

Year 2002

VCE

Further Mathematics Written Examination 2

Detailed Suggested Solutions

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Question 1

a. The independent variable is the year, because the salary depends upon the year.

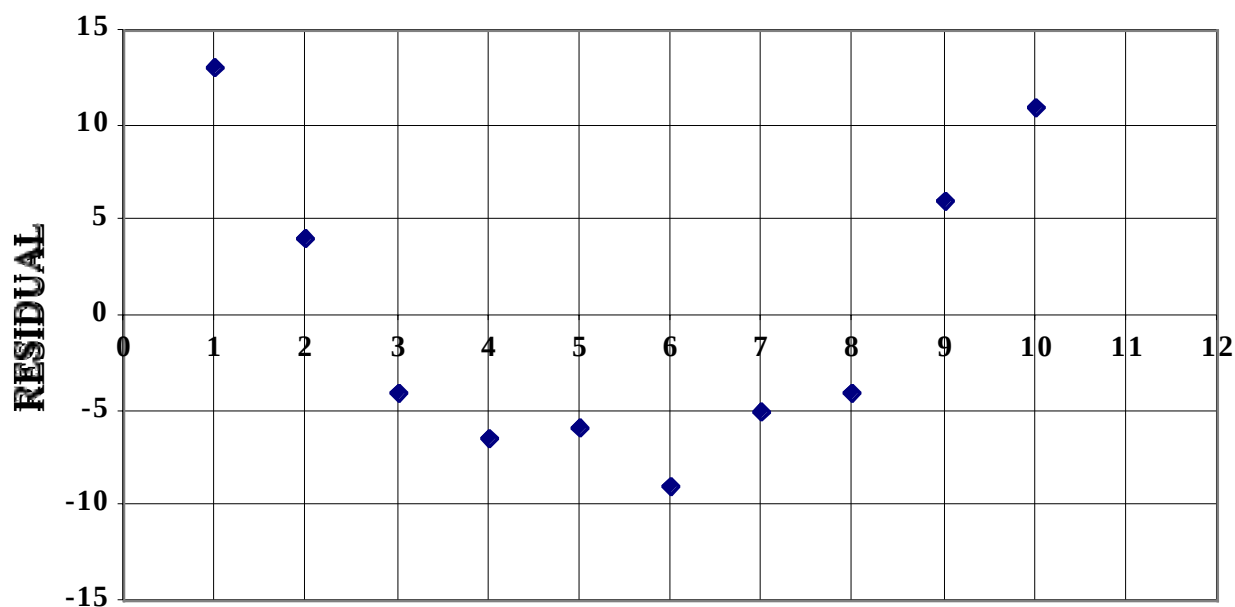
b.(i) The average salary increase is the gradient of the line given by the regression equation, $y = 20.9 + 10.99x$. The gradient is the coefficient of the x .
Hence, average salary increase = 10.99 thousands of dollars per year.
= \$10990 per year

b.(ii) Year 2005 is $x = 15$
 $y = 20.9 + 10.99 \times 15$
 $y = 185.75$ thousands of dollars
 $y = \$185,750$

c.(i) For 1995, $x = 5$
 $y = 20.9 + 10.99 \times 5 = 75.85$

~~Difference between this value and the value on the time series plot which is $70 = 70 - 75.85 = -5.85$~~

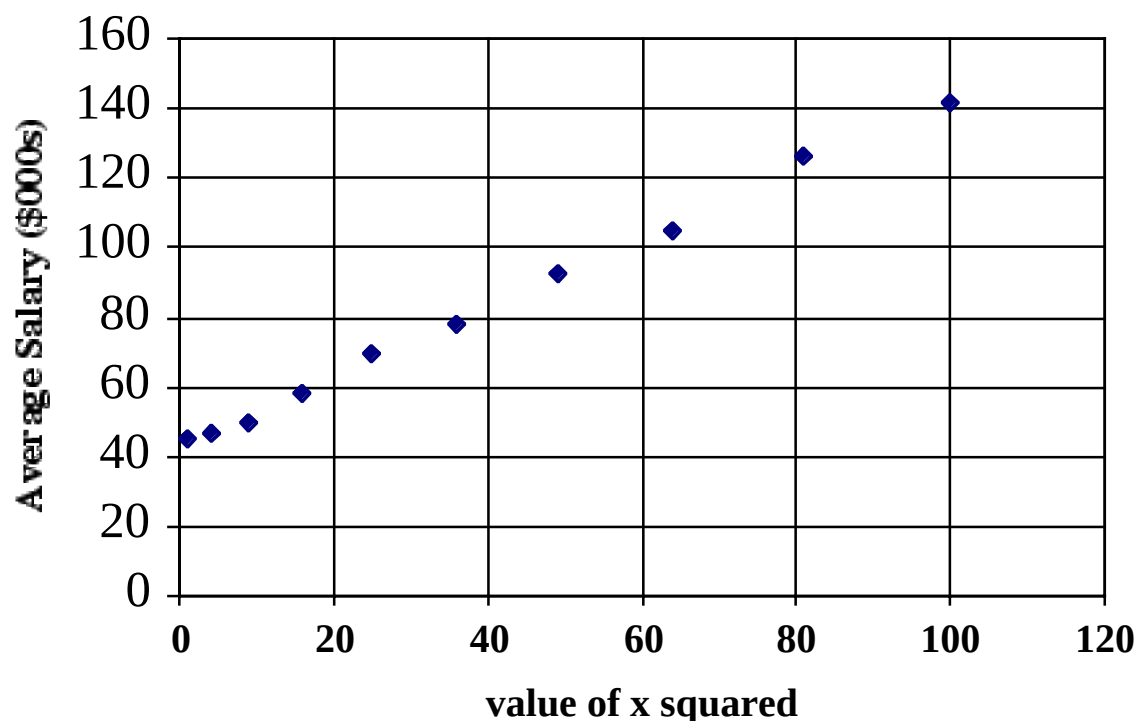
c.(ii)



d.(i)

Year (x)	1	2	3	4	5	6	7	8	9	10
Year ² (x ²)	1	4	9	16	25	36	49	64	81	100
Average salary (y) (\$000s)	45	47	50	58	70	78	93	105	126	142

d.(ii)



d.(iii)

Using the graphics calculator, Stat menu, and putting x^2 into L_1 and average salary into y gives the regression line, $y = 1x^2 + 42.89$

i.e. $y = 42.89 + 1x^2$

d.(iv)

$$y = 42.89 + x^2$$

In 2005, $x = 15$

$$\Rightarrow x^2 = 225$$

$$\therefore y = 42.89 + 225 = 267.89 \text{ thousand dollars}$$

$$\therefore y = \$267,890$$

d.(v)

The points on this residual plot are randomly scattered above and below the line $y = 0$, which suggests that the x^2 transformation has been successful in linearising the time series plot.

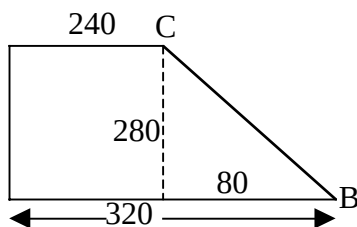
Question 1(a) Arithmetic sequence with $a = 2$ $n = 3$ $d = 3$ $t_3 = a + 2d$ $t_3 = 2 + 2 \times 3 = 8$	(b) $S_n = \frac{n}{2}[2a + (n-1)d]$ $a = 2$ $n = 10$ $d = 3$ $S_{10} = \frac{10}{2}[4 + 9 \times 3]$ $S_{10} = 5[4 + 27]$ $S_{10} = 5 \times 31$ $S_{10} = 155$
(c) $t_n = 50 = a + (n-1)d$ $50 = 2 + (n-1) \times 3$ $48 = (n-1) \times 3$ $16 = n-1$ $17 = n$ In the 17th week	Question 2 dragonscale:dogwort:honey=3:2:4 Total of these parts = $3+2+4 = 9$ Fraction dogwort = $\frac{2}{9}$ So for 27 grams, need $\frac{2}{9}$ of 27 = 6 grams of dogwort.
Question 3(a) $T_{n+1} = 2T_n - 11$ $T_2 = T_{1+1} = 2T_1 - 11 = 27$ $\Rightarrow 2T_1 = 38$ $\Rightarrow T_1 = 19$ But $T_1 = k$ $\therefore k = 19$	(b) $T_3 = 2T_2 - 11$ $T_3 = 2 \times 27 - 11$ $T_3 = 43$ $T_4 = 2T_3 - 11$ $T_4 = 2 \times 43 - 11$ $T_4 = 75$ $T_5 = 2T_4 - 11$ $T_5 = 2 \times 75 - 11$ $T_5 = 139$ $T_6 = 2T_5 - 11$ $T_6 = 2 \times 139 - 11$ $T_6 = 267$ There are 267 sick dragons in week 6

Question 3(c) The sequence is 19, 27, 43, 75, 139, 267..... This sequence is neither an arithmetic sequence nor a geometric sequence because it does not have a common difference or a common ratio.	Question 4(a) This is a geometric sequence with $a = 80$ $r = \frac{1}{2}$ $t_n = ar^{n-1}$ $t_5 = 80 \times \left(\frac{1}{2}\right)^4$ $t_5 = \frac{80}{16} = 5$ They added 5 km² in the fifth year.
Question 4(b) $S_6 = \frac{a(1-r^6)}{1-r}$ $S_6 = \frac{80(1-\left(\frac{1}{2}\right)^6)}{1-\frac{1}{2}}$ $S_6 = 160(1-\left(\frac{1}{2}\right)^6)$ $S_6 = 157.5 \text{ km}^2$	Question 4(c) Total area = 230 + S_∞ of a geometric series $= 230 + \frac{a}{1-r}$ $= 230 + \frac{80}{1-\frac{1}{2}}$ $= 230 + 160$ $= 390 \text{ km}^2$

Question 1(a) $\text{Area} = \frac{1}{2}bc\sin A$ $\text{Area} = \frac{1}{2} \times 1203 \times 951 \sin 80^\circ$ $\text{Area} = 563336.13$ $\text{Area} = 563336 \text{ square metres to the nearest sq m}$	(b) Using the cosine rule $(PR)^2 = 1203^2 + 951^2 - 2 \times 1203 \times 951 \times \cos 80^\circ$ $(PR)^2 = 1954284.563$ $PR = \sqrt{1954284.563}$ $PR = 1397.957$ $PR = 1398 \text{ m to the nearest metre.}$
(c) Using the sine rule $\frac{951}{\sin \angle RPQ} = \frac{1397.957}{\sin 80^\circ}$ $\sin \angle RPQ = \frac{951 \sin 80^\circ}{1397.957}$ $\sin \angle RPQ = 0.6699$ $\angle RPQ = \sin^{-1} 0.6699$ $\angle RPQ = 42.06^\circ$ $\text{Bearing of R from P} = 90^\circ - 42.06^\circ$ $\text{Bearing of R from P} = 47.94^\circ$ $\text{Bearing of R from P} = 048^\circ \text{ to the nearest degree}$	(d) $\text{Area} = \frac{1}{2}bc\sin A \text{ where } A \text{ is the angle } SPR$ $\angle SPR = 48^\circ - 15^\circ = 33^\circ$ $\text{Area} = \frac{1}{2} \times 1048 \times 1397.957 \sin 33^\circ$ $\text{Area} = 398964 \text{ sq metres}$ $\text{Area} = 398960 \text{ m}^2$ to the nearest 10 square metres. (e) $\text{Area of camping ground} = 398964 + 563336$ $\text{Area of camping ground}$ $= 962300 \text{ square metres}$ to the nearest 10 square metres.

Question 2

(a)(i)



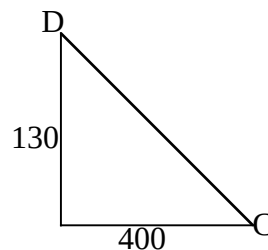
Using Pythagoras' Theorem

$$BC^2 = 80^2 + 280^2 = 84800$$

$$BC = \sqrt{84800} = 291.2$$

= 291m to the nearest metre

(a)(ii)



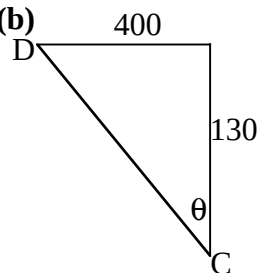
Using Pythagoras' Theorem

$$CD^2 = 130^2 + 400^2 = 176900$$

$$CD = \sqrt{176900} = 420.6$$

CD = 421 m to the nearest metre.

(b)



$$\tan \theta = \frac{400}{130}$$

$$\theta = 72^\circ$$

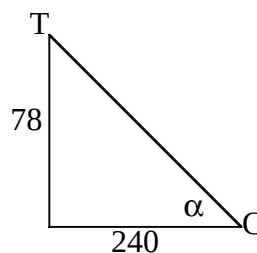
Hence, bearing of D from C is $360 - 72 = 288^\circ T$

Vertical height of the tree, T, above C = $450 - 280 - 92 = 78$

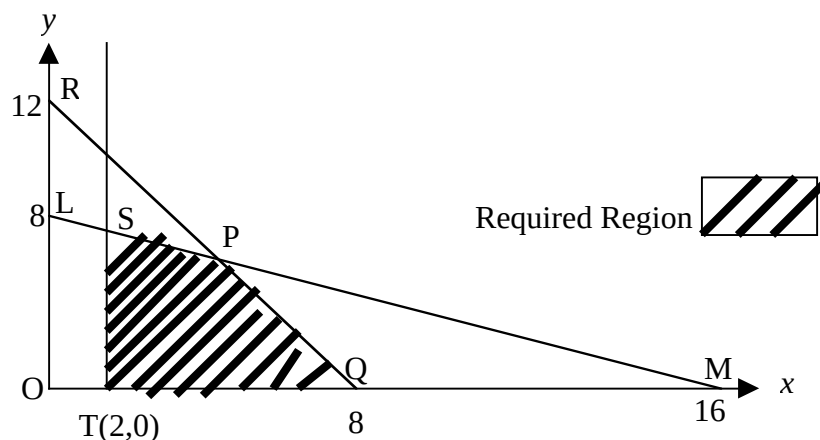
$$\tan \alpha = \frac{78}{240} \Rightarrow \alpha = 18^\circ$$

Bearing of T from C = $270 + 18 = 288^\circ T$

Since both D and the tree are on the same bearing from C, the pine tree is on the proposed road from C to D



Question 1 (a) Stops at station CD for 2 mins Stops at station GH for $1\frac{1}{2}$ mins. Total time stopped at both stations = $2 + 1\frac{1}{2}$ Total time stopped at both stations = $3\frac{1}{2}$ mins	(b)(i) Train goes fastest when the graph is steepest. By eye this could be AB or EF. For the section EF the train goes 600 m in 3 minutes. For the section AB the train only goes 500m in a longer period of time than 3 minutes, namely, 3.5 minutes. Hence, the train goes fastest in the line segment EF
(b)(ii) Speed for EF section = $600 \div 3 = 200$ metres/minute	
Question 2(a) The line RQ contains the points (0,12) and (8,0) For the line $2x + 4y = 32$ When $x = 0$ $4y = 32 \Rightarrow y = 8$ So (0,8) lies on this line. For the line $30x + 20y = 240$ When $x = 0$ $20y = 240 \Rightarrow y = 12$ $\therefore$ (0,12) lies on this line. When $y = 0$ $30x = 240 \Rightarrow x = 8$ $\therefore$ (8,0) lies on this line Hence, RQ is the line $30x + 20y = 240$ i.e. $3x + 2y = 24$	(b) Solving the two linear equations $30x + 20y = 240$ and $2x + 4y = 32$ simultaneously, gives $3x + 2y = 24(1)$ $x + 2y = 16(2)$ $(1) - (2) \rightarrow 2x = 8$ $\Rightarrow x = 4$ Substituting $x = 4$ in (2) $2y = 12$ $\Rightarrow y = 6$ $\therefore$ P is the point(4,6)
c(i) $T = 5x + 7y$	
c(ii) Coordinates of boundary intersections (0,0) (0,8) (4,6) (8,0) Maximum thrill time = 62 minutes.	Thrill time $0 + 0 = 0$ $5 \times 0 + 7 \times 8 = 56$ $5 \times 4 + 7 \times 6 = 62$ $5 \times 8 + 7 \times 0 = 40$



2(e)

At S or $x = 2$

$$2x + 4y = 32$$

$$\therefore 4y = 28$$

$$\Rightarrow y = 7$$

S is point (2,7)

At (2,0) $T = 8$

At (2,7) $T = 8 + 56 = 64$

At (4,6) $T = 16 + 48 = 64$

At (8,0) $T = 32$

Hence, number of rides to maximise thrill time are the points that lie on the line SP and between S and P, or S or P, and where the coordinates are integer values because you cannot have a fraction of a ride. The only integer x value between 2 and 4 is 3 when $x = 3$

$$2x + 4y = 32 \text{ becomes } 6 + 4y = 32$$

$$\Rightarrow 4y = 26 \Rightarrow y = 6\frac{1}{2} \text{ which is not possible}$$

So the only possible number of rides of each type are
 2 on the Xceletron and 7 on the Yellevator
 or 4 on the Xceletron and 6 on the Yellevator

Module 4 Business-related mathematics. Suggested solutions.

Question 1 a.(i) the balance went from \$3600 to \$3000 so \$600 was withdrawn on 2 February.	a.(ii) There was a balance of \$3000 and then \$85.50 was deposited so this amount was added to the previous balance. So account balance for 14 May = \$3085.50
b. Her balance up to 15 December was \$2400. This was the minimum balance for the month. Interest = $2400 \times \frac{0.3}{100}$ Interest = \$7.20	
Question 2(a) $I = \frac{PRT}{100}$ $416 = \frac{4000 \times R \times 2}{100}$ $R = \frac{416 \times 100}{4000 \times 2}$ $R = 5.2\%$	2(b) $4.8\% = \frac{4.8}{100} = 0.048$ $c = 1 + 0.048 = 1.048$
c. $A = PR^n$ $4416 = 4000R^2$ $R^2 = 1.104$ $R = \sqrt{1.104} = 1.050714$ $1 + \frac{r}{100} = 1.050714$ $\Rightarrow \frac{r}{100} = 0.050714$ $\Rightarrow r = 5.07\% \text{ to 2 decimal places}$	
Question 3(a) Amount depreciation = 23600 - 7000 Amount depreciation = 16600 $16600 = \frac{23600 \times R \times 5}{100}$ $R = \frac{16600 \times 100}{23600 \times 5}$ $R = 14.0678$ $R = 14.1\% \text{ to one decimal place}$	(b) $A = P\left(1 - \frac{r}{100}\right)^n$ $7000 = 23600\left(1 - \frac{r}{100}\right)^5$ $0.2966101695 = \left(1 - \frac{r}{100}\right)^5$ $(0.2966101695)^{\frac{1}{5}} = 1 - \frac{r}{100}$ $0.7842187242 = 1 - \frac{r}{100}$ $\frac{r}{100} = 0.2157812758$ $r = 21.6\% \text{ to one decimal place}$

Question 4(a)(i) $r = 8\% \text{ per annum} = 2\% \text{ per quarter}$ $R = 1 + \frac{2}{100} = 1.02$	4(a)(ii) 4 quarters in each year $\times$ 4 years = 16 quarters $n = 16$
Question 4(b) $A = 20000(1.02)^{16} - \frac{1500(1.02^{16} - 1)}{0.02}$ $A < 0$ Hence, the loan can be repaid in 4 years.	

Question 1(a)

Routes from Amity to Bevin

$$A \rightarrow D \quad 3 + 10 = 13$$

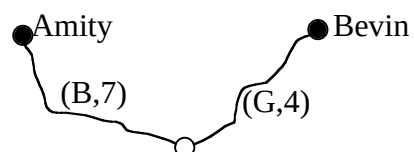
$$B \rightarrow F \quad 7 + 5 = 12$$

$$B \rightarrow G \quad 7 + 4 = 11$$

$$C \rightarrow E \rightarrow F \quad 5 + 5 + 5 = 15$$

$$C \rightarrow E \rightarrow G \quad 5 + 5 + 4 = 14$$

Hence, length of shortest path is 11 km.

(b)**Question 2(a)**

Each node is of even degree except for the node at Amity and the node connecting P and S.

When all nodes are even except for two, then it is possible to pass over every line once as long as you start at one of the nodes of odd degree and finish at the other node of odd degree.

(b)

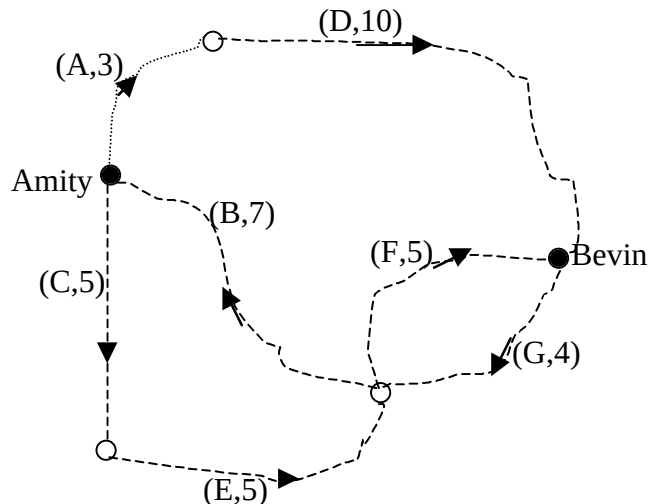
You start at Amity which is a node of odd degree so must finish at the other node of odd degree, namely, the node connecting P and S

(c)

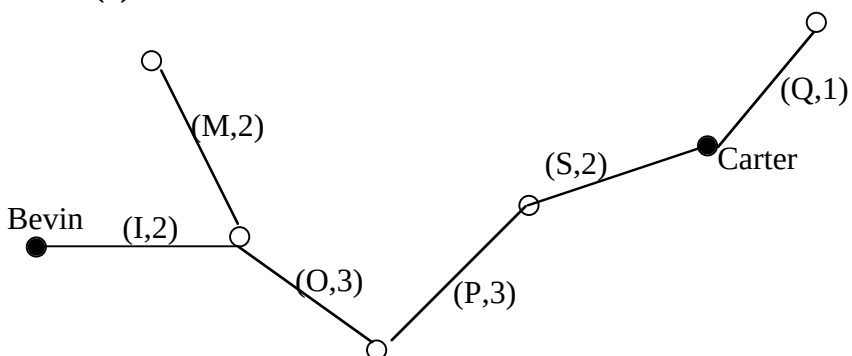
The competitor must avoid road J at the end of road H, otherwise he will not be able to pass along the roads ABC and E and end up at the node joining P and S.

(d)

After traveling along all roads from Amity to P, then travel along road S again to get to Carter.

(e)

Question 3(a)



(b)(i)

(B) I O P S Q K H (B)

(b)(ii)

Total length = $2 + 3 + 3 + 2 + 1 + 8 + 3 = 22$ km.

End of suggested solutions 2002 VCE Further Mathematics Examination 2

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