

Year 12 Trial Exam Paper
2018
SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided above.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

This trial examination produced by Insight Publications is NOT an official VCAA paper for the 2018 Year 12 Specialist Mathematics 1 written examination. The Publishers assume no legal liability for the opinions, ideas or statements contained in this trial examination. This examination paper is licensed to be printed, photocopied or placed on the school intranet and used only within the confines of the purchasing school for examining their students. No trial examination or part thereof may be issued or passed on to any other party, including other schools, practising or non-practising teachers, tutors, parents, websites or publishing agencies without the written consent of Insight Publications.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 (4 marks)

Consider the complex numbers $z = 2 + i$ and $v = 1 + 3i$.

a. Show that $\frac{v}{z} = \sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$.

2 marks

b. Find $\text{Arg}\left(\frac{\bar{v}}{\bar{z}}\right)$ and $\text{Arg}\left(\frac{v}{zi}\right)$.

2 marks

Consider the relation defined below.

a. Show that $(1, 1)$ is a point on the curve defined by this relation.

- 3 marks

[illegible]

Question 3 (6 marks)

The waiting time, in minutes, for customers entering the Royal Melbourne Show is normally distributed with mean μ .

In 2016, a random sample of 16 customers entering the Royal Melbourne Show gave an approximate 95% confidence interval for μ as $5.02 < \mu < 6.98$.

- a.** Find the mean $\bar{x}$ and hence show that the sample standard deviation is 2.

2 marks

In 2017, the number of inspectors at entry points to the Royal Melbourne Show was increased and the waiting time, in minutes, for customers was claimed to be normally distributed with a mean waiting time of 4.5 minutes and a standard deviation of 1.5 minutes.

A random sample of 36 customers entering the Royal Melbourne Show had a mean waiting time of 4 minutes. Assume that the population standard deviation remains at 1.5 minutes.

- b.** Test the hypothesis that the mean waiting time is less than that claimed by finding an approximate p value for this test. Hence explain whether or not the null hypothesis should be rejected at the 5% level of significance.

4 marks

Question 4 (3 marks)

Evaluate $\int_0^{\sqrt{e^2-1}} \frac{4x \log_e(x^2+1)}{x^2+1} dx$.

Question 5 (3 marks)

A parallelogram $OABC$ is defined by the vectors $\vec{OA} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{OC} = 2\hat{i} - \hat{j} - 2\hat{k}$.

N is the point on OA that is closest to the point C .

- a.** Find the scalar resolute of $\vec{OC}$ in the direction of $\vec{OA}$.

1 mark

- b.** Hence find the length of CN .

2 marks

A lift is travelling downwards at a constant speed of $v \text{ ms}^{-1}$. A woman standing in the lift drops her cup of coffee from a height of h metres above the floor of the lift. The coffee cup hits the floor of the lift after 0.5 seconds.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 7 (3 marks)

Solve the differential equation $\frac{dy}{dx} = \frac{e^{0.5x}}{3\sqrt{y}}$ for y given that $y = 1$ when $x = 0$.

Question 8 (4 marks)

The acceleration of a particle at time t seconds is given by $\ddot{\mathbf{r}}(t) = 2\mathbf{i} + 4\mathbf{j} \text{ ms}^{-2}$.

Find the distance of the particle from its initial position when $t = 2$, given that

$\dot{\mathbf{r}}(0) = -2\mathbf{i} - 3\mathbf{k} \text{ ms}^{-1}$ and $\mathbf{r}(0) = \mathbf{i} - \mathbf{j} + 2\mathbf{k} \text{ m}$.

Question 9 (4 marks)

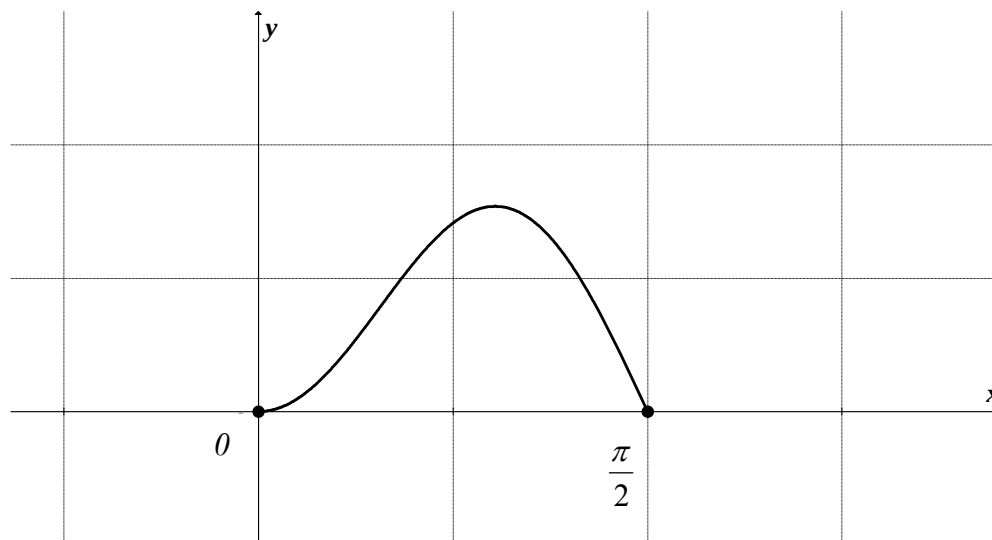
Given that $\sin(x + y) = \frac{1}{2}$ and $\frac{\tan(x)}{\tan(y)} = 3$, find $\sin(x - y)$.

Question 10 (5 marks)

The curve shown below can be defined parametrically by

$$x = \arcsin(t)$$

$$y = 2t^2\sqrt{1-t^2}, \text{ where } 0 \leq t \leq 1.$$



- a. Find the cartesian equation of the curve in the form $y = f(x)$.

1 mark

The area bounded by the curve and the x -axis is rotated about the x -axis.

- b.** Find the volume of the solid generated.

4 marks

END OF QUESTION AND ANSWER BOOK