

Year 2002

VCE

Further Mathematics Written Examination 1

Detailed Suggested Solutions

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Question 1 A Both work type and gender are categorical variables.	Question 2 D % of females who are clerical workers = $\frac{143}{198} \times 100 = 72.2\%$ which is closest to 72%
Question 3 E Total number surveyed = $205 + 198 = 403$ Number clerical or manual workers = $21 + 143 + 72 + 31 = 267$ % school leavers who become clerical or manual workers = $\frac{267}{403} \times 100 = 66.25\%$ which is closest to 66%	Question 4 C The statement which supports the suggestion that the type of work undertaken is associated with gender has to have something to do with different percentages of males and females doing a particular work. Only alternative C does this.
Question 5 C Since the graph is fairly symmetrical about the middle number of 60, then the median would be close to 60.	Question 6 B About 99.7% lie within ± 3 standard deviations of the mean. 99.7% is close to 100%. Because of the symmetry of the graph, the mean will be close to 60. The largest value, $90 - 60 = 30 = 3$ standard deviations so 1 standard deviation = $30 / 3 = 10$
Question 7 D The median is the middle of the ages. Median = $\frac{37 + 39}{2} = 38$ Lower Quartile = 32 Upper Quartile = $\frac{41 + 43}{2} = 42$ IQR = $42 - 32 = 10$	Question 8 B Weight of heaviest man = 117 $\mu = 81.1$ $\sigma = 17.9$ $\mu + 2\sigma = 81.1 + 2 \times 17.9 = 116.9 \approx 117$ Probability that population lies within $\pm 2\sigma$ of the mean = 95% $\therefore$ 5% lie at the two extremes. Because of the symmetry of the bell shaped curve, 2.5% are bigger than the heaviest man and 2.5% are lighter than the lightest man
Question 9 B Using the graphics calculator, enter waist in L_1 for x and weight in L_2 for y This gives the regression line $y = 1.11x - 20$ Weight = $1.11 \times \text{waist} - 20$	Question 10 D Using the same data on the calculator as for Question 9, $r = 0.9314$

Question 11 D

Divide the data into 3 groups and take the median point.

For the lower third, the median x value is half way between 165 and 166 i.e. 165.5

For the lower third, the median y value is half way between 62 and 67 i.e.

$$\frac{62 + 67}{2} = \frac{129}{2} = 64.5$$

For lower third, median point is (165.5, 64.5)

For the upper third, the median x value is half way between 178 and 182 i.e. 180

For the upper third, the median y value is half way between 97 and 98 i.e. 97.5

For upper third, median point is (180, 97.5)

$$\text{slope} = \frac{y_u - y_L}{x_u - x_L} = \frac{97.5 - 64.5}{180 - 165.5} = \frac{33}{14.5}$$

slope = 2.28 which is closest to 2.3

Question 12 B

The coefficient of determination is a measure of how well the linear rule linking the x and y values can predict the value of y when we are given the value of x . When the outlier is removed, the prediction will be better. Hence, the coefficient of determination will increase.

Question 13 E

Equation of line is of the form

$$y = mx + c$$

c is the y intercept so $c = 17$

Two points on the line are (2, 15) and (11, 5)

$$m = \frac{5 - 15}{11 - 2} = \frac{-10}{9} = -1.1$$

$$\text{So } y = -1.1x + 17$$

$$\text{Rate} = 17 - 1.1 \times \text{year}$$

Question 1 D This is an arithmetic sequence with $a = 24$, $d = -4$, $n = 6$. $S_n = \frac{n}{2}[2a + (n - 1)d]$ $S_n = 3(48 + 5 \times (-4))$ $S_n = 3(48 - 20)$ $S_n = 3 \times 28$ $S_n = 84$	Question 2 E This is a geometric sequence. $a = 512$ $r = (100 - 12.5)\% = 87.5\% = 0.875$ $n = 4$ $t_n = ar^{n-1}$ $t_4 = 512(0.875)^3$ $t_4 = 343$
Question 3 D $15000 + [15000 + 0.05(15000)] + \dots$ $= 15000 + 15000(1.05) + \dots$ This a geometric series where $a = 15000$ $r = 1.05$ $n = 5$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_5 = \frac{15000(1.05^5 - 1)}{1.05 - 1} = 82884.5$ which is closest to \$82 884	Question 4 D This is an arithmetic sequence with $a = 13$, $d = 4$ $n = 8$ $t_n = a + (n - 1)d$ $t_4 = 13 + 7 \times 4$ $t_7 = 13 + 28$ $= 41$
Question 5 D $2 + 0.8 + (0.2 \times 0.8) + (0.2^2 \times 0.8) + (0.2^3 \times 0.8)$ $= 2.8 + S_\infty$ of Geometric Sequence where $a = 0.2 \times 0.8$ $r = 0.2$ $2.8 + S_\infty = 2.8 + \frac{0.2 \times 0.8}{1 - 0.2} = 2.8 + 0.2 = 3$	Question 6 B $t_{n+1} = t_n + 4$ $t_1 = 3$ $t_2 = t_1 + 4 = 3 + 4 = 7$ $t_3 = t_2 + 4 = 7 + 4 = 11$ This is an arithmetic sequence with $a = 3$ $d = 4$ $t_n = a + (n - 1)d$ $t_n = 3 + (n - 1) \times 4$ $t_n = 3 + 4n - 4$ $t_n = 4n - 1$

Question 7 C

5000 baht costs \$223.21

1 baht costs $\frac{\$223.21}{5000}$ 7000 baht costs $\frac{223.21}{5000} \times 7000 = \312.49 **Question 8 E**

Increase each year means that the graph must be rising. It is a geometric increase, hence the graph will be curved upwards, with the slope becoming greater with time.

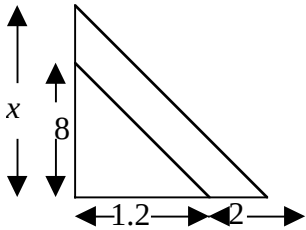
Question 9 A A_1 is the amount of money invested in the first year = \$250000

$$A_2 = 250000 + 0.0454 \times 250000 - 15000$$

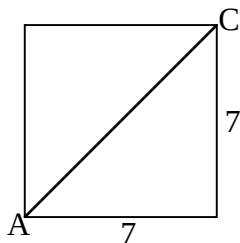
$$A_2 = 250000(1.0454) - 15000$$

$$A_2 = 1.0454A_1 - 15000$$

$$A_{n+1} = 1.0454A_n - 15000 \text{ where } A_1 = 250000$$

Question 1 B $\tan \theta = \frac{10}{17}$ $\theta = \tan^{-1}\left(\frac{10}{17}\right)$ $\theta = 30.47^{\circ} \text{ which is closest to } 30^{\circ}$	Question 2 B $\text{Area of triangle} = \frac{1}{2}bc \sin \theta$ $\text{Area of triangle} = \frac{1}{2} \times 3 \times 3 \times \sin 60^{\circ}$ $\text{Area of triangle} = 3.897$ $\text{which is closest to } 3.9 \text{ cm}^2$
Question 3 C $\cos 22^{\circ} = \frac{40}{AC}$ $AC = \frac{40}{\cos 22^{\circ}}$ $AC = 43.14 \text{ which is closest to } 43\text{m}$	Question 4 E  Similar triangles $\frac{x}{8} = \frac{3.2}{1.2}$ $x = \frac{3.2 \times 8}{1.2}$ $x = 21.33$ $\text{which is closest to } 21 \text{ cm.}$
Question 5 C Volume of smaller box = $L \times W \times H = LWH$ Volume of larger box = $2L \times 2W \times 2H = 8LWH$ Ratio of Volume smaller box: Volume of larger box = $LWH:8LWH = 1:8$	Question 6 C Using cosine rule: $(GH)^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 19^{\circ}$ $(GH)^2 = 9.23$ $GH = \sqrt{9.23}$ $GH = 3.03$ $\text{which is closest to } 3.0 \text{ m}$

Question 7 B

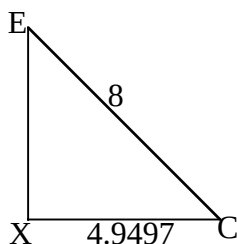


$$AC = \sqrt{7^2 + 7^2}$$

$$AC = \sqrt{98}$$

$$AC = 9.8995$$

$$\frac{1}{2}AC = 4.9497$$



$$64 = h^2 + 4.9497^2$$

$$h^2 = 64 - 24.5$$

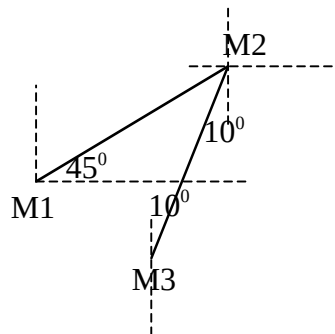
$$h^2 = 39.5$$

$$h = \sqrt{39.5}$$

$$h = 6.28$$

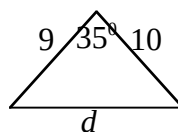
which is closest to 6.3cm

Question 8 A



M2 is 10° true from M3

Question 9 B



$$d^2 = 9^2 + 10^2 - 2 \times 9 \times 10 \cos 35^\circ$$

$$d^2 = 81 + 100 - 180 \cos 35^\circ$$

$$d^2 = 33.5526$$

$$d = \sqrt{33.5526}$$

$$d = 5.79$$

which is closest to 5.8km

Question 1 D Two points on line are (0,5) and (7,0) Equation of line is of form $y = mx + c$ c is y intercept, $c = 5$ So $y = mx + 5$ $m = \frac{0 - 5}{7 - 0} = -\frac{5}{7}$ $\therefore y = -\frac{5}{7}x + 5$ $\Rightarrow 7y = -5x + 35$ $\Rightarrow 7y + 5x = 35$ $\Rightarrow 5x + 7y = 35$	Question 2 C The open circle indicates that the value is not included and the closed circle indicates that the value is included. Hence, $C = 10$ for $0 < x \leq 10$ $C = 20$ for $10 < x \leq 20$ $C = 40$ for $20 < x \leq 40$
Question 3 E In 1997 the line is almost horizontal, indicating that the number of cattle remained relatively unchanged.	Question 4 E $4x + y = 7$(1) $5x + 7y = 3$(2) $(1) \times 7 \rightarrow 28x + 7y = 49$(1a) $5x + 7y = 3$(2) $(1a) - (2) \rightarrow 23x = 46$ $\Rightarrow x = 2$ Substituting $x = 2$ in (1) $8 + y = 7$ $y = 7 - 8$ $y = -1$ So $x = 2, y = -1$

Question 5 A At (0,0) $4x - 2y = 0$ At (0,10) $4x - 2y = -20$ At (2,9) $4x - 2y = -10$ At (8,5) $4x - 2y = 22$ At (11,0) $4x - 2y = 44$ Minimum of these values is - 20	Question 6 D The value of the objective function does not have to be positive.
Question 7 B The relation $y = 2x^2$ is not a straight line if the axes are y and x When y versus x^2 we have a straight line. When $x^2 = 9, y = 2 \times 9 = 18$ When $x^2 = 3, y = 2 \times 3 = 6$ So(3,6) and (9,18) are points on the line.	Question 8 B For the line $2x + 3y = 24$ When $x = 0, y = 8$ When $y = 0, x = 12$ $\therefore 2x + 3y = 24$ cuts the axes at (0,8) and (12,0) $\therefore$ not D or E $2x + 3y \leq 24$ $\therefore$ not C $2y - x = 12$ When $x = 0, y = 6$ When $y = 0, x = -12$ $\therefore 2y - x = 12$ cuts the axes at (0,6) and (-12,0) $2y \geq x + 12$ $\therefore$ shading is above this line..
Question 9 D Number of trainee inspectors = x Number items inspected by trainee inspectors per hour = $15x$ Number items inspected by trainee inspectors in 8 hours = $8 \times 15x = 120x$ Number of qualified inspectors = y Number items inspected by qualified inspectors per hour = $25y$ Number items inspected by qualified inspectors in 8 hours = $8 \times 25y = 200y$ Total items inspected = $120x + 200y \geq 1800$	

Question 1 D $SI = \frac{PRT}{100}$ $1800 = \frac{P \times 3 \times 3}{100}$ $180000 = 9P$ $P = \$20000$	Question 2 B $A = PR^n$ $P = 16000$ $R = 1.04$ $n = 2$ $A = 16000(1.04)^2$ $A = \$17305.60$
Question 3 B Interest per quarter = $\frac{6}{4} = 1.5\%$ Value of investment at beginning of fourth year = Value of investment at end of twelfth quarter $A = 8500(1.015)^{12} = \$10162.75$ Value of investment at end of fourth year = Value of investment at end of sixteenth quarter $A = 8500(1.015)^{16} = \$10786.37$ Interest earned in 4th year = $10786.37 - 10162.75$ = \$623.62	Question 4 C $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$ $A = 0$ $P = 172000$ $r = 6\% \text{ per annum} = 0.5\% \text{ per month}$ $R = 1.005$ $n = 20 \times 12 = 240$ $0 = 172000(1.005)^{240} - \frac{Q(1.005^{240} - 1)}{0.005}$ $462.041Q = 569355.1698$ $Q = \$1232.26$
Question 5 B Effective rate of interest = $\frac{2n}{n+1} \times \text{flat rate}$ Amount repaid = $480 \times 48 = 23040$ Interest = $23040 - 18000 = 5040$ $I = \frac{PRT}{100}$ $5040 = \frac{18000 \times r \times 4}{100}$ $r = \frac{5040 \times 100}{18000 \times 4} = 7$ Effective rate = $\frac{96}{49} \times 7 = 13.7\% \text{ per annum}$	Question 6 C Purchase Price = \$36000 Number units produced in 1 year = 24000 Number units produced in 6 years = 24000×6 = 144000 Depreciation over 6 years = 144000×0.16 = 23040 Value of machine after 6 years = $36000 - 23040$ = 12960

Question 7 C

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

$$r = \frac{7}{12} = 0.5833\%$$

$$R = 1.005833$$

$$A = 0$$

$$P = 135000$$

$$Q = 1200$$

$$0 = 135000(1.005833)^n - \frac{1200(1.005833^n - 1)}{0.005833}$$

$$\frac{1200(1.005833^n - 1)}{0.005833} = 135000(1.005833)^n$$

$$205726.0415(1.005833^n - 1) = 135000(1.005833)^n$$

$$1.5239(1.005833^n - 1) = (1.005833)^n$$

$$1.5239 \times 1.005833^n - 1.005833^n = 1.5239$$

$$0.5239 \times 1.005833^n = 1.5239$$

$$1.005833^n = 2.9088$$

$$\log_{10}(1.005833)^n = \log_{10} 2.9088$$

$$n \log_{10} 1.005833 = 0.4637$$

$$n \times 0.0025 = 0.4637$$

$$n = 185.48 \text{ months} = 15.45 \text{ years}$$

Amount of time saved = $20 - 15.45 = 4.55$ years
which is closest to 5 years

Question 8 A

This is not a straight line, so not C or D
The graph is increasing so not B
Substituting a value, say $n = 8$, into A and E
will show A is closest to the graph value
of 3200.

Question 9 C

Let the original price be \$100

Increase of 8% = $100 + 0.08 \times 100$

$$= 100(1.08)$$

$$= \$108$$

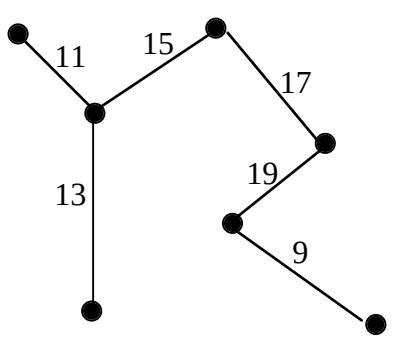
Discount \$108 by 3% = $108 - 0.3 \times 108$

$$= 108 \times 0.97$$

$$= \$104.76$$

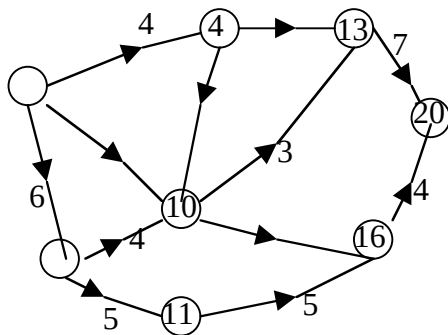
Increase from original \$100 = \$4.76

$$\% \text{ increase} = \frac{4.76}{100} \times 100 = 4.76\%$$

Question 1 C $V + F - E = 2$ $V + 8 - 15 = 2$ $V = 2 + 15 - 8$ $V = 9$	Question 2 D Rik did defeat Yunis. (true) Grant defeated Merek and was defeated by Ben, so Grant played only two matches. (true) Ben defeated Grant, Mauricio and Yunis and was not defeated by anyone, so Ben won all his matches. (true) Merek defeated Rik so statement D is false. 8 matches have been played since there are 8 lines, so E is true.
Question 3 A $1 \rightarrow 1$ is 0 $1 \rightarrow 2$ is 1 $1 \rightarrow 3$ is 1 $1 \rightarrow 4$ is 0 $1 \rightarrow 5$ is 0 so first row is 0 1 1 0 0 $\therefore$ A or C $2 \rightarrow 1$ is 1 $2 \rightarrow 2$ is 0 $2 \rightarrow 3$ is 1 $2 \rightarrow 4$ is 0 $2 \rightarrow 5$ is 0 so second row is 1 0 1 0 0	Question 4 C An Eulerian circuit uses each and every edge only once, and begins and ends at the same point. For an Eulerian circuit to exist each vertex must be of even degree. A has vertices of degree 1 and 3 so not an Eulerian circuit. B has vertices of degree 3 and 5 so not an Eulerian circuit. C only has vertices of even degree so C is an Eulerian circuit. D has vertices of degree 3 so not an Eulerian circuit. E has vertices of degree 3 and 5 so not an Eulerian circuit.
Question 5 E A costs $10 + 8 + 6 + 5 = \\$29$ B costs $2 + 4 + 7 + 8 = \\$21$ C costs $5 + 3 + 7 + 7 = \\$22$ D costs $6 + 3 + 7 + 5 = \\$21$ E costs $2 + 4 + 7 + 7 = \\$20$ Hence, E minimises the total cost.	Question 6 D  Weight of minimum weight spanning tree $= 11 + 13 + 15 + 17 + 19 + 9 = 84$

Question 7 D

The earliest time to finish each activity is given in the circles.



Hence, the minimum time to finish the project is 20 days.

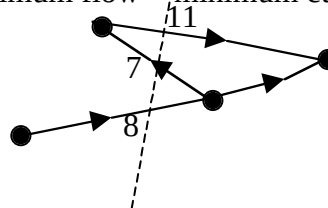
Question 8 A

If float time = 0, then the activity is critical.
Float time = latest start time – earliest start time.

Float time = 0 for activities B,C,D,F, and G

Question 9 A

Maximum flow = minimum cut capacity



When the line marked 8 is cut, there is no flow through the line marked 7

Hence, minimum cut capacity = 11 + 8 = 19

Hence, maximum flow = 19

End of suggested solutions 2002 VCE Further Mathematics Examination 1

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