

Year 12 Trial Exam Paper
2018
MATHEMATICAL METHODS
Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring blank sheets of paper, notes of any kind or correction fluid/tape into the examination.
- Calculators are NOT permitted in this examination.

Materials provided

- Question and answer book of 11 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

- a.** Let $y = \cos(x) \cos(4x)$.

Find $\frac{dy}{dx}$.

2 marks

- b.** Let $f(x) = 2e^{2x}$.

Evaluate $f'(\log_e(2))$.

2 marks

Question 2 (5 marks)

Let $y = \frac{\log_e(x)}{x}$.

a. Find $\frac{dy}{dx}$.

2 marks

b. Hence, calculate $\int_1^2 \frac{\log_e(x)}{x^2} dx$.

3 marks

Question 3 (3 marks)

Let $f: D \rightarrow \mathbb{R}$, $f(x) = \log_e(x) + \log_e(5-2x)$.

- a.** State the maximal domain of f .

1 mark

- b.** Solve the equation $f(x) = 0$.

2 marks

Question 4 (5 marks)

Lily enjoys seeing birds in a tree outside her house. The random variable X represents the number of birds in the tree when she looks outside each morning.

The distribution of X is given in the table below.

X	0	1	2	3
$\Pr(X = x)$	0.45	0.3	0.15	0.1

- a.** Find $\Pr(X = 0 | X < 2)$.

1 mark

- b.** Find $E(X)$.

1 mark

- c.** Lily records how many birds she sees each morning before school for a week. Let $\hat{P}$ represent the sample proportion of days that Lily sees at least two birds in a particular five day week.

- i.** Find $E(\hat{P})$.

1 mark

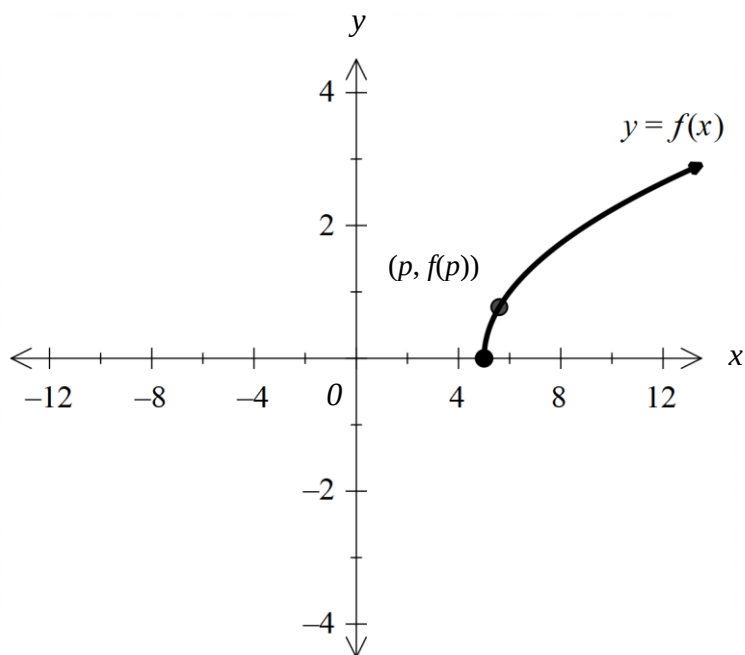
- ii.** Find $\Pr(\hat{P} = 0.2)$.

Express your answer in the form $\frac{a}{b^c}$, where a , b and c are positive integers.

2 marks

Question 5 (4 marks)

Let $f:[5,\infty)\rightarrow\mathbb{R}$, where $f(x)=\sqrt{x-5}$.



The point, P , with coordinates $(p, f(p))$, is on the curve defined by $y = f(x)$.

- a.** Find the gradient of the tangent to the curve at the point P in terms of p .

1 mark

- b.** Find the value of p such that the tangent to the curve at P passes through the point $(0, 0)$.

3 marks

Question 6 (2 marks)

The events A and B from a sample space are independent. If $\Pr(A' \cap B) = 0.2$ and $\Pr(A) = 0.4$, find $\Pr(A \cap B)$.

Question 7 (3 marks)

Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} kx^3(1-x)^2 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

where k is a real number.

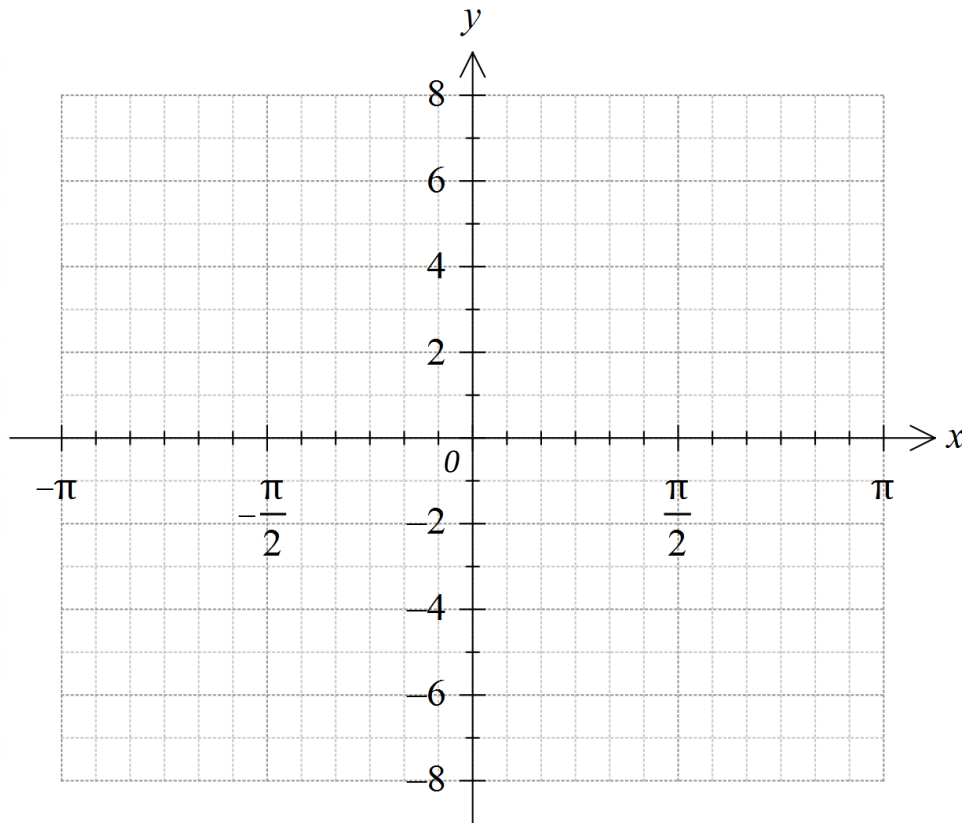
Find the value of k .

Question 8 (7 marks)

Let $f: \left[-\frac{7\pi}{12}, \frac{\pi}{4}\right] \rightarrow \mathbb{R}$, where $f(x) = 2 - 4\sin(2x)$.

- a.** Sketch the graph of $y = f(x)$ on the axes below. Label all end points and axes intercepts with their coordinates.

4 marks



- b.** Find the area enclosed by the graph of $y = f(x)$ and the x -axis over the interval $x \in \left[-\frac{7\pi}{12}, \frac{\pi}{12}\right]$.

3 marks

Question 9 (7 marks)

Let $f: R \setminus \{b\} \rightarrow R$, $f(x) = \frac{a}{x-b}$, where a and b are real numbers and $a \neq 0$.

- a.** Find the domain and rule of the inverse function f^{-1} .

2 marks

The transformation $T: R^2 \rightarrow R^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$, where g, h and k are integers, maps the graph of $y = f(x)$ onto the graph of $y = f^{-1}(x)$.

- b.** Find the values g, h and k in terms of a and b .

2 marks

- c. Find the values of a , in terms of b , for which the equation $f(x) = f^{-1}(x)$ has no real solutions.

3 marks

END OF QUESTION AND ANSWER BOOK