

Year 2002

VCE

Mathematical Methods (CAS) Written Examination 1

Detailed Suggested Solutions

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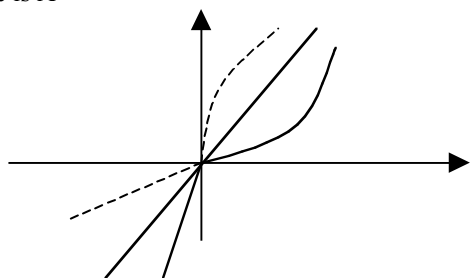


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Question 1 E Period = $\frac{2\pi}{n} = 4\pi$ $4\pi n = 2\pi$ $n = \frac{1}{2}$ Amplitude = 1 Translated up 1 It is a cos graph since none of the alternatives are translated to the left or right, and the graph is a maximum when $x = 0$ Hence the equation of the graph is $y = 1 + \cos \frac{1}{2}x$ <i>i.e.</i> $y = 1 + \cos \frac{x}{2}$	Question 2 E $\sin 2x = 1 \quad 0 \leq x \leq 4\pi$ $0 \leq 2x \leq 8\pi$ $2x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$ $x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$ Sum of solutions = $\frac{\pi}{4} + \frac{5\pi}{4} + \frac{9\pi}{4} + \frac{13\pi}{4}$ $= \frac{28\pi}{4} = 7\pi$
Question 3 D Maximum temperature when $\sin \frac{\pi t}{12} = -1$ $\frac{\pi t}{12} = \frac{3\pi}{2}$ $\frac{t}{12} = \frac{3}{2}$ $t = \frac{36}{2} = 18$ Time = 18 hrs. after midnight = 6.00p.m.	Question 4 E The equation of the function is of the form $y = a(x+2)(x+1)(x-1)(x-3)$ When $x = 0$ $y = a \times (+) \times (+) \times (-) \times (-) = \text{negative}$ Hence, a is negative A possible equation is $y = -(x+2)(x+1)(x-1)(x-3)$ $y = (x+2)(x+1)(x-1) \times (-)(x-3)$ $y = (x+2)(x+1)(x-1)(3-x)$
Question 5 C A vertical asymptote exists when $x = -b$ $\therefore -b = 3$ $\therefore b = -3$ A horizontal asymptote exists when $y = c$ $\therefore c = 2$ $y = \frac{a}{x-3} + 2$ When $x = 4, y = 0$ $0 = \frac{a}{4-3} + 2$ $0 = a + 2$ $a = -2$ $a = -2, b = -3, c = 2$	Question 6 A $y = f(-x)$ is a reflection in the Y axis of $y = f(x)$ Hence, the vertical asymptote will be $x = -1$

Question 7 A

The inverse function is a reflection in the line $y=x$
The only graph that is a reflection of the given graph in this line is A



Question 8 C

$y = \tan \frac{x}{2}$, not possible because there is an

asymptote at $x = \frac{\pi}{4}$

$y = \log_e \left(\frac{x}{2} \right)$, not possible at $x = 0$

$y = \frac{x}{2} + 1$ is linear but the values given in the table are not linear

$y = \frac{x^2}{2}$, when $x = 0, y = 0$

$y = e^{\frac{x}{2}}$ is the best fit for the given data

Question 9 B

$$\begin{aligned} x^4 + x^3 - 3x^2 - 3x &= x(x^3 + x^2 - 3x - 3) \\ &= x[x^2(x+1) - 3(x+1)] \\ &= x(x+1)(x^2 - 3) \\ &= x(x+1)(x - \sqrt{3})(x + \sqrt{3}) \end{aligned}$$

Hence, the linear factors are

$$x, x+1, x - \sqrt{3}, x + \sqrt{3}$$

Question 10 C

The graph of f is a reflection of the graph of g in the line $y = x$

Both f and g are increasing

$$f(g(x)) = f(\log_e x) = e^{\log_e x}$$

$$\text{Let } y = e^{\log_e x}$$

$$\log_e y = \log_e e^{\log_e x}$$

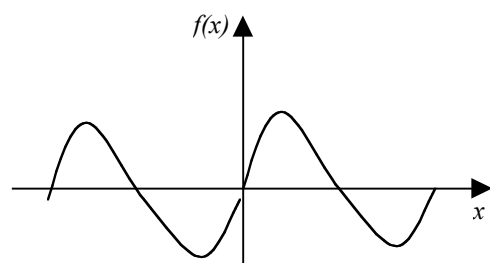
$$\log_e y = \log_e x \log_e e$$

$$\log_e y = \log_e x$$

$$\Rightarrow y = x$$

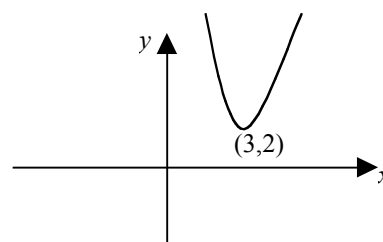
$$\therefore f(g(x)) = x$$

Question 11 D



The graph of $f(x) = \sin(x)$, where $x \in \mathbb{R}$ is as shown above.
For one value of y there are many values of x .
Hence it is not a one to one function.

Question 12 D



This graph will have an inverse function if it is a one to one function. Hence, restrict the domain so that $x \leq 3$ or $x \geq 3$
From the given alternatives, the answer would be D.

Question 13 C

$f'(x)$ is the gradient of the given function.
For the given function which is always increasing, the gradient is always positive. this means the gradient graph is always above the X axis. Hence the only possibilities are B or C.
The given graph increases by less and less as it goes from left to right. Hence, the gradient becomes less and less positive. Thus C is the graph of $f'(x)$.

Question 14 B

$$y = \log_e(\cos(2x))$$

$$\text{Let } u = \cos(2x)$$

$$\frac{du}{dx} = -2 \sin(2x)$$

$$y = \log_e u$$

$$\frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{u} \times -2 \sin(2x)$$

$$\frac{dy}{dx} = \frac{-2 \sin(2x)}{\cos(2x)}$$

$$\frac{dy}{dx} = -2 \times \frac{\sin(2x)}{\cos(2x)}$$

$$\frac{dy}{dx} = -2 \tan(2x)$$

Question 15 D

$$y = x \sin(x)$$

$$\text{When } x = \pi, y = \pi \sin \pi = \pi \times 0 = 0$$

$$\therefore \text{ given point is } (\pi, 0)$$

$$\frac{dy}{dx} = x \cos(x) + \sin(x)$$

$$\text{When } x = \pi,$$

$$\frac{dy}{dx} = \pi \cos(\pi) + \sin(\pi)$$

$$\frac{dy}{dx} = \pi \times -1 + 0$$

$$\frac{dy}{dx} = -\pi$$

$$\text{Gradient of normal} = \frac{-dx}{dy} = \frac{1}{\pi}$$

$$\text{Equation of normal is of form } y = mx + c$$

$$\text{So } y = \frac{1}{\pi}x + c$$

$$\text{When } x = \pi, y = 0$$

$$\text{So } 0 = \frac{1}{\pi} \times \pi + c$$

$$0 = 1 + c$$

$$c = -1$$

$$\text{Hence, equation of normal is}$$

$$y = \frac{1}{\pi}x - 1$$

$$y = \frac{1}{\pi}(x - \pi)$$

Question 16 C

$$\text{Rate of change with respect to } x \text{ is } \frac{dy}{dx}$$

$$y = e^{-x} - 1$$

$$\frac{dy}{dx} = -e^{-x}$$

$$\text{When } x = 0$$

$$\frac{dy}{dx} = -e^{-0} = -1$$

Question 17 A

$$f(x+h) \approx f(x) + hf'(x)$$

$$f(x) = x^2$$

$$3.02^2 = f(3.02)$$

$$= f(3 + 0.02) \approx f(3) + 0.02 f'(3)$$

Question 18 B

$f(0) = 0$ is point (0,0)

$f(-3) = 0$ is the point (-3,0)

$f'(0) = 0$ means there is a stationary point at (0,0)

Hence not C

$f'(-1) = 0$ means there is a stationary point

when $x = -1$

$f'(x) > 0$ for $x < -1$ means there is a positive gradient when $x < -1$

This means that the graph is rising when $x < -1$

Hence, not A, C or E

$f'(x) < 0$ for $x > -1$ except $x = 0$ means there is a negative gradient when $x > -1$ except $x = 0$

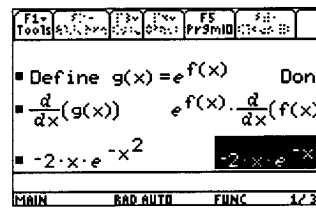
This means that the graph is falling when $x > -1$ except when $x = 0$. Hence, not D

Question 19 A

$$g(x) = e^{f(x)} \Rightarrow g'(x) = f'(x)e^{f(x)} = -2xe^{-x^2}$$

$$\therefore f'(x) = -2x$$

$$f(x) = -x^2$$



Question 20 B

$$f'(x) = 2 \cos(5x)$$

$$f(x) = \int 2 \cos(5x) dx$$

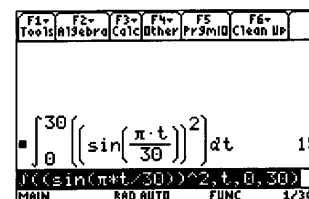
$$f(x) = 2 \int \cos(5x) dx$$

$$f(x) = 2 \times \left[\frac{1}{5} \sin(5x) \right] + c$$

$$f(x) = \frac{2}{5} \sin(5x) + c$$

Question 21 D

$$x = \int_0^{30} \sin^2\left(\frac{\pi t}{30}\right) dt$$



Using graphics calculator use

Math9

$$\text{fnInt}\left(\left(\sin\left(\frac{\pi x}{30}\right)\right)^2, x, 0, 30\right)$$

This gives $x = 15$

Question 22 D

$$A = \int_a^b f(x) dx + \left| \int_b^c f(x) dx \right|$$

$$A = \int_a^b f(x) dx - \int_b^c f(x) dx$$

$$A = \int_a^b f(x) dx + \int_c^b f(x) dx$$

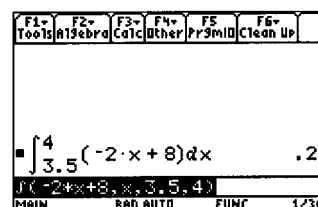
Question 23 B

$$\Pr(X > 3.5) = \int_{3.5}^4 (-2x + 8) dx$$

Using graphics calculator enter

Matl

fnIn



Question 24 A

Without replacement is hypergeometric.

$$\Pr(X = x) = \frac{{}^D C_x {}^{N-D} C_{n-x}}{{}^N C_n}$$

$$x \geq 0$$

$$N = 20$$

$$n = 4$$

$$D = 5$$

$$\Pr(x \geq 0) = 1 - \Pr(x = 0)$$

$$\Pr(x \geq 0) = 1 - \frac{{}^5 C_0 {}^{15} C_4}{{}^{20} C_4}$$

$$\Pr(x \geq 0) = 1 - \frac{1 \times {}^{15} C_4}{{}^{20} C_4}$$

$$\Pr(x \geq 0) = 1 - \frac{{}^{15} C_4}{{}^{20} C_4}$$

Question 25 A

$\mu = np$ where p is the probability defective.

$$\mu = 10$$

$$\sigma = \sqrt{npq} = \sqrt{\mu q} = \sqrt{10q}$$

$$3 = \sqrt{10q}$$

$$9 = 10q$$

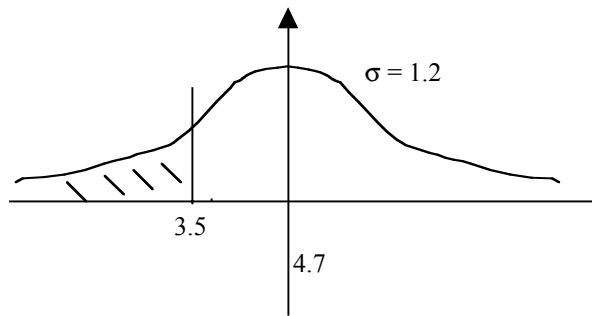
$$q = \frac{9}{10} = 0.9$$

$$p = 1 - q$$

$$p = 1 - 0.9$$

$$p = 0.1$$

Question 26 B



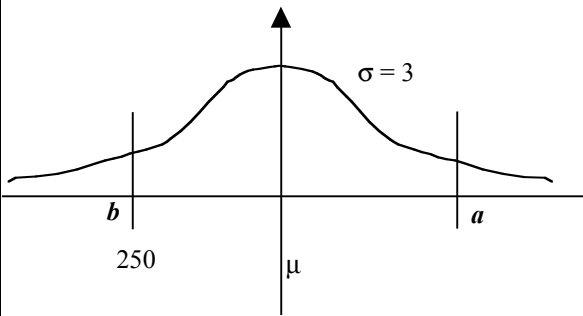
$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{3.5 - 4.7}{1.2}$$

$$Z = -1$$

$$\Pr(X < 3.5) = \Pr(Z < -1) = \Pr(Z > 1)$$

Question 27 E



$$\Pr(X < 250) = 0.01$$

$$\Pr(Z > a) = 0.01$$

$$\Pr(Z < a) = 1 - 0.01 = 0.99$$

$$a = 2.33 \text{ (from tables)}$$

$$\text{So } b = -2.33$$

$$Z = \frac{x - \mu}{\sigma}$$

$$-2.33 = \frac{250 - \mu}{3}$$

$$-6.99 = 250 - \mu$$

$$\mu = 256.99 \approx 257$$

Question 1.

F1- Tools	F2- Algebra	F3- Calc	F4- Other	F5- Pr&MIO	F6- Clean Up
$\int_{100}^{\infty} \left(\frac{c}{x^2} \right) dx = \frac{c}{100}$ $\text{solve}\left(\frac{c}{100} = 1, c\right) \quad c = 100$ $\text{solve}(c/100=1, c)$					
MAIN		RAD AUTO		FUNC 2/30	

F1- Tools	F2- Algebra	F3- Calc	F4- Other	F5- Pr&MIO	F6- Clean Up
$\int_m^{\infty} \left(\frac{100}{x^2} \right) dx = \frac{100}{m}$ $\text{solve}\left(\frac{100}{m} = 1/2, m\right) \quad m = 200$ $\text{solve}(100/m=1/2, m)$					
MAIN		RAD AUTO		FUNC 2/30	

(a)

$$\int_{100}^{\infty} \frac{c}{x^2} dx = 1 \text{ for pdf}$$

$$\therefore -\frac{c}{x} \Big|_{100}^{\infty} = 1$$

$$-\frac{c}{\text{very large number}} + \frac{c}{100} = 1$$

$$\frac{c}{100} \approx 1$$

$$\therefore c = 100$$

(b)

Median, m is $F(m) = \frac{1}{2}$

$$F(m) = \int_{100}^m \frac{100}{x^2} dx = \frac{1}{2}$$

$$\Rightarrow -\frac{100}{x} \Big|_{100}^m = \frac{1}{2}$$

$$\Rightarrow -\frac{100}{m} + \frac{100}{100} = \frac{1}{2}$$

$$\Rightarrow \frac{100}{m} = \frac{1}{2}$$

$$\therefore m = 200$$

Question 2.

(a)

Total number slots, $n(s) = 37$
 Number red slots, $n(R) = 18$
 $\Pr(R) = \frac{18}{37} = 0.486$ to 3 dp

(b)

$$\Pr(R'R'R'R) = \frac{19}{37} \times \frac{19}{37} \times \frac{19}{37} \times \frac{19}{37} \times \frac{18}{37}$$

$$\Pr(R'R'R'R) = \left(\frac{19}{37}\right)^4 \times \frac{18}{37} = 0.0338$$

$$\Pr(R'R'R'R) = 0.034$$
 to 3 dp

(c)

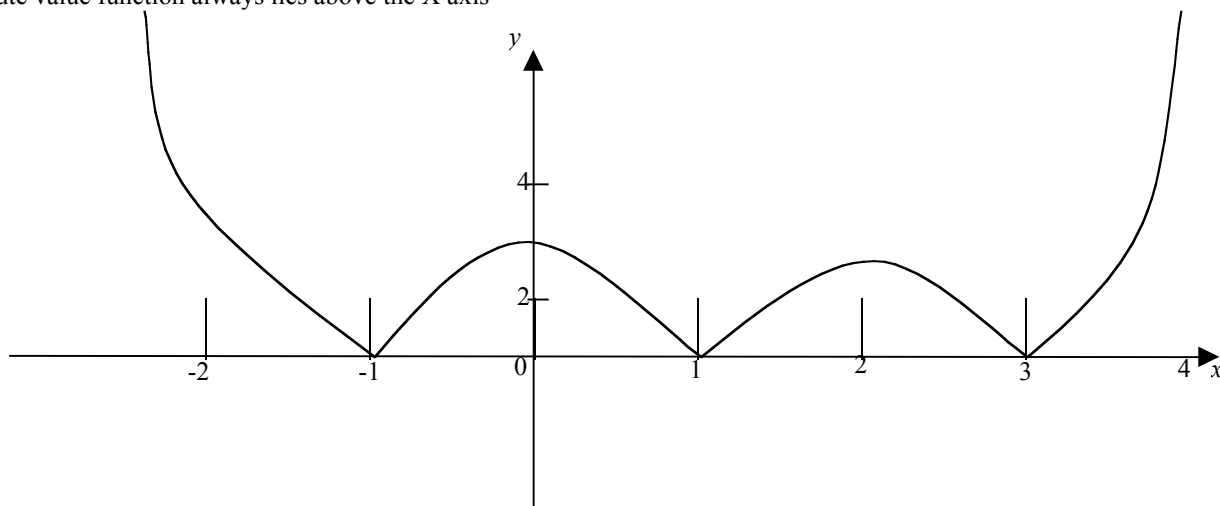
$$\Pr(R) \text{ or } \Pr(R'R) \text{ or } \Pr(R'R'R) \text{ or } \Pr(R'R'R'R) = \frac{18}{37} + \frac{19}{37} \times \frac{18}{37} + \left(\frac{19}{37}\right)^2 \times \frac{18}{37} + \left(\frac{19}{37}\right)^3 \times \frac{18}{37}$$

$$\Pr(R) \text{ or } \Pr(R'R) \text{ or } \Pr(R'R'R) \text{ or } \Pr(R'R'R'R) = \frac{18}{37} \left[1 + \frac{19}{37} + \left(\frac{19}{37}\right)^2 + \left(\frac{19}{37}\right)^3 \right]$$

$$\Pr(R) \text{ or } \Pr(R'R) \text{ or } \Pr(R'R'R) \text{ or } \Pr(R'R'R'R) = 0.930$$
 to 3 dp

Question 3.

The absolute value function always lies above the X axis



Question 4.

F1 Tools	F2 Edit	F3 View	F4 List	F5 PySimd	F6 Help
■ E -1 1 -1 1 1 1 1 1 3 2 1 0 0 0 1 0					
-1 1 -1 1 1 -1 1 1 ans(1)^(-1)*([1][5/3][1]-2) MAIN BAD AUTO FUNC 2/2					

F1- Tools	F2- W19ebrd	F3- Calc	F4- Other	F5 Pr3m	F6- Clean Up
[0]	[0]	[1]	[0]	[0]	[1/3 -3/2 0 17/6]
ans(1)^(-1)*[1][1][5/3][1][2]					
MAIN	BAD AUTO	FUNC		2/30	

(a)

$$a(-1)^3 + b(-1)^2 + c(-1) + d = 1$$

$$\Rightarrow -a + b - c + d = 1$$

$$a(1)^3 + b(1)^2 + c(1) + d = \frac{5}{3}$$

$$\Rightarrow a + b + c + d = \frac{5}{3}$$

$$f'(x) = 3ax^2 + 2bx + c$$

$$f'(1) = 3a + 2b + c = -2$$

$$f'(0) = c = 0$$

(b)

$$X = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \quad K = \begin{bmatrix} 1 \\ 5 \\ 3 \\ -2 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Question 4.(continued)

(c)

$$AX = K$$

$$A^{-1}AX = A^{-1}K$$

$$X = A^{-1}K = \begin{bmatrix} \frac{1}{3} \\ -\frac{3}{2} \\ 0 \\ \frac{17}{6} \end{bmatrix}$$

$$a = \frac{1}{3} \quad b = -\frac{3}{2} \quad c = 0 \quad d = \frac{17}{6}$$

$$f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{17}{6}$$

Question 5.

(a)

Dilation : $y = \frac{1}{2} = \frac{1}{2x}$

Reflection in Y axis: $y = -\frac{1}{2x}$

Translation + 3 parallel to X axis: $y = -\frac{1}{2(x-3)}$

Translation +1 parallel to Y axis: $y = -\frac{1}{2(x-3)} + 1$

(b)

$$x - 3 \neq 0$$

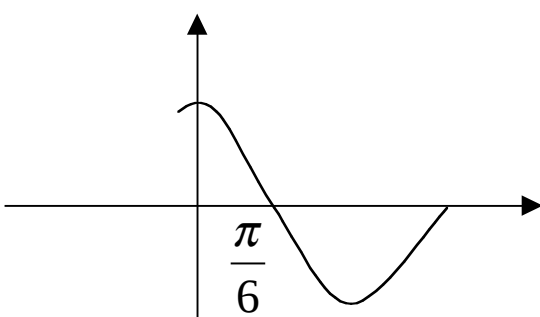
Hence, $x \neq 3$

Domain : $R \setminus \{3\}$

There is a horizontal asymptote, $y = 1$

Hence, Range : $R \setminus \{1\}$

Question 6.

(a) $2\sin(3x) - \frac{3}{2} = -\frac{1}{2} \quad 0 \leq x \leq \frac{\pi}{2}$ $2\sin(3x) = 1 \quad 0 \leq 3x \leq \frac{3\pi}{2}$ $\sin(3x) = \frac{1}{2}$ $3x = \frac{\pi}{6}, \frac{5\pi}{6}$ $x = \frac{\pi}{18}, \frac{5\pi}{18}$	(b) $f'(x) = 6\cos(3x)$ $f'(1) = 6 \times \cos 3 = -5.93995$ $f'(1) = -5.940 \text{ to 3 dp.}$
(c) Want $6\cos(3x) > 0$ $\Rightarrow \cos(3x) > 0$  $f'(x) > 0$ for $0 \leq x < \frac{\pi}{6}$ i.e. $[0, \frac{\pi}{6})$	

End of suggested solutions 2002 VCE Mathematical Methods (CAS) Examination 1

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