

Year 2003

VCE

Further Mathematics Trial Examination 2

Detailed Suggested Solutions

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These solutions are suggested solutions only. Teachers and students should carefully read the answers and comments supplied by the Mathematics Examiners.
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Question 1

a. The qualitative variable is the name of the towns. (1 mark)

b. The quantitative variable is the cost of the houses. (1 mark)

c.

Boomtown		Newtown
169, 165, 161, 160, 160, 156	150 – 169	155
189, 188, 187, 185, 182, 179, 170	170 – 189	170, 175, 180, 182, 185
196, 195, 195, 194, 194	190 – 209	192, 203, 206, 208
218, 215, 210	210 – 229	210, 210, 214, 215, 220, 225, 229
	230 – 249	230, 233, 240, 245

(1 mark)

d. \$187,000 (1 mark)

e. \$195,000 (1 mark)

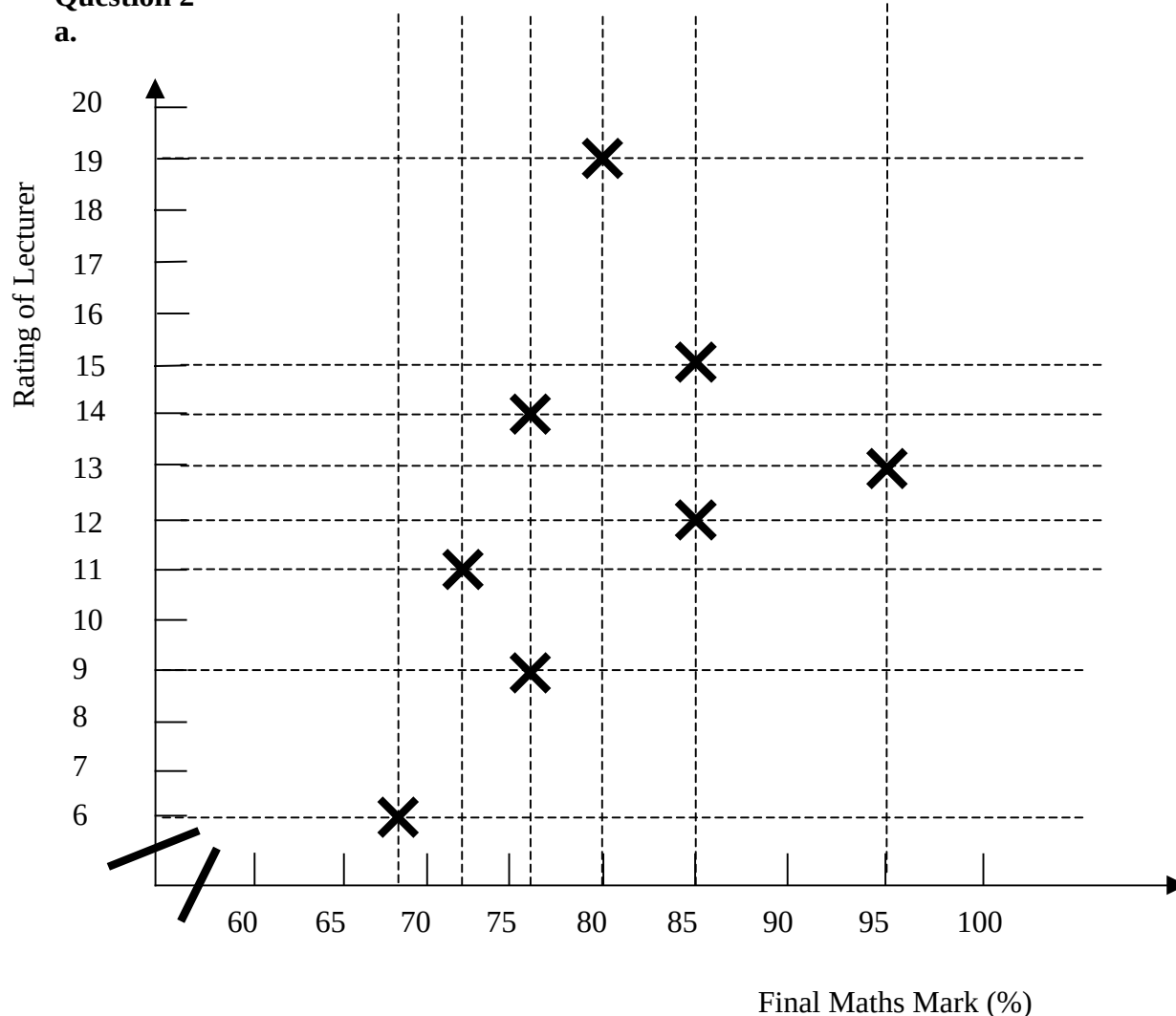
f. Use a graphics calculator \$206,000 (1 mark)

g. Use the same screen from the graphics calculator as for the mean \$24,500 (1 mark)

h. $206,000 \pm 2 \times 24,500 = 206,000 \pm 49,000$ Range of values = \$157,000 to \$255,000 (1 mark)

Question 2

a.



(2 marks. Subtract $\frac{1}{2}$ mark for each incorrectly positioned point to a maximum of 4 points)

b. The students with lower scores tend to rate the lecturer lower and the students with higher scores tend to rate the lecturer higher. (1 mark)	c. Use a graphics calculator. Stat/Edit then Stat/Calc $y = 0.22x - 5.46$ (1 mark) y is the rating of the lecturer and x is the maths mark. (1 mark)
d. Use the same screen of the graphics calculator as in c. Pearson's correlation coefficient = 0.5 (1 mark)	e. $8 = 0.22x - 5.46$ $\Rightarrow 13.46 = 0.22x$ $\Rightarrow x = \frac{13.46}{0.22}$ $\Rightarrow x = 61\%$ (1 mark)

End of Core

Module 1 Number patterns and applications. Suggested Solutions

Question 1

a. Arithmetic sequence $t_n = a + (n-1)d$ $a = 2000 \quad n = 5 \quad d = -16$ $\Rightarrow t_5 = 2000 + 4(-16)$ $\Rightarrow t_5 = 2000 - 64 = 1936$ (1 mark)	b. $S_n = \frac{n}{2}[2a + (n-1)d]$ $\Rightarrow S_7 = \frac{7}{2}[4000 + 6 \times (-16)]$ $\Rightarrow S_7 = \frac{7}{2}[4000 - 96]$ $\Rightarrow S_7 = 13,664$ (1 mark)
c. This is a geometric sequence. The tenth day is the third day of the geometric sequence. $a = 1,048,576, \quad r = \frac{1}{2}, \quad n = 3$ $t_n = ar^{n-1}$ $\Rightarrow t_3 = 1,048,576 \times \left(\frac{1}{2}\right)^2$ $\Rightarrow t_3 = 262,144$ (1 mark)	d. $1048576 \times \left[\frac{1}{2}\right]^{n-1} = 128$ $\Rightarrow \left[\frac{1}{2}\right]^{n-1} = 0.00012207$ $\Rightarrow \log_{10}\left[\frac{1}{2}\right]^{n-1} = \log_{10} 0.00012207$ $\Rightarrow (n-1)\log_{10}(0.5) = -3.913389944$ $\Rightarrow n-1 = \frac{-3.913389944}{\log_{10}(0.5)} = 13$ $\Rightarrow n = 14$ (1 mark) That is, 14 days counting from when the advertising campaign began or 21 days from the beginning of sales. (1 mark)
e. First week sales = 13,664 Second week sales $S_7 = \frac{a(1-r^7)}{1-r}$ $\Rightarrow S_7 = \frac{1048576(1-\left(\frac{1}{2}\right)^7)}{\frac{1}{2}}$ $\Rightarrow S_7 = 2080768$ (1 mark) Total sales $= 13,664 + 2,080,768$ $= 2,094,432$ books (1 mark)	f. $13,664 : 2,080,768$ $= 427 : 65,024$ (1 mark) g. $427 : 65,024$ $x : 390,144$ $x = \frac{390144}{65024} \times 427 = 2562$ books

Module 1 Number patterns and applications. Suggested Solutions

Question 2

a. $T_1 = 200$ $T_2 = 5 \times 200 - 600 = 400$ (1 mark) $T_3 = 5 \times 400 - 600 = 1400$ $T_4 = 5 \times 1400 - 600 = 6400$ (1 mark)	b. There is no common difference. $400 - 200 \neq 1400 - 400$ (1 mark)
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Question 3 a. Arithmetic sequence $t_5 = a + 4d = 27 \quad \dots(1)$ $t_{15} = a + 14d = 10.5 \quad \dots(2)$ Subtract (1) from (2) $\Rightarrow 10d = -16.5$ $\Rightarrow d = -1.65$ (1 mark) $\Rightarrow t_5 = a - 4 \times 1.65 = 27$ $\Rightarrow t_5 = a - 6.6 = 27$ $\Rightarrow a = 33.6$ cm (1 mark)	b. $S_{15} = \frac{15}{2} [2 \times 33.6 + 14 \times (-1.65)]$ $\Rightarrow S_{15} = 330.75$ cm (1 mark)
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End of Module 1

Module 2 Geometry and trigonometry. Suggested solutions.

Question 1

a.Sum of angles in triangle = 180°

Therefore,

$$\angle XTY = 180 - (36 + 15) = 180 - 51 = 129^\circ$$

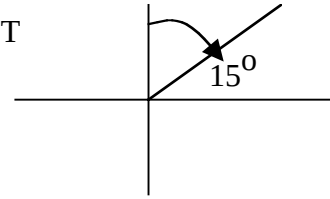
(1 mark)

b.

True bearing is measured in a clockwise direction from north.

$$90 - 15 = 075^\circ \text{ T}$$

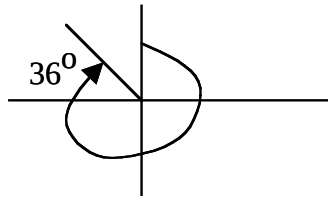
(1 mark)

**c.**

True bearing is measured in a clockwise direction from north.

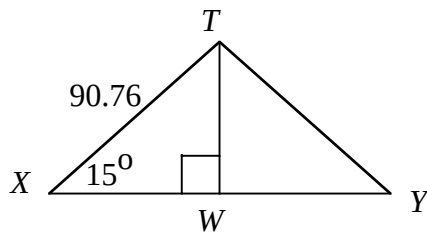
$$270 + 36 = 306^\circ \text{ T}$$

(1 mark)

**d.**

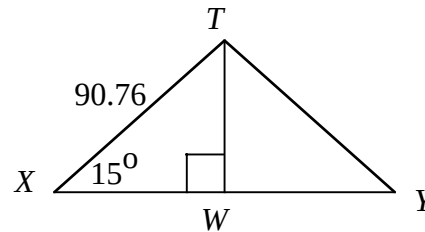
$$\frac{XT}{\sin 36^\circ} = \frac{120}{\sin 129^\circ} \quad (1 \text{ mark})$$

$$\Rightarrow XT = \frac{120 \times \sin 36^\circ}{\sin 129^\circ} = 90.76 \text{ m} \quad (1 \text{ mark})$$

e.Shortest path is where $\angle TWX = 90^\circ$ (1 mark)

$$\sin 15^\circ = \frac{TW}{90.76}$$

$$\Rightarrow TW = 90.76 \times \sin 15^\circ = 23.49 \text{ m} \quad (1 \text{ mark})$$

f.

$$\cos 15^\circ = \frac{XW}{90.76}$$

$$\Rightarrow XW = 90.76 \times \cos 15^\circ = 87.667 \text{ m} \quad (1 \text{ mark})$$

$$\Rightarrow WY = 120 - 87.667 = 32.33 \text{ m} \quad (1 \text{ mark})$$

g.Area of path = 234.9 m^2

Area of triangle

$$= \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} \times 120 \times 90.76 \times \sin 15^\circ$$

$$= 1409.424992 \text{ m}^2 \quad (1 \text{ mark})$$

 $\Rightarrow$ Area to be sown with grass

$$= 1409.424992 - 234.9$$

$$= 1174.5 \text{ m}^2 \quad (1 \text{ mark})$$

Question 2

a.

Using Pythagoras

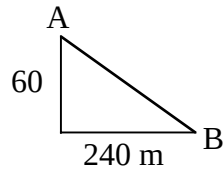
$$AB^2 = 240^2 + 60^2 \quad (1 \text{ mark})$$

$$\Rightarrow AB = \sqrt{57600 + 3600}$$

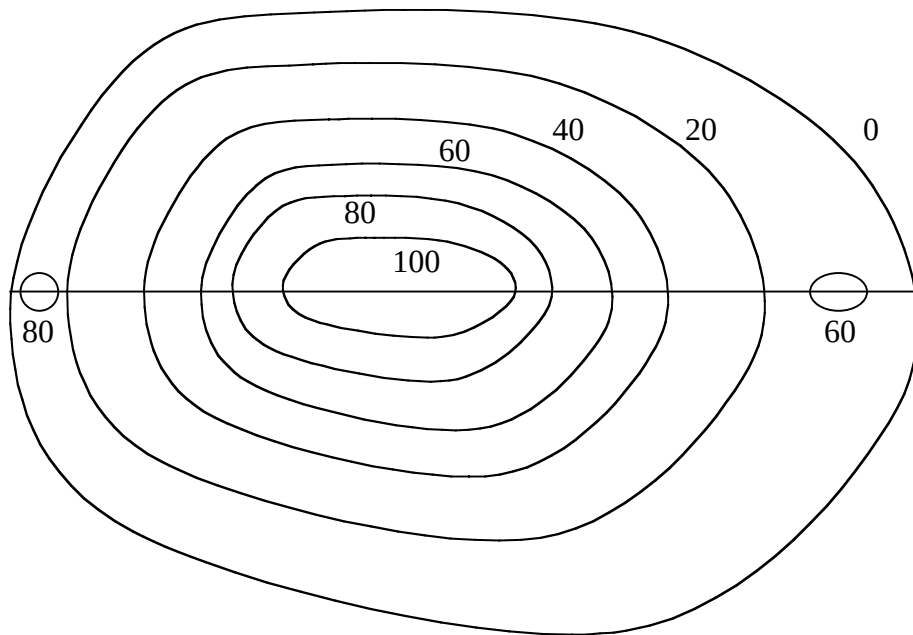
$$\Rightarrow AB = \sqrt{61200}$$

$$\Rightarrow AB = 247.386$$

$$\Rightarrow AB = 247 \text{ m} \quad (1 \text{ mark})$$



b.



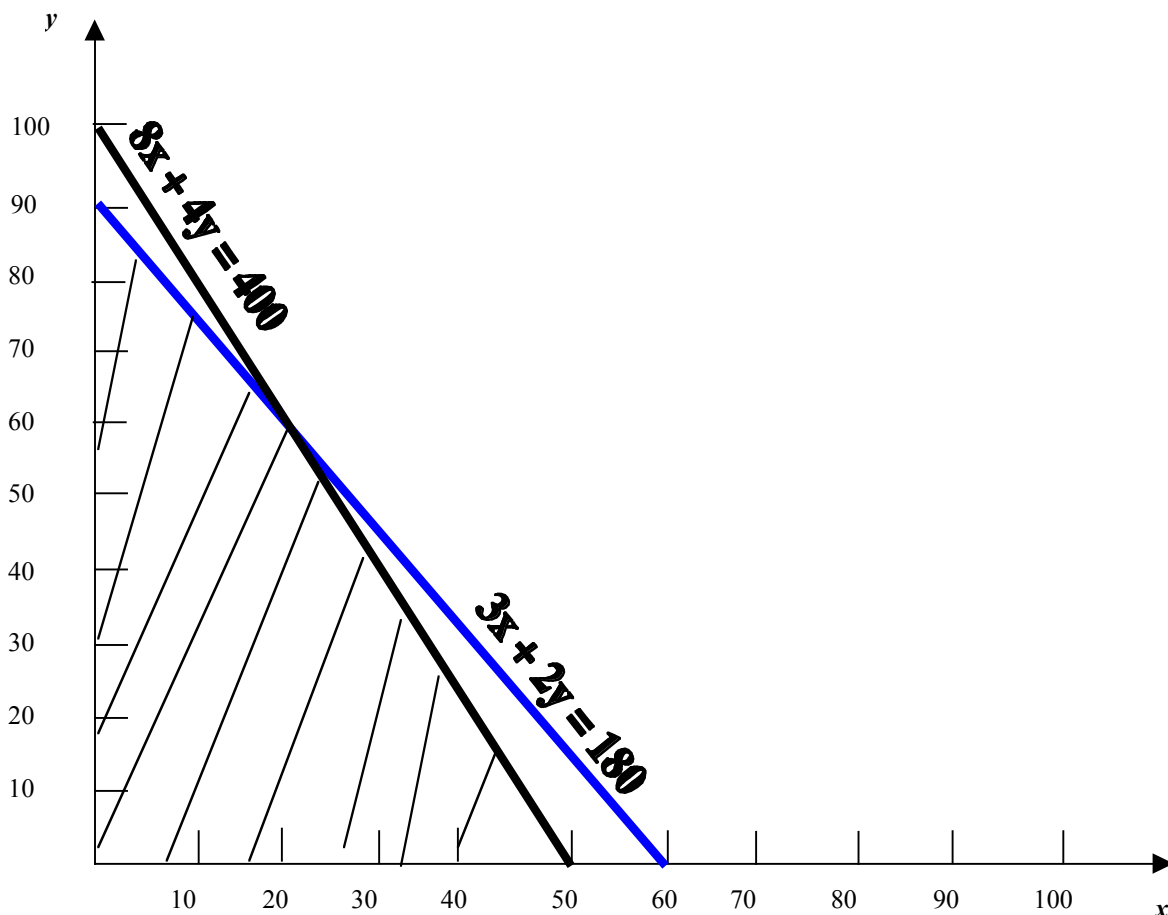
End of Module 2

Question 1	b.
a. C to D , G to H , K to L (1 mark)	10 minutes + 20 minutes = 30 minutes (1 mark)
c. 4 metres (1 mark)	d. A to B (1 mark)
e. $\frac{4}{10} = 0.4 \text{ m/min}$ (1 mark)	f. $95 - 55 = 40 \text{ minutes}$ (1 mark)

g.
 $V = 10 \times 4 = 40 \text{ m}^3 = 40,000,000 \text{ cm}^3$
 $1 \text{ cm}^3 = 1 \text{ millilitre}$
 $\Rightarrow 40,000,000 \text{ cm}^3 = 40,000,000 \text{ millilitres}$
 $\Rightarrow 40,000,000 \text{ cm}^3 = 40,000 \text{ litres}$
(1 mark)

Question 2	b.
a. $B = 400$ (1 mark)	$3x + 2y \leq 180$ (1 mark)

c. and d. (2 marks + 1 mark)



Module 3 Graphs and relations. Suggested solutions.

Question 2 e. $8x + 4y = 400$ $\Rightarrow 4x + 2y = 200 \quad (1)$ $3x + 2y = 180 \quad (2)$ By subtraction $x = 20$ (1 mark) Substituting $x = 20$ in (1) $\Rightarrow 80 + 2y = 200$ $\Rightarrow 2y = 120$ $\Rightarrow y = 60 \quad (1 \text{ mark})$ Point of intersection is (20,60) (1 mark)	b. Corner points are (0,90) , (20,60) and (50,0) At (0,90), $P = 0 + 10,800 = \\$10,800$ At (20,60), $P = 1,800 + 7,200 = \\$9,000$ At (50,0), $P = 4,500 + 0 = \\$4,500$ Therefore, the company should make zero Bass Guitars and 90 Electric Guitars each week (1 mark)
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Total = 15 marks

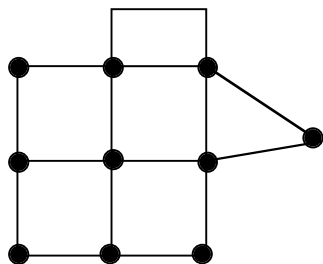
End of Module 3

Question 1 a. $1120 \times 12 \times 16 = \\$215,040$ (1 mark)	b. Interest paid = $215,040 - 70,000 = 145,040$ (1 mark) $I = \frac{PRt}{100}$ $\Rightarrow 145040 = \frac{70000 \times R \times 16}{100}$ $\Rightarrow 145040 = 11200R$ $\Rightarrow R = 12.95\% \text{ (1 mark)}$
c. Effective interest rate $= \frac{2n}{n+1} \times \text{flat rate}$ $= \frac{2 \times 16 \times 12}{(16 \times 12) + 1} \times 12.95$ $= \frac{2 \times 192}{193} \times 12.95$ $= 25.77\% \text{ (1 mark)}$	Question 2 a. Annual income = $320 \times 52 = \\$16,640$ (1 mark) Payment to agent $= 7\frac{1}{2}\% \text{ of } 16640$ $= \frac{15}{200} \times 16640 = 1248$ Actual earnings = $16640 - 1248$ = $\\$15,392$ (1 mark)
b. Value at 1st Jan 2003 $= PR^3$ $= 220000(1 + 0.15)^3$ $= 220000(1.15)^3$ $= 334592.5 \text{ (1 mark)}$ Value at 1st Jan 2004 $= 334592.5(1.18)$ $= 394819.15$ $= \$394,819 \text{ (1 mark)}$	c. $A = 9600(1 - 0.12)^3$ (1 mark) $\Rightarrow A = 9600(0.88)^3$ $\Rightarrow A = 6542.13$ $\Rightarrow A = \\$6542$ (1 mark)
Question 3 a. $\frac{10.8}{4} = 2.7\% \text{ per quarter (1 mark)}$	b. Four repayments per year for 10 years = $4 \times 10 = 40$ repayments (1 mark)
c. $0 = PR^n - \frac{Q(R^n - 1)}{R - 1} \text{ (1 mark)}$ $\Rightarrow 80000(1.027)^{40} = \frac{Q(1.027^{40} - 1)}{0.027} \text{ (1 mark)}$ $\Rightarrow 232224.3366 = Q \times 70.4742299$ $\Rightarrow Q = \frac{232224.3366}{70.4742299} = 3295.166715$	She would have to repay \$3295.17 per quarter (1 mark)

Module 5 Networks and decision mathematics. Suggested solutions.

Question 1

a.



(1 mark)

b.

One possible route is:

Start at the corner of Albatross and Parrot.

Walk along Parrot – Hawk – Peacock – Pelican – Dove – Brolga – Sparrow – Eagle – Swan – Crow – Galah – Honeysucker – Albatross – Kookaburra – Rosella.

Finish at the corner of Rosella and Honeysucker.

(1 mark for starting at Albatross and Parrot **or** at Rosella and Honeysucker.)

(1 mark for completing a correct route.)

(2 marks)

c.

This is an Eulerian network

(1 mark)

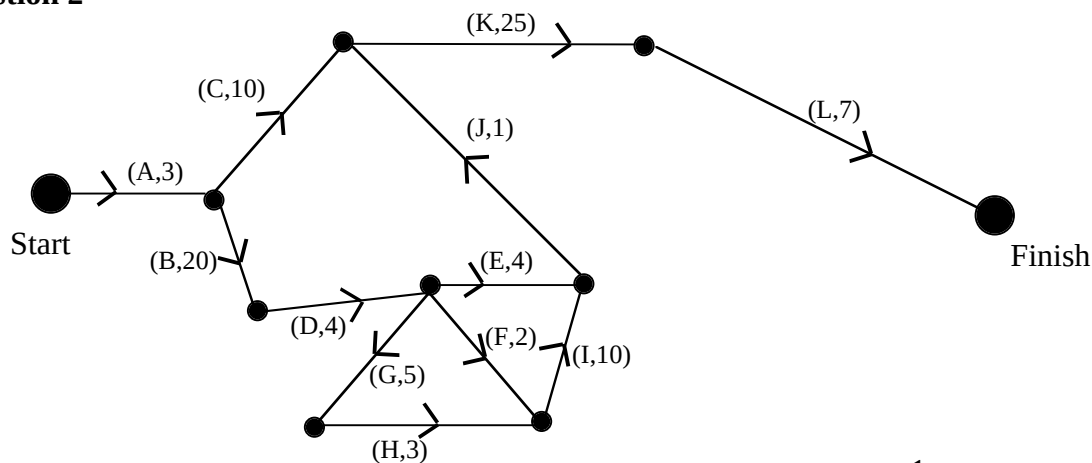
d. i. and ii.

Aroula is right because an Eulerian circuit only exists if each vertex is of even degree. Here there are two vertices of odd degree.

(2 marks)

Question 2

a.

(2 marks with $-\frac{1}{2}$ mark for each error)

Module 5 Networks and decision mathematics. Suggested solutions.

Question 2(continued) b. The critical path is the longest path to each connecting node. A – B – D – G – H – I – J – K – L (1 mark)	c. Minimum time $= 3 + 20 + 4 + 5 + 3 + 10 + 1 + 25 + 7$ $= 78$ minutes (1 mark)
d. P = 35 Q = 46 R = 71 S = $41 - 27 = 14$ T = $33 - 27 = 6$ (1 mark each = 5 marks)	

End of Module 5

End of suggested solutions 2003 Further Mathematics Trial Examination 2

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