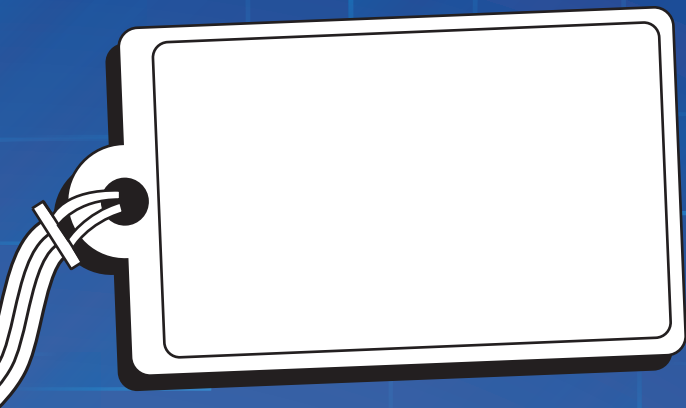




Interest

INTEREST

SERIES **K**



www.mathletics.com.au



INTEREST

The whole point of Financial Maths is to predict what will happen to money over time. This is so you can be prepared by knowing how much money you will have earned or need to pay after **interest** is added to an amount of money over a certain amount of time.



Answer these questions, *before* working through the chapter.

I used to think:

What do the terms 'principal value', 'future value' and 'interest' mean and how are they related?

If an interest rate is 11% pa, what does this mean? Can an interest rate only be per year?

What are the three different types of interest rates?



Answer these questions, *after* working through the chapter.

But now I think:

What do the terms 'principal value', 'future value' and 'interest' mean and how are they related?

If an interest rate is 11% pa, what does this mean? Can an interest rate only be per year?

What are the three different types of interest rates?



What do I know now that I didn't know before?

When money is invested it grows. The 'extra amount' earned is called the **interest** on the investment. There are two different kinds of interest that can be earned called **simple interest** and **compound interest**.

Finding Simple Interest

Simple interest is also called 'flat interest'. The initial amount invested is called the principal. Remember, interest is the extra amount earned on the principal. The formula to find simple interest is:

$$I = PRT$$

Diagram showing the components of the formula $I = PRT$:

- I is labeled "Simple interest"
- P is labeled "Principal"
- R is labeled "Rate (as a decimal)"
- T is labeled "Time period"

The interest rate r is often a percentage. In the above formula R is the percentage written in decimal form, so $R = \frac{r}{100}$

Find the simple interest if you invest \$2500 at a rate of 5% pa (per annum) for 4 years

This means: $P = \$2500$

$$r = 5\%$$

$$T = 4 \text{ years}$$

First, R must be found from r

$$R = \frac{r}{100} = \frac{5}{100} = 0.05$$

Use the above formula to find the simple interest

$$\begin{aligned}
 I &= PRT \\
 &= (\$2500)(0.05)(4) \\
 &= \$500
 \end{aligned}$$

R is a decimal (pointing to 0.05)

This means after 4 years the 'extra amount' earned will be \$500, so the total amount after 4 years is:

$$\text{Total} = \$2500 + \$500 = \$3000$$

Since $R = \frac{r}{100}$, this version of the formula could also be used:

$$I = P \times \frac{r}{100} \times T$$

When using this version, you don't need to find R separately. Either formula will work, so use the one you think is easier.

Time Periods

The interest rate, r , does not have to be per year. It can be per month, per week or even daily. **It is very important that the time periods and the rate match!** This means, that if the rate is per month then the time periods must be months, or if the rate is per week, then time periods must be weeks. Otherwise the formulas won't work.

In the example on the previous page, the time periods are years and the interest rate is pa (per year), so they match.

Find the simple interest if you invest \$1000 at an interest rate of 10% per month for 2 years

Notice that the interest rate is in months and the time periods are in years. These do not match! We have two options:

- Method 1: Convert the time periods to months.
- Method 2: Convert the rate to a yearly rate.

Both methods will work!

Method 1: There are 12 months in a year. So in 2 years there are 24 months. So

$$P = \$1000 \quad r = 10\% \text{ per month} \quad T = 24 \text{ months}$$

Now use the formula to find simple interest

$$\begin{aligned} I &= P \times \frac{r}{100} \times T \\ &= \$1000 \times \frac{10}{100} \times 24 \\ &= \$2400 \end{aligned}$$

Method 2: There are 12 months in a year. So if $r = 10\%$ per month, the yearly interest rate is
 $10\% \times 12 = 120\%$

$$\text{So } P = \$1000 \quad r = 120\% \text{ per year} \quad T = 2 \text{ years}$$

$$\begin{aligned} I &= P \times \frac{r}{100} \times T \\ &= \$1000 \times \frac{120}{100} \times 2 \\ &= \$2400 \end{aligned}$$

Both methods find the same answer. So use whichever method you think is easier. But remember – the time periods and the rate must match.

Always assume that 1 year is also:

- 12 months or
- 52 weeks or
- 365 days

unless the question specifies otherwise.

Harder Questions with the Simple Interest Formula

Sometimes the interest formula is used to find the value of P , R , r or T .

A certain principal amount earned \$2310 in interest at a rate of 11% pa over 7 years. What was the principal amount?

$$I = \$2310 \quad r = 11\% \text{ pa} \quad T = 7 \text{ years}$$

Write the formula and substitute in the given values

$$I = P \times \frac{r}{100} \times T$$
$$\$2310 = P \times \frac{11}{100} \times 7$$

Now, solve for P

$$\therefore P \times 0.77 = \$2310$$
$$\therefore P = \frac{\$2310}{0.77}$$
$$= \$3000$$

So the principal amount (the initial amount) is \$3000.

When finding the interest rate, remember that it must match the time periods.

\$5700 was invested over 36 months and returned \$8208 in simple interest. What was the interest rate?

$$P = \$5700 \quad I = \$8208 \quad T = 36 \text{ months}$$

Write the formula and substitute in the given values

$$I = P \times \frac{r}{100} \times T$$
$$\$8208 = \$5700 \times \frac{r}{100} \times 36$$

Now, solve for r

$$\frac{r}{100} = \frac{\$8208}{\$5700 \times 36}$$
$$\therefore r = 4\%$$

Since the time periods and the interest rate must match, the interest rate must have the same time periods as T

$$\therefore r = 4\% \text{ per month}$$

1. Find R as a decimal if:

a $r = 12\%$ pa.

b $r = 22\%$ pa.

2. Find the simple interest for the following:

a $P = \$1200$, $R = 0.08$, $T = 12$.

b $P = \$2000$, $r = 12\%$, $T = 8$.

3. Find the simple interest for the following:

a \$4550 at a rate of 20% pa for 4 years.

b \$32 000 for 2 years at a rate of 16% pa.

c \$50 000 for 3 months at a rate of 4.2% per month.

4. Convert the rate or the time periods to ensure they match.

a $r = 3\% \text{ pa}, T = 12 \text{ months}.$

b $r = 7.8\% \text{ pa}, T = 156 \text{ weeks}.$

c $r = 9\% \text{ per month}, T = 2 \text{ years}.$

d $r = 0.05\% \text{ per week}, T = 2.08 \text{ years}.$

e $r = 1\% \text{ per day}, T = 1 \text{ year}.$

f $r = 6\% \text{ pa}, T = 1095 \text{ days}.$

5. You invest \$4000 at a rate of 1.5% per month for 3 years.

a Find the simple interest.

b What is the total amount of money you now own?
(remember: total amount = principal + interest)

6. A principal of \$7650 returns simple interest of \$3672 after 4 years.

- a What is the interest rate?

- b Convert this interest rate to a monthly interest rate.

7. Let's say you have \$5000 to invest for 5 years. Which of these rates returns the most interest?

- 2.7% per month.
- 32% per year.
- 0.7% per week.

8. Your friend Jesse took out a loan of \$9840 to pay for a car. Jesse paid back the money by making \$250 payments each month for 4 years on a simple interest rate.

- a What is the total amount Jesse paid back to the bank over the 4 years?
- b How much extra did Jesse pay on top of the loan?
(Hint: This is the interest)
- c What was the monthly interest rate used to pay off the interest in b ?
(Hint: $P = \$9840$)
- d Convert this interest rate to an annual interest rate.

Interest = Future Value – Principal

Remember that the interest is the 'extra amount' added to the principal after a certain amount of time. The future value of an investment is the total invested plus the interest after a certain amount of time. So, if a principal value, (P) earns simple interest (I) then the future value (A) is found using this equation:

$$P + I = A \quad \leftarrow \quad A = \text{future value}$$

This means that:

$$I = A - P$$

Or Interest = Final Amount – Principal.

A principal amount earns simple interest of \$400 and the final amount after 3 years is \$1200. What was the principal?

$$\begin{aligned} I &= A - P \\ \therefore \$400 &= \$1200 - P \\ \therefore P &= \$800 \end{aligned}$$

The final amount A can also be found in this way:

$$\begin{aligned} A &= P + I \quad \leftarrow \quad I = PRT \\ \therefore A &= P + PRT \\ \therefore A &= P(1 + RT) \quad \leftarrow \quad R = \frac{r}{100} \end{aligned}$$

Suppose you invest \$12 500 to earn simple interest at a rate of 4% per month for 1 year. Find the future value.

Step 1: Write down the values we know.

$$P = \$12\,500$$

$$r = 4\% \text{ per month}$$

$$T = 1 \text{ year}$$

Step 2: Make sure that the time periods and the interest rate match.

The rate has been given **per month** and the time periods in **years**. Let $T = 1 \text{ year} = 12 \text{ months}$.

Step 3: Find

$$R = \frac{r}{100}$$

$$R = \frac{4}{100} = 0.04 \text{ per month}$$

Step 4: Use the above formula to find the future value.

$$\begin{aligned} A &= P(1 + RT) \\ &= \$12\,500(1 + 0.04 \times 12) \\ &= \$18\,500 \end{aligned}$$

Step 3 in the above example can be skipped if you use this version of the formula: $A = P\left(1 + \frac{r}{100} \times T\right)$.

This is called the simple interest formula. Either method will work, so use the one you think is easier.

Compound Interest

Simple interest is called 'flat interest' because the interest added at the end of each time period is the same.

Compound interest is different because the amount of interest at each period depends on the amount of interest added at the previous time period.

Think about it like this: Imagine you are investing a principal P at a rate of $r\%$ compounded at each time period (T).

- At the end of the 1st time period (beginning of 2nd time period) the future value will be:

$$\begin{aligned}
 &P\left(1 + \frac{r}{100} \times 1\right) \quad \text{After 1 time period, } T = 1 \\
 &= P\left(1 + \frac{r}{100}\right)
 \end{aligned}$$

This value becomes the principal value of the 2nd time period. Let's call this P_2 .

- At the end of the 2nd time period (beginning of 3rd time period) the future value will be:

$$\begin{aligned}
 &P_2\left(1 + \frac{r}{100}\right) \\
 &= \left[P\left(1 + \frac{r}{100}\right)\right]\left(1 + \frac{r}{100}\right) \\
 &= P\left(1 + \frac{r}{100}\right)^2
 \end{aligned}$$

P is the original principal value

This value becomes the principal value of the 3rd time period. Let's call this P_3 .

- At the end of the 3rd time period (beginning of 4th time period) the future value will be

$$\begin{aligned}
 &P_3\left(1 + \frac{r}{100}\right) \\
 &= \left[P\left(1 + \frac{r}{100}\right)^2\right]\left(1 + \frac{r}{100}\right) \\
 &= P\left(1 + \frac{r}{100}\right)^3
 \end{aligned}$$

P is the original principal value

This value becomes the principal value of the 4th time period.

- From this pattern we can see that after n time periods the future value of the investment, A , is:

$$A = P\left(1 + \frac{r}{100}\right)^n \quad \text{Compound interest formula}$$

This is the formula to find a future value A at a compound interest rate $r\%$ for n time periods.

This formula could also be written as $A = P(1 + R)^n$. Use the version you find easier.

$$R = \frac{r}{100}$$

Here is an example which uses the compound interest formula.

Let's say you invest \$5000 at 4% pa compounded annually. What is the future value after 7 years?

Using the compound interest formula $A = P\left(1 + \frac{r}{100}\right)^n$.

$$P = \$5000 \quad r = 4\% \quad n = 7 \text{ years}$$

$$\begin{aligned} \therefore A &= \$5000\left(1 + \frac{4}{100}\right)^7 \\ &= \$5000(1.04)^7 \\ &= \$6579.658... \\ &= \$6579.66 \text{ (nearest cent)} \end{aligned}$$

When working with compound interest, the interest rate AND the time periods need to match the compounded rate.

\$19700 is invested for 4 years at a rate of 15% pa compounded monthly. Find the total value of the investment.

Step 1: Convert the annual interest rate to a monthly interest rate (to match **compounded monthly**).

Be careful, although the interest rate is compounded monthly, it is still per year ('pa'). This must be converted to a **monthly** interest rate to match the fact that it is **compounded monthly**.

Divide the annual interest rate by 12.

$$r = \frac{15\%}{12} = 1.25\% \text{ per month compounded monthly.}$$

Step 2: Make sure that the time periods are in months to match **compounded monthly**.

$$4 \text{ years} = 48 \text{ months. So let } n = 48.$$

Step 3: Use the above formula to find the future value.

$$\begin{aligned} A &= P\left(1 + \frac{r}{100}\right)^n \\ &= \$19700\left(1 + \frac{1.25}{100}\right)^{48} \\ &= \$19700(1.0125)^{48} \\ &= \$35762.49 \text{ (nearest cent)} \end{aligned}$$

1. A principal amount of \$4500 earns \$2100 interest over time.

a Write down the values of I and P .

b Find the value of the final amount A .

2. \$6800 earns simple interest at a rate of 3.2% per month for 2 years.

a Use the simple interest formula to find the total value of this investment to the nearest cent.

b What was the interest earned on this investment to the nearest cent?

3. Find the value of the total investment (future value) and the interest I after one year to the nearest cent if:

a \$1200 is invested at a rate of 5% pa compounded annually.

b \$16 700 is invested at 12% pa compounded quarterly.

c \$20 000 is invested at 8% pa compounded every 6 months.

4. You invest your \$375 for 2 years. Use the compound interest formula to calculate how much money you will have (to the nearest cent) if the interest rate is:

- a 7.2% pa compounded annually.
- b 7.2% pa compounded monthly.
- c 7.2% pa compounded every 6 months.
- d Which is a better investment?

Applying Compound Interest

The compound interest formula can be used to find amounts other than the total investment A . It could be used to find a principal required to return a certain total investment. Here is an example.

How much money do you need to invest to have \$10 000 after 5 years if the interest rate is 18% pa compounded monthly?

Step 1: Convert the annual interest rate to a monthly interest rate (to match **compounded monthly**).

$$r = \frac{18\%}{12} = 1.5\% \text{ per month compounded monthly}$$

Step 2: Make sure that the time periods are in months to match **compounded monthly**.

In 5 years there are 60 months. So let $n = 60$.

Step 3: Use the compound interest formula to solve for the principal.

$$\begin{aligned} A &= P\left(1 + \frac{r}{100}\right)^n \\ \therefore \$10\,000 &= P\left(1 + \frac{1.5}{100}\right)^{60} \\ \therefore \$10\,000 &= P(1.015)^{60} \\ \therefore P &= \frac{\$10\,000}{(1.015)^{60}} \\ \therefore P &= \$4092.96 \text{ (nearest cent)} \end{aligned}$$



Remember, interest rate AND the time periods need to match the compounded rate

You would need to invest \$4092.96 to have \$10 000 at the end of 5 years at this compounded interest rate.

The compound interest formula can be used to find any of A , P , r or n .

Find the annual interest rate, compounded annually required to have \$10 000 in 4 years if you invest \$6000.

Step 1: Make sure that the time periods are in months to match **compounded monthly**.

The time periods (4 years) is already annual, so no adjustment is necessary.

Step 2: Use the compound interest formula to solve for the interest rate.

$$\begin{aligned} A &= P\left(1 + \frac{r}{100}\right)^n \\ \therefore \$10\,000 &= \$6000\left(1 + \frac{r}{100}\right)^4 \\ \therefore \left(1 + \frac{r}{100}\right)^4 &= 1.6666... \\ \therefore 1 + \frac{r}{100} &= \sqrt[4]{1.6666...} = (1.13621...) \\ \therefore \frac{r}{100} &= 0.13621... \\ \therefore r &= 13.62\% \text{ pa (2 decimal places)} \end{aligned}$$

Using the Compound Interest Formula to Find n

Using the compound interest formula to find n (the amount of time periods) is done by trial and error.

How long will it take \$25 000 invested at 14% pa compounded annually to grow to an amount greater than \$100 000?

Step 1: Use the compound interest formula.

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$\therefore A = \$25000\left(1 + \frac{14}{100}\right)^n$$

$$= \$25000(1.14)^n$$

Step 2: Try different values for n until A is greater than \$100 000.

- If $n = 9$ years $\rightarrow$ then $A = \$25000(1.14)^9 = \$81298.713...$ This is less than \$100 000
- If $n = 10$ years $\rightarrow$ then $A = \$25000(1.14)^{10} = \$92680.532...$ This is less than \$100 000
- If $n = 11$ years $\rightarrow$ then $A = \$25000(1.14)^{11} = \$105655.807...$ **This is greater than \$100 000**

So it will take $n = 11$ years for the investment to grow to greater than \$100 000.

Remember, the interest rate and the time period both need to match the compounded rate. Here is an example.

Let's say you have \$2100 to invest at a rate of 12% pa compounded monthly. How long would it take to have a total investment between \$3000 and \$3010?

Step 1: Convert the annual interest rate to a monthly interest rate (to match compounded monthly).

$$r = \frac{12\%}{12} = 1\% \text{ per month compounded monthly}$$

Step 2: Use the compound interest formula.

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$\therefore A = \$2100\left(1 + \frac{1}{100}\right)^n$$

$$= \$2100(1.01)^n$$



Remember, interest rate AND the time periods need to match the compounded rate

Step 3: Try different values for n until A is between \$3000 and \$3010.

- If $n = 34$ months $\rightarrow$ then $A = \$2100(1.01)^{34} = \$2945.411...$ This is less than \$3000
- If $n = 35$ months $\rightarrow$ then $A = \$2100(1.01)^{35} = \$2974.865...$ This is less than \$3000
- If $n = 36$ months $\rightarrow$ then $A = \$2100(1.01)^{36} = \$3004.614...$ **This is between \$3000 and \$3010**
- If $n = 37$ months $\rightarrow$ then $A = \$2100(1.01)^{37} = \$3034.660...$ This is less than \$3010

So for the total investment to be between \$3000 and \$3010, the principal of \$2100 must be invested for 36 months (3 years).

1. How much money do you need to invest if you're aiming to have \$13 500 after 5 years and:

a the interest rate is 2.4% pa compounded **annually**?

b the interest rate is 2.4% pa compounded **monthly**?



Remember,
interest rate AND
the time periods
need to match the
compounded rate

2. To have \$5000 in 7 years after investing \$2360, calculate the required annual interest rate (to 2 decimal places) if the rate is:

a Compounded annually.

b Compounded monthly.



Remember,
interest rate AND
the time periods
need to match the
compounded rate

3. A principal investment P is invested for 10 years. Answer these questions correct to 2 decimal places.

- a Use the simple interest formula to find the simple interest rate required for the investment to double.
(Hint: $A = 2P$)
- b Use the compound interest formula to find the interest rate compounded annually required for it to double the principal.

4. How long will it take to earn a total investment greater than \$60 000 if you invest \$17 000 at interest rate of:

- a 18% pa compounded annually?
- b 18% pa compounded monthly?
- c 18% pa compounded quarterly (4 times a year)?
- d Which is quicker?

Loans

Banks and other organisations which lend money always charge interest on their loan. The loan is always the principal (P) and the borrower will have to pay the loan back and pay interest in addition.

Dylan took out a loan of \$7200 from a bank to help pay for a car. Dylan is required to pay the loan off over the next 3 years at an interest rate of 8% pa compounded quarterly. What is the total amount that Dylan will pay for the loan?

Step 1: Convert the annual interest rate to a quarterly interest rate (to match compounded quarterly).

Compounded quarterly means every 3 months (4 times each year). So, divide the annual interest rate by 4.

$$r = \frac{8\%}{4} = 2\% \text{ per quarter compounded quarterly}$$

Step 2: Make sure that the time periods are in quarters to match compounded quarterly.

$$3 \times 4 \text{ quarters} = 12 \text{ quarters. So let } n = 12.$$

Step 3: Use the compound interest formula to find the future value.

$$\begin{aligned} A &= P\left(1 + \frac{r}{100}\right)^n \\ &= \$7200\left(1 + \frac{2}{100}\right)^{12} \\ &= \$7200(1.02)^{12} \\ &= \$9131.34 \text{ (nearest cent)} \end{aligned}$$

So Dylan is actually required to pay the organisation that lent him money **more than the loan**. This is how banks and money lending organisations make a profit. To find the bank's profit, just find the interest using the formula $I = A - P$

$$I = \$9131.34 - \$7200 = \$1931.34$$

Often the borrower has to pay back the total amount in equal payments. These are usually each month or each week. Here is what happens in the case of the example above.

Dylan must pay the loan off in equal monthly payments. What is the value of each monthly payment?

$$\begin{aligned} \text{Monthly payments} &= \frac{\text{Total amount paid}}{\text{Months in 3 years}} \\ &= \frac{\$9131.34}{36} \\ &= \$253.65 \text{ (nearest cent)} \end{aligned}$$

This means Dylan will pay \$253.65 each month for the next 3 years to pay off the loan.

Converting Reducible Interest to Flat Interest

Sometimes a money lender makes the borrower pay off a loan using 'reducible interest'. This is different to simple and compound interest because the interest charged reduces as the amount owing decreases. Reducible interest is more complicated to work with, so it is important to be able to convert reducible interest to simple interest.

The good news is that there is a table which converts reducible interest to simple interest (flat interest). It converts annual or monthly reducible interest rates to annual flat rates. This table is on the last page of this book.

Convert the following reducible interest rates into equivalent flat interest rates using the reducible interest table

- | | |
|---|--|
| <p>a Reducible rate of 9% pa over 2 years
Equivalent flat rate is 4.82% per year</p> | <p>b Reducible rate of 5% pa over 4 years
Equivalent flat rate is 2.64% per year</p> |
| <p>c Reducible rate of 1% per month over 5 years
This is a reducible rate of 12% pa over 5 years
Equivalent flat rate is 6.69% per year</p> | <p>d Reducible rate of 1.25% per month over 10 years
This is a reducible rate of 15% pa over 10 years
Equivalent flat rate is 9.36% per year</p> |

Here is an example how to use this table in calculations.

You are required to pay off a loan of \$4000 at a reducible interest rate of 15% pa over 6 years

- a** Find the equivalent simple interest rate from the table

This reducible interest rate is equivalent to a flat rate of 8.71%

- b** Use the flat rate to calculate the total amount you will pay back after 6 years.

This is a flat rate so the simple interest formula is used

$$\begin{aligned}
 A &= P\left(1 + \frac{r}{100} \times T\right) \\
 &= \$4000\left(1 + \frac{8.71}{100} \times 6\right) \\
 &= \$4000(1.5226) \\
 &= \$6090.40
 \end{aligned}$$

- c** What is the interest on your loan?

$$\begin{aligned}
 I &= A - P \\
 &= \$6090.40 - \$4000 \\
 &= \$2090.40
 \end{aligned}$$

1. Use the table converting annual reducible interest rates to equivalent annual flat rates to answer these questions.

- a Convert the 7% pa reducible interest rate over 6 years to a flat rate.
- b What is the equivalent annual flat interest rate to 9% pa reducible interest over 3 years?
- c What is the equivalent annual flat interest rate to 15% pa reducible interest over 8 years?
- d What is the equivalent annual flat interest rate to 1.25% per month reducible interest over 5 years?
- e What is the equivalent annual flat interest rate to 0.75% per month reducible interest over 9 years?

2. If to pay a loan off, you are required to pay a total amount of \$11 232 over 6 years, then:

- a If you plan to pay yearly, what is your annual repayment?
- b If you plan to pay monthly, what is your monthly repayment?
- c If you plan to pay weekly, what is your weekly repayment?

3. A bank loans Gary \$32 000. He must pay the loan off over the next 8 years with interest charged at a rate of 11 % pa compounded annually.

- a Do the time periods match the rate the interest is compounded?
- b Does the interest rate match the the rate the interest is compounded?
- c Find the total amount Gary will have paid after he has paid off his loan.
- d If Gary is required to pay this loan off monthly, how much will he have to pay each month?
- e If Gary is required to pay this loan off quarterly, how much will he have to pay each quarter?

4. Let's say you borrow \$100 000 from a bank to put a deposit on a house. The bank charges you interest at 16.8% pa compounded monthly for the next 8 years.

- Convert the annual interest rate compounded monthly to a monthly rate compounded monthly. Why is this necessary?
- Find the total amount you will have paid to the bank after 8 years.
- How much profit did the bank make?
- What is the value of each monthly payment?

5. Ashley takes out a loan of \$3600. She is required to pay the loan back over 3 years at a reducible interest rate of 9% pa.

- a Find the equivalent annual flat interest rate from the table.
- b Use the flat interest rate to find the total Ashley will have paid over the 3 years in paying off her loan.
- c If Ashley is required to make quarterly payments, then how much must she pay each quarter?
- d If Ashley is required to make monthly payments, then how much is Ashley required to pay each month?

6. Find your monthly payments if you take out a loan of \$8500 at a reducible interest rate of 11% pa over 10 years.

Table to convert reducible interest rates to equivalent flat rates

Annual Reducible rate r (%)	Annual Reducible rate as a decimal $(R = \frac{r}{100})$	Monthly Reducible rate as a decimal $(\frac{r}{12})$	Equivalent annual flat rate (%) per year (rounded to 2 d.p.)									
			1	2	3	4	5	6	7	8	9	10
3	0.03	0.0025	1.63%	1.58%	1.56%	1.56%	1.56%	1.57%	1.57%	1.58%	1.58%	1.59%
4	0.04	0.00333	2.18%	2.11%	2.10%	2.09%	2.10%	2.11%	2.12%	2.13%	2.14%	2.15%
5	0.05	0.00417	2.73%	2.65%	2.63%	2.64%	2.65%	2.66%	2.67%	2.69%	2.71%	2.73%
6	0.06	0.005	3.28%	3.18%	3.17%	3.18%	3.20%	3.22%	3.24%	3.27%	3.30%	3.32%
7	0.07	0.00583	3.83%	3.73%	3.72%	3.74%	3.76%	3.79%	3.83%	3.86%	3.90%	3.93%
8	0.08	0.00667	4.39%	4.27%	4.27%	4.30%	4.33%	4.37%	4.42%	4.46%	4.51%	4.56%
9	0.09	0.0075	4.94%	4.82%	4.83%	4.86%	4.91%	4.96%	5.02%	5.08%	5.14%	5.20%
10	0.1	0.00833	5.50%	5.37%	5.39%	5.44%	5.50%	5.56%	5.64%	5.71%	5.78%	5.86%
11	0.11	0.00917	6.06%	5.93%	5.95%	6.01%	6.09%	6.17%	6.26%	6.35%	6.44%	6.53%
12	0.12	0.01	6.62%	6.49%	6.52%	6.60%	6.69%	6.79%	6.90%	7.00%	7.11%	7.22%
15	0.15	0.0125	8.31%	8.18%	8.27%	8.40%	8.55%	8.71%	8.87%	9.03%	9.20%	9.36%
18	0.18	0.015	10.02%	9.91%	10.05%	10.25%	10.47%	10.70%	10.94%	11.17%	11.40%	11.62%
20	0.2	0.01667	11.16%	11.07%	11.26%	11.52%	11.79%	12.08%	12.36%	12.64%	12.92%	13.19%

Basics:

1. **a** $R = 0.12$ **b** $R = 0.22$
.....
2. **a** $I = \$1152$ **b** $I = \$1920$
.....
3. **a** $I = \$3640$ **b** $I = \$10\,240$
c $I = \$6300$
.....
4. **a** $T = 1$ year OR $r = 0.25\%$ per month
b $T = 3$ years OR $r = 0.15\%$ per week
c $T = 24$ months OR $r = 108\%$ pa
d $T = 108.16$ weeks OR $r = 2.6\%$ pa
e $T = 365$ days OR $r = 365\%$ pa
f $T = 3$ years OR $r = 0.016\%$ per day
.....
5. **a** $I = \$2160$
b Total money now own is \$6160
.....
6. **a** $r = 12\%$ pa
b $r = 1\%$ per month
.....
7. 0.7% per week gives the highest rate of return per annum, it will be the rate that returns the most interest.
.....
8. **a** Total amount paid = \$12 000
b Extra paid = \$2160
c $r = 0.46\%$ per month
d $r = 5.5\%$ pa

Knowing More:

1. **a** $I = \$2100$
 $P = \$4500$
b Final amount = \$6600
.....
2. **a** $A = \$12\,022.40$
b $I = \$5222.40$
.....
3. **a** $A = \$1260$
 $I = \$60$
b $A = \$18\,796$
 $I = \$2096$
c $A = \$21\,632$
 $I = \$1632$
.....
4. **a** $A = \$430.94$
b $A = \$432.90$
c $A = \$431.99$
d 7.2% pa compounded monthly is a better investment since there is a higher return.

Using Our Knowledge:

1. **a** $P = \$11\,990.41$
b $P = \$11\,974.86$
.....
2. **a** $r = 11.32\%$ pa
b $r = 10.77\%$ pa
.....
3. **a** $r = 10.00\%$ pa
b $r = 7.18\%$ pa (to 2 d.p.) compounded annually

Using Our Knowledge:

4. **a** It will take 8 years for the investment to be greater than \$60 000
- b** It will take 85 months to earn more than \$60 000
- c** It will take $7\frac{1}{4}$ years to earn more than \$60 000
- d** It would be quicker to invest in an interest rate that is compounded monthly in order to reach the \$60 000 faster.

Thinking More:

1. **a** 3.79% pa **b** 4.83% pa
c 9.03% pa **d** 8.55% pa
e 5.14% pa
-
2. **a** \$1872 pa **b** \$156 per month
c \$36 per week
-
3. **a** Yes, since the interest rate is compounded annually and the time period specifies a year figure the time period and the period the interest rate is compounded do match.
- b** Yes, the interest rate is a per annum figure and the interest rate is compounded annually then the interest rates match up.
- c** $A = \$73\,745.21$
- d** \$768.18 per month
- e** \$2304.54 per quarter

Thinking More:

4. **a** 14% per month

This is necessary since the formulas only work when units are consistent. For example, since the interest rate is compounded monthly it wouldn't make sense to use years as your time period since the interest rate is not compounded annually.

- b** $A = \$379\,877.59$
- c** $I = \$279\,877.59$
- d** Each payment = \$3957.06

5. **a** 4.83% pa
b Total = \$4121.64
c \$343.47 per quarter
d \$114.49 per month

6. \$117.09 per month

