

Year 2002

VCE

Specialist Mathematics Examination 1

Detailed Suggested Solutions

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Question 1 E PQ is north $\angle PQR = 40^\circ$ (alternate angle with the given 40° angle) $\angle PRQ = 110^\circ$ (sum of angles in triangle = 180°) Using sine rule $\frac{x}{\sin 110^\circ} = \frac{6}{\sin 40^\circ}$	Question 2 C $y = 1 + \frac{4}{x^2}$ This graph has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 1$
Question 3 D y intercept when $x = 0$ $y^2 = 1$ $y = \pm 1$ x intercept when $y = 0$ $0 = 4x^2 + 1$ $-1 = 4x^2$ But this is not possible so no x intercepts As x becomes large $y^2 \rightarrow 4x^2$ $y \rightarrow \pm 2x$ $\therefore$ asymptotes are $y = \pm 2x$	Question 4 A $\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$ $\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x \sin^2 x}$ $\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x} \times \frac{1}{\sin^2 x}$ $\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \sec^2 x \times \operatorname{cosec}^2 x$ $\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \sec^2 x \operatorname{cosec}^2 x$

Question 5 E

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{4} \times \frac{\pi}{3}$$

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi^2}{12}$$

Question 6 A

$$Z = x + yi$$

$$\bar{Z} = x - yi$$

$$Z - \bar{Z} = x + yi - (x - yi)$$

$$Z - \bar{Z} = x + yi - x + yi$$

$$Z - \bar{Z} = 0x + yi + yi$$

$$Z - \bar{Z} = 2yi$$

Question 7 B

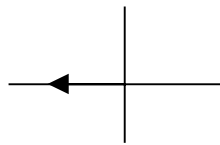
$$Z = -2i$$

$$Z^2 = 4i^2$$

$$Z^2 = -4$$

$$|Z^2| = 4$$

$$\text{Arg}(Z^2) = \pi \quad -\pi < \text{Arg}(Z) \leq \pi$$



Question 8 D

$$P(2 - i) = 0 \text{ since } 2 - i \text{ is a root}$$

$\therefore P(2 + i) = 0$ since if $2 - i$ is a root then $2 + i$ is also a root

$$(z - 2 + i)(z - 2 - i) = z^2 - 4z + 1$$

$$\begin{array}{r} 2z + 1 \\ z^2 - 4z + 1 \overline{) 2z^3 - 7z^2 + 6z + 5} \\ \underline{2z^3 - 8z^2 + 10z} \\ z^2 - 4z + 5 \\ \underline{z^2 - 4z + 5} \end{array}$$

$\therefore 2z + 1$ is a factor

$\therefore z = -\frac{1}{2}$ is a root

$\therefore$ one real root and two complex roots.

Question 9 B

$$z = x + iy$$

$$\bar{z} = x - iy$$

$$z\bar{z} = x^2 + y^2 \text{ which is a circle}$$

Shaded region lies between and on the circles

$$x^2 + y^2 = 1 \text{ and } x^2 + y^2 = 9$$

$$\therefore 1 \leq z\bar{z} \leq 9$$

Question 10 C

$$\frac{3x + 4}{(x - 4)^2} = \frac{A}{x - 4} + \frac{B}{(x - 4)^2}$$

$$\frac{3x + 4}{(x - 4)^2} = \frac{A(x - 4) + B}{(x - 4)^2}$$

$$3x + 4 = A(x - 4) + B$$

$$\text{Let } x = 4$$

$$16 = B$$

$$\therefore 3x - 12 = A(x - 4)$$

$$\text{Let } x = 5$$

$$A = 3$$

Question 11 A

$$\frac{d}{dx}(x\cos^{-1}x) = \cos^{-1}x - \frac{x}{\sqrt{1-x^2}}$$

$$x\cos^{-1}x = \int \cos^{-1}x - \int \frac{x}{\sqrt{1-x^2}}$$

$$\int \cos^{-1}x dx = x\cos^{-1}x + \int \frac{x}{\sqrt{1-x^2}} dx$$

Question 12 E

Let $u = 2e^{2x} - 1$

$$du = 4e^{2x} dx$$

$$\int \frac{du}{u} = \int \frac{4e^{2x} dx}{2e^{2x} - 1}$$

$$\int \frac{e^{2x} dx}{2e^{2x} - 1} = \frac{1}{4} \int \frac{du}{u}$$

$$\int \frac{e^{2x} dx}{2e^{2x} - 1} = \frac{1}{4} \log_e u$$

$$\int \frac{e^{2x} dx}{2e^{2x} - 1} = \frac{1}{4} \log_e (2e^{2x} - 1)$$

Question 13 C

Let $u = 2x + 1$

$\therefore 2x + 3 = u + 2$ and $\frac{du}{dx} = 2 \Rightarrow du = 2dx$

When $x = 0, u = 1$

When $x = 4, u = 9$

$$\frac{1}{2} \int_0^4 (2x + 3) \sqrt{2x + 1} (2) dx = \frac{1}{2} \int_1^9 (u + 2) \sqrt{u} du$$

Question 14 A

Make sure calculator is in radian mode.

Press Maths9 to get fnInt

then enter $((x^2 - 2x)/(2 \cos(x)), x, 0, 1)$

Press enter

This gives the answer - 0.4369

Question 15 B

$$V = \int_0^4 \pi x^2 dy$$

$$V = \pi \int_0^4 (4 - y) dy$$

$$V = \pi \left[4y - \frac{y^2}{2} \right]_0^4$$

$$V = \pi(16 - 8 - 0)$$

$$V = 8\pi$$

Question 16 E

$$y = e^{kx}$$

$$\frac{dy}{dx} = ke^{kx}$$

$$\frac{d^2y}{dx^2} = k^2 e^{kx}$$

$$\therefore k^2 e^{kx} = 5ke^{kx} - 6e^{kx}$$

$$\Rightarrow k^2 = 5k - 6$$

$$\Rightarrow k^2 - 5k + 6 = 0$$

$$(k - 3)(k - 2) = 0$$

$$k = 3 \text{ or } 2$$

Question 17 D

$$\frac{dr}{dt} = 3$$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt} = 4\pi r^2 \times 3$$

$$\frac{dV}{dt} = 12\pi r^2$$

$$\text{When } r = 8$$

$$\frac{dV}{dt} = 12 \times \pi \times 64$$

$$\frac{dV}{dt} = 768\pi$$

Question 18 B

$$-2 \cos(30)^\circ \mathbf{j} + 2 \sin(30)^\circ \mathbf{k}$$

$$= -2 \times \frac{\sqrt{3}}{2} \mathbf{j} + 2 \times \frac{1}{2} \mathbf{k}$$

$$= -\sqrt{3} \mathbf{j} + \mathbf{k}$$

Question 19 D

Taking the 5 possible answers in turn and checking the dot product which $= 0$ for perpendicular lines.

A. $2 + 4 + 6 \neq 0$

B. $2 + 4 - 6 = 0$

C. $2 + 4 - 6 = 0$

D. $-4 - 2 + 6 = 0$

E. $-4 - 2 + 6 = 0$

Checking the answers B, C D, and E for magnitude.

B. $\frac{4}{9}\sqrt{1+4+4} = \frac{12}{9} \neq 4$

C. $4\sqrt{1+4+4} = 12 \neq 4$

D. $\frac{4}{3}\sqrt{4+1+4} = 4$

E. $\frac{4}{9}\sqrt{4+1+4} = \frac{4}{3} \neq 4$

$\therefore$ answer is D

Question 20 A

Let $\mathbf{a} = -\hat{i} - \hat{j} + 3\hat{k}$

Let $\mathbf{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$(\mathbf{a} \cdot \mathbf{b})\hat{\mathbf{b}} = -\hat{i} - 2\hat{j} + 2\hat{k}$

$\mathbf{a} - (\mathbf{a} \cdot \mathbf{b})\hat{\mathbf{b}} = -\hat{i} - \hat{j} + 3\hat{k} - (-\hat{i} - 2\hat{j} + 2\hat{k})$

$\mathbf{a} - (\mathbf{a} \cdot \mathbf{b})\hat{\mathbf{b}} = \hat{j} + \hat{k}$ which is the vector resolute of $\mathbf{a}$ perpendicular to $\mathbf{b}$

Question 21 B

Must show that $\vec{PQ} = k\vec{QS}$

where $k \in \mathbb{R}$

$\vec{PQ} = \vec{PO} + \vec{OQ}$

$\vec{PQ} = \vec{OQ} - \vec{OP}$

$\vec{PQ} = \mathbf{q} - \mathbf{p}$

$\vec{QS} = \vec{QO} + \vec{OS}$

$\vec{QS} = \vec{OS} - \vec{OQ}$

$\vec{QS} = \mathbf{s} - \mathbf{q}$

$\therefore$ Must show that $\mathbf{q} - \mathbf{p} = k(\mathbf{s} - \mathbf{q})$

Question 22 A

$\mathbf{a} = \frac{dv}{dt} = \cos(t-1)\hat{i} + 3e^{1-t}\hat{k}$

When $t = 1$

$\mathbf{a} = \cos 0\hat{i} + 3e^0\hat{k}$

$\mathbf{a} = 1\hat{i} + 3\hat{k}$

$\mathbf{a} = \hat{i} + 3\hat{k}$

Question 23 C

Height refers to $\hat{j}$ component

Maximum height when

$$2t - 6 = 0$$

$$\Rightarrow 2t = 6$$

$$\Rightarrow t = 3$$

Question 24 C

$$v = \int 2 \sin t \hat{i} \, dt$$

$$v = -2 \cos t \hat{i} + \vec{c}$$

$$2\hat{i} + 2\hat{j} = -2 \cos \pi \hat{i} + \vec{c}$$

$$2\hat{i} + 2\hat{j} = 2\hat{i} + \vec{c}$$

$$\vec{c} = 2\hat{j}$$

$$\therefore v = -2 \cos t \hat{i} + 2\hat{j}$$

$$v(0) = -2 \cos 0 \hat{i} + 2\hat{j}$$

$$v(0) = -2\hat{i} + 2\hat{j}$$

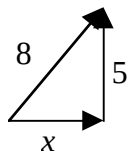
Question 25 B

Using Pythagoras Theorem

$$x^2 = 8^2 - 5^2$$

$$x = \sqrt{64 - 25}$$

$$x = \sqrt{39}$$



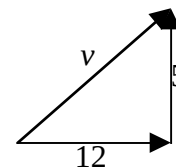
Question 26 D

$$\vec{p} = m \vec{v}$$

$$\vec{v} = 13$$

$$\vec{p} = 2 \times 13$$

$$\vec{p} = 26$$



Question 27 E

In E the resultant force (without F) is acting down the plane. Friction always opposes the motion, so F would have to be acting up the plane.

Question 28 C

$$T = m_2 g$$

$$T = m_1 g \sin \theta$$

$$m_2 g = m_1 g \sin \theta$$

$$m_2 = m_1 \sin \theta$$

$$\frac{1}{\sin \theta} = \frac{m_1}{m_2}$$

Question 29 C

Dropped from rest means $v = 0$ when $t = 0$

Hence, not A or B

The velocity increases and becomes more and more positive until the ball hits the floor.

When the ball rebounds the velocity is negative and becomes less and less negative.

Hence, not E

In D the same change of velocity takes the same time and so the height reached would be the same.

Hence, C.

Question 30 D

$$\text{When } t = 0, P = \frac{20}{100} \times 150 = 30$$

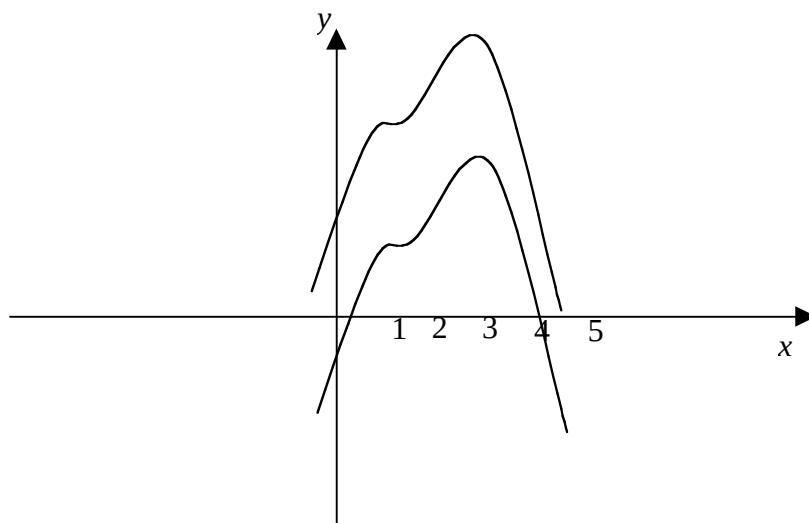
$$\frac{dP}{dt} = \text{Input} - \text{Output}$$

$$\frac{dP}{dt} = 10 - \frac{P}{150} \times 10$$

$$\frac{dP}{dt} = 10 - \frac{P}{15} \quad t = 0, P = 30$$

Question 1 a. Centre $(-1, 2)$ Equation of ellipse of form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$where $h = -1, k = 2, a = 1, b = 3$ $\therefore \frac{(x+1)^2}{1} + \frac{(y-2)^2}{9} = 1$	Question 1 b. Translate 1 unit to the left parallel to the X axis. Translate 2 units up parallel to the Y axis. Dilate 3 units parallel to the Y axis.
Question 2 Distance traveled by car in 60 secs. = Distance traveled by truck in 60 secs. = 17×60 $\therefore 24T + \frac{24(60 - T)}{2} = 1020$$24T + 720 - 12T = 1020$$12T = 300$$T = 25 \text{ sec.}$	Question 3 Let $u = \cos(\frac{x}{2})$ $du = -\frac{1}{2} \sin(\frac{x}{2}) dx$$\int \frac{\sin \frac{x}{2}}{\cos^2 \frac{x}{2}} dx = -2 \int \frac{\frac{-1}{2} \sin \frac{x}{2}}{u^2} dx$$\int \frac{\sin \frac{x}{2}}{\cos^2 \frac{x}{2}} dx = -2 \int \frac{du}{u^2}$$\int \frac{\sin \frac{x}{2}}{\cos^2 \frac{x}{2}} dx = -2 \int u^{-2} du$$\int \frac{\sin \frac{x}{2}}{\cos^2 \frac{x}{2}} dx = -2 \frac{u^{-1}}{-1} = \frac{2}{u} + c \text{ where } c \text{ is a constant}$$\int \frac{\sin \frac{x}{2}}{\cos^2 \frac{x}{2}} dx = \frac{2}{\cos \frac{x}{2}} + c$ An antiderivative is $2 \sec(\frac{x}{2})$

Question 4



Turning Point at $x = 1$ and $x = 3$

Gradient negative when $x > 3$

Gradient positive when $x < 3$ except when $x = 1$

Maximum when $x = 3$

Stationary point of inflexion when $x = 1$

Question 5

$$|\mathbf{u}| = \sqrt{1+1+1} = \sqrt{3}$$

$$|\mathbf{v}| = \sqrt{1+a^2+1} = \sqrt{2+a^2}$$

$$\mathbf{u} \cdot \mathbf{v} = 1 - a - 1 = -a$$

$$\mathbf{u} \cdot \mathbf{v} = \sqrt{3} \times \sqrt{2+a^2} \cos 60^\circ = -a$$

$$a = -\frac{\sqrt{3} \times \sqrt{2+a^2}}{2}$$

$\therefore a$ is a negative number

$$2a = -\sqrt{3} \times \sqrt{2+a^2}$$

$$4a^2 = 3(2+a^2)$$

$$a^2 = 6$$

$$a = -\sqrt{6} \text{ since } a \text{ is negative.}$$

Question 6

$$\frac{dv}{dt} = -2 + \sqrt{v^2 + 5}$$

$$\frac{dt}{dv} = \frac{1}{-2 + \sqrt{v^2 + 5}}$$

$$t = \int_3^{10} \frac{1}{-2 + \sqrt{v^2 + 5}} dv$$

Use graphics calculator

Math9 fnInt(Integrand, variable, 3, 10)

$t = 1.7$ secs to 2 significant figures

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