sqrt{4a^2}}{10}$$

$$x = \frac{10a}{10} \text{ or } \frac{6a}{10}$$

$$x = a \text{ or } 0.6a \quad (1 \text{ mark})$$

**Question 6**

**a.**

Hypergeometric

$$N = 20, D = 4, n = 6 \quad x \geq 1$$

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) \quad (1 \text{ mark})$$

$$= 1 - \frac{\binom{4}{0} \binom{16}{6}}{\binom{20}{6}} = 0.7934 \quad (1 \text{ mark})$$

**b.**

Pr not defective and defective and not defective

$$\text{and defective} = \frac{16}{20} \times \frac{4}{20} \times \frac{16}{20} \times \frac{4}{20} = 0.0256 \quad (1 \text{ mark})$$

**c.**

$$\Pr \text{ rejects} = \Pr(X \geq 1) = 1 - \Pr(X = 0)$$

$$= 1 - \binom{4}{0} \left(\frac{4}{20}\right)^0 \left(\frac{16}{20}\right)^4 = 1 - \left(\frac{16}{20}\right)^4$$

$$\Pr \text{ accepts batch} = 1 - \Pr \text{ rejects}$$

$$= 1 - \left[ 1 - \left(\frac{16}{20}\right)^4 \right] \\ = \left(\frac{16}{20}\right)^4 = 0.4096$$

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