



YEAR 11 MATHEMATICAL METHODS A (CAS)

Semester 1 Technology Examination June 2009

NAME: _____ Class: JLB / PHE / PM

TIME: 90 minutes plus 15 minutes reading time.

Section A: Multiple Choice Questions

There are **20 questions** in this section. Choose the **BEST answer** and circle the letter A, B, C, D or E on the Multiple Choice Answer Sheet on the last page of this booklet. You may tear this Multiple Choice Answer Sheet off but ensure that your name is written on it. Each question is worth two marks. Answer all questions. Total 40 marks. **Suggested time 40 minutes.**

Section B: Analysis Questions

There are **4 questions** in this section. You must answer all these questions. Follow the directions given in each question. Answer in the space provided in this BOOKLET. Total 42 marks. **Suggested time 50 minutes.**

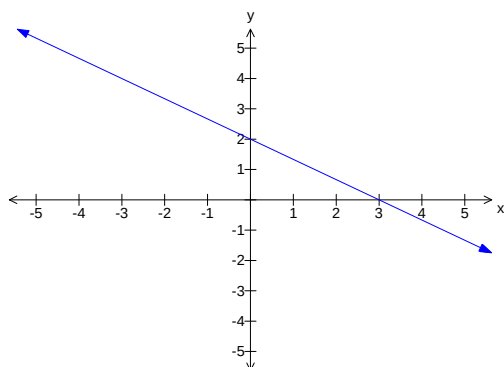
Section	Number of questions	Marks	Minutes
A: Multiple choice	20	40	40
B: Analysis	4	42	50
Total		82	90

- V200 calculators are allowed.
- DICTIONARIES of any type are not allowed.
- You may bring seven double-sided A4 sheets of notes and calculator notes.
- Unless otherwise stated, the diagrams in this booklet are not to scale.
- *Marks will be awarded for correct and logical working in the extended-response section.*

Section A: Multiple choice questions (2 marks each)

Question 1

The gradient of the graph shown is:



- A** -2 **B** $\frac{3}{2}$ **C** $\frac{2}{3}$ **D** $-\frac{2}{3}$ **E** $-\frac{3}{2}$

Question 2

The magnitude of the angle that the line $3y - 6x = 18$ makes with the x-axis is:

- A** $71^{\circ}34'$ **B** $63^{\circ}26'$ **C** $-26^{\circ}34'$ **D** $80^{\circ}32'$ **E** $26^{\circ}34'$

Question 3

The value of x in $2(1 - 9x) \geq 4(1 - 3x)$ is:

- A** $x \geq \frac{1}{3}$ **B** $x \geq -\frac{1}{3}$ **C** $x \geq \frac{1}{10}$ **D** $x \leq \frac{1}{3}$ **E** $x \leq -\frac{1}{3}$

Question 4

The turning point of the graph with equation $y = (x - 2)^2 + 4$ is:

- A** $(2, 4)$ **B** $(-2, 4)$ **C** $(2, -4)$ **D** $(4, 4)$ **E** $(-2, -4)$

Question 5

The number of solutions for the equation $0 = 2x^2 + 3x - 1$ is:

- A** 0 **B** 1 **C** 2 **D** 3 **E** unknown

Question 6

$f(1+a)$ of the function $f(x) = 4x^2 + x + 2$ is:

A $4a^2 + 9a + 11$

B $4a^2 + 9a + 7$

C $4a^2 + 2a + 11$

D $4a^2 + 2a + 7$

E $4a^2 + 2a + 2$

Question 7

The value of x for $f(x) = 5x^2 + 2$, when $f(x) = 22$ is:

A ± 4

B $\pm\sqrt{10}$

C ± 2

D 2422

E 2

Question 8

The radian angle equivalent of 150° is:

A $\frac{5}{6\pi}$

B $\frac{27000}{\pi}$

C $\frac{6}{5\pi}$

D $\frac{5\pi}{6}$

E $\frac{6\pi}{5}$

Question 9

For $x^2 - x + a$ to be a perfect square, a is equal to:

A 1

B $\frac{1}{4}$

C $-\frac{1}{4}$

D $-\frac{1}{2}$

E $\frac{1}{2}$

Question 10

If $y = x^2 - 6x + 11$ is changed to the form $y = a(x-h)^2 + k$, then the values of a , h and k are:

A $a=1, h=-6, k=2$

B $a=1, h=-3, k=-2$

C $a=1, h=3, k=2$

D $a=1, h=3, k=-15$

E $a=1, h=-3, k=2$

Question 11

The equations of the asymptotes of the graph of $y = -\frac{1}{2x-3}$ are:

- A** $x = -3, y = 0$ **B** $x = 3, y = 0$ **C** $x = \frac{3}{2}, y = 0$
D $x = -\frac{3}{2}, y = -9$ **E** $x = -3, y = 2$

Question 12

The circle with equation $x^2 + 4x + y^2 - 10y + 20 = 0$ has centre coordinates, C , and radius, r , respectively of:

- A** $C = (-2, 5), r = 9$ **B** $C = (2, -5), r = 9$ **C** $C = (2, -5), r = 3$
D $C = (-2, 5), r = \sqrt{20}$ **E** $C = (-2, 5), r = 3$

Question 13

The transformations that the graph of $y = \sqrt{x}$ has undergone to produce the graph of $y = 3\sqrt{x+1} - 2$ are:

- A** Dilated by factor 3 from x -axis, translated right 1 unit & 2 units down.
B Dilated by factor $\frac{1}{3}$ from y -axis, translated right 1 unit & 2 units down.
C Dilated by factor 3 from x -axis, translated left 1 unit & 2 units down.
D Dilated by factor $\frac{1}{3}$ from y -axis, translated left 1 unit & 2 units up.
E Dilated by factor 3 from y -axis, translated left 1 unit & 2 units up.

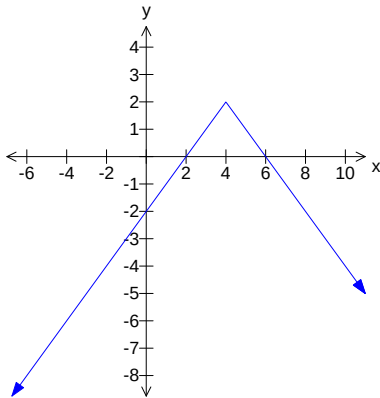
Question 14

The inverse function $f^{-1}(x)$ of $f(x) = \frac{3}{x-2} + 5$ is:

- A** $f^{-1}(x) = \frac{1}{3}(x+2) - 5$ **B** $f^{-1}(x) = \frac{3}{x-5} + 2$ **C** $f^{-1}(x) = \frac{2}{x-3} - 5$
D $f^{-1}(x) = \frac{3}{x-2} + 5$ **E** $f^{-1}(x) = 5(x-2) + 3$

Question 15

The equation of the graph shown below is given by:



A $y = -|x - 4| + 2$

B $y = |x - 4| + 2$

C $y = -|x - 4| - 2$

D $y = -|x + 4| + 2$

E $y = |-x + 4| + 2$

Question 16

The equation of the line perpendicular to the line with equation $-6y - 3x = 12$ that passes through the point $(4, 4)$ is:

A $y = \frac{1}{2}x + 2$

B $y = -2x + 12$

C $y = -\frac{1}{2}x + 6$

D $y = 2x - 4$

E $y = -\frac{x}{2} - 2$

Question 17

If $-2A + \begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -2 & -9 \\ 9 & -15 \end{bmatrix}$ then the matrix $A =$

A $\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$

B $\begin{bmatrix} -2 & -4 \\ 4 & -6 \end{bmatrix}$

C undefined

D $\begin{bmatrix} 1 & -2 \\ 3 & -7 \end{bmatrix}$

E $\begin{bmatrix} 2 & 4 \\ -4 & 6 \end{bmatrix}$

Question 18

The maximum value of the graph of $y = -\sin x + 2$ is:

- A** 1 **B** -1 **C** 2 **D** 3 **E** 0

Question 19

The graph of $y = a \sin x + k$, where a and k are real numbers, has a minimum value of -2 . Possible values of a and k are:

- A** $a = 5, k = 3$ **B** $a = 2, k = 2$ **C** $a = 3, k = 5$
D $a = -1, k = 1$ **E** $a = -2, k = 2$

Question 20

The function $f : R \rightarrow R, f(x) = -4\sin(\pi x)$ has:

- A** amplitude 4 and period 2
B amplitude 4 and period 2π
C amplitude 4 and period π
D amplitude -4 and period 2π
E amplitude -4 and period π

Section B: ANALYSIS QUESTIONS

1 For $f : [1, \infty) \rightarrow \mathbb{R}$, $f(x) = (x-1)^2 - 3$,

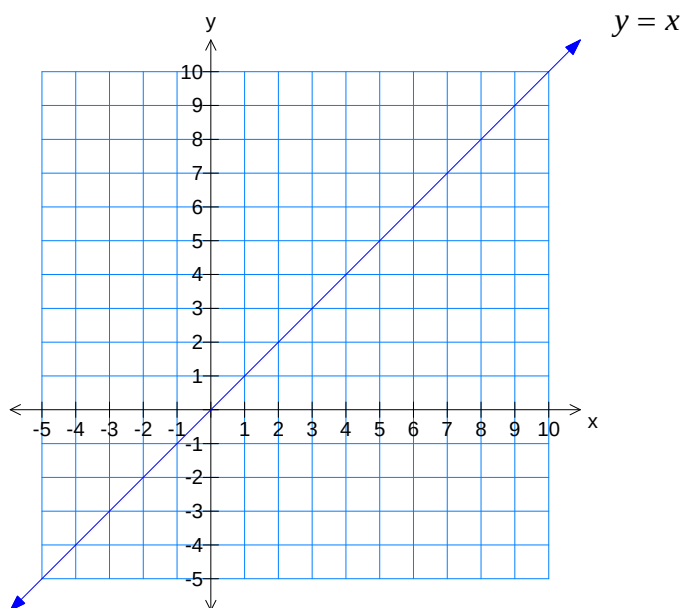
1a) Explain why f is a one-to-one function.

1b) State the domain and range of f . 1c) State the domain and range of f^{-1} , the inverse of f .

1d) Find the inverse of f and write f^{-1} using function notation.

1e) Find the point of intersection between f and f^{-1} , rounded to **2 decimal places**.

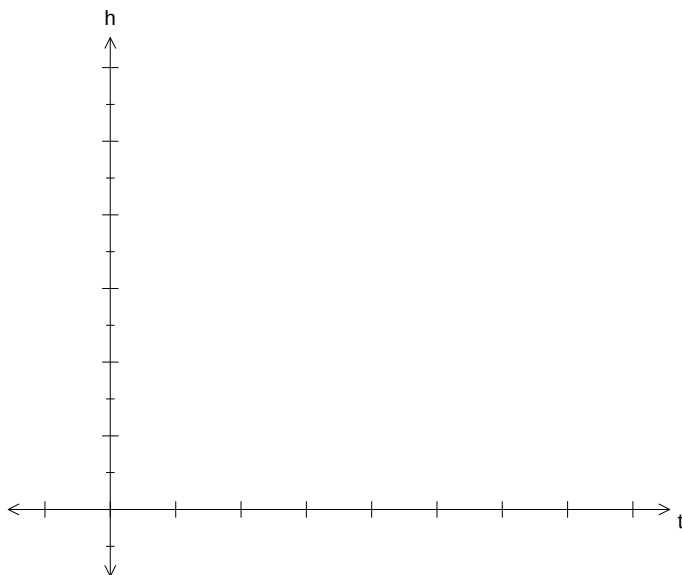
1f) Sketch f and f^{-1} below. Label all endpoints and intercepts.



1+2+2+2+1+4 = 12 marks

2 A projectile is fired upwards and to the right and its path can be modelled by the equation $h = 24t - 3t^2$. If the projectile reaches a height of h metres above the ground after t seconds:

2a) Sketch a graph of the path of the projectile over an appropriate domain on the set of axes below, labelling all intercepts and turning points.



2b) Find the time taken to reach the maximum height.

2c) Find the maximum height the ball reaches above the ground.

2d) Find the time taken to first reach a height of 24 metres, correct to **1 decimal place**.

3+2+1+2= 8 marks

3 If $A = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$.

a) Find $\det(A)$.

b) Find the inverse matrix of A , A^{-1}

c) Find the matrix X , if $AX = B$

d) Find the matrix X , if $XA = B$

e) If $C = \begin{bmatrix} x & 3 \\ 2 & x \end{bmatrix}$, find the value(s) of x such that $C + A$ is a singular matrix.

f) Solve the simultaneous equations $2x + y = 4$ and $4x + y = 16$ using *matrices*.

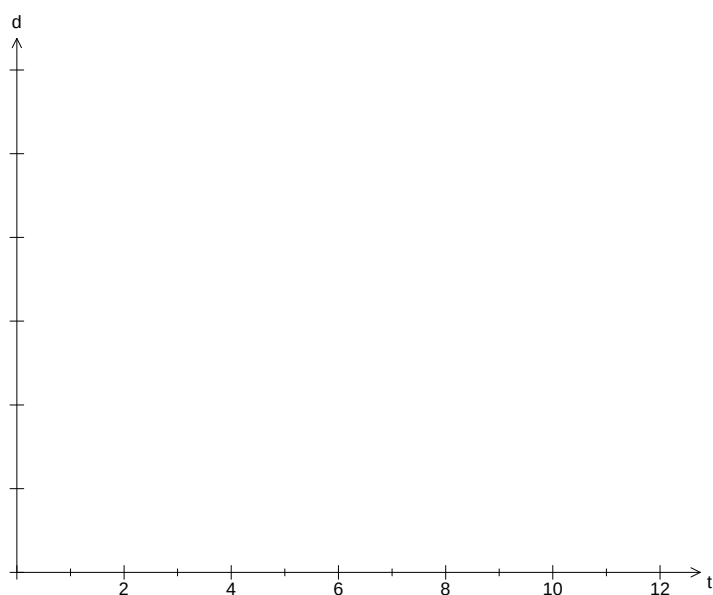
1+1+2+2+3+3 = 12 marks

4 The depth of water, d metres, in a harbour can be modelled by the equation

$d(t) = 3 + 2\cos\left(\frac{\pi t}{6}\right)$, where t is the number of hours after midnight.

a) State the depth of the water at high and low tides in the harbour.

b) Sketch the graph of $d(t) = 3 + 2\cos\left(\frac{\pi t}{6}\right)$ for $0 \leq t \leq 12$. Label all intercepts, endpoints and turning points.



c) A ferry enters the harbour at 10am. What is the depth of the water at this time?

d) The ferry can only stay in the harbour if the depth is at least 3.5m. What is the latest time it must leave the harbour in the afternoon (*correct to the nearest minute*)?

2+4+2+2 = 10 marks

END OF QUESTION AND ANSWER BOOK ☺

11MMA CAS Formula Sheet

Gradient of a straight line joining two points: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

General linear equation is $y = mx + c$, where m is the gradient and c is the y-intercept.

Equation of a straight line of gradient m , passing through (x_1, y_1) is $y - y_1 = m(x - x_1)$.

If two straight lines are **perpendicular** to each other the product of their gradients is -1 .
Eg. $m_1 \times m_2 = -1$.

Distance between two points $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

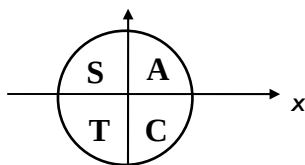
Midpoint of a line joining (x_1, y_1) and (x_2, y_2) is: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

A **quadratic function** is defined by $y = ax^2 + bx + c$ where a , b and c are constants.

The turning point of $y = ax^2 + bx + c$ is $x = -\frac{b}{2a}$, $y = f\left(-\frac{b}{2a}\right)$.

General quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Signs of trigonometric functions:



A = all positive, **S** = only sin positive, **T** = only tan positive, **C** = only cos positive.

Using symmetrical properties of trigonometric functions, it can be seen that:

$\sin(\pi - \theta) = \sin\theta$,	$\cos(\pi - \theta) = -\cos\theta$,	$\tan(\pi - \theta) = -\tan\theta$	for the second quadrant.
$\sin(\pi + \theta) = -\sin\theta$,	$\cos(\pi + \theta) = -\cos\theta$,	$\tan(\pi + \theta) = \tan\theta$	for the third quadrant.
$\sin(2\pi - \theta) = -\sin\theta$,	$\cos(2\pi - \theta) = \cos\theta$,	$\tan(2\pi - \theta) = -\tan\theta$	for the fourth quadrant.
$\sin(-\theta) = -\sin\theta$,	$\cos(-\theta) = \cos\theta$,	$\tan(-\theta) = -\tan\theta$	for the fourth quadrant.

11MMA CAS Multiple choice answer sheet. Circle the best answer.

You may remove this page.

NAME: _____

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E