

# **Year 2002**

## **VCE**

### **Mathematical Methods Written Examination 1**

### **Detailed Suggested Solutions**

© Kilbaha Pty Ltd 2003

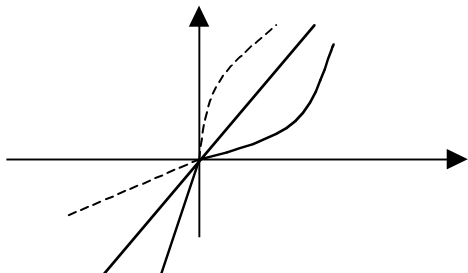


Kilbaha Pty Ltd Publishers ABN 47 065 111 373  
PO Box 2227  
Kew Vic 3101  
Australia  
Tel: 03 9817 5374  
Fax: 03 9817 4334  
[chemas@chemas.com](mailto:chemas@chemas.com)  
[www.chemas.com](http://www.chemas.com)

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>Question 1 E</b></p> <p>Period = <math>\frac{2\pi}{n} = 4\pi</math></p> <p><math>4\pi n = 2\pi</math></p> <p><math>n = \frac{1}{2}</math></p> <p>Amplitude = 1</p> <p>Translated up 1</p> <p>It is a cos graph since none of the alternatives are translated to the left or right, and the graph is a maximum when <math>x = 0</math></p> <p>Hence the equation of the graph is</p> <p><math>y = 1 + \cos \frac{1}{2}x</math></p> <p>i.e. <math>y = 1 + \cos \frac{x}{2}</math></p>         | <p><b>Question 2 E</b></p> <p><math>\sin 2x = 1 \quad 0 \leq x \leq 4\pi</math></p> <p><math>0 \leq 2x \leq 8\pi</math></p> <p><math>2x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}</math></p> <p><math>x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}</math></p> <p>Sum of solutions = <math>\frac{\pi}{4} + \frac{5\pi}{4} + \frac{9\pi}{4} + \frac{13\pi}{4}</math></p> <p><math>= \frac{28\pi}{4} = 7\pi</math></p>             |
| <p><b>Question 3 D</b></p> <p>Maximum temperature when</p> <p><math>\sin \frac{\pi t}{12} = -1</math></p> <p><math>\frac{\pi t}{12} = \frac{3\pi}{2}</math></p> <p><math>\frac{t}{12} = \frac{3}{2}</math></p> <p><math>t = \frac{36}{2} = 18</math></p> <p>Time = 18 hrs. after midnight = 6.00p.m.</p>                                                                                                                                                                                          | <p><b>Question 4 E</b></p> <p>The equation of the function is of the form</p> <p><math>y = a(x+2)(x+1)(x-1)(x-3)</math></p> <p>When <math>x = 0</math></p> <p><math>y = a \times (+) \times (+) \times (-) \times (-) = \text{negative}</math></p> <p>Hence, <math>a</math> is negative</p> <p>A possible equation is</p> <p><math>y = -(x+2)(x+1)(x-1)(x-3)</math></p> <p><math>y = (x+2)(x+1)(x-1) \times (-)(x-3)</math></p> <p><math>y = (x+2)(x+1)(x-1)(3-x)</math></p> |
| <p><b>Question 5 C</b></p> <p>A vertical asymptote exists when <math>x = -b</math></p> <p><math>\therefore -b = 3</math></p> <p><math>\therefore b = -3</math></p> <p>A horizontal asymptote exists when <math>y = c</math></p> <p><math>\therefore c = 2</math></p> <p><math>y = \frac{a}{x-3} + 2</math></p> <p>When <math>x = 4, y = 0</math></p> <p><math>0 = \frac{a}{4-3} + 2</math></p> <p><math>0 = a + 2</math></p> <p><math>a = -2</math></p> <p><math>a = -2, b = -3, c = 2</math></p> | <p><b>Question 6 A</b></p> <p><math>y = f(-x)</math> is a reflection in the Y axis of <math>y = f(x)</math></p> <p>Hence, the vertical asymptote will be <math>x = -1</math></p>                                                                                                                                                                                                                                                                                             |

**Question 7 A**

The inverse function is a reflection in the line  $y=x$   
 The only graph that is a reflection of the given graph in this line is A

**Question 8 C**

Using a graphical calculator, use the STAT button and edit  
 Enter the  $x$  values in  $L_1$  and the  $y$  values in  $L_2$ .  
 Press the STAT PLOT button and plot.  
 The shape of the graph is exponential, hence, the given data would be best modeled using an exponential function.

**Question 9 B**

$$\begin{aligned}
 x^4 + x^3 - 3x^2 - 3x &= x(x^3 + x^2 - 3x - 3) \\
 &= x[x^2(x+1) - 3(x+1)] \\
 &= x(x+1)(x^2 - 3) \\
 &= x(x+1)(x - \sqrt{3})(x + \sqrt{3})
 \end{aligned}$$

Hence, the linear factors are

$$x, x+1, x+\sqrt{3}, x-\sqrt{3}$$

**Question 10 A**

$$\text{Let } y = 2^{\log_2(x+5)}$$

$$\text{Then } \log_2 y = \log_2 2^{\log_2(x+5)}$$

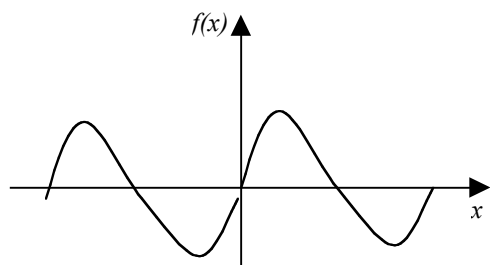
$$\log_2 y = \log_2(x+5) \log_2 2$$

$$\text{But } \log_2 2 = 1$$

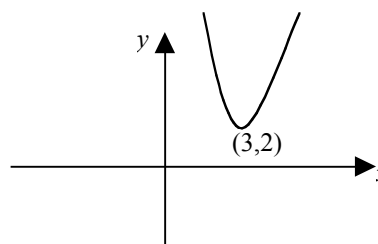
$$\text{So } \log_2 y = \log_2(x+5)$$

$$\text{So } y = x+5$$

$$\text{So } 2^{\log_2(x+5)} = x+5$$

**Question 11 D**

The graph of  $f(x)=\sin(x)$ , where  $x \in \mathbb{R}$  is as shown above. For one value of  $y$  there are many values of  $x$ . Hence it is not a one to one function.

**Question 12 D**

This graph will have an inverse function if it is a one to one function. Hence, restrict the domain so that  $x \leq 3$  or  $x \geq 3$ . From the given alternatives, the answer would be D.

**Question 13 C**

$f'(x)$  is the gradient of the given function.

For the given function which is always increasing, the gradient is always positive. this means the gradient graph is always above the X axis. Hence the only possibilities are B or C.

The given graph increases by less and less as it goes from left to right. Hence, the gradient becomes less and less positive. Thus C is the graph of  $f'(x)$ .

**Question 14 B**

$$y = \log_e(\cos(2x))$$

$$\text{Let } u = \cos(2x)$$

$$\frac{du}{dx} = -2\sin(2x)$$

$$y = \log_e u$$

$$\frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{u} \times -2\sin(2x)$$

$$\frac{dy}{dx} = \frac{-2\sin(2x)}{\cos(2x)}$$

$$\frac{dy}{dx} = -2 \times \frac{\sin(2x)}{\cos(2x)}$$

$$\frac{dy}{dx} = -2 \tan(2x)$$

**Question 15 D**

$$y = x \sin(x)$$

$$\text{When } x = \pi, y = \pi \sin \pi = \pi \times 0 = 0$$

$$\therefore \text{ given point is } (\pi, 0)$$

$$\frac{dy}{dx} = x \cos(x) + \sin(x)$$

$$\text{When } x = \pi,$$

$$\frac{dy}{dx} = \pi \cos(\pi) + \sin(\pi)$$

$$\frac{dy}{dx} = \pi \times -1 + 0$$

$$\frac{dy}{dx} = -\pi$$

$$\text{Gradient of normal} = \frac{-dx}{dy} = \frac{1}{\pi}$$

$$\text{Equation of normal is of form } y = mx + c$$

$$\text{So } y = \frac{1}{\pi}x + c$$

$$\text{When } x = \pi, y = 0$$

$$\text{So } 0 = \frac{1}{\pi} \times \pi + c$$

$$0 = 1 + c$$

$$c = -1$$

$$\text{Hence, equation of normal is}$$

$$y = \frac{1}{\pi}x - 1$$

$$y = \frac{1}{\pi}(x - \pi)$$

**Question 16 C**

$$\text{Rate of change with respect to } x \text{ is } \frac{dy}{dx}$$

$$y = e^{-x} - 1$$

$$\frac{dy}{dx} = -e^{-x}$$

$$\text{When } x = 0$$

$$\frac{dy}{dx} = -e^{-0} = -1$$

**Question 17 A**

$$f(x+h) \approx f(x) + hf'(x)$$

$$f(x) = x^2$$

$$3.02^2 = f(3.02)$$

$$= f(3 + 0.02) \approx f(3) + 0.02 f'(3)$$

**Question 18 B**

$$f(0) = 0 \text{ is point } (0,0)$$

$$f(-3) = 0 \text{ is the point } (-3,0)$$

$$f'(0) = 0 \text{ means there is a stationary point at } (0,0)$$

Hence not C

$$f'(-1) = 0 \text{ means there is a stationary point when } x = -1$$

$$f'(x) > 0 \text{ for } x < -1 \text{ means there is a positive gradient when } x < -1$$

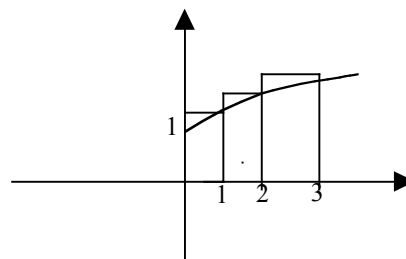
This means that the graph is rising when  $x < -1$

Hence, not A, C or E

$$f'(x) < 0 \text{ for } x > -1 \text{ except } x = 0 \text{ means there is a negative gradient when } x > -1 \text{ except } x = 0$$

This means that the graph is falling when  $x > -1$  except when  $x = 0$

Hence, not D

**Question 19 A**

Right rectangle approximation

$$= f(1) \times 1 + f(2) \times 1 + f(3) \times 1$$

$$f(1) = \sqrt{1+1} = \sqrt{2}$$

$$f(2) = \sqrt{1+2} = \sqrt{3}$$

$$f(3) = \sqrt{1+3} = \sqrt{4} = 2$$

$$\text{So right rectangle approximation} = \sqrt{2} + \sqrt{3} + 2$$

**Question 20 B**

$$f'(x) = 2 \cos(5x)$$

$$f(x) = \int 2 \cos(5x) dx$$

$$f(x) = 2 \int \cos(5x) dx$$

$$f(x) = 2 \times \left[ \frac{1}{5} \sin(5x) \right] + c$$

$$f(x) = \frac{2}{5} \sin(5x) + c$$

**Question 21 B**

$$\frac{dy}{dx} = 3(2x+1)^{-\frac{1}{2}}$$

$$y = 3 \int (2x+1)^{-\frac{1}{2}} dx$$

$$y = 3 \left[ \frac{\frac{1}{2}(2x+1)^{\frac{1}{2}}}{\frac{1}{2}} \right] + c$$

$$y = 3(2x+1)^{\frac{1}{2}} + c$$

**Question 22 D**

$$A = \int_a^b f(x)dx + \left| \int_b^c f(x)dx \right|$$

$$A = \int_a^b f(x)dx - \int_b^c f(x)dx$$

$$A = \int_a^b f(x)dx + \int_c^b f(x)dx$$

**Question 23 D**

Of the given alternatives, the discrete random variable would be the one that could only have integer values. Hence, the number of goals scored in a football match is a discrete random variable.

**Question 24 A**

Without replacement is hypergeometric.

$$\Pr(X = x) = \frac{{}^D C_x {}^{N-D} C_{n-x}}{{}^N C_n}$$

$$x \geq 0$$

$$N = 20$$

$$n = 4$$

$$D = 5$$

$$\Pr(x \geq 0) = 1 - \Pr(x = 0)$$

$$\Pr(x \geq 0) = 1 - \frac{{}^5 C_0 {}^{15} C_4}{{}^{20} C_4}$$

$$\Pr(x \geq 0) = 1 - \frac{1 \times {}^{15} C_4}{{}^{20} C_4}$$

$$\Pr(x \geq 0) = 1 - \frac{{}^{15} C_4}{{}^{20} C_4}$$

**Question 25 A**

$\mu = np$  where  $p$  is the probability defective.

$$\mu = 10$$

$$\sigma = \sqrt{npq} = \sqrt{\mu q} = \sqrt{10q}$$

$$3 = \sqrt{10q}$$

$$9 = 10q$$

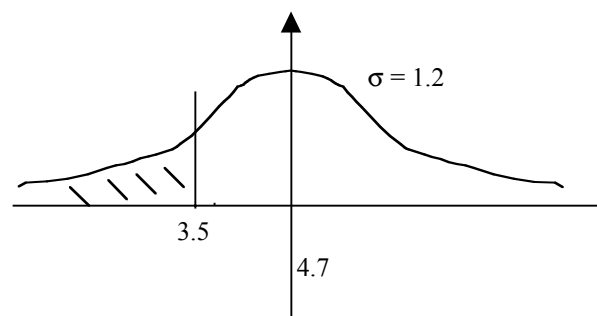
$$q = \frac{9}{10} = 0.9$$

$$p = 1 - q$$

$$p = 1 - 0.9$$

$$p = 0.1$$

## Question 26 B



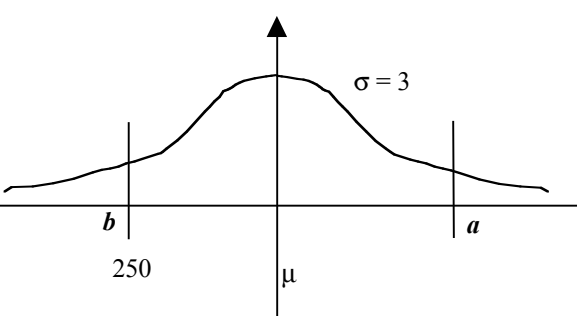
$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{3.5 - 4.7}{1.2}$$

$$Z = -1$$

$$\Pr(X < 3.5) = \Pr(Z < -1) = \Pr(Z > 1)$$

## Question 27 E



$$\Pr(X < 250) = 0.01$$

$$\Pr(Z > a) = 0.01$$

$$\Pr(Z < a) = 1 - 0.01 = 0.99$$

$$a = 2.33 \text{ (from tables)}$$

$$\text{So } b = -2.33$$

$$Z = \frac{x - \mu}{\sigma}$$

$$-2.33 = \frac{250 - \mu}{3}$$

$$-6.99 = 250 - \mu$$

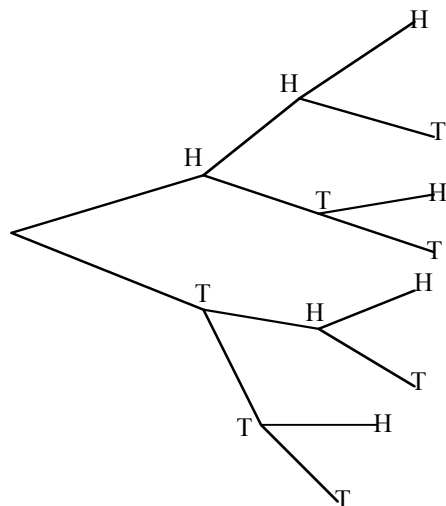
$$\mu = 256.99 \approx 257$$

**2002 Mathematical Methods Written Examination 1  
Suggested Solutions**

**End of Multiple-Choice Questions**

**Question 1.**

(a) Outcome for 3 tosses



Outcomes are HHH, HHT, HTH, HTT, THH, THT, TTH, TTT  
Total outcomes = 8

| No. Heads | Pr            |
|-----------|---------------|
| 0         | $\frac{1}{8}$ |
| 1         | $\frac{3}{8}$ |
| 2         | $\frac{3}{8}$ |
| 3         | $\frac{1}{8}$ |

(b)

$$\begin{aligned}
 E(X) &= \sum x \Pr(X = x) \\
 &= (0 \times \frac{1}{8}) + (1 \times \frac{3}{8}) + (2 \times \frac{3}{8}) + (3 \times \frac{1}{8}) \\
 &= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} \\
 &= \frac{12}{8} = 1.5
 \end{aligned}$$

### Question 2.

Without replacement is hypergeometric. There are 95 good and 5 defective.

Pr accepted = Pr number of defectives in sample = 0

$$\begin{aligned}\Pr(X = x) &= \frac{{}^D C_x {}^{N-D} C_{n-x}}{{}^N C_n} \text{ where } D = 5, x = 0, N = 100, n = 5 \\ &= \frac{{}^5 C_0 {}^{95} C_5}{{}^{100} C_5} = 0.770 \text{ to 3 d.p.}\end{aligned}$$

### Question 3.

(a) Period is the distance along the  $X$  axis before the graph repeats its pattern. Period = 8

(b) Amplitude is the distance above and below the central horizontal line, which in this case is the line  $y = -0.5$ . The amplitude is thus equal to 1.

### Question 4.

|                                                                                                                                                                                                                                                                                     |                                                                                                        |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|
| <p>(a)</p> $\begin{aligned}2 \times 2^{-2x} &= 2002 \\ 2^{-2x} &= 1001 \\ \log_{10} 2^{-2x} &= \log_{10} 1001 \\ -2x \log_{10} 2 &= \log_{10} 1001 \\ -2x \times 0.30103 &= 3.0004 \\ -2x &= \frac{3.0004}{0.30103} \\ -2x &= 9.9672 \\ x &= -4.984 \text{ to 3 d.p.}\end{aligned}$ | <p>(b)</p> $2 \log_e (3x + 1) - \log_e x = \log_e (3x + 1)^2 - \log_e x = \log_e \frac{(3x + 1)^2}{x}$ |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|

### Question 5.

|                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>(a)</p> $\begin{aligned}\text{Dilation : } y &= \frac{\frac{1}{2}}{x} = \frac{1}{2x} \\ \text{Reflection in } Y \text{ axis : } y &= -\frac{1}{2x} \\ \text{Translation +3 parallel to } X \text{ axis : } y &= -\frac{1}{2(x-3)} \\ \text{Translation +1 parallel to } Y \text{ axis : } y &= -\frac{1}{2(x-3)} + 1\end{aligned}$ | <p>(b)</p> $\begin{aligned}x - 3 &\neq 0 \\ \text{Hence, } x &\neq 3 \\ \text{Domain : } R \setminus \{3\} \\ \text{There is a horizontal asymptote, } y &= 1 \\ \text{Hence, Range : } R \setminus \{1\}\end{aligned}$ |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

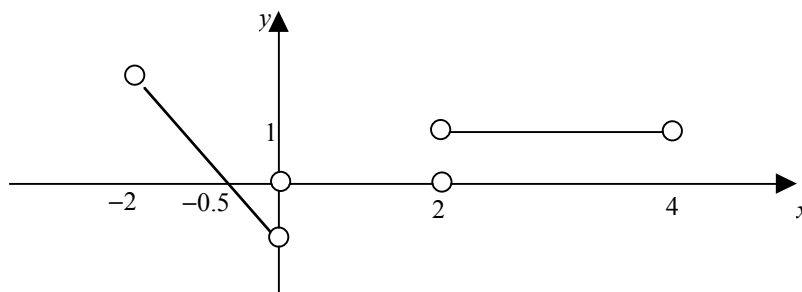
**Question 6.**

|                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                 |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
| <p>(a)</p> $2 \sin(3x) - 1 = -0.5 \quad 0 \leq x \leq \frac{\pi}{2}$ $2 \sin(3x) = 0.5 \quad 0 \leq 3x \leq \frac{3\pi}{2}$ $\sin(3x) = 0.25 \quad 0 \leq 3x \leq 4.7124$ $3x = 0.25268, \pi - 0.25268$ $3x = 0.25268, 2.8889$ $x = 0.084, 0.963$                                                                                                                                                                          | <p>(b)</p> $f'(x) = 6 \cos(3x)$ <p>When <math>x = 1</math>, <math>6 \cos(3) = -5.940</math></p> |
| <p>(c)</p> <p>Want <math>f'(x) &gt; 0</math><br/>         Want <math>6 \cos(3x) &gt; 0</math><br/>         The graph of <math>6 \cos(3x)</math> has a period of <math>\frac{2\pi}{3}</math></p> <div data-bbox="597 604 1318 1054" data-label="Figure"> </div> <p>From the graph it can be seen that the rate of change is positive for <math>0 &lt; x &lt; \frac{\pi}{6}</math> i.e. <math>0 &lt; x &lt; 0.524</math></p> |                                                                                                 |

### Question 7.

(a)

For a function to have a derivative at a point, the graph must be continuous at that point. For  $-2 < x < 0$ , the graph is continuous. The gradient is positive in this region up to  $x = -0.5$  and then it is negative. No derivative exists at  $x = 0$  because the graph is not continuous at this point. For  $0 < x < 2$ , the gradient is zero. No gradient exists at  $x = 2$  because a tangent cannot be drawn at this point. Here the graph is continuous but not differentiable. For  $2 < x < 4$  the gradient is a positive constant value.



(b)

The domain of  $f'(x)$  can be seen from the graph to be  $\{x: -2 < x < 4\} \setminus \{0, 2\}$

### Question 8.

(a)

$$2x^2 + 4x - 5 = 3x + 1$$

$$2x^2 + x - 6 = 0$$

$$(2x - 3)(x + 2) = 0$$

$$x = \frac{3}{2}, x = -2$$

(b)

$$A = \int_{-2}^{\frac{3}{2}} (3x + 1 - 2x^2 - 4x + 5) dx$$

$$A = \int_{-2}^{\frac{3}{2}} (6 - x - 2x^2) dx$$

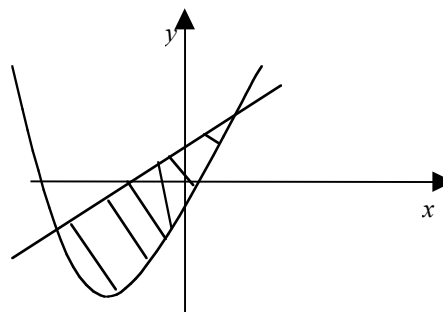
$$A = 6x - \frac{x^2}{2} - \frac{2x^3}{3} \Big|_{-2}^{\frac{3}{2}}$$

$$A = \left(9 - \frac{9}{8} - \frac{9}{4}\right) - \left(-12 - 2 + \frac{16}{3}\right)$$

$$A = 9 - \frac{27}{8} + 14 - \frac{16}{3}$$

$$A = 23 - 8.7083$$

$$A = 14.292 \text{ sq units to 3 d.p.}$$



End of suggested solutions 2002 VCE Mathematical Methods Examination 1

KILBAHA PTY LTD (Publishers in Education)  
PO BOX 2227 KEW  
VIC 3101  
AUSTRALIA

TEL: (03) 9817 5374  
FAX: (03) 9817 4334  
chemas@chemas.com  
[www.chemas.com](http://www.chemas.com)