

Year 2003

VCE

Further Mathematics Trial Examination 1

Suggested Solutions

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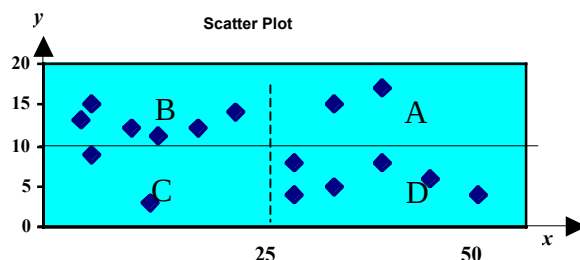
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These solutions are suggested solutions only. Teachers and students should carefully read the answers and comments supplied by the Mathematics Examiners.
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Question 1 D There would be more demand for the ski equipment in the winter season than at other times of the year, hence the trend is most likely seasonal.	Question 2 C 40% of Candor's votes go to De Jong $40\% \text{ of } 90 = \frac{40}{100} \times \frac{90}{1} = 36$ De Jong now has $181 + 36 = 217$ votes.
Question 3 D Female votes = $313 + 90 = 403$ Total votes = $204 + 313 + 90 + 181 = 788$ $\% \text{ female votes} = \frac{403}{788} \times \frac{100}{1} = 51.14\%$	Question 4 D This graph shows a positive correlation. Correlation has nothing to do with cause and effect.
Question 5 C The median is half way between the 15th and the 16th numbers, i.e. between 24 and 26. $\therefore 25$	Question 6 B Lower Quartile, LQ, is the middle of the lowest 15 numbers, i.e. 14 Upper Quartile, UQ, is the middle of the upper 15 numbers, i.e. 38 Interquartile range, IQR, = $UQ - LQ$ $IQR = 38 - 14 = 24$
Question 7 C Mr. Smith's class has a range of $60 - 10 = 50$ Ms. Johnson's class has a range of $70 - 10 = 60$ Mrs. Young's class has a range of $80 - 0 = 80$ $\therefore$ Mrs. Young's class has the largest range. The standard deviation measures the difference between each score and the mean. Ms. Johnson's class will have the largest standard deviation because there are more scores at the extremes so they will give a greater difference from the mean.	Question 8 E $\mu = \\$70,000$ $99.7\% \Rightarrow 3\sigma$ from the mean $85,600 - 70,000 = 15,600 = 3\sigma$ $\therefore \sigma = 5,200$ 95% lie within $\mu \pm 2\sigma$ $2\sigma = 10,400$ $\mu \pm 2\sigma = 70,000 \pm 10,400$ $\therefore$ between \$59,600 and \$80,400

Question 9 B

$$q = \frac{(a + c) - (b + d)}{a + b + c + d}$$

$$q = \frac{(2 + 2) - (6 + 6)}{2 + 6 + 2 + 6}$$

$$q = \frac{4 - 12}{16}$$

$$q = -\frac{8}{16}$$

$$q = -0.5$$

Question 10 E

$$\text{Seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$$

$$78.57\% = \frac{5260}{\text{deseasonalised figure}}$$

$$0.7857 \times \text{deseasonalised figure} = 5260$$

$$\text{deseasonalised figure} = \frac{5260}{0.7857}$$

$$\text{deseasonalised figure} = \$6,695$$

Question 11 B

Median of first two x values is midway between 20 and 40 i.e. 30

Median of first two y values is midway between 20 and 60 i.e. 40

$\therefore$ Lower Median is the point (30,40)

Median of last two x values is midway between 90 and 100 i.e. 95

Median of last two y values is midway between 80 and 90 i.e. 85

$\therefore$ Upper Median is the point (95,85)

$$\text{gradient} = \frac{y_u - y_l}{x_u - x_l} = \frac{85 - 40}{95 - 30} = \frac{45}{65} = 0.7$$

Question 12 A

Using graphics calculator, use stat edit to enter data with x values in L_1 and y values in L_2
Go to stat calc 4 and read the equation
Closest to $y = 0.3x + 0.4$

Question 13 E

From the same graphical display on the calculator as for question 12, read off the r value
 $r = 0.95$

Module 1 Number patterns and applications. Suggested solutions.

Question 1 D

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$n = 8$$

$$a = 0.025$$

$$r = \frac{0.0125}{0.025} = 5$$

$$S_8 = \frac{0.025(5^8 - 1)}{4} = 2441.4$$

Question 2 B

This is a geometric sequence.

$$t_n = ar^{n-1}$$

$$n = 12$$

$$a = 6$$

$$r = \frac{-12}{6} = -2$$

$$t_{12} = 6 \times (-2)^{11} = -12288$$

Question 3 D

2003 is the ninth year.

This is an arithmetic sequence

$$n = 9$$

$$a = 28$$

$$d = 7$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_9 = \frac{9}{2}[56 + 8 \times 7]$$

$$S_n = \frac{9}{2}[56 + 56] = 504$$

Question 4 A

$$t_n = 3n + 7$$

$$t_1 = 10$$

$$t_2 = 13$$

$$t_3 = 16$$

Sequence is 10,13,16

$$\therefore \text{common difference} = 13 - 10 = 3$$

Question 5 C

$$A = PR^n$$

$$R = 1 + \frac{r}{100} = 1.045$$

$$A = 30,000 \times (1.045)^{10} = \$46,589$$

This is closest to \$46,600

Question 6 D

$$a + d = -1 \quad (1)$$

$$a + 4d = 17 \quad (2)$$

$$(2) - (1) \rightarrow 3d = 18$$

$$\therefore d = 6$$

Substituting in (1)

$$a + 6 = -1$$

$$a = -7$$

$$t_7 = a + 6d$$

$$t_7 = -7 + 36$$

$$t_7 = 29$$

Module 1 Number patterns and applications. Suggested solutions.

Question 7 A

$$S_{\infty} = \frac{a}{1-r}$$

$$a = \frac{2}{5}$$

$$S_{\infty} = \frac{4}{15}$$

$$\frac{4}{15} = \frac{\frac{2}{5}}{1-r}$$

$$1-r = \frac{2}{5} \div \frac{4}{15}$$

$$1-r = \frac{3}{2}$$

$$r = 1 - 1\frac{1}{2}$$

$$r = -\frac{1}{2}$$

Question 8 B

This represents a geometric sequence

$$t_n = ka^n$$

If $a > 1$, the graph is an increasing graph which it is not.

$\therefore$ not A

If a is negative, then t_n would be sometimes positive and sometimes negative, depending on the value of n .

$\therefore$ not C, D or E

Question 9 D

$$24 \text{ hours} = 24 \times 60 \text{ minutes} = 1440$$

$$\text{Number of 20 minute periods in this time} = 1440 \div 20 = 72$$

$$\text{Number of bacteria at end of period} = 8 \times 2^{72} = 3.8 \times 10^{22} \text{ which is closest to } 4.0 \times 10^{22}$$

Module 2 Geometry and trigonometry. Suggested solutions.

Question 1 E

$$\cos \theta = \frac{12}{21}$$

$$\theta = \cos^{-1}\left(\frac{12}{21}\right)$$

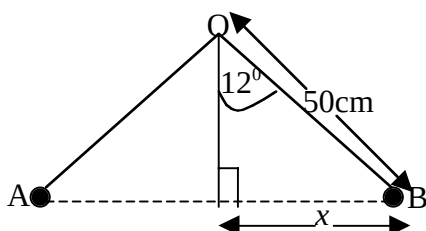
$$\theta = 55.15^\circ$$

which is closest to 55°

Question 2 C

True bearing are measured in a clockwise direction from North.

$$\therefore 270^\circ - 12^\circ = 258^\circ \text{T}$$

Question 3 C

$$\sin 12^\circ = \frac{x}{50}$$

$$x = 50 \sin 12^\circ$$

$$AB = 2x = 2 \times 50 \sin 12^\circ$$

$$AB = 20.79 \text{ cm which is closest to } 21 \text{ cm.}$$

Question 4 B

$$\text{Area} = \frac{1}{2}bc \sin A$$

$$\text{Area} = \frac{1}{2} \times 50 \times 50 \times \sin 24^\circ$$

$$\text{Area} = 508.4 \text{ which is closest to } 500 \text{ cm}^2$$

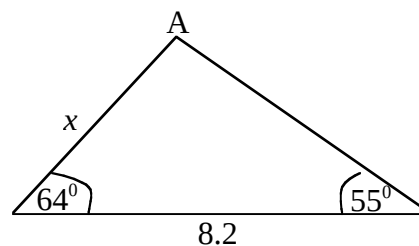
Question 5 D

$$b^2 = c^2 + a^2 - 2ca \cos B$$

$$b^2 = 230^2 + 110^2 - 2 \times 230 \times 110 \cos 25^\circ$$

$$b = \sqrt{230^2 + 110^2 - 2 \times 230 \times 110 \cos 25^\circ}$$

$$b = 138.4 \text{ which is closest to } 138 \text{ m}$$

Question 6 C

$$\angle A = 180 - (64 + 55) = 61^\circ$$

$$\frac{8.2}{\sin 61^\circ} = \frac{x}{\sin 55^\circ}$$

$$\therefore x = \frac{8.2 \sin 55^\circ}{\sin 61^\circ} = 7.7 \text{ km}$$

Module 2 Geometry and trigonometry. Suggested solutions.

Question 7 C

Similar figures

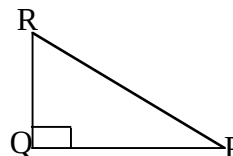
$$\therefore \text{scale factor for volume} = \frac{270}{10} = 27$$

$$\therefore \text{scale factor for linear measure} = \sqrt[3]{27} = 3$$

$$\therefore \text{scale factor for area measure} = 3^2 = 9$$

$$\text{TSA of larger cylinder} = 9 \times 26.4 = 237.6$$

which is closest to 240cm^2

Question 8 A

$$ST = RP$$

$$\therefore RP = 30$$

$$\sin 28^\circ = \frac{RQ}{30}$$

$$RQ = 30 \sin 28^\circ = 14.084$$

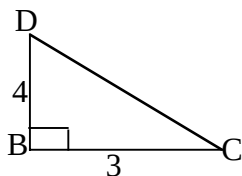
$$\cos 28^\circ = \frac{QP}{30}$$

$$QP = 30 \cos 28^\circ = 26.488$$

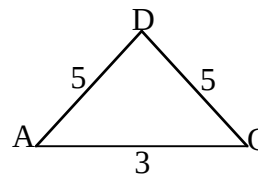
$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$

$$\text{Area of triangle} = \frac{1}{2} \times 26.488 \times 14.084 = 186.5$$

which is closest to 187 cm^2

Question 9 E

In the right angled triangle, DBC,
using the Pythagorean triad 3:4:5 $DC = 5$
Similarly, $DA = 5$



Using the cosine rule

$$5^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos \angle DCA$$

$$0 = 9 - 30 \cos \angle DCA$$

$$30 \cos \angle DCA = 9$$

$$\cos \angle DCA = 0.3$$

$$\angle DCA = \cos^{-1}(0.3)$$

$$\angle DCA = 72.5^\circ \text{ which is closest to } 72^\circ$$

Question 1 E Two points on the line are (0,2) and (4,1) Equation of the line is of the form $y = mx + c$ c is the y intercept, $\therefore c = 2$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{1 - 2}{4 - 0} = \frac{-1}{4}$ $y = -\frac{1}{4}x + 2$ $4y = -x + 8$ $4y + x - 8 = 0$	Question 2 C To find the point of intersection, solve the two equations simultaneously. $y = 2x + 1$ $y = -x + 4$ $\therefore 2x + 1 = -x + 4$ $\Rightarrow 3x = 3$ $\Rightarrow x = 1$ Substituting $x = 1$ in the first equation $y = 2 + 1 = 3$
Question 3 D A is true because Janet returns home when distance = 0 and she leaves for the shop when the distance = 0 B is true because the time of the journey extended from time = 0 to time = 60 minutes i.e. one hour C is true because the furthest distance traveled from home = 12 km. D is false because the total length of actual riding time to get to the shop = $10 + 5 = 15$ minutes and the total length of actual riding time to get home from the shop = $5 + 5 + 5 = 15$ minutes E is true because the number of stops to go to the shop = 1 and the number of stops on the homeward journey = 2	Question 4 E $y = m \times \frac{1}{x} + c$ When $x = \frac{1}{2}, y = 10$ When $x = \frac{1}{10}, y = 2$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\Rightarrow m = \frac{10 - 2}{\frac{1}{2} - \frac{1}{10}}$ $\Rightarrow m = \frac{8}{\frac{4}{10}}$ $\Rightarrow m = 20$
Question 5 C I and II are increasing. The graph then decreases more rapidly at first and then less rapidly $\therefore$ III	Question 6 C Here the shaded region is under the line $y = 2x$, and includes the line $\therefore y \leq 2x$ Also the shaded region is under the line $y = 6 - x$, and includes the line $\therefore y \leq 6 - x$ Also the shaded region is above the line $y = 0$, and includes the line $\therefore y \geq 0$

Module 3 Graphs and relations. Suggested solutions.

Question 7 D Point of intersection of $y = 2x$ and $y = 6 - x$ $2x = 6 - x$ $3x = 6$ $x = 2$ When $x = 2$, $y = 2 \times 2 = 4$ $\therefore$ Point of intersection is (2,4) Corner points of shaded region are (0,0), (2,4) and (6,0) $P = 100x + 150y$ At (0,0) $P = 0$ At (2,4) $P = 200 + 600 = 800$ At (6,0) $P = 600 + 0 = 600$ $\therefore$ Maximum Profit is \$800	Question 8 C For $0 \leq W \leq 20$, $C = 0$ For $20 < W \leq 30$, C is linear with gradient of 30 So $C = 30W + k$ When $C = 0, W = 20$ $\therefore 0 = 600 + k$ $\therefore k = -600$ $\therefore$ For $20 < W \leq 30$, $C = 30W - 600$ For $30 < W \leq 40$, C is linear with gradient of 40 So $C = 40W + d$ When $C = 800, W = 40$ $\therefore 800 = 1600 + d$ $\therefore d = -800$ $\therefore$ For $30 < W \leq 40$, $C = 40W - 800$
Question 9 B It takes 6 people to make one stained glass window $\therefore$ it takes $6x$ people to make x stained glass windows. It takes 2 people to make one double glazed window $\therefore$ it takes $2y$ people to make y stained glass windows. The maximum number of employees is 24. $\therefore 6x + 2y \leq 24$ $\therefore 3x + y \leq 12$	

Module 4 Business-related mathematics. Suggested solutions.

Question 1 A $P = 60,000$ $R = 1 + \frac{5}{100} = 1.05$ $n = 3$ $A = PR^n$ $A = 60,000(1.05)^3$ $A = 69,457.5$ which is closest to \$69,000	Question 2 B The graph is decreasing so it represents depreciation $\therefore$ not E The rate at which the graph is decreasing is getting smaller with time. This suggests the depreciation is calculated on a smaller amount each year $\therefore$ not C Simple interest would give a straight line graph $\therefore$ not A The depreciation over the whole four years = 20% in total $\therefore$ not D
Question 3 E Total amount borrowed = $10.30 \times 3000 = 30,900$ $I = \frac{PRT}{100} = \frac{30,900 \times 5.5 \times 4}{100}$ $I = \\$6,798$	Question 4 B $I = \frac{PRT}{100}$ $20 = \frac{3000 \times R}{100} \times \frac{1}{4}$ $8000 = 3000R$ $R = 2\frac{2}{3}\%$ which is closest to 3%
Question 5 B Deposit = $\frac{40}{100} \times 16,800 = 6,720$ Balance = $16,800 - 6,720 = 10,080$ Amount actually paid on time payment = $66 \times 52 \times 5 = 17,160$ Interest = $17,160 - 10,080$ Interest = \$7,080	Question 6 C Effective rate of interest = $\frac{2n}{n+1}$ x flat rate = $\frac{2 \times 36}{36+1} \times 9$ = $\frac{72}{37} \times 9$ = 17.5%

Module 4 Business-related mathematics. Suggested solutions.

Question 7 B

$$A = 502,840$$

$$P = 250,000$$

$$n = 5$$

$$A = PR^n$$

$$502,840 = 250,000 \times R^5$$

$$R^5 = 2.011357$$

$$R = \sqrt[5]{2.011357}$$

$$R = 1.15$$

$$1 + \frac{r}{100} = 1.15$$

$$\frac{r}{100} = 0.15$$

$$r = 15\%$$

Question 8 B

$$P = 30,000$$

$$\text{Interest per quarter} = \frac{4.2}{4} = 1.05$$

$$R = 1 + \frac{1.05}{100} = 1.0105$$

$$n = 10 \times 4 = 40$$

$$A = PR^n$$

$$A = 30,000 \times 1.0105^{40} = 45,559$$

$$I = 45,559 - 30,000$$

$$I = 15,559$$

which is closest to \$15,600

Question 9 C

$$\text{Interest per fortnight} = \frac{5.2}{26} = 0.2$$

$$R = 1 + \frac{0.2}{100} = 1.002$$

$$5000 = 4000(1.002)^n$$

$$1.25 = 1.002^n$$

$$\log_{10} 1.25 = n \log_{10} 1.002$$

$$n = \frac{\log_{10} 1.25}{\log_{10} 1.002}$$

$$n = 111.68 \text{ fortnights}$$

$$n = \frac{111.68}{26} = 4.29 \text{ years}$$

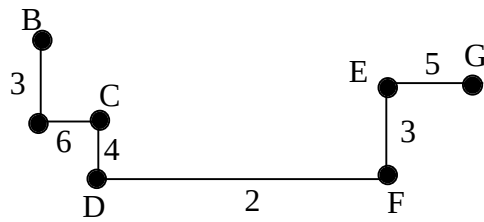
which is closest to 5 years.

Module 5 Networks and decision mathematics. Suggested solutions.

Question 1 C A shakes hands with 4 people B shakes hands with 3 other people C shakes hands with 2 other people D shakes hands with 1 other person Total number of handshakes = $4 + 3 + 2 + 1 = 10$	Question 2 D A and B are Eulerian circuits because you can traverse the network, starting and finishing at the same point. C and E are Eulerian paths because you can traverse the network but you do not finish at the point where you started.
Question 3 E The circle is considered of degree 2 plus there are 3 other paths which all adds to degree 5.	Question 4 C F to F has 2 paths (a circle). Hence, the last element in the matrix must be 2 A to B has 1 path, so the second element in the first row of the matrix is 1.
Question 5 E A road linking A to C would enable the salesperson to go from I to H to G to F to E to B to D to C to A.	Question 6 D The critical path is the path with the longest duration. B — F — I = $20 + 15 + 1 = 53$ B — G = $20 + 36 = 56$ C — D — F — I = $14 + 16 + 15 + 18 = 63$ C — D — G = $14 + 16 + 36 = 66$ C — H — J = $14 + 22 + 24 = 60$

Module 5 Networks and decision mathematics. Suggested solutions.

Question 7 E



Length of path = $3 + 6 + 4 + 2 + 3 + 5 = 23$

Question 8 C

4 ③ 6 5
 ② 7 3 4
 ③ 5 7 9
 5 ② 4 3

1 ① 3 2
 ① 5 ① 2
 0 2 4 6
 3 0 2 ①

1 0 2 1
 0 5 0 1
 0 2 3 5
 3 0 1 0

Brian does task 1
 Alan does task 4
 Jo does task 2
 Meg does task 3

Total time = $3 + 3 + 3 + 3 = 12$

Question 9 A

Float time = latest possible start time – earliest possible start time.

A critical path is A D F I J. Earliest time D can start is day 9

Latest time D can start is day 9.

Float time = $9 - 9 = 0$

End of suggested solutions 2003 Further Mathematics Trial Examination 1

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