

SPECIALIST MATHEMATICS

UNIT 3

Written examination 1

FORMULA SHEET

Directions to students

Remove this formula sheet during reading time.

This formula sheet is provided as a reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2} (a+b) h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3} \pi r^2 h$

volume of a pyramid: $\frac{1}{3} Ah$

volume of a sphere: $\frac{4}{3} \pi r^3$

area of a triangle: $\frac{1}{2} bc \sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{x-h^2}{a^2} + \frac{y-k^2}{b^2} = 1$

hyperbola: $\frac{x-h^2}{a^2} - \frac{y-k^2}{b^2} = 1$

Circular (trigometric) functions

$\cos^2 x + \sin^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$\sin x + y = \sin x \cos y + \cos x \sin y$

$\cos x + y = \cos x \cos y - \sin x \sin y$

$\tan x + y = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\sin 2x = 2 \sin x \cos x$

$\cot^2 x + 1 = \operatorname{cosec}^2 x$

$\sin x - y = \sin x \cos y - \cos x \sin y$

$\cos x - y = \cos x \cos y + \sin x \sin y$

$\tan x - y = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$0, \pi$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r \cos \theta + i \sin \theta = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis} \theta_1 + \theta_2$$

$$z^n = r^n \operatorname{cis} n\theta \quad \text{de Moivre's theorem}$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis} \theta_1 - \theta_2$$

Calculus

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} e^{ax} = ae^{ax}$$

$$\frac{d}{dx} \log_e x = \frac{1}{x}$$

$$\frac{d}{dx} \sin ax = a \cos ax$$

$$\frac{d}{dx} \cos ax = -a \sin ax$$

$$\frac{d}{dx} \tan ax = a \sec^2 ax$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \cos^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e x + c$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax + c$$

$$\int \cos ax dx = \frac{1}{a} \sin ax + c$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1} \left(\frac{x}{a} \right) + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1} \left(\frac{x}{a} \right) + c$$

product rule: $\frac{d}{dx} uv = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$

constant (uniform) acceleration: $v = u + at$ $s = ut + \frac{1}{2} at^2$ $v^2 = u^2 + 2as$ $s = \frac{1}{2} (u + v)t$

TURN OVER

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\underline{i} + \frac{dy}{dt}\underline{j} + \frac{dz}{dt}\underline{k}$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

Mechanics

momentum:

$$\underline{p} = m\underline{v}$$

equation of motion:

$$\underline{R} = m\underline{a}$$

friction:

$$F \leq \mu N$$

END OF FORMULA SHEET