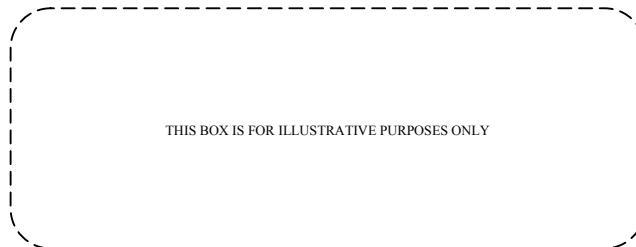




2010 Trial Examination



THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER

Figures

Words

Letter

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SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 10 pages.
- Working space is provided throughout the book.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

- a. Let vectors $\mathbf{a} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = m\mathbf{i} + n\mathbf{j} - 2\mathbf{k}$ be the position vectors of points A and B relative to the origin. Find all possible values of m and n such that vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular and $|\vec{AB}| = 2\sqrt{14}$.

4 marks

- b. The acute angle between the vectors $\mathbf{a} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\vec{AB} = 4\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}$ is $\theta = \cos^{-1}\left(\frac{p}{q\sqrt{7}}\right)$. Determine the values of p and q , where $\frac{p}{q}$ is in simplest form.

2 marks

TURN OVER

Question 2

- a. If $z = i$ is a solution of the equation $z^3 - cz^2 + 3iz + 1 - i = 0$, show that $c = 2 + 2i$.

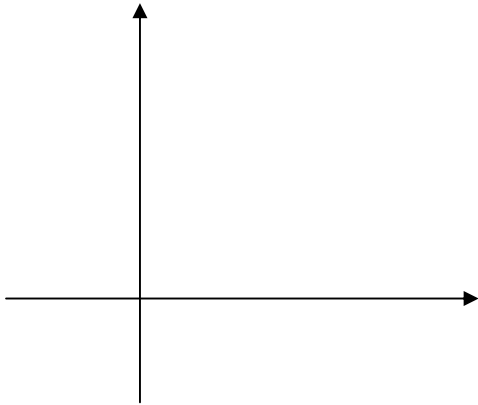
1 mark

- b. Find the other two solutions of the equation $z^3 - (2 + 2i)z^2 + 3iz + 1 - i = 0$.

4 marks

Question 3

Draw an accurate graph of $y = \frac{4}{\pi} \sin^{-1}(2x-1) + 3$. State the domain and the range and give the coordinates of the inflection point.



3 marks

Question 4

If $\tan \alpha = \frac{1}{12}$, $\tan \beta = \frac{2}{5}$ and $\tan \gamma = \frac{1}{3}$, where α, β and γ are acute angles, show that

$$\alpha + \beta + \gamma = \frac{\pi}{4}.$$

3 marks

TURN OVER

Question 5

Let $y^2 = xy - \log_e y$.

a. Find $\frac{dy}{dx}$.

2 marks

b. Find $\frac{dx}{dy}$.

2 marks

c. Use the results from part **a.** and **b.** to show that $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$.

1 mark

Question 6

The region bounded by the curve $y = \frac{x+1}{\sqrt{x^2-1}}$, the lines $y=0$, $x=2$ and $x=4$ is rotated about the x axis. Find the exact value of the volume of this solid of revolution.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

3 marks

TURN OVER

Question 7

- a.** Differentiate $x \sin^2 x$ with respect to x .

1 mark

- b. Hence, find the exact value of $\int_0^{\frac{3\pi}{2}} x \sin(2x) dx$.**

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4 marks

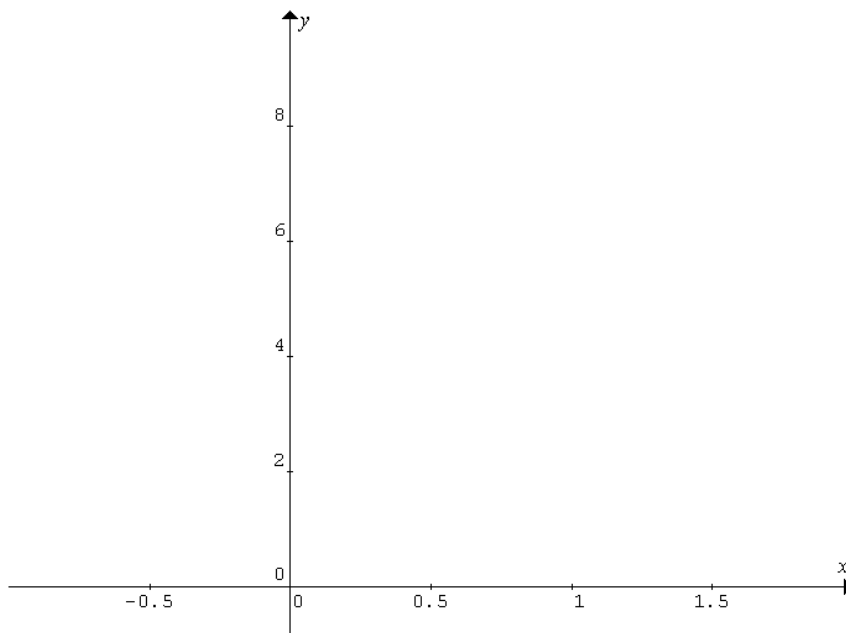
Question 8

The position of a particle at time t seconds is given by $\mathbf{r}(t) = e^{-2t}\mathbf{i} + (2e^t + 1)\mathbf{j}$, $t \geq 0$.

- a. Find the Cartesian equation of the path of the particle. State the domain and the range of the path.

3 marks

- b. Sketch the graph of the path. Label the asymptote(s) and end point(s).



2 marks

TURN OVER

Question 9

Three particles of masses 2kg, 4kg and 6kg are placed on a rough horizontal surface and are connected by two inextensible strings. A horizontal pulling force of 24 N is applied to the heaviest particle, as shown in the diagram. The coefficient of friction between the particles and the surface is $\mu = \frac{1}{10}$.



- a. Label the diagram above with all the forces acting in the system. Hence, show that the acceleration of the system is $a = 2 - \frac{g}{10}$.

3 marks

- b. Find the tensions between the particles.

2 marks

END OF QUESTION AND ANSWER BOOK