



INSIGHT

Trial Exam Paper

2009

MATHEMATICAL METHODS (CAS)

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes

Writing time: 2 hours

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved **graphics** calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring the following items into the examination: blank sheets of paper and/or white out liquid/tape.

Materials provided

- The question and answer book of 21 pages, with a separate sheet of miscellaneous formulas.
- An answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the box provided and on the answer sheet for multiple-choice questions.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam

- Place the answer sheet for multiple-choice questions inside the front cover of this question book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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SECTION 1**Instructions for Section 1**

Answer **all** questions in pencil on the multiple-choice answer sheet.

Select the response that is **correct** for the question.

A correct answer scores 1 mark, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers.

If more than one answer is selected no marks will be awarded.

Question 1

The function $f : [0, 2\pi] \rightarrow \mathbb{R}$, $f(x) = 1 - 3\sin(2x - \frac{\pi}{2})$ has a range and period respectively of

- A. $[0, 3]$ and 2
- B. $[-1, 3]$ and 2π
- C. $[-2, 4]$ and π
- D. $[-3, 3]$ and π
- E. $[-2, 4]$ and 2π

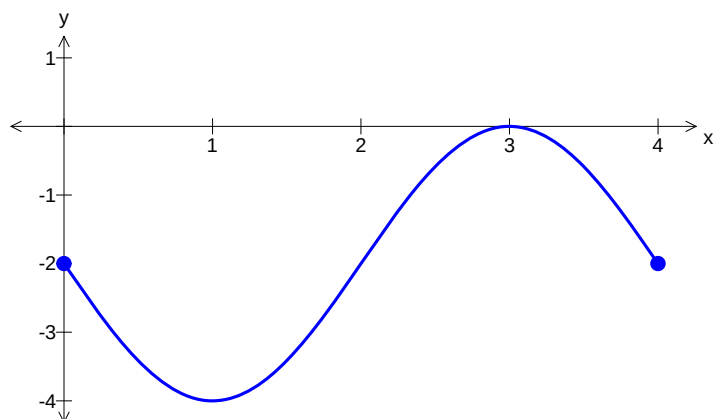
Question 2

The range of the function $f(x) = e^{|x|} - 3$ is

- A. $[-2, \infty)$
- B. $\mathbb{R}^+$
- C. $(-2, 0]$
- D. $(-2, 0)$
- E. $(-3, \infty)$

Question 3

The diagram below shows one cycle of the graph of a circular function.



A possible equation for the function whose graph is shown is

- A. $y = 2 - 2 \sin(4x)$
- B. $y = -2 - 2 \sin(4x)$
- C. $y = 4 - 2 \sin(4\pi x)$
- D. $y = -2 - 2 \sin\left(\frac{\pi}{2} x\right)$
- E. $y = -2 - 2 \sin\left(\frac{1}{2} x\right)$

Question 4

One cycle of the graph of the function with the equation $y = \tan(ax)$ has successive vertical asymptotes at $x = \frac{3}{8}$ and $x = \frac{5}{8}$.

A possible value for a is

- A. 2
- B. 2π
- C. 4
- D. 4π
- E. 8

Question 5

For the system of simultaneous linear equations

$$z = -3$$

$$x - y = 5$$

$$x + y = 1$$

an equivalent matrix equation is

A.
$$\begin{bmatrix} 0 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} z \\ y \\ x \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}$$

B.
$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -3 \end{bmatrix}$$

C.
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}$$

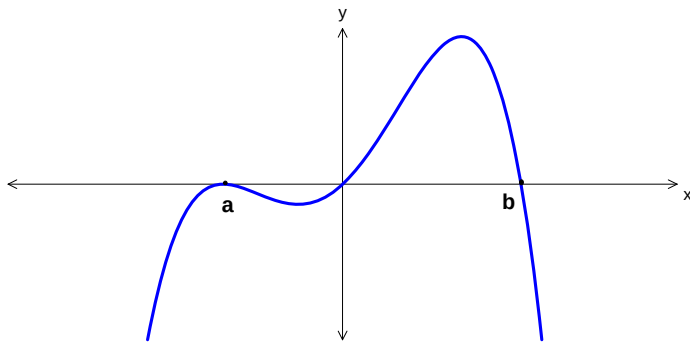
D.
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -3 \end{bmatrix}$$

E.
$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}$$

Question 6

The transformation $T : R^2 \rightarrow R^2$ which maps the curve with the equation $y = e^x$ to the curve with the equation $y = e^{2x-4} + 3$ could have the rule

- A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix}$
- B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$
- C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix}$
- D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix}$
- E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

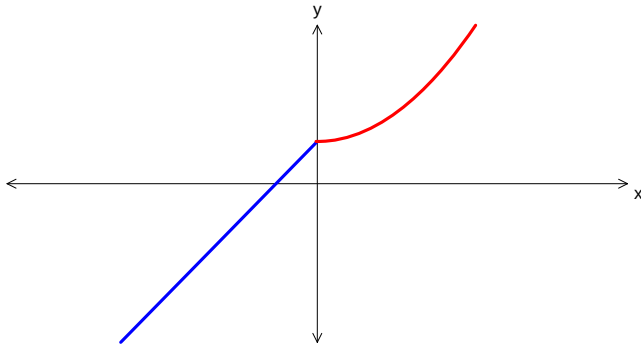
Question 7

The graph shown could be that of a function f with the equation

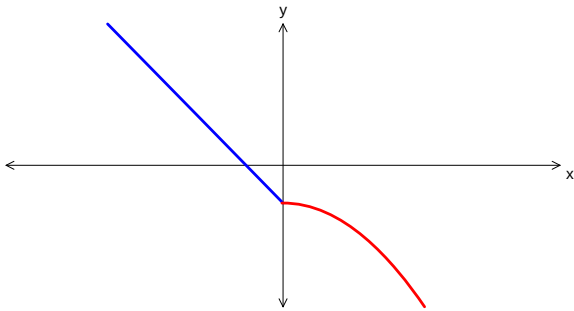
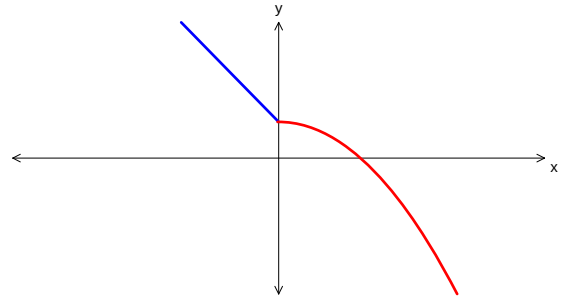
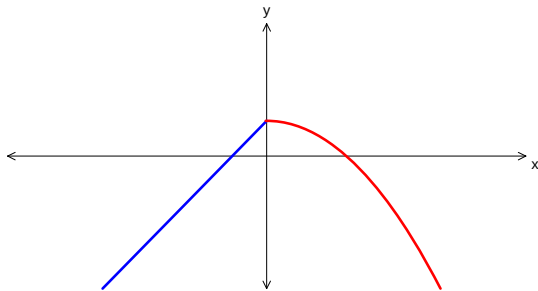
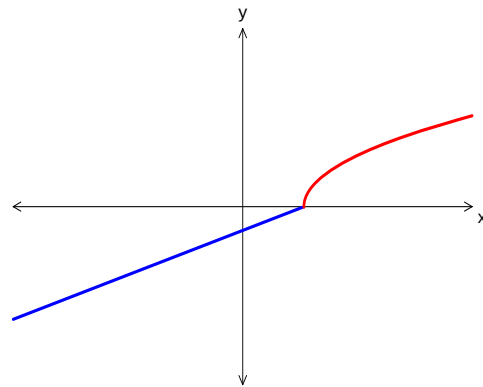
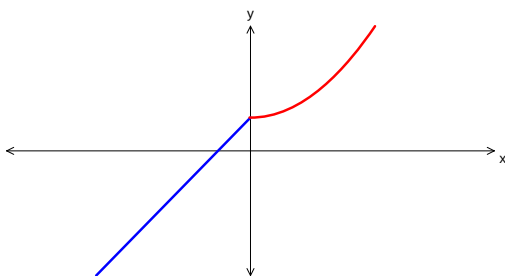
- A. $y = -x(x+a)^2(x-b)$
- B. $y = -x(x+a)(x-b)$
- C. $y = x(x-a)^2(x-b)$
- D. $y = -x(x+a)^2(b-x)$
- E. $y = x(x-a)^2(b-x)$

Question 8

The graph of the function with the equation $y = f(x)$ is shown below.

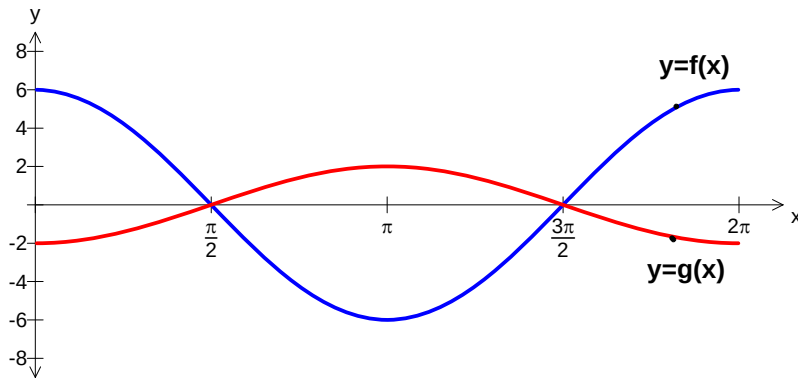


Which of the following is most likely to be the graph of the inverse function?

A.**B.****C.****D.****E.**

Question 9

The diagram below shows the graphs of two circular functions, f and g .



The graph of the function with the equation $y = f(x)$ is transformed into the graph of the function with the equation $y = g(x)$ by

- A. a dilation by a scale factor of $\frac{1}{3}$ from the y -axis followed by a reflection in the x -axis.
- B. a dilation by a scale factor of 3 from the y -axis followed by a reflection in the x -axis.
- C. a dilation by a scale factor of 3 from the y -axis followed by a reflection in the y -axis.
- D. a dilation by a scale factor of $\frac{1}{3}$ from the x -axis followed by a reflection in the x -axis.
- E. a dilation by a scale factor of 3 from the x -axis followed by a reflection in the y -axis.

Question 10

The function defined by $f : A \rightarrow R$, $f(x) = e^{(x-b)^2}$, $b \in R$, will have an inverse function for all values of b , if its domain A is

- A. R
- B. $R \setminus \{b\}$
- C. $[b, \infty)$
- D. R^+
- E. $[-b, \infty)$

Question 11

If $f(x) = e^{2x}$ and $g(x) = \frac{1}{\sqrt{x}}$ then $g(f(1))$ is

- A. $\frac{1}{e}$
- B. $-e$
- C. $\sqrt{e}$
- D. 1
- E. -1

Question 12

If $y = \log_e(\sin(3x))$ then $\frac{dy}{dx}$ is equal to

- A. $\frac{3\cos(3x)}{\sin(3x)}$
- B. $\frac{1}{\sin(3x)}$
- C. $\frac{1}{3\cos(3x)}$
- D. $3\tan(3x)$
- E. $-3\tan(3x)$

Question 13

The equation of the normal to the curve with equation $y = x \cos(x)$, at the point on the curve with x -coordinate 2π is

- A. $y = \frac{1}{x \sin(x)}$
- B. $y = -x$
- C. $y = x$
- D. $y = -x + 4\pi$
- E. $y = x - 4\pi$

Question 14

If $y = 2e(e^x - 1)$ then the rate of change of y with respect to x when $x = 0$ is

- A. $2e^2$
- B. 0
- C. $2e - 1$
- D. $2e^2 - 2e$
- E. $2e$

Question 15

The interval $[0, 5]$ is divided into n equal subintervals by the points $x_0, x_1, \dots, x_{n-1}, x_n$ where $0 = x_0 < x_1 < \dots < x_{n-1} < x_n = 5$. Let $\delta x = x_i - x_{i-1}$ for $i = 1, 2, \dots, n$.

Then $\lim_{\delta x \rightarrow 0} \sum_{i=1}^n (2x_i \delta x)$ is equal to

- A. $\int_5^0 2x \, dx$
- B. $\int_0^5 x^2 \, dx$
- C. 25
- D. 12.5
- E. 10

Question 16

If $f'(x) = 5e^{2x}$ and c is a real constant, then $f(x)$ is equal to

- A. $\frac{5}{2}e^{2x} + c$
- B. $10e^{2x} + c$
- C. $\frac{5}{2}e^{3x} + c$
- D. $\frac{5}{2}e^{x^2} + c$
- E. $5e^{x^2} + c$

Question 17

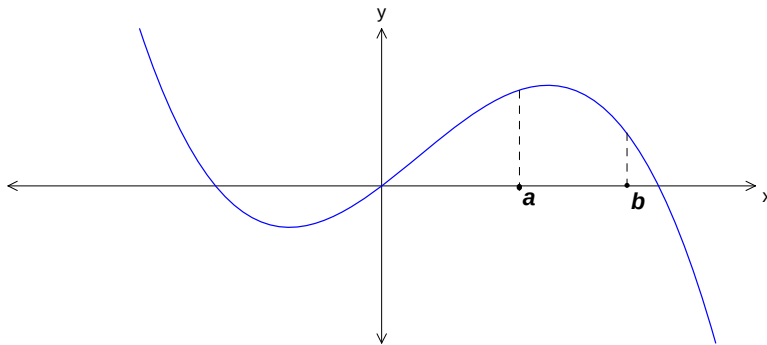
Let p be a function defined on the interval $[3, 5]$ and q a function such that $q'(x) = p(x)$, for all $x \in [3, 5]$

$\int_3^5 p(x) dx$ is equal to

- A. $q(x) + c$
- B. $p(5) - p(3)$
- C. $q(5) - q(3)$
- D. $q'(5) - q'(3)$
- E. $p(x) + c$

Question 18

The graph of the function with equation $y = f(x)$ is shown below.



Let g be a function such that $g'(x) = f(x)$.

On the interval (a, b) , the graph of g will have a

- A. maximum turning point.
- B. minimum turning point.
- C. negative gradient.
- D. positive gradient.
- E. stationary point of inflection.

Question 19

The number of defective batteries in a box of batteries ready for sale is a random variable with a binomial distribution with mean 8.1 and standard deviation 0.9.

If a battery is drawn at random from the box, the probability that it is not defective is

- A. 0.1
- B. 0.9
- C. 0.09
- D. 0.3
- E. 0.7

Question 20

Let X be a normally distributed random variable with mean μ and standard deviation σ . Which one of the following is not always true?

- A. $\Pr(X > \mu) = 0.5$
- B. $\Pr(X > a) = 1 - \Pr(X < a)$
- C. $\Pr(\mu - \sigma < X < \mu + \sigma) \approx 0.68$
- D. $\Pr(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.95$
- E. $\Pr(a < X < b) = \Pr(X > b) - \Pr(X > a)$

Question 21

The random variable X has the following probability distribution.

X	1	2	3
$\Pr(X=x)$	a	b	0.1

If the mean of X is 1.3, then the values of a and b respectively are—

- A. 0.5, 0.4
- B. 0.8, 0.1
- C. 0.1, 0.8
- D. 0.3, 0.7
- E. 0.4, 0.5

Question 22

Juicy Giant orange juice is packed in small glass bottles labelled as containing 250 *ml*. The packing process produces bottles that are normally distributed with a standard deviation of 3 *ml*. In order to guarantee that only 1% of bottles are under-volume, the actual mean volume, in *ml*, would be required to be closest to

- A. 256
- B. 257
- C. 258
- D. 250.01
- E. 247

END OF SECTION 1
TURN OVER

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal answer will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise stated diagrams are not drawn to scale.

Question 1

Grace shops each day at one of two supermarkets, Costless and Spendway. Her choice of supermarket each day depends only on which supermarket she has shopped at on the previous day. If she shops at Costless one day then the chance of her shopping at Costless the next day is $\frac{3}{5}$.

The transition matrix for the probabilities of Grace shopping at either supermarket given the supermarket she shopped at on the previous day is

$$\begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix}$$

- a.** If she shops at Spendway one day, what is the chance she shops at Costless the next day?

1 mark

- b.** Suppose she shops at Costless one Friday.

What is the exact probability she shops at Costless the next Monday?

3 marks

- c.** In the long term, what percentage of days, to the nearest percent, will Grace shop at Spendway?

2 marks

The time, t in minutes, she spends in the supermarket is independent of the type of supermarket she shops at and is a random variable with probability density function

$$f(t) = \begin{cases} at(60-t) & \text{if } 0 \leq t \leq 60 \\ 0 & \text{otherwise} \end{cases}$$

- d.** Show that $a = \frac{1}{36000}$.

2 marks

- e.** What is the probability, correct to 3 decimal places, that Grace spends longer than 45 minutes shopping in the supermarket?

2 marks

- f.** On 5% of occasions Grace spends between n and $2n$ minutes shopping. Find the value of n , correct to 3 decimal places.

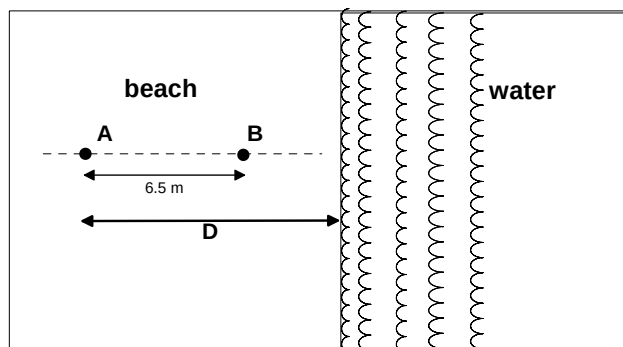
2 marks

Total 12 marks

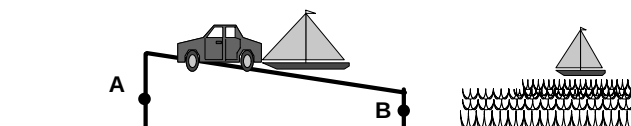
Question 2

Keith likes to holiday at Point Roadknight beach where the waves roll on to the beach at regular intervals.

The diagrams below show the beach with two markers, A and B at the start and end of the boat ramp, where A is 6.5 metres further up the beach than B . The line AB is perpendicular to the water's edge.



View from above

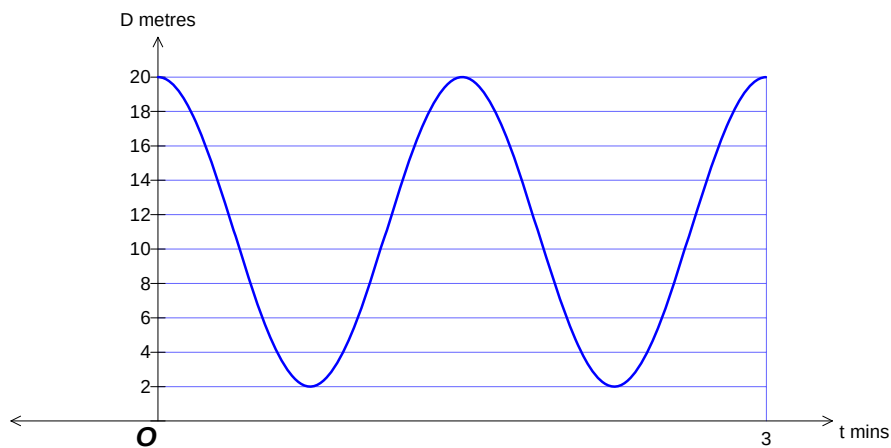


Side view

Keith records the distance of the water's edge from the top marker A . He calculates that on one particular day the distance D metres of the water's edge from the top marker A is a function of time t (in minutes from when he starts to observe the waves). It can be modelled exactly by the equation

$$D = a \cos(bt) + c$$

where a , b and c are positive constants. The graph of D as a function of time t is shown below:



- a. State the maximum and minimum distances of the water's edge from marker A on this day.

2 marks

- b.** Find the number of waves that hit the beach in one hour on this day.

2 marks

- c.** Find the values of a , b and c .

3 marks

- d. i.** Write down an equation, the solution of which gives the times when the marker at B is in the water on this day.

1 mark

- ii.** Find the exact percentage of time that the marker at B is in the water.

3 marks

Due to winds, tides and currents, on some days the waves come further up the beach and are closer together. Keith observes that on such a day the distance of the water's edge from marker A can be described by the equation

$$D = (8 + S) \cos\left(\frac{8\pi}{3}\right) + 11 \quad \text{where } S \text{ metres is the seasonal tidal factor which varies}$$

with the factors described above. S is normally distributed with a mean of 2 and a standard deviation of 0.3.

- e. On a particular day the waves just reach the top of the boat ramp at marker A. Find the value of S on this day.

2 marks

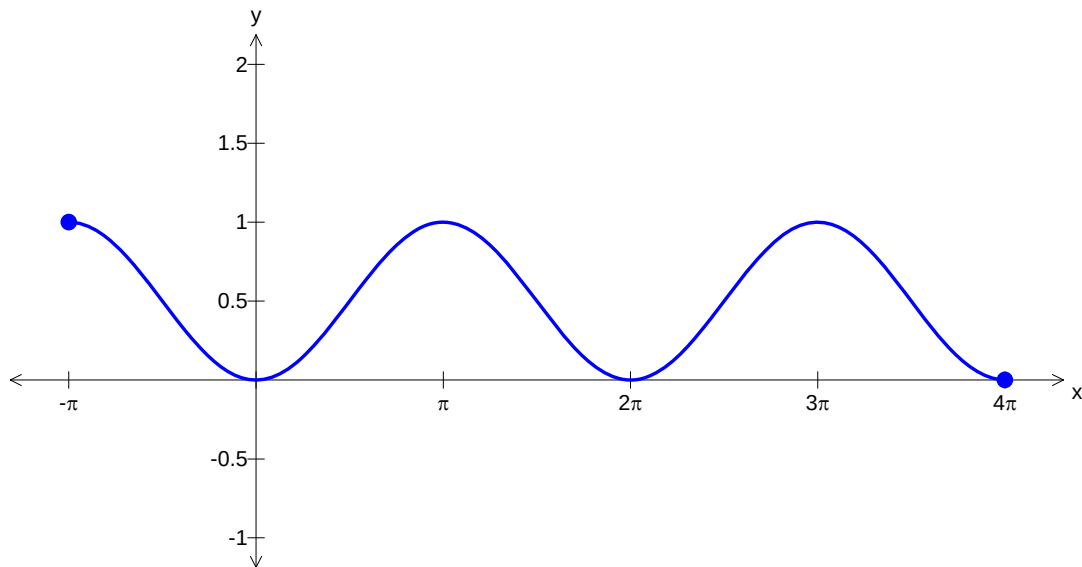
- f. Marcus decides to go fishing from the boat ramp at Point Roadknight beach, however he does not know the value of S . How many metres up the boat ramp from marker B should he stand so that he has an 80% chance of not getting wet from the waves? Answer correct to two decimal places.

3 marks

Total 16 marks

Question 3

The graph of $f : [-\pi, 4\pi] \rightarrow \mathbb{R}$, $f(x) = \sin^2\left(\frac{x}{2}\right)$ is shown below.



- a.** Find $f'\left(\frac{3\pi}{2}\right)$ and $f'\left(\frac{5\pi}{2}\right)$.

2 marks

- b. i.** Find the gradient of the normal to the curve at any point x .

2 marks

- ii.** Find exact coordinates of the points on the curve $y = f(x)$ where the gradient of the normal to the curve is equal to 2.

2 marks

- iii. Find the exact equations of the **normals** to the curve at the points where $x = \frac{3\pi}{2}$ and $x = \frac{5\pi}{2}$.

2 marks

- c. The graph of $y = f(x)$ is transformed to give the graph of $y = f(x) + b$. Find the exact value of b , such that the graph of the normal at $x = \frac{3\pi}{2}$ and the graph of the normal at $x = \frac{5\pi}{2}$ intersect on the x -axis.

2 marks

Let $g : [-\pi, 4\pi] \rightarrow \mathbb{R}$, $g(x) = \sin^2\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) + 1$.

- d. i. Find $g'(x)$.

1 mark

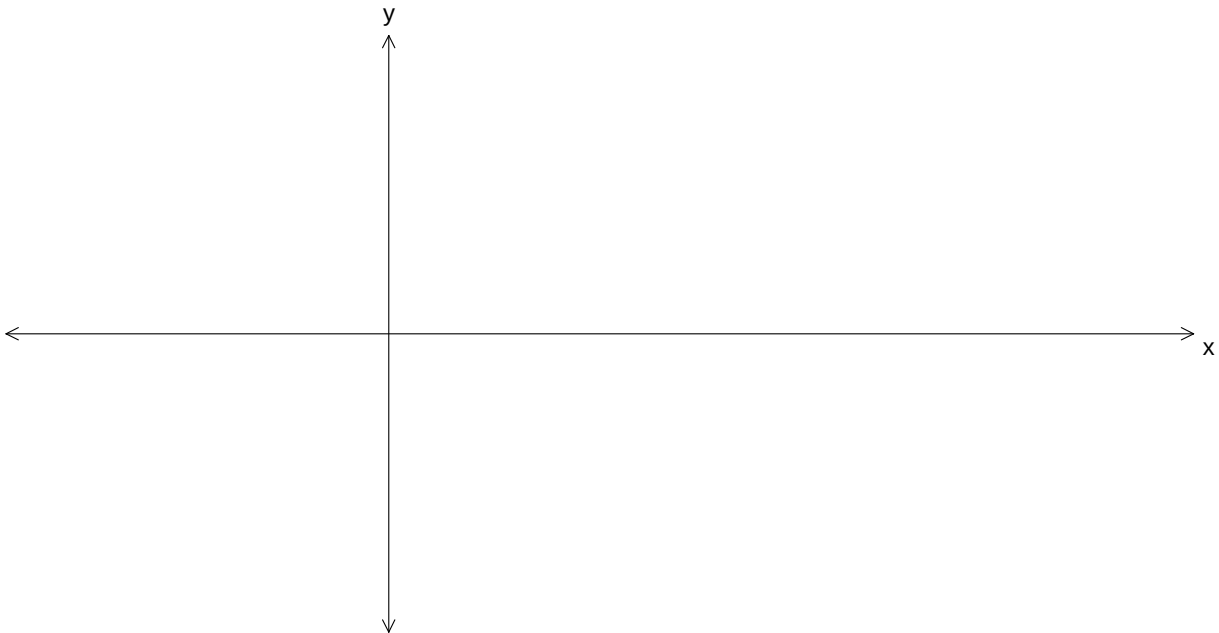
- ii. Write down an equation, the solution of which gives the values of x for which the gradient is zero.

1 mark

- iii. Solve the equation for $x \in [-\pi, 4\pi]$ to find the exact coordinates on the curve where the gradient is zero.

3 marks

- iv. Sketch the graph of $y = g(x)$ on the axes below, labeling intercepts, stationary points and endpoints with exact coordinates.



2 marks

Total 17 marks

Question 4

Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 9e^{-x}$.

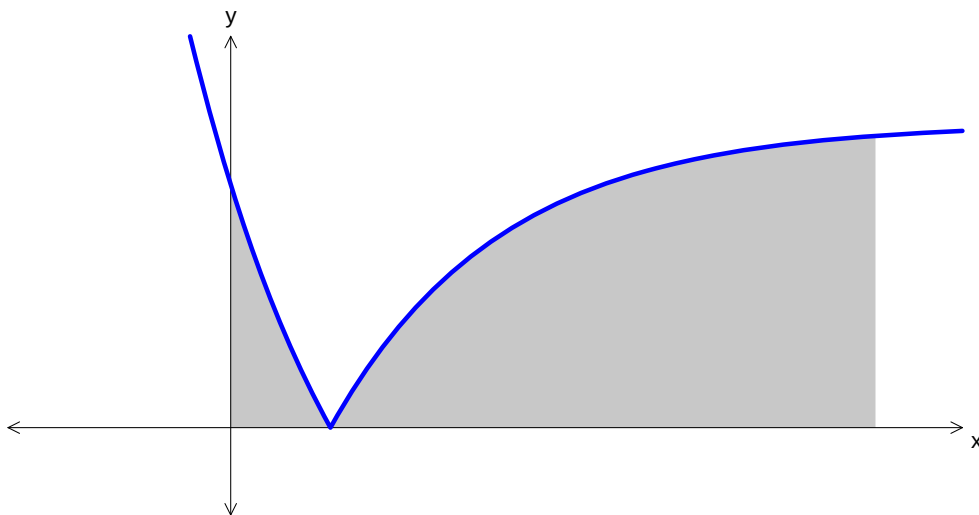
- a. i. Show that $f(u)f(-u) = 81$, where u is a real number.

1 mark

- ii. Show that $f(u+v) = \frac{1}{9} f(u)f(v)$, where u and v are real numbers.

2 marks

A panel of a stained glass window has a section of lead outline described by the graph with the equation $g(x) = |9e^{-x} - 5|$ and is shown below. The shaded area is red-coloured glass.



- b. The x -intercept of the graph occurs at $x = \log_e(a)$. Find the exact value of a .

2 marks

- c. The red-coloured glass is bounded by the area under the graph, the x -axis, the y -axis and the line $x = 4$. Find the exact area of the red-coloured glass.

3 marks

- d. Find the value of a , correct to 3 decimal places, such that the area of the red-coloured glass enclosed by the graph, the x -axis, the y -axis and the line $x = a$ is equal to 35 square units.

2 marks

- e. In producing the leadlight panel, the craftsman creates a sketch of the red-coloured panel and notices when positioning the curve that two points on the curve describing the red-coloured glass are exactly 1 unit horizontally apart. Find the coordinates of the two points, correct to 3 decimal places.

3 marks

Total 13 marks