

Year 2003

VCE

Further Mathematics

Trial Examination 2



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STUDENT NUMBER**Letter**

Figures									
Words									

VICTORIAN CERTIFICATE OF EDUCATION 2003

FURTHER MATHEMATICS

Trial Written Examination 2 (Analysis task)

Reading time: 15 minutes

Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core

Number of questions	Number of questions to be answered
2	2

Modules

Number of modules	Number of modules to be answered
5	3

Directions to students

Materials

Question and answer book of 26 pages.

Working space is provided throughout the book.

There is a detachable sheet of miscellaneous formula supplied.

You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.

You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task

Detach the formula sheet from the book during reading time.

Please ensure that your **student number** is written in the space provided on the front cover of this book.

Answer **all** questions in the core and in each of the three chosen modules.

The marks allotted to each part of each question are indicated at the end of the part.

The core is worth 15 marks. Each module is worth 15 marks. There is a total of 60 marks available for the examination.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π , e , surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

All written responses should be in English.

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Business-related mathematics

simple interest: $I = \frac{\text{Pr } T}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest = $\frac{2n}{n+1}$ x flat rate

annuities: $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$, where $R = 1 + \frac{r}{100}$

Geometry and trigonometry

area of a triangle: $\frac{1}{2}bh$

area of a triangle: $\frac{1}{2}bc \sin A$

area of circle: πr^2

volume of sphere: $\frac{4}{3}\pi r^3$

volume of cone: $\frac{1}{3}\pi r^2 h$

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Graphs and relations

Straight line graphs

gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation $y - y_1 = m(x - x_1)$ gradient-point form

$y = mx + c$ gradient-intercept form

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ two-point form

Number patterns and applications

arithmetic series: $a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \dots = \frac{a}{1 - r}, |r| < 1$

linear difference equations: $t_n = at_{n-1} + b = a^{n-1}t_1 + b\frac{(a^{n-1} - 1)}{a - 1}, a \neq 1$
 $= a^n t_0 + b\frac{(a^n - 1)}{a - 1}$

Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Statistics

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

END OF FORMULA SHEET

Specific Instructions

This task paper consists a core and five modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

		Page
Core		2
Module		
Module 1:	Number patterns and applications	6
Module 2:	Geometry and trigonometry	10
Module 3:	Graphs and relations	14
Module 4:	Business-related mathematics	19
Module 5:	Networks and decision mathematics	23

Question 1

The table below gives the price of houses in two cities, Boomtown and Newtown taken from the times Daily Newspaper. The prices are quoted in thousands of dollars.

Boomtown	Newtown
156	155
160	170
160	175
161	180
165	182
169	185
170	192
179	203
182	206
185	208
187	210
188	210
189	214
194	215
194	220
195	225
195	229
196	230
210	233
215	240
218	245

- a.** What is the qualitative variable in this set of data?

1 mark

- b.** What is the quantitative variable in this set of data?

1 mark

Question 1 (continued)

- c. Complete the back to back stem and leaf for the given data.

Boomtown		Newtown
	150 – 169	
	170 – 189	
	190 – 209	
	210 – 229	
	230 – 249	

1 mark

- d. Find the median price for Boomtown.

1 mark

- e. What is the upper quartile for Boomtown?

1 mark

- f. Find the mean price for Newtown houses to the nearest thousand.

1 mark

- g. Find the standard deviation for Newtown to the nearest five hundred dollars.

1 mark

- h. Between what values would you expect 95% of all the houses in Newtown to lie?

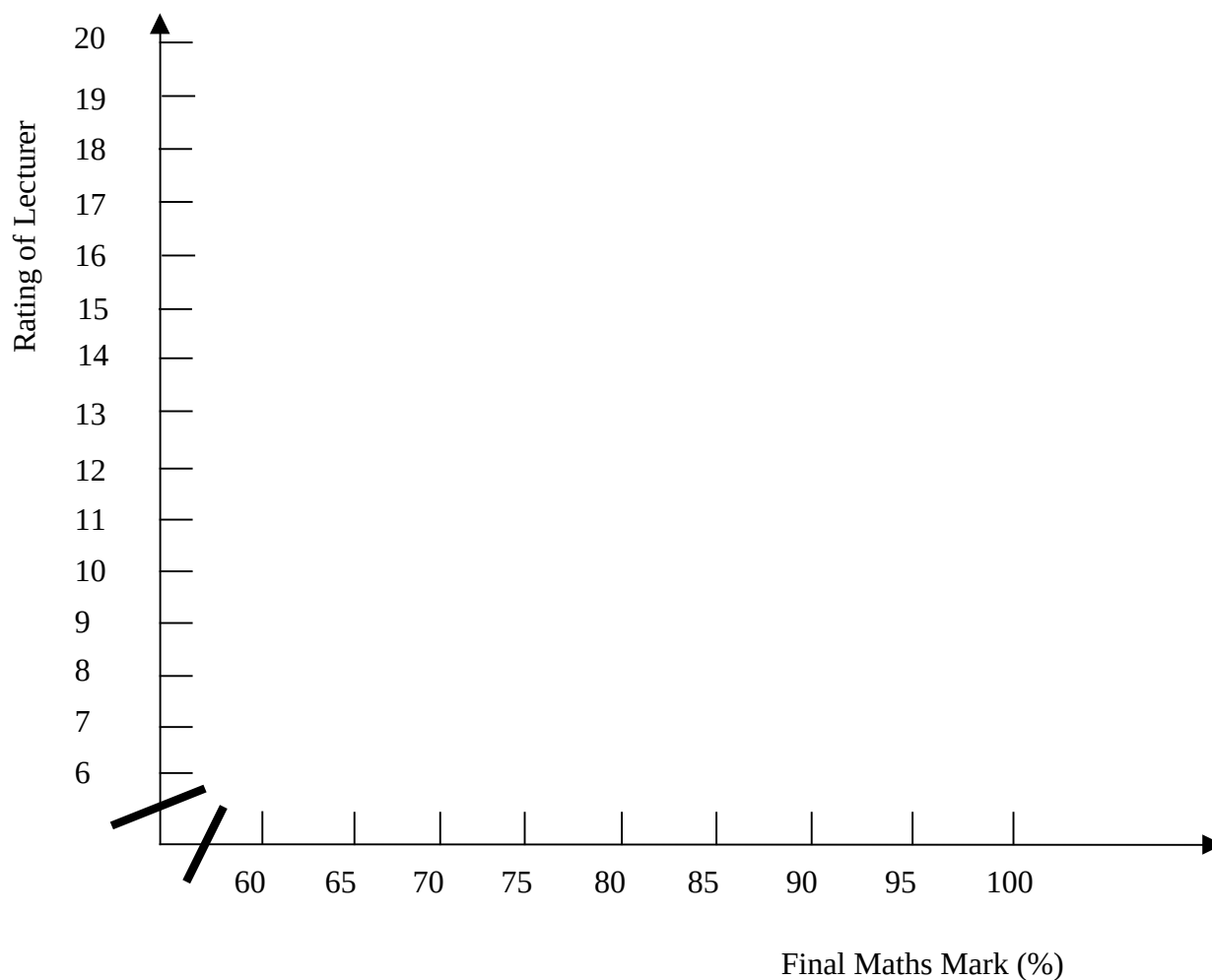
1 mark

Question 2

Students in a final year university mathematics class were asked to rate their lecturer on a scale of 1 to 20 where 20 meant that the lecturer was held in the highest regard. The following table gives this rating as well as the corresponding mark the student received in the final mathematics examination.

Final maths mark	76	85	72	76	68	80	95	85
Rating of lecturer	14	12	11	9	6	19	13	15

- a. Draw a scatter plot of these data on the axes below.



2 marks

Question 2 (continued)

- b.** What does the graph tell you about the relationship between the students' scores and their rating of the lecturer?

1 mark

- c.** Find the equation of the least squares regression line and give meaning to the x and y in your equation.

2 marks

- d.** Find Pearson's correlation coefficient to one decimal place.

1 mark

- e.** What maths score would you expect a student to get if the student gave a rating of 8 to the lecturer? Give your answer to the nearest whole number.

1 mark

Total marks = 15

End of Core

If you choose this module, all questions are to be answered.

Question 1

A book shop has a new book for sale called “*The Adventures of Gary Hotter*”. The book shop sells 2000 of these books on the first day and then 16 books less each day for the next six days.

- a. How many of these books will be sold by this shop on the fifth day?

1 mark

- b. How many of these books will have been sold by this shop at the end of the first seven day week?

1 mark

After seven days, all book shops in Australia heavily discount “*The Adventures of Gary Hotter*” so that on the **eighth day** 1,048,576 of these books are sold nationwide. On **each** successive day, the book shops sell $\frac{1}{2}$ of the number of these books sold on the previous day.

- c. How many of these books are sold nationwide on the tenth day?

1 mark

Question 1. (continued)

- d. For how many days could the sales continue in this way until there were only 128 copies of “*The Adventures of Gary Hotter*” sold nationwide?

2 marks

- e. At the beginning of the third week, the book shops around Australia are restocked by ordering exactly the total number of books they have sold in the first two seven day weeks of sales. How many books are ordered?

2 marks

- f. What is the ratio of books sold in the first week to books sold in the second week?

1 mark

Question 1. (continued)

- g.** The ratio of books sold in week one to books sold in week two equals the ratio of books sold in week five to books sold in week four. If 390,144 books are sold in week four, how many books are sold in week five?

1 mark

Question 2

Another book, “Super Bear” has sales such that the number of sales T_n in week n obeys the difference equation $T_{n+1} = 5T_n - 600$ where $T_1 = 200$

- a.** Find the number of “super Bear” books sold in week four.

2 marks

- b.** Explain why the sequence generated by this difference equation is not an arithmetic sequence.

1 mark

Question 3

To get the books from the top shelf of a bookstore, the salesperson must climb a ladder. The rungs of this ladder diminish uniformly in length from the bottom to the top. The fifth bottom rung is 27 cm long and the top rung is 10.5 cm long. There are fifteen rungs altogether.

- a.** How long is the bottom rung?

2 marks

- b.** What length of wood was required to make all of the rungs in the ladder?

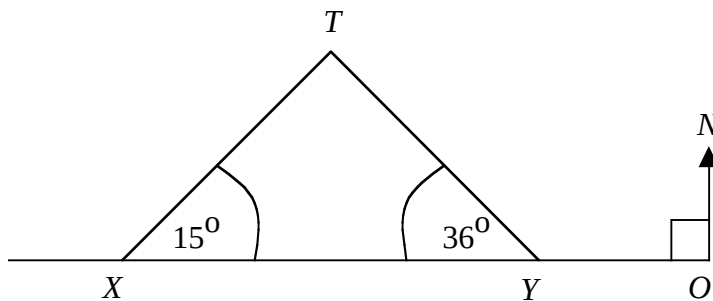
1 mark

Total marks = 15

End of Module1

If you choose this module, all questions are to be answered.

Question 1



In the diagram above, ON is due north and X and Y are two points on the verandah of a large resort hotel. When I look at a tree, T , from position X , $\angle TXY = 15^\circ$ and when I look at the same tree from position Y , $\angle TYX = 36^\circ$. The distance from X to Y is 120 m.

a. Find $\angle XTY$

1 mark

b. What is the true bearing of the tree from X ?

1 mark

c. What is the true bearing of the tree from Y ?

1 mark

Question 1 (continued)

- d.** Find the distance from X to the tree, T . Give your answer to two decimal places.

2 marks

- e.** I wish to make a path from the verandah to the tree. What is the length of the shortest path, TW , that can be made, where W is a point on the verandah?
Give your answer to two decimal places.

2 marks

- f.** How far is W from Y ? Give your answer to two decimal places.

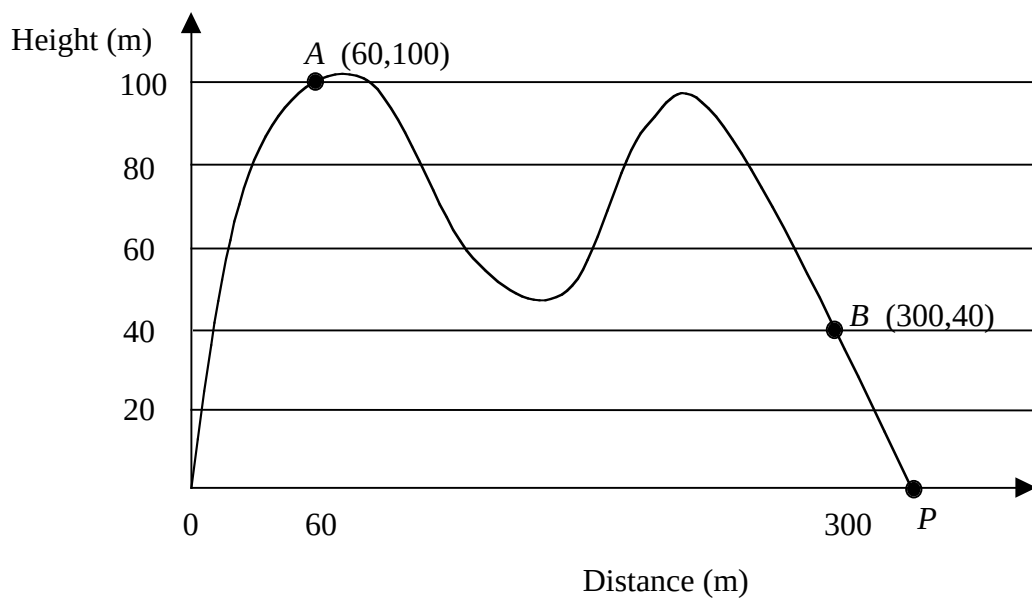
2 marks

Question 1 (continued)

- g.** The resort owner wishes to have the area around the path leading from the verandah to the tree and inside the triangle XTY , sown with grass seed. The area of the path is 234.9 m^2 . Calculate the area to be sown with grass seed. Give your answer to one decimal place.

2 marks

Question 2



The above graph is a profile of the contour of a piece of land on a hillside

- a. Find the direct straight line distance from A to B to the nearest metre

2 marks

- b. Construct an appropriate contour map for this piece of land.

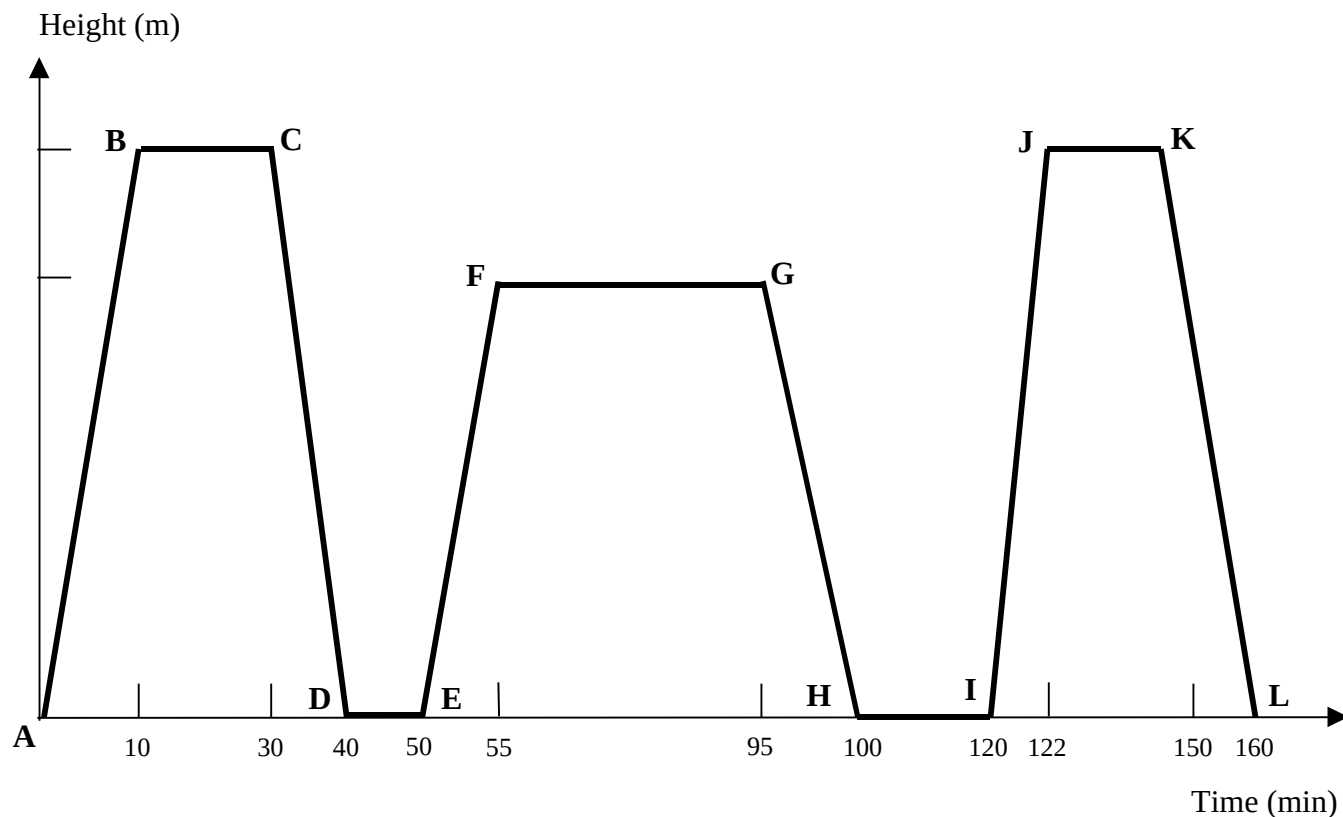
2 marks

Total marks = 15

End of Module 2

If you choose this module, all questions are to be answered.

Question 1



In a factory a vat is being filled with water for use in a chemical reaction. The graph above shows what happens when the vat is being filled and emptied over a 160 minute period.

- a. Using the letters in the graph above, state which sections show when the vat is being emptied.

1 mark

- b. For how long was the vat left empty in the first 160 minutes?

1 mark

Question 1 (continued)

c. What is the maximum height of water in the vat?

1 mark

d. In which section of the graph is the rate of filling slowest?

1 mark

e. What is the slowest rate of filling of the vat?

1 mark

f. For how long was the vat left at $\frac{3}{4}$ full before it was emptied?

1 mark

h. The volume of water in the vat, in m^3 , is 10 times the height of water in the vat.
How many litres were in the vat when $t = 20$ minutes? (1 millilitre = 1 cm^3)

1 mark

Question 2

A manufacturer of musical instruments produces Bass Guitars and Electric Guitars. The time required to construct and fine tune one of these instruments in hours is given in the table below.

Instrument	Construction time (hours)	Fine tune time (hours)
Bass Guitar	8	3
Electric Guitar	4	2

The maximum number of hours in a week that can be spent by the workers who construct these instruments is 400 hours. The maximum number of hours that the fine tuners can work in a week is 180 hours.

Let the number of Bass Guitars made in a week = x .

Let the number of Electric Guitars made in a week = y .

The constraint for the number of hours spent constructing these musical instruments is

$$8x + 4y \leq B$$

a. What is the value of B ?

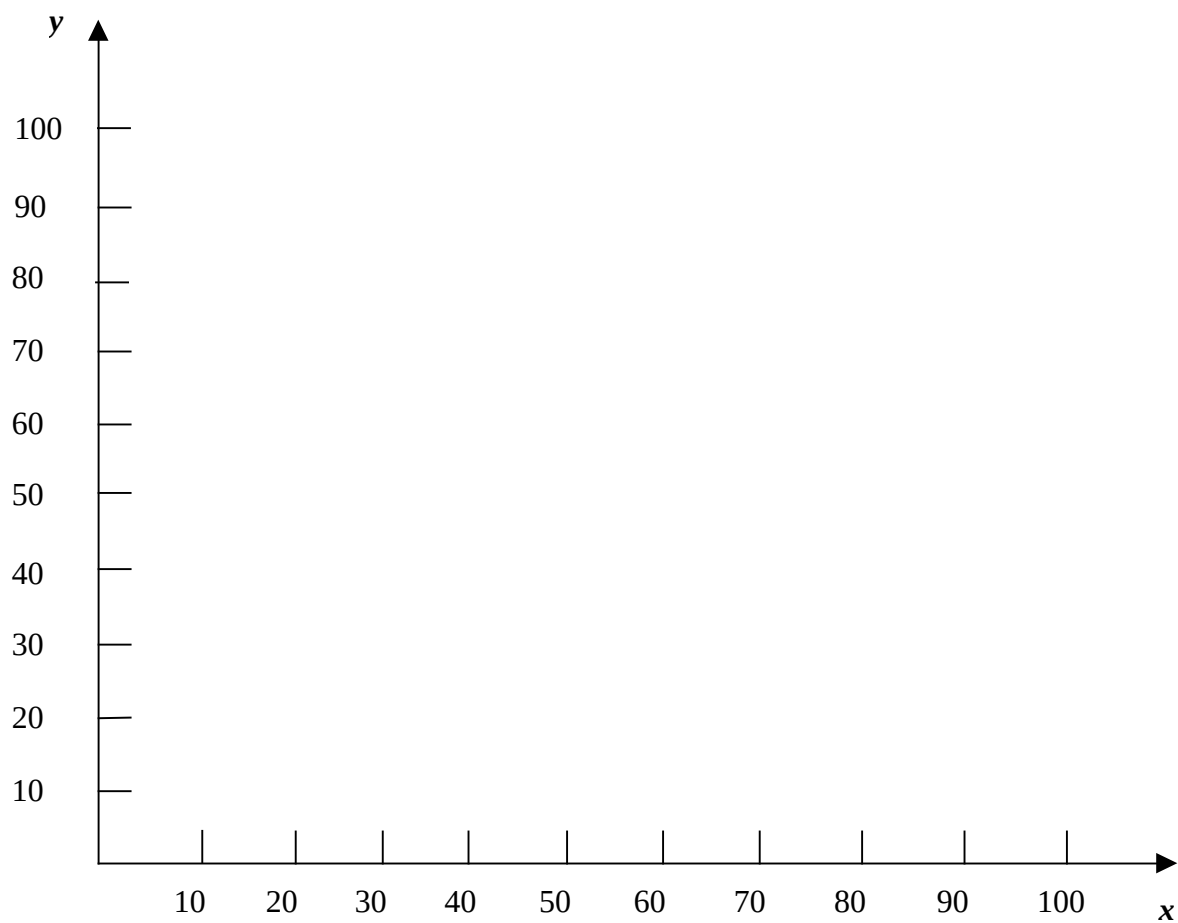
1 mark

b. What is the constraint for the number of hours spent fine tuning the musical instruments?

1 mark

Question 2 (continued)

- c. On the axes below draw the straight lines showing the two constraints found in parts **a.** and **b.**



2 marks

- d. On the above graph, shade the feasible region defined by the constraints of the problem.

1 mark

Question 2 (continued)

- e. Find the point of intersection of the two lines drawn in c.

2 marks

- f. If the profit equation for the sale of these guitars is $P = 90x + 120y$, how many of each type of guitar should the company make each week to maximise its profit?

1 mark

End of Module 3

Module 4 Business-related mathematics

If you choose this module, all questions are to be answered.

Question 1

Meg decides to buy a Unit. She takes out a \$70,000 loan with the Australasia Bank and repays \$1,120 per month for 16 years.

- a. How much will Meg pay the Bank altogether?

1 mark

- b. What is the equivalent flat interest rate per annum for the loan? Give your answer to two decimal places.

2 marks

- c. What is the effective interest rate for the loan? Give your answer to two decimal places.

1 mark

Question 2

- a. Meg decides to go home and live with her parents. She rents the Unit for \$320 per week. She pays an estate agent $7\frac{1}{2}\%$ of these earnings to manage the renting of her Unit. How much does she actually earn per year for the rental of this property?

2 marks

- b. The Unit was valued at \$220,000 on 1st January 2000 and it has increased in value at 15% of the previous year's value each year until 1st January 2003. It is expected to increase in value by 18% during 2003. What is the expected value of the Unit on 1st January 2004? Give your answer to the nearest dollar.

2 marks

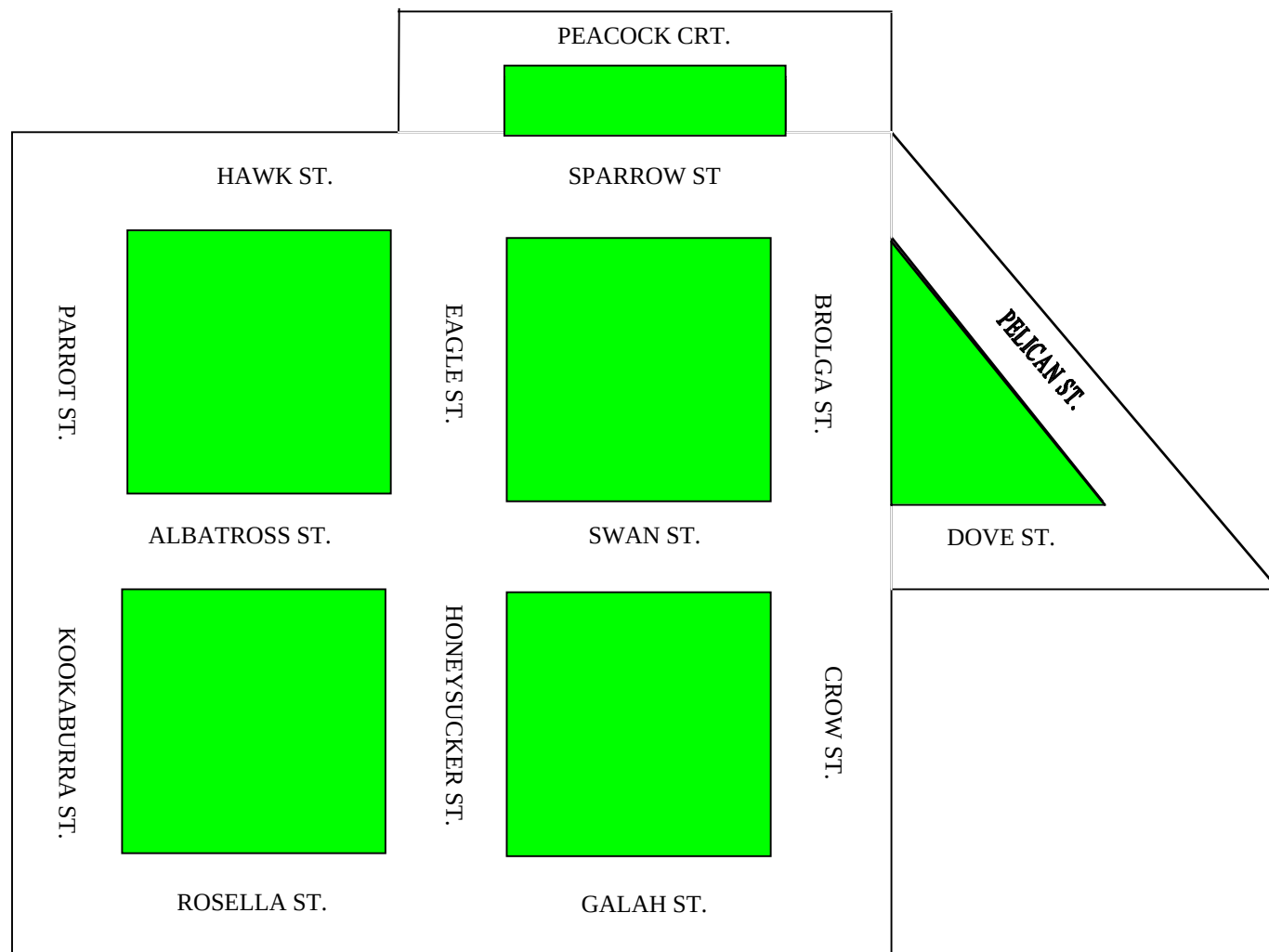
Question 2 (continued)

- c. Meg spent \$9,600 furnishing her Unit but the furniture has depreciated compoundly at 12% per annum. The furniture was purchased on 1st January 2000. What was the value of the furniture on 1st January 2003? Give your answer to the nearest dollar.

2 marks

If you choose this module, all questions are to be answered.

Question 1



Aroula and Janet are selling *Gosmuretic Beauty Products* and wish to go to every house in the streets shown above. They have agreed that Aroula will visit the houses on the even numbered side of every street and Janet will visit the houses on the opposite side of the street.

- Draw a network diagram showing the streets along which Aroula and Janet will work selling their product.

Question 1 (continued)

- b.** Aroula and Janet wish to travel along all these streets without walking along any street twice. Suggest a route they could take.

2 marks

- c.** What is the name given to the type of network they will have walked when they have been to every home?

1 mark

- d.** Aroula and Janet want to start and finish at the same point and travel along each street only once. Aroula says that this is not possible. Janet says it is possible if they start and finish at the intersection of Honeysucker and Rosella Streets.

- i.** Who is correct?

1 mark

- ii.** Give a reason for your answer.

1 mark

Question 2

Andrew wants to make a chocolate cake. The following table shows the activities that Andrew must complete, the time that each activity takes and the order of the activities.

Symbol	Activity	Time in minutes	Immediate Predecessor
A	Read recipe	3	-
B	Purchase ingredients	20	A
C	Preheat oven	10	A
D	Collect ingredients and equipment	4	B
E	Grease cake tin	4	D
F	Melt butter	2	D
G	Measure dry ingredients in basin	5	D
H	Add milk and eggs while mixing	3	G
I	Add melted butter and mix	10	F , H
J	Pour cake mixture into cake tin	1	E , I
K	Bake cake	25	C , J
L	Ice cake	7	K

- a. Draw a network to represent the activities involved in making this chocolate cake.

2 marks

- b. Find the critical path.

1 mark

Question 2 (continued)

- c. What is the minimum time required to make the chocolate cake?

1 mark

- d. In the table below, find the values of **P** , **Q** , **R** , **S** and **T**.

Activity	Earliest start time	Latest start time	Float time
A	0	0	0
B	3	3	
C		Q	43
D	23		
E			S
F	27		T
G			
H			
I	P		
J		45	
K			0
L		R	0

5 marks

Total marks = 15

End of Module 5

END OF QUESTION AND ANSWER BOOK

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