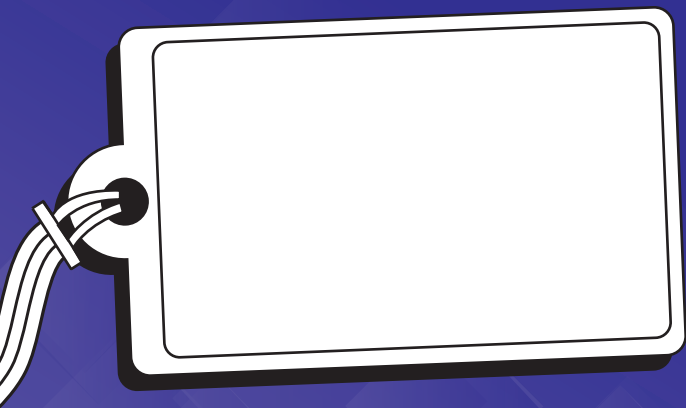




Trigonometry &
Polar Coordinates

TRIGONOMETRY & POLAR COORDINATES

SERIES **M**



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TRIGONOMETRY & POLAR COORDINATES

Up until now points on an axes have always had coordinates of the form (x, y) . These are called 'rectangular' coordinates. There is another way to label points using the distance of the point from the origin, and an angle θ .

Answer these questions, *before* working through the chapter.

I used to think:

If a point P has polar coordinates $P(r, \theta) = P(3, 30^\circ)$, what does this mean?

How are radians different from degrees?

Answer these questions, *after* working through the chapter.

But now I think:

If a point P has polar coordinates $P(r, \theta) = P(3, 30^\circ)$, what does this mean?

How are radians different from degrees?

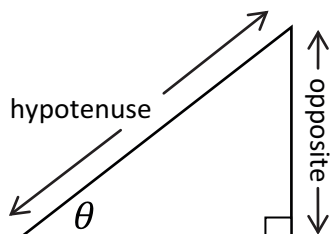


What do I know now that I didn't know before?



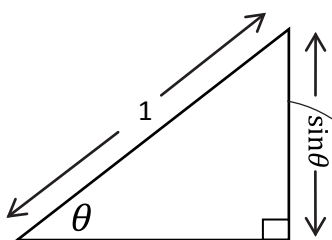
$$y = \sin\theta$$

Here's a right angle triangle. θ is an angle within the triangle.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

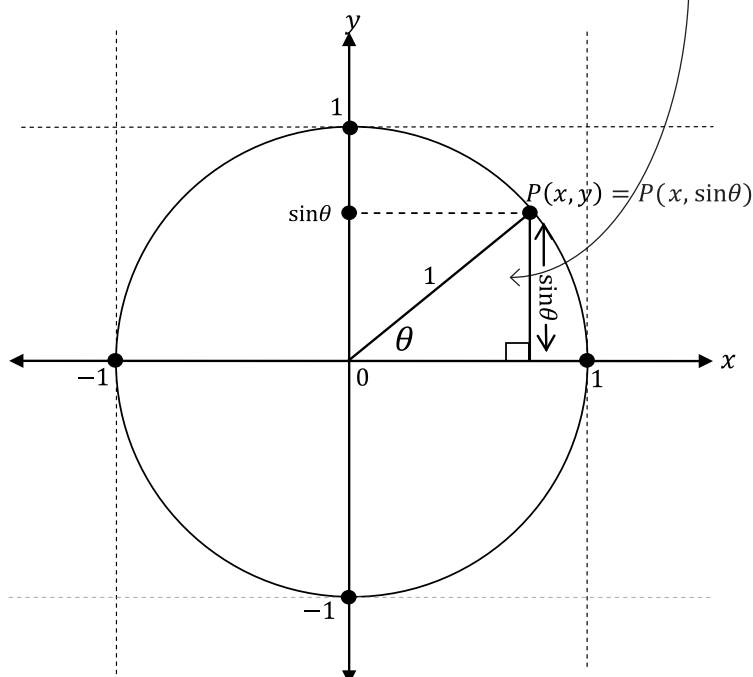
If the hypotenuse has a length of 1 unit, then the opposite side has length $\sin\theta$



$$\sin\theta = \frac{\text{opposite}}{1}$$

$$\therefore \sin\theta = \text{opposite}$$

The same triangle (above), with hypotenuse 1 unit, can be drawn into a circle with a radius 1 and centre at the origin.



Point P on the circle has coordinates $P(x, y)$.

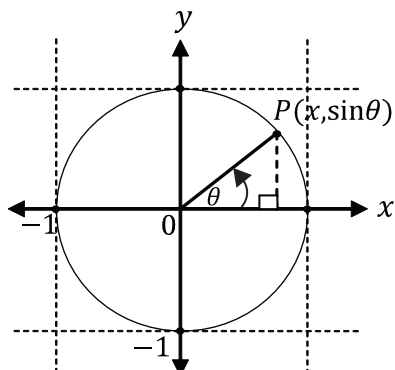
The y -coordinate at point P is $\sin\theta$ since the opposite side's height is $\sin\theta$.

So point P has coordinates $P(x, \sin\theta)$.

$$y = \sin\theta$$

When is $\sin\theta$ positive and negative?

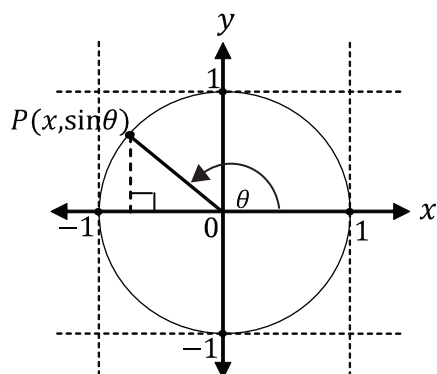
As θ increases, point P will move around the circle.



θ is between 0° and 90° (1st Quadrant)

The y-coordinate is $\sin\theta$ and **above** the x-axis

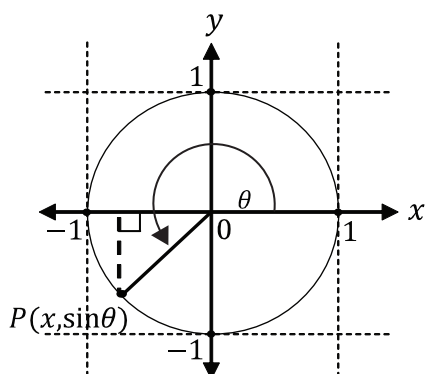
$\therefore \sin\theta$ is **positive** when θ is between 0° and 90°



θ is between 90° and 180° (2nd Quadrant)

The y-coordinate is $\sin\theta$ and **above** the x-axis

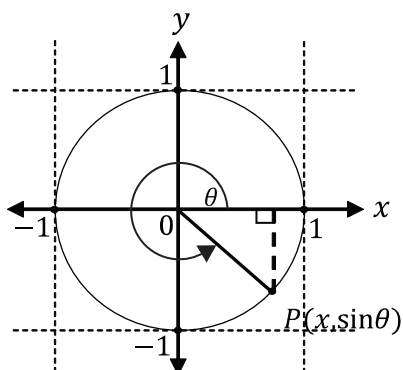
$\therefore \sin\theta$ is **positive** when θ is between 90° and 180°



θ is between 180° and 270° (3rd Quadrant)

The y-coordinate is $\sin\theta$ and **below** the x-axis

$\therefore \sin\theta$ is **negative** when θ is between 180° and 270°



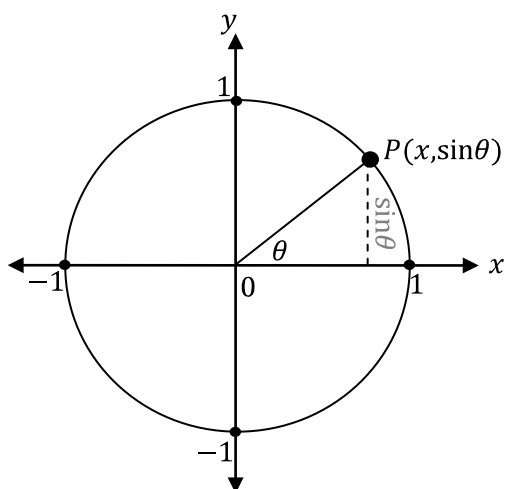
θ is between 270° and 360° (4th Quadrant)

The y-coordinate is $\sin\theta$ and **below** the x-axis

$\therefore \sin\theta$ is **negative** when θ is between 270° and 360°

1. Identify whether the values for $\sin\theta$ will be positive or negative. Circle the correct answer:

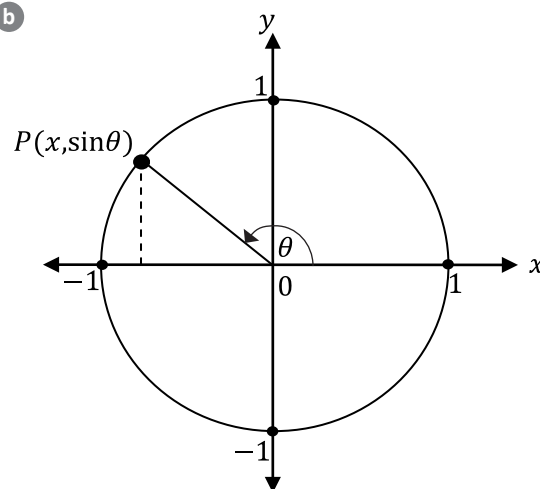
a



$\sin\theta$ is above / below the x -axis

$\sin\theta$ is positive / negative

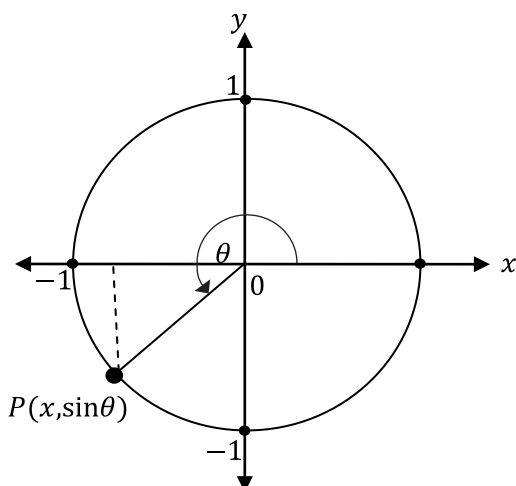
b



$\sin\theta$ is above / below the x -axis

$\sin\theta$ is positive / negative

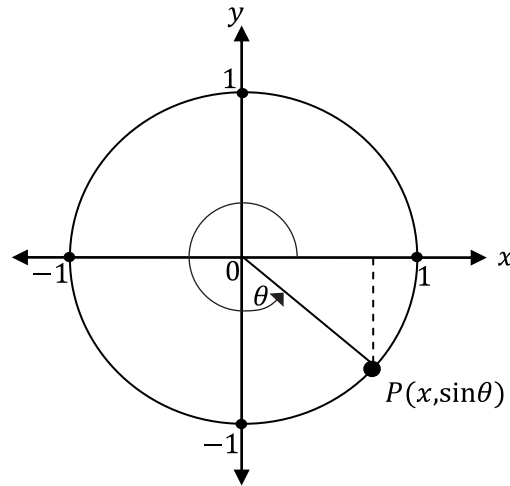
c



$\sin\theta$ is above / below the x -axis

$\sin\theta$ is positive / negative

d



$\sin\theta$ is above / below the x -axis

$\sin\theta$ is positive / negative

2. Circle the correct answers:

a

If $\theta = 70^\circ$ then θ is between 0 and 180° / 180° and 360° and $\sin\theta$ is positive / negative.

b

If $\theta = 110^\circ$ then θ is between 0 and 180° / 180° and 360° and $\sin\theta$ is positive / negative.

c

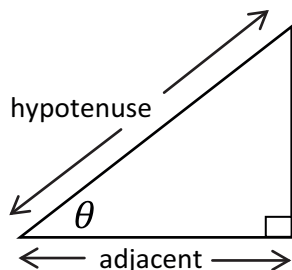
If $\theta = 300^\circ$ then θ is between 0 and 180° / 180° and 360° and $\sin\theta$ is positive / negative.

d

If $\theta = 240^\circ$ then θ is between 0 and 180° / 180° and 360° and $\sin\theta$ is positive / negative.

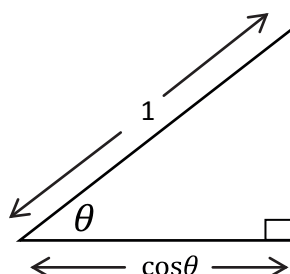
$$x = \cos\theta$$

Here's a right angle triangle. θ is an angle within the triangle.



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

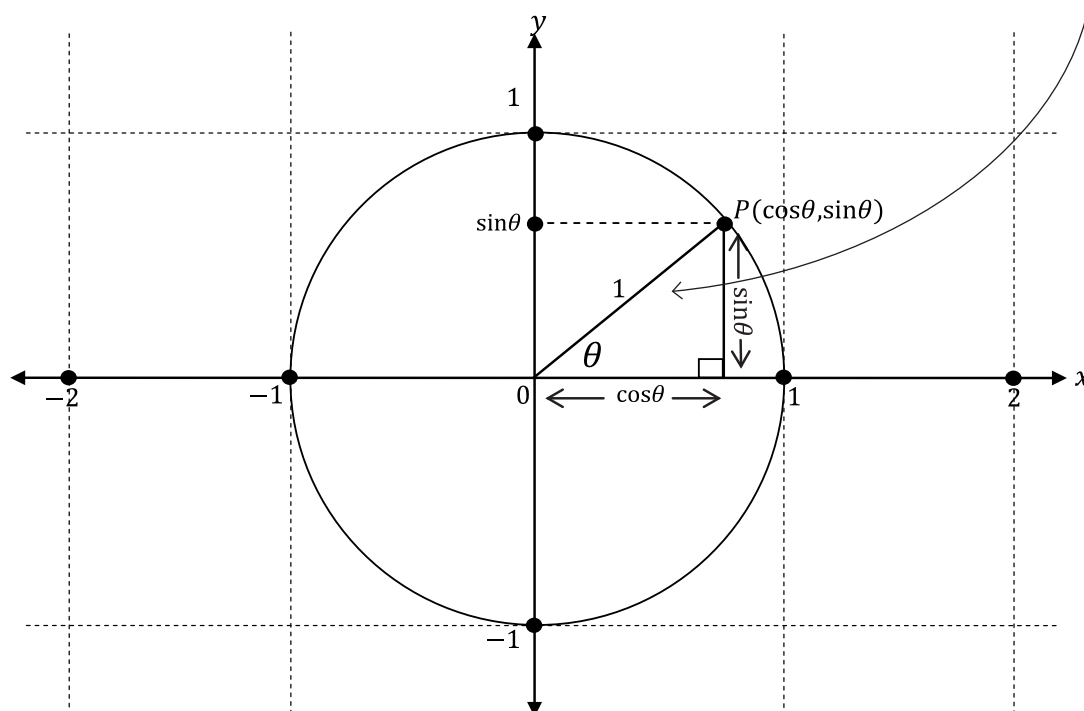
If the hypotenuse has a length of 1 unit, then the adjacent side has length $\cos\theta$



$$\cos\theta = \frac{\text{adjacent}}{1}$$

$$\therefore \cos\theta = \text{adjacent}$$

The same triangle (above), with hypotenuse 1 unit, can be drawn into a circle with a radius 1 and centre at the origin.



Point P on the circle has coordinates $P(x, y)$.

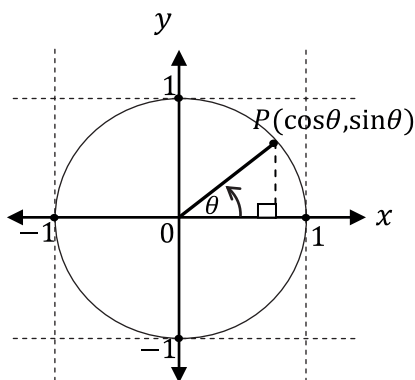
We already know that the y -coordinate of P is $\sin\theta$.

The x -coordinate at point P is $\cos\theta$ since the adjacent side's length is $\cos\theta$.

So point P has coordinates $P(\cos\theta, \sin\theta)$.

When is $\cos\theta$ positive and negative?

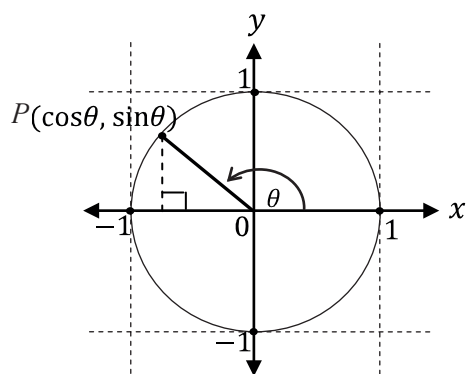
As θ increases, point P will move around the circle.



θ is between 0° and 90° (1st Quadrant)

The x-coordinate is $\cos\theta$ and **positive**

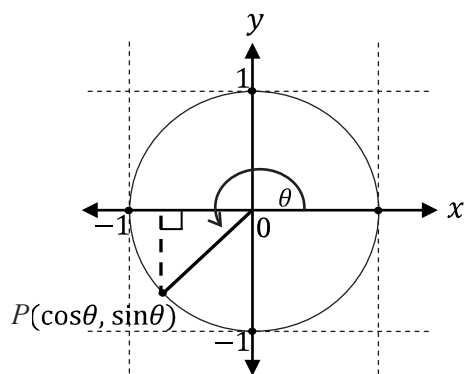
$\therefore \cos\theta$ is **positive** when θ is between 0° and 90°



θ is between 90° and 180° (2nd Quadrant)

The x-coordinate is $\cos\theta$ and **negative**

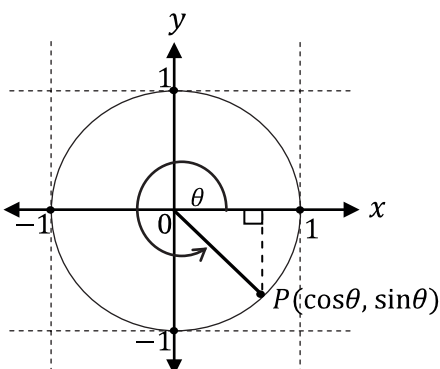
$\therefore \cos\theta$ is **negative** when θ is between 90° and 180°



θ is between 180° and 270° (3rd Quadrant)

The x-coordinate is $\cos\theta$ and **negative**

$\therefore \cos\theta$ is **negative** when θ is between 180° and 270°

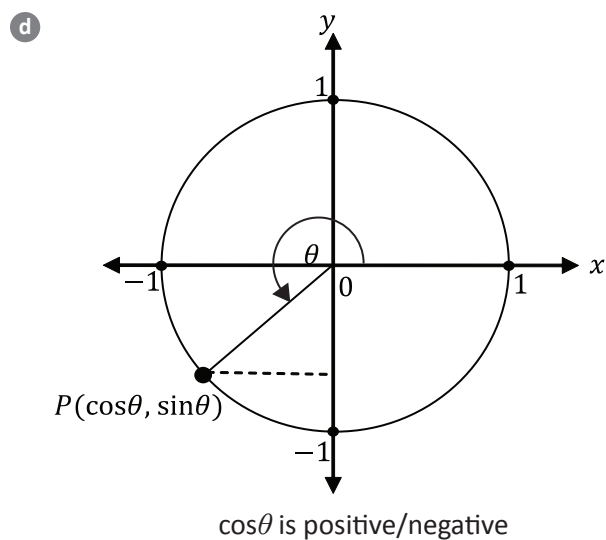
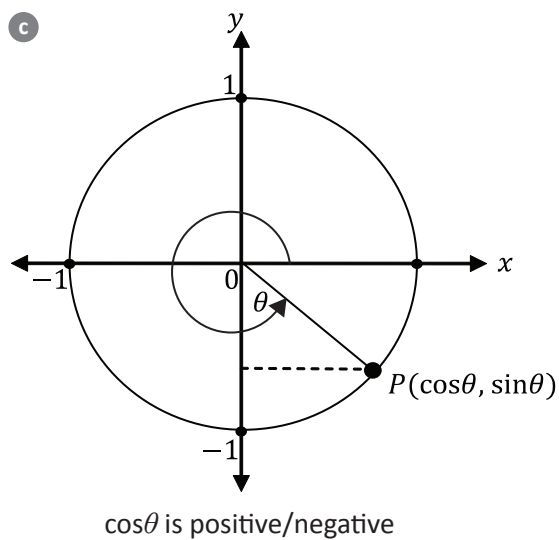
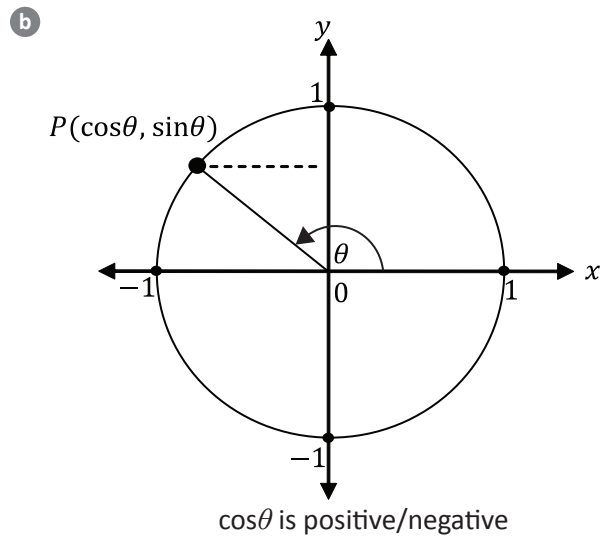
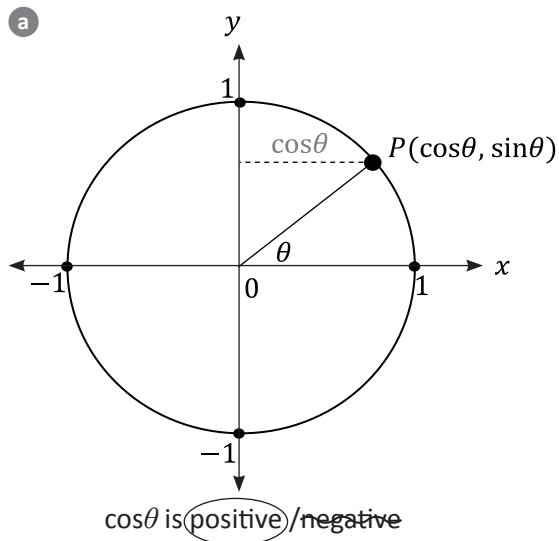


θ is between 270° and 360° (4th Quadrant)

The x-coordinate is $\cos\theta$ and **positive**

$\therefore \cos\theta$ is **positive** when θ is between 270° and 360°

3. Identify whether the values for $\cos\theta$ will be positive or negative. Circle the correct answer:



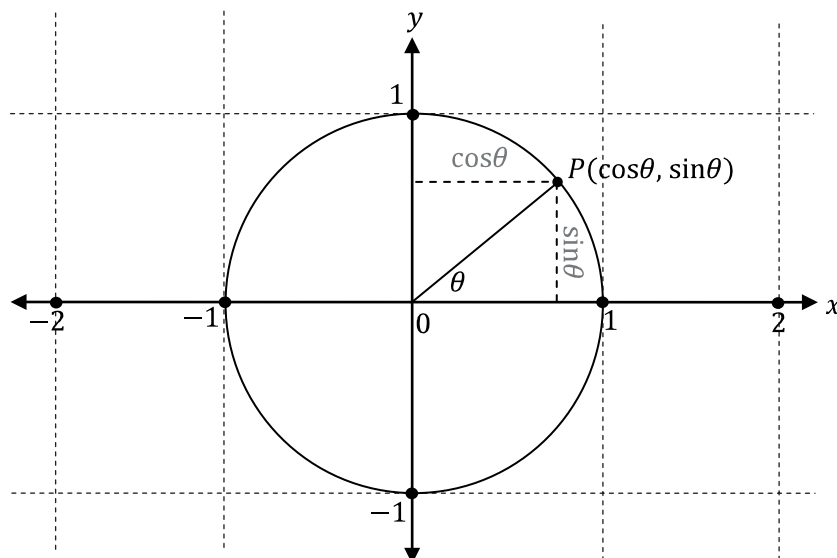
4. Circle the correct answers:

- a If $\theta = 65^\circ$ then θ is between 0 and 90° / 90° and 180° / 180° and 270° / 270° and 360° so $\cos\theta$ is positive/negative.
- b If $\theta = 250^\circ$ then θ is between 0 and 90° / 90° and 180° / 180° and 270° / 270° and 360° so $\cos\theta$ is positive/negative.
- c If $\theta = 135^\circ$ then θ is between 0 and 90° / 90° and 180° / 180° and 270° / 270° and 360° so $\cos\theta$ is positive/negative.
- d If $\theta = 290^\circ$ then θ is between 0 and 90° / 90° and 180° / 180° and 270° / 270° and 360° so $\cos\theta$ is positive/negative.

Coordinates using θ

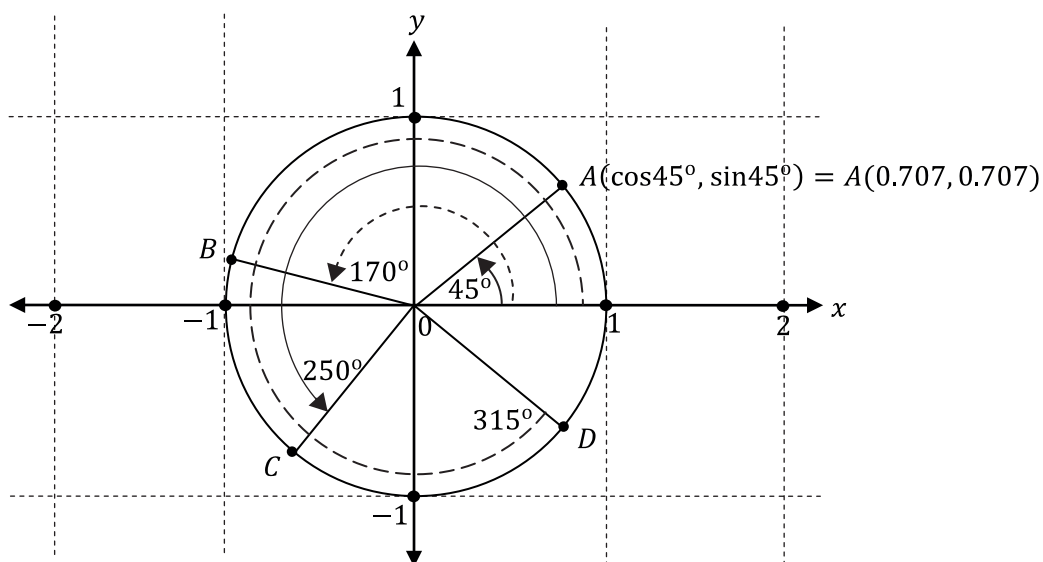
P is the point at the end of the radius with x -coordinate $x = \cos\theta$ and y -coordinate $y = \sin\theta$.

So, $P(x, y) = P(\cos\theta, \sin\theta)$.



So, on a circle with $r = 1$, both the x and y coordinates can be found using θ only.

5. Find the coordinates of the points A, B, C and D correct to 3 decimal places.



a A: $\theta = 45^\circ$

$$x = \cos 45^\circ \quad y = \sin 45^\circ$$

$$A(x, y) = A(\cos 45^\circ, \sin 45^\circ) = A(0.707, 0.707)$$

b B: $\theta = 170^\circ$

$$x = \quad y =$$

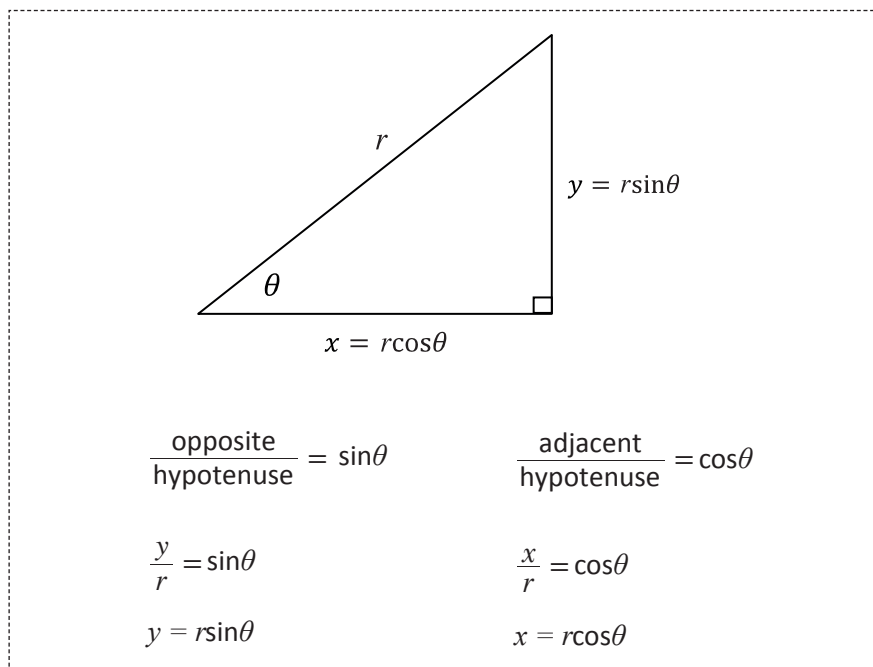
$$B(x, y)$$

c C:

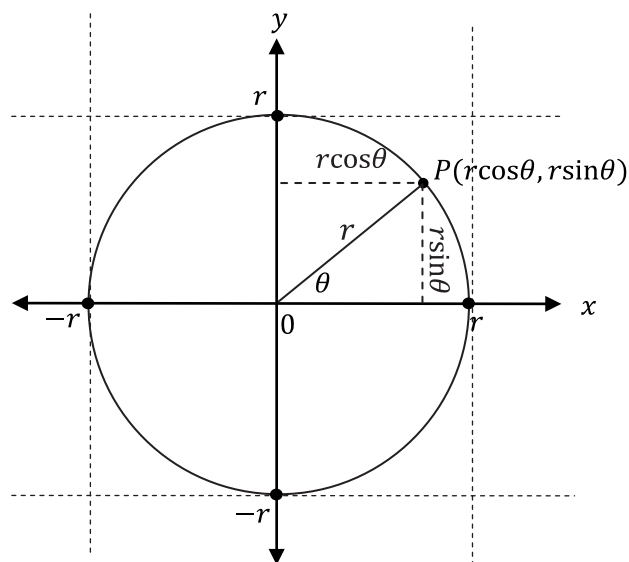
d D:

Polar coordinates

So far we have been restricted to a circle with a radius of 1. What are the polar coordinates of a point if it lies on a circle which has a radius of 2, 3 or 100? Look at the diagram below with hypotenuse r .



The circle below has a radius of r and its centre at the origin. The same triangle is drawn on the axes below.

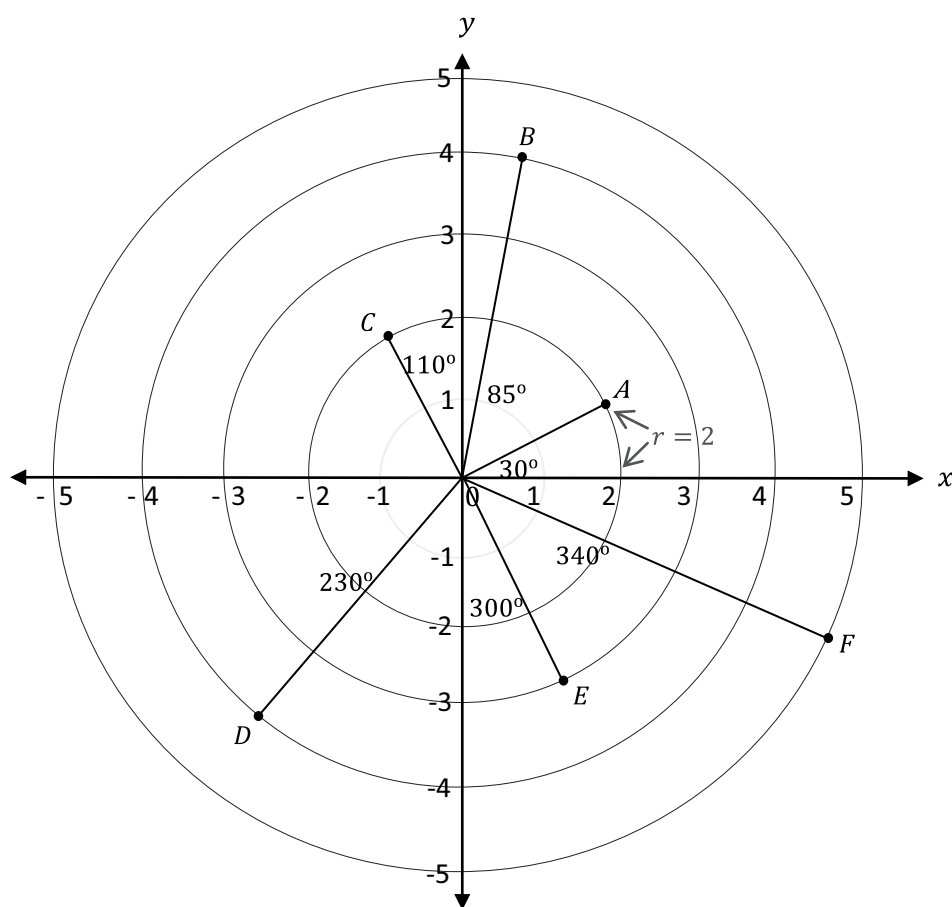


P is the point at the end of the radius. $P(x, y) = P(r \cos \theta, r \sin \theta)$. Both the x and y coordinates can be found using r and θ .

So instead of using normal rectangular coordinates $P(x, y)$, in polar coordinates we write $P(r, \theta)$.

r is the radius of the circle on which P lies or r is the distance between P and $(0, 0)$.
 θ is the angle between the line connecting the origin to P and the positive x -axis.

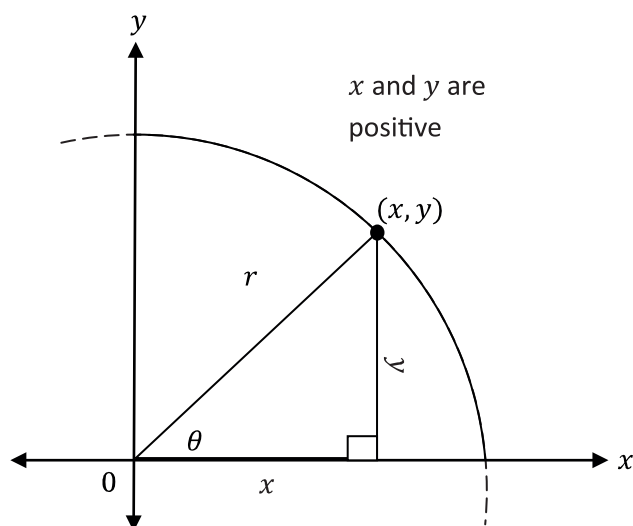
6. Complete the table for the following points rounded to 2 d.p. where appropriate:



Point	r	θ	Polar coordinates (r, θ)	$x = r\cos\theta$	$y = r\sin\theta$	(x, y) Coordinates
A	2	30°	$A(2, 30^\circ)$	$2\cos 30^\circ = \sqrt{3}$	$2 \sin 30^\circ = 1$	$A(\sqrt{3}, 1)$
B	4					
C		110°				
D		230°				
E						
F						

Changing positive rectangular coordinates (x, y) to polar coordinates (r, θ)

In this section we're only going to work with positive coordinates. Negative coordinates can get a little tricky, we'll save them for later. If a point has positive coordinates then it is in the first quadrant.



Remember, for polar coordinates we just need to find r and θ . Use Pythagoras to find r :

$$r = \sqrt{x^2 + y^2}$$

From the diagram we also say that $\tan \theta = \frac{y}{x}$. If we make θ the subject then:

$$\theta = \tan^{-1} \left(\frac{y}{x} \right)$$

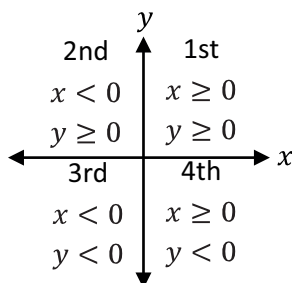
θ will be between $0^\circ - 90^\circ$ in first quadrant

7. Change the following rectangular coordinates (x, y) to polar coordinates (r, θ) by completing the table (to 2 d.p.):

(x, y)	$r = \sqrt{x^2 + y^2}$	$\theta = \tan^{-1} \left(\frac{y}{x} \right)$	(r, θ)
$(3, 4)$	$\sqrt{3^2 + 4^2} = 5$	$\tan^{-1} \left(\frac{4}{3} \right) = 53.13^\circ$	$(5, 53.13^\circ)$
$(5, 12)$	$\sqrt{5^2 + 12^2} = 13$		
$(4\sqrt{2}, 4\sqrt{2})$		$\tan^{-1} \left(\frac{4\sqrt{2}}{4\sqrt{2}} \right) =$	
$(1, \sqrt{3})$			
$(3\sqrt{3}, 3)$			

Changing coordinates by quadrant

Up until now we have only converted coordinates to polar form from the first quadrant, where x and y are positive. In the other quadrants, θ is calculated differently.



2nd Quadrant ($x < 0, y \geq 0$)

As usual we need to find r and θ . r is found in the usual way:

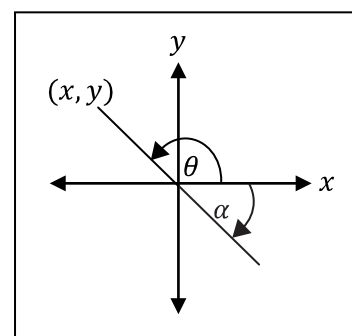
$$r = \sqrt{x^2 + y^2}$$

θ can't be found using a formula directly. First we must calculate **reference angle** α

$$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$$

α will be negative. We then find θ by adding 180° to α .

$$\theta = \alpha + 180^\circ$$



3rd Quadrant ($x < 0, y < 0$)

As usual we need to find r and θ . r is found in the usual way:

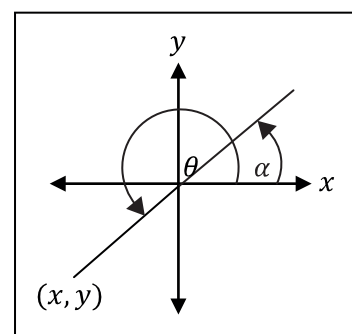
$$r = \sqrt{x^2 + y^2}$$

θ can't be found using a formula directly. First we must calculate **reference angle** α

$$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$$

α will be positive. We then find θ by adding 180° to α .

$$\theta = \alpha + 180^\circ$$



4th Quadrant ($x \geq 0, y < 0$)

As usual we need to find r and θ . r is found in the usual way:

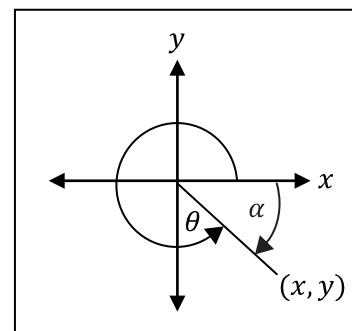
$$r = \sqrt{x^2 + y^2}$$

θ can't be found using a formula directly. First we must calculate **reference angle** α

$$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$$

α will be negative. We then find θ by adding 360° to α .

$$\theta = \alpha + 360^\circ$$

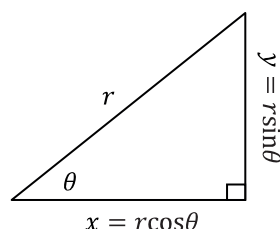


8. Use the rules for each quadrant to complete the following table (to 2 d.p.):

(x, y)	Quadrant	$r = \sqrt{x^2 + y^2}$	$a = \tan^{-1}\left(\frac{y}{x}\right)$	θ	(r, θ)
$(3, -4)$	4th	$r = \sqrt{3^2 + (-4)^2} = 5$	$\tan^{-1}\left(\frac{-4}{3}\right) = -53.13^\circ$	$a + 360^\circ = 306.87^\circ$	$(5, 306.87^\circ)$
$(-1, \sqrt{3})$	2nd	$r = \sqrt{(-1)^2 + (\sqrt{3})^2} =$	$\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) =$	$a + 180^\circ =$	
$(-5, -12)$	3rd				
$(-\sqrt{2}, \sqrt{2})$					
$(15, 8)$					
$(7, -7)$					
$(-9, -12)$					
$(-\sqrt{3}, 1)$					

Changing polar coordinates (r, θ) to rectangular coordinates (x, y)

If we have a point in polar coordinates (r, θ) then we need to find x and y . Use these formulas (we used them earlier too!)

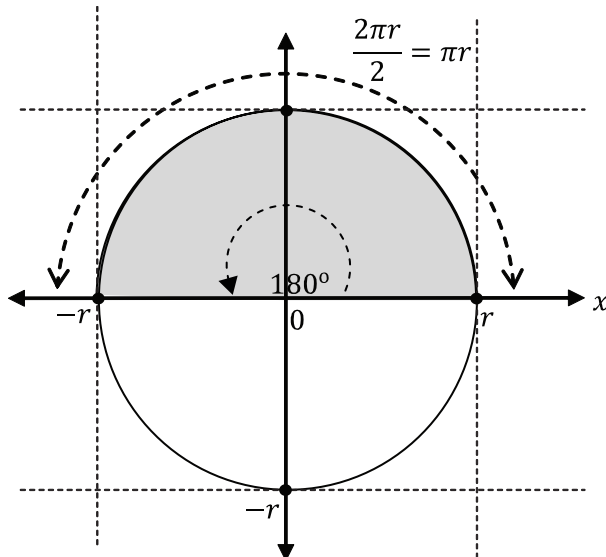


9. Complete the table below to find the rectangular coordinates (to 2 d.p.):

(r, θ)	$x = r \cos \theta$	$y = r \sin \theta$	(x, y)
$(\sqrt{2}, 45^\circ)$	$x = \sqrt{2} \cos 45^\circ = 1$	$y = \sqrt{2} \sin 45^\circ = 1$	$(1, 1)$
$(3, 60^\circ)$	$x = 3 \cos 60^\circ =$		
$(5, 130^\circ)$		$y = 5 \sin 60^\circ =$	
$(4, 200^\circ)$			
$(6, 315^\circ)$			
$(7, 270^\circ)$			

Arc length covered by an angle

The diagram shows a circle with radius r whose centre is at the origin. How much of the circumference is covered by 180° ?



It is easy to see that **for any radius r** , an angle of 180° will always cover half of the circle's circumference.

$$\text{Arc length} = \frac{1}{2} \times \text{circumference} = \frac{1}{2} \times 2\pi r = \pi r$$

In general, the arc length covered by any angle θ is:

$$\text{Arc length} = \frac{\theta}{360^\circ} \times \text{circumference} = \frac{\theta}{360^\circ} \times 2\pi r$$

10. Use the circle and the above formula to complete the table below (ignore the last column for now):

θ	$\frac{\theta}{360^\circ}$	Arc length covered by θ	θ in Radians
30°	$\frac{30^\circ}{360^\circ} = \frac{1}{12}$	$\frac{1}{12} \times 2\pi r = \frac{\pi}{6} r$	$\frac{\pi}{6} r \div r = \frac{\pi}{6}$
60°			
90°			
120°			
240°			
270°			
300°			
360°			

Radians

Radians are a different unit for measuring angles (instead of degrees).

Look at the circle on the previous page. Radians are defined as **arc length covered by an angle divided by the radius of the circle**. Ok, that sounds complicated, but look at the example below to convert 180° to radians.

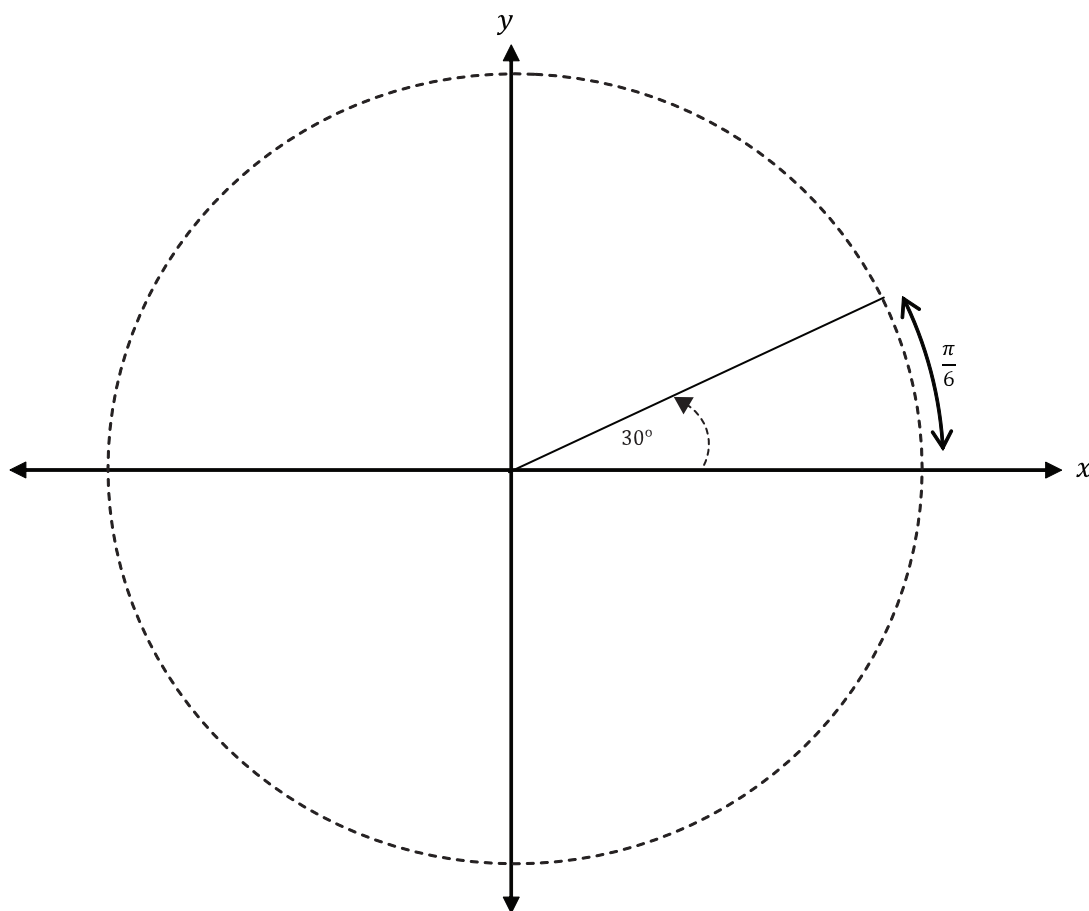
$$180^\circ = \frac{\text{Arc length covered by } 180^\circ}{\text{radius}} = \frac{\pi r}{r} = \pi$$

So we find the important expression:

$$180^\circ = \pi \text{ radians}$$

This would be written as $180 = \pi \text{ rad}$
where '**rad**' is the unit for radians

11. Complete the last column of the table on the previous page. Use the table to draw all the angles in radians on the axes below:



Converting between radians and degrees

Let's develop a formula to convert degrees to radians based on the fact that $180^\circ = \pi$ 'radians'. To convert θ degrees to radians use the formula:

$$\theta(\text{radians}) = \theta(\text{degrees}) \times \frac{\pi}{180^\circ}$$

12. Use this formula to convert the following angles from degrees to radians (leave answers as fractions):

a 60°

b 90°

$$= 60^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{3} \text{ rad}$$

c 40°

d 100°

e 225°

f 315°

13. To develop a formula to convert radians to degrees, make θ° the subject of the formula above:

$$\theta(\text{degrees}) =$$

14. Use this formula to convert the following angles from radians to degrees (1 decimal place):

a $\frac{2\pi}{3}$

b $\frac{\pi}{4}$

$$= \frac{2\pi}{3} \times \frac{180^\circ}{\pi} = 120^\circ$$

c $\frac{5\pi}{6}$

d $\frac{3\pi}{2}$

e $\frac{4\pi}{3}$

f $\frac{7\pi}{4}$

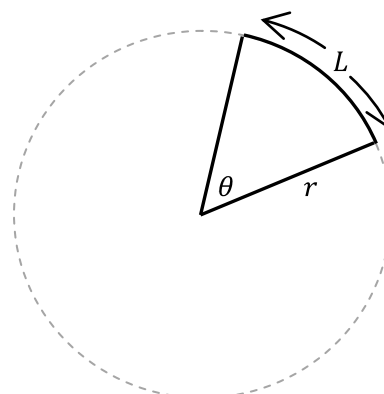
Arc length

L is used to represent the length of an arc created with an angle θ and radius r . θ (in radians) is the **ratio of the arc length covered by the angle and the radius**.

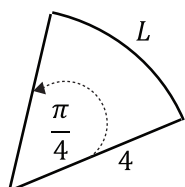
In maths this means: $\theta = \frac{L}{r}$

If we solve for L then we find the formula for arc length:

$$L = r\theta$$

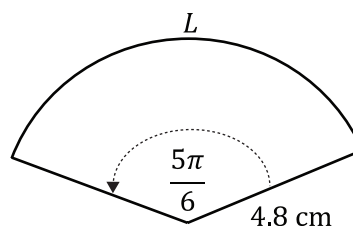


15. a Find the arc length L .

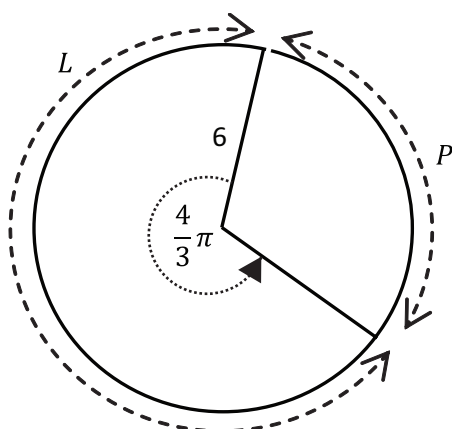


$$\begin{aligned} L &= r\theta \\ &= 4 \times \frac{\pi}{4} = \pi \text{ cm} \end{aligned}$$

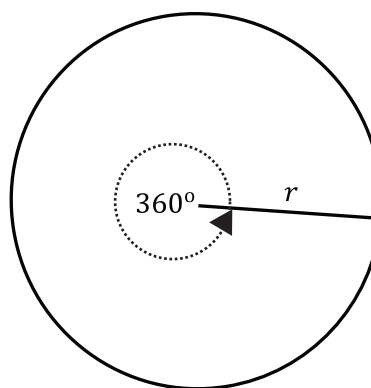
b Find the perimeter of the following sector.



c Find L and P in the following diagram.



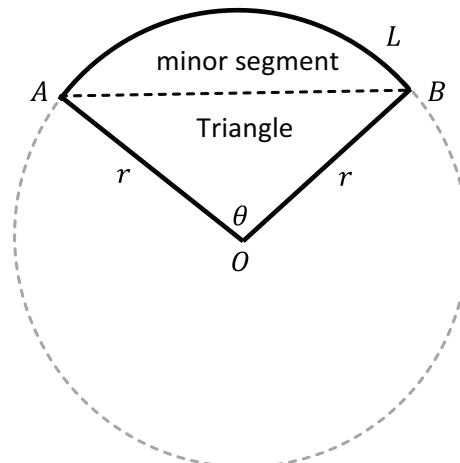
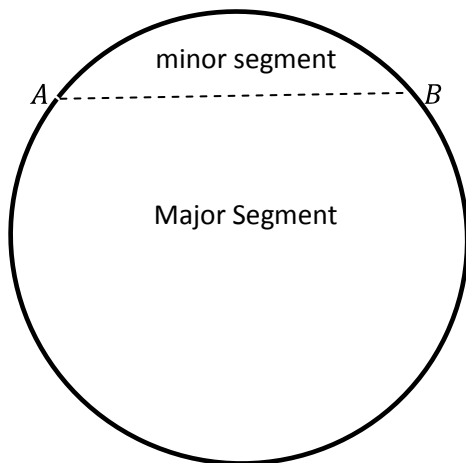
d Find the arc length of the entire circle below. (Hint: Convert the angle to radians first)



Does this sound familiar?

Area of a sector

A chord divides a circle into a **minor segment** and a **major segment**, and a sector into a **triangle** and **minor segment**.



An angle of θ in radians covers $\frac{\theta}{2\pi}$ of the area of a circle. So, if θ is in radians, the area of the sector can be found using the formula:

$$\text{Area of sector} = \frac{\theta}{2\pi} \times (\text{area of circle}) = \frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2} r^2 \theta$$

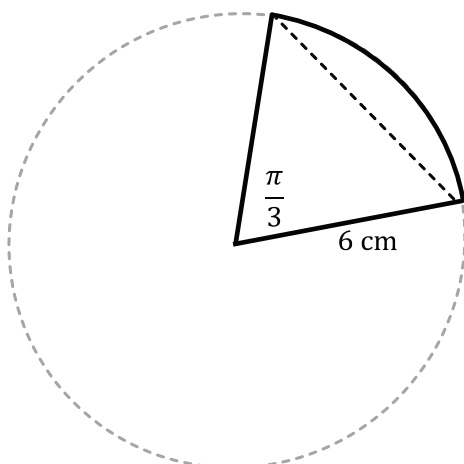
In $\triangle OAB$, we can use the area rule (area of $\triangle ABC = \frac{1}{2} ab \times \sin C$) to find its area:

$$\text{Area of triangle} = \frac{1}{2} r^2 \sin \theta$$

To find the area of the minor segment, simply subtract the area of the triangle from the area of the sector:

$$\text{Area of minor segment} = \text{area of sector} - \text{area of triangle} = \frac{1}{2} r^2 (\theta - \sin \theta)$$

16. Use the diagram on the left to answer the following questions:



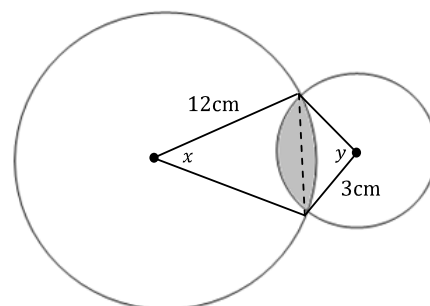
- Identify the major and minor segments on the diagram
- Find the area of the sector: $\frac{1}{2} r^2 \theta = \frac{1}{2} (6)^2 \times \frac{\pi}{3} = 6\pi \text{ cm}^2$
- Find the area of the triangle
- Find the area of the minor segment

Let's see what you can do

17. Find the arc length of a sector subtending an angle of $\frac{5}{6}\pi$ and a radius of 12cm.

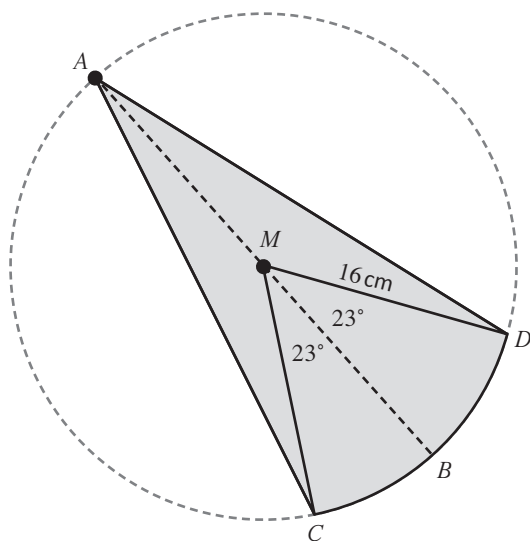
18. A sector with a radius of 4cm has an area of $4\pi \text{ cm}^2$. What is the angle inside the sector?

19. Find the overlapping area of the 2 circles below if $x = \frac{\pi}{3}$ and $y = \frac{2\pi}{3}$ (to 2 d.p.):



20. What is the perimeter of the overlapping area in the previous question?

21. Find the area of the region below if M is the centre of the circle and AB is the diameter:



Quadrants in radians

Exact ratios reminder (first quadrant)

$$\sin 30^\circ = \frac{1}{2}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 90^\circ = 1$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\cos 90^\circ = 0$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\tan 45^\circ = 1$$

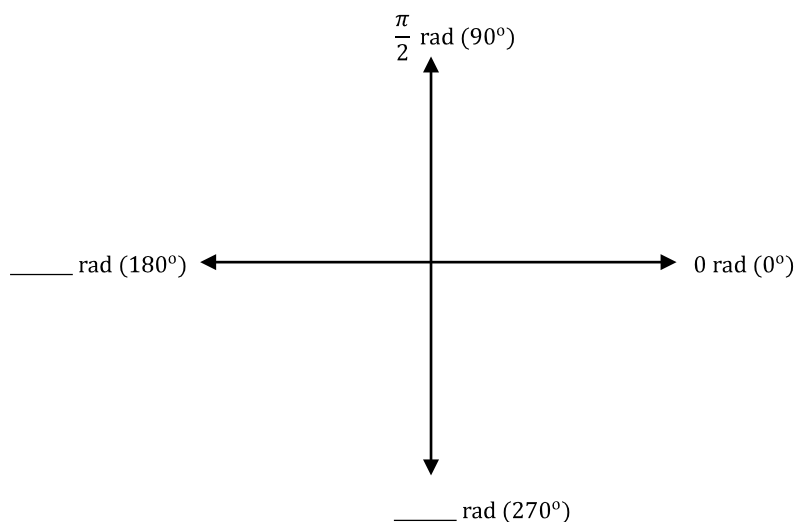
$$\tan 60^\circ = \sqrt{3}$$

$$\tan 90^\circ = \text{undefined}$$

22. Use the exact ratios above to complete the table:

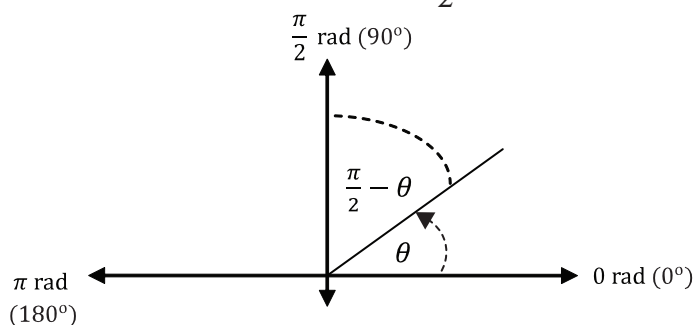
θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0			
$\frac{\pi}{6}$	$\frac{1}{2}$		
$\frac{\pi}{3}$			$\sqrt{3}$
	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{2}$			
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$		
$\frac{3\pi}{2}$			undefined

23. Complete the radian values for the quadrants on the axes:



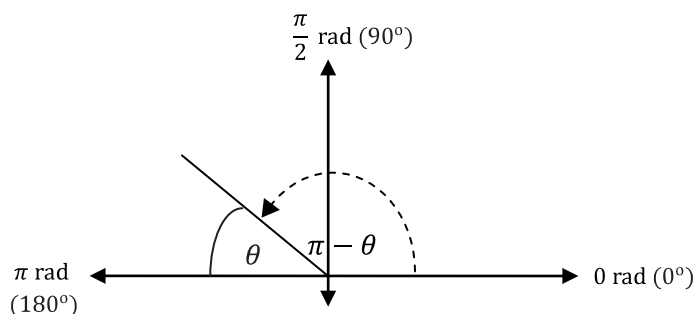
24. Complete the following tables:

a 1st Quadrant (between 0 and $\frac{\pi}{2}$)



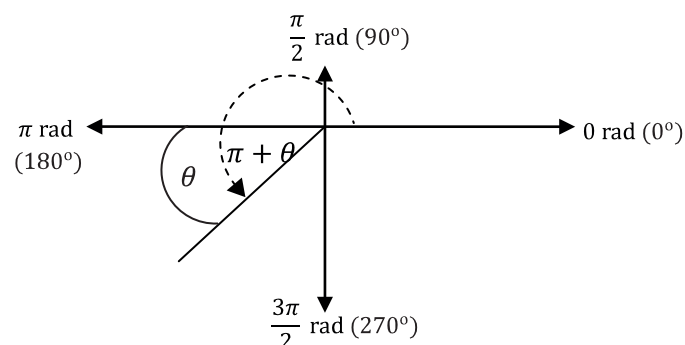
Rule in degrees	Rule in radians
$\sin(90^\circ - \theta) = \cos \theta$	$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$
$\cos(90^\circ - \theta) = \sin \theta$	
$\tan(90^\circ - \theta) = \cot \theta$	

b 2nd Quadrant (between $\frac{\pi}{2}$ and π)



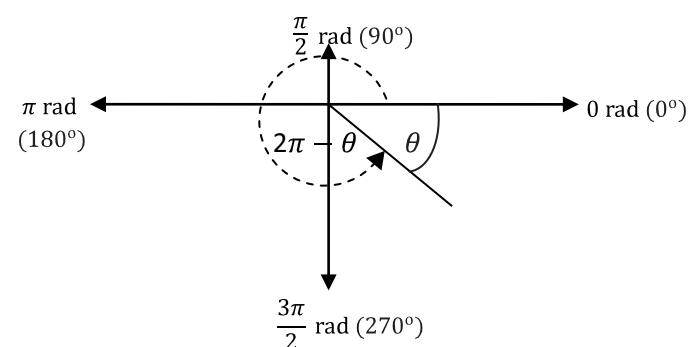
Rule in degrees	Rule in radians
$\sin(180^\circ - \theta) = \sin \theta$	$\sin(\pi - \theta) = \sin \theta$
$\cos(180^\circ - \theta) = -\cos \theta$	
$\tan(180^\circ - \theta) = -\tan \theta$	

c 3rd Quadrant (between π and $\frac{3\pi}{2}$)



Rule in degrees	Rule in radians
$\sin(180^\circ + \theta) = -\sin \theta$	
$\cos(180^\circ + \theta) = -\cos \theta$	$\cos(\pi + \theta) = -\cos \theta$
$\tan(180^\circ + \theta) = \tan \theta$	

d 4th Quadrant (between $\frac{3\pi}{2}$ and 2π)



Rule in degrees	Rule in radians
$\sin(360^\circ - \theta) = -\sin \theta$	
$\cos(360^\circ - \theta) = \cos \theta$	
$\tan(360^\circ - \theta) = -\tan \theta$	

25. Use what you know about quadrants to complete the following table:

Expression of θ	θ (radians)	Quadrant of θ	Simplification Rule
$\sin \frac{\pi}{6}$	$\frac{\pi}{6}$	1st	
$\cos \frac{\pi}{2}$		1st	
$\sin \frac{3\pi}{4}$	$\frac{3\pi}{4}$	2nd	$\sin \frac{3\pi}{4} = \sin\left(\pi - \frac{3\pi}{4}\right) = \sin \frac{\pi}{4}$
$\tan \frac{11\pi}{12}$			
$\cos \frac{5\pi}{4}$			
$\tan \frac{2\pi}{3}$			
$\cos \frac{9\pi}{10}$	$\frac{9\pi}{10}$		
$\sin \frac{19\pi}{10}$			
$\tan \frac{7\pi}{4}$		4th	$\tan \frac{7\pi}{4} = \tan\left(2\pi - \frac{\pi}{4}\right) = -\tan \frac{\pi}{4}$
$\sin \frac{\pi}{10}$			$\cos \frac{8\pi}{7} = \cos\left(\pi + \frac{\pi}{7}\right) = -\cos \frac{\pi}{7}$
$\cos \frac{8\pi}{7}$		3rd	
$\tan \frac{15\pi}{8}$		4th	

Sometimes radians are written in decimal form rather than fractions. For example π radians = 3.14159... radians.

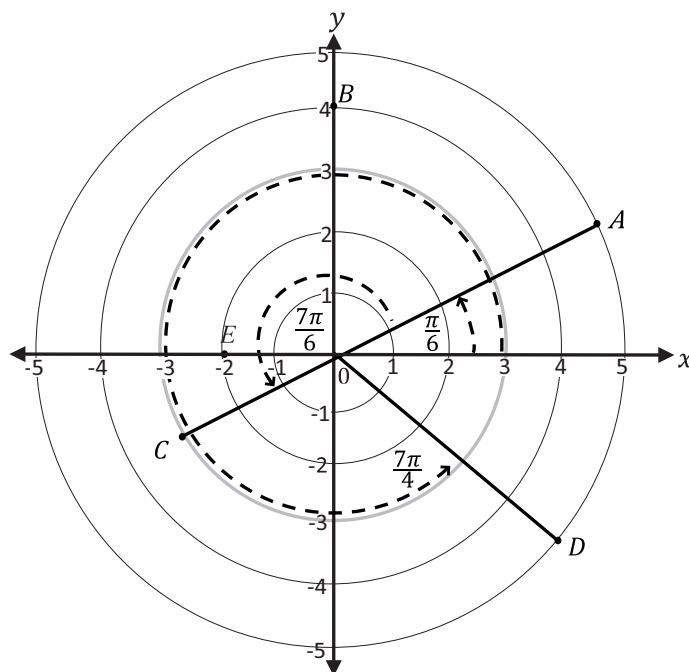
26. Can you complete the table below?

θ as a fraction	θ as a decimal	Quadrant
$\frac{\pi}{2}$	1.5707...	1st
$\frac{\pi}{6}$		1st
$\frac{4\pi}{3}$		
$\frac{3\pi}{4}$		
$\frac{7\pi}{4}$		

Polar coordinates in radians

Polar coordinates are treated in exactly the same way as with degrees. The values for θ are in radians instead of degrees.

27. Complete the table for the following points:



Point	r	θ	Polar coordinates (r, θ)	$x = r\cos\theta$	$y = r\sin\theta$	(x, y) Coordinates
A						
B	4	$\frac{\pi}{2}$	$(4, \frac{\pi}{2})$	$4\cos\frac{\pi}{2} = 0$	$4\sin\frac{\pi}{2} = 4$	$(0, 4)$
C						
D						
E		π				

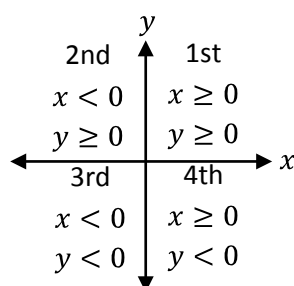
Converting coordinates by quadrant with radians (set your calculator to radian mode)

These conversions work the same as degrees just with radian values. This means we substitute π for 180° and 2π for 360° .

If we are given rectangular coordinates (x, y) we need to find r and θ , where θ is in radians. r is ALWAYS calculated the same way, using Pythagoras:

$$r = \sqrt{x^2 + y^2}$$

Before we find θ , we need to calculate the reference angle $A = \tan^{-1}\left(\frac{y}{x}\right)$. Then θ is calculated differently in each quadrant.



1st Quadrant	2nd Quadrant	3rd Quadrant	4th Quadrant
$x \geq 0 \quad y \geq 0$	$x < 0 \quad y \geq 0$	$x < 0 \quad y < 0$	$x \geq 0 \quad y < 0$
$r = \sqrt{x^2 + y^2}$	$r = \sqrt{x^2 + y^2}$	$r = \sqrt{x^2 + y^2}$	$r = \sqrt{x^2 + y^2}$
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$	$A = \tan^{-1}\left(\frac{y}{x}\right)$	$A = \tan^{-1}\left(\frac{y}{x}\right)$	$A = \tan^{-1}\left(\frac{y}{x}\right)$
	A will be negative	A will be positive	A will be negative
	$\theta = A + \pi$	$\theta = A + \pi$	$\theta = A + 2\pi$

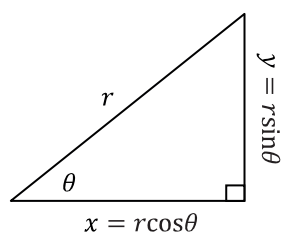
28. Use the rules for each quadrant to complete the following table (to 2 d.p.):

(x, y)	Quadrant	$r = \sqrt{x^2 + y^2}$	$A = \tan^{-1}\left(\frac{y}{x}\right)$	θ (radians)	(r, θ)
$(3, -4)$	4th	$r = \sqrt{3^2 + (-4)^2} = 5$	$\tan^{-1}\left(\frac{-4}{3}\right) = -0.93$	$A + 2\pi = 5.36$	$(5, 5.36)$
$(\sqrt{3}, \sqrt{6})$	1st	$r = \sqrt{(\sqrt{3})^2 + (\sqrt{6})^2} =$	$\tan^{-1}\left(\frac{\sqrt{6}}{\sqrt{3}}\right) =$		
$(-6, 8)$	2nd				
$(-12, -9)$	3rd				
$(-5, 0)$	2nd				
$(0, 5)$	4th				
$(-1, -\sqrt{3})$	3rd				
$(\sqrt{2}, -\sqrt{7})$	4th				

Converting polar coordinates (r, θ) to rectangular coordinates (x, y)

The process is EXACTLY the same as with degrees.

If we have a point in polar coordinates (r, θ) then we need to find x and y .



29. Complete the table below to find the rectangular coordinates (Set calculators to radian mode):

(r, θ)	$x = r \cos \theta$	$y = r \sin \theta$	(x, y)
$(4, \frac{\pi}{3})$	$4 \cos \frac{\pi}{3} = 4\left(\frac{1}{2}\right) = 2$	$4 \sin \frac{\pi}{3} = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$	$(2, 2\sqrt{3})$
$(\sqrt{2}, \frac{\pi}{4})$	$\sqrt{2} \cos \frac{\pi}{4} =$	$\sqrt{2} \sin \frac{\pi}{4} =$	
$(3, \frac{5\pi}{4})$			
$(2, \frac{15\pi}{8})$			
$(1, \frac{\pi}{2})$			
$(3, \pi)$			
$(5, \frac{3\pi}{2})$			

Basics

1.
 - a $\sin\theta$ is above the x -axis
 $\sin\theta$ is positive
 - b $\sin\theta$ is above the x -axis
 $\sin\theta$ is positive
 - c $\sin\theta$ is below the x -axis
 $\sin\theta$ is negative
 - d $\sin\theta$ is below the x -axis
 $\sin\theta$ is negative
2.
 - a If $\theta = 70^\circ$ then θ is between 0 and 180° and $\sin\theta$ is positive.
 - b If $\theta = 110^\circ$ then θ is between 0 and 180° and $\sin\theta$ is positive.
 - c If $\theta = 300^\circ$ then θ is between 180° and 360° and $\sin\theta$ is negative.
 - d If $\theta = 240^\circ$ then θ is between 180° and 360° and $\sin\theta$ is negative.
3.
 - a $\cos\theta$ is positive
 - b $\cos\theta$ is negative
 - c $\cos\theta$ is positive
 - d $\cos\theta$ is negative
4.
 - a If $\theta = 65^\circ$ then θ is between 0 and 90° so $\cos\theta$ is positive.
 - b If $\theta = 250^\circ$ then θ is between 180° and 270° so $\cos\theta$ is negative.
 - c If $\theta = 135^\circ$ then θ is between 90° and 180° so $\cos\theta$ is negative.
 - d If $\theta = 290^\circ$ then θ is between 270° and 360° so $\cos\theta$ is positive.

Knowing More

5.
 - a $A(0.707, 0.707)$ (to 3 d.p.)
 - b $B(-0.985, 0.174)$ (to 3 d.p.)
 - c $C(-0.342, -0.940)$ (to 3 d.p.)
 - d $D(0.707, -0.707)$ (to 3 d.p.)

6.

Point	r	θ	Polar coordinates (r, θ)	$x = r\cos\theta$	$y = r\sin\theta$	(x, y) Coordinates
A	2	30°	$A(2, 30^\circ)$	$2\cos 30^\circ = 1.73$	$2\sin 30^\circ = 1$	$A(1.73, 1)$
B	4	85°	$B(4, 85^\circ)$	$4\cos 85^\circ = 0.35$	$4\sin 85^\circ = 3.98$	$B(0.35, 3.98)$
C	2	110°	$C(2, 110^\circ)$	$2\cos 110^\circ = -0.68$	$2\sin 110^\circ = 1.88$	$C(-0.68, 1.88)$
D	4	230°	$D(4, 230^\circ)$	$4\cos 230^\circ = -2.57$	$4\sin 230^\circ = -3.06$	$D(-2.57, -3.06)$
E	3	300°	$E(3, 300^\circ)$	$3\cos 300^\circ = 1.50$	$3\sin 300^\circ = -2.60$	$E(1.50, -2.60)$
F	5	340°	$F(5, 340^\circ)$	$3\cos 340^\circ = 2.82$	$3\sin 340^\circ = -1.03$	$F(2.82, -1.03)$

Knowing More

7.	(x, y)	$r = \sqrt{x^2 + y^2}$	$\theta = \tan^{-1}\left(\frac{y}{x}\right)$	(r, θ)
	$(3, 4)$	$\sqrt{3^2 + 4^2} = 5$	$\tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$	$(5, 53.13^\circ)$
	$(5, 12)$	$\sqrt{5^2 + 12^2} = 13$	$\tan^{-1}\left(\frac{12}{5}\right) = 67.38^\circ$	$(13, 67.38^\circ)$
	$(4\sqrt{2}, 4\sqrt{2})$	$\sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2} = 8$	$\tan^{-1}\left(\frac{4\sqrt{2}}{4\sqrt{2}}\right) = \tan^{-1} 1 = 45^\circ$	$(8, 45^\circ)$
	$(1, \sqrt{3})$	$\sqrt{1^2 + (\sqrt{3})^2} = 2$	$\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 60^\circ$	$(2, 60^\circ)$
	$(3\sqrt{3}, 3)$	$\sqrt{(3\sqrt{3})^2 + 3^2} = 6$	$\tan^{-1}\left(\frac{3}{3\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$	$(6, 30^\circ)$

8.	(x, y)	Quadrant	$r = \sqrt{x^2 + y^2}$	$\theta = \tan^{-1}\left(\frac{y}{x}\right)$	θ	(r, θ)
	$(3, -4)$	4 th	$r = \sqrt{3^2 + (-4)^2} = 5$	$\tan^{-1}\left(\frac{-4}{3}\right) = -53.13^\circ$	$\alpha + 360^\circ = 306.87^\circ$	$(5, 306.87^\circ)$
	$(-1, \sqrt{3})$	2 nd	$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$	$\tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) = -60^\circ$	$\alpha + 180^\circ = 120^\circ$	$(2, 120^\circ)$
	$(-5, -12)$	3 rd	$\sqrt{(-5)^2 + (-12)^2} = 13$	$\tan^{-1}\left(\frac{12}{5}\right) = 67.38^\circ$	$\alpha + 180^\circ = 247.38^\circ$	$(13, 247.38^\circ)$
	$(-\sqrt{2}, \sqrt{2})$	2 nd	$r = \sqrt{(-\sqrt{2})^2 + (\sqrt{2})^2} = 2$	$\tan^{-1}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right) = -45^\circ$	$\alpha + 180^\circ = 135^\circ$	$(2, 135^\circ)$
	$(15, 8)$	1 st	$r = \sqrt{(15)^2 + (8)^2} = 17$	$\tan^{-1}\left(\frac{8}{15}\right) = 28.07^\circ$	$\alpha = 28.07^\circ$	$(17, 28.07^\circ)$
	$(7, -7)$	4 th	$r = \sqrt{(7)^2 + (-7)^2} = 7\sqrt{2}$	$\tan^{-1}\left(\frac{-7}{7}\right) = -45^\circ$	$\alpha + 360^\circ = 315^\circ$	$(7\sqrt{2}, 315^\circ)$
	$(-9, -12)$	3 rd	$r = \sqrt{(-9)^2 + (-12)^2} = 15$	$\tan^{-1}\left(\frac{-12}{-9}\right) = 53.13^\circ$	$\alpha + 180^\circ = 233.13^\circ$	$(15, 233.13^\circ)$
	$(-\sqrt{3}, 1)$	2 nd	$r = \sqrt{(-\sqrt{3})^2 + (1)^2} = 2$	$\tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = -30^\circ$	$\alpha + 180^\circ = 150^\circ$	$(2, 150^\circ)$

9.

(r, θ)	$x = r \cos \theta$	$y = r \sin \theta$	(x, y)
$(\sqrt{2}, 45^\circ)$	$x = \sqrt{2} \cos 45^\circ = 1$	$y = \sqrt{2} \sin 45^\circ = 1$	$(1, 1)$
$(3, 60^\circ)$	$x = 3 \cos 60^\circ = \frac{3}{2} = 1.5$	$y = 3 \sin 60^\circ = 3\left(\frac{\sqrt{3}}{2}\right) = 2.60$	$(1.50, 2.60)$
$(5, 130^\circ)$	$x = 5 \cos 130^\circ = -3.21$	$y = 5 \sin 130^\circ = 3.83$	$(-3.21, 3.83)$
$(4, 200^\circ)$	$x = 4 \cos 200^\circ = -3.76$	$y = 4 \sin 200^\circ = -1.37$	$(-3.76, -1.37)$
$(6, 315^\circ)$	$x = 6 \cos 315^\circ = 6 \cos 45^\circ = \frac{6}{\sqrt{2}} = 4.24$	$y = 6 \sin 315^\circ = -4.24$	$(4.24, -4.24)$
$(7, 270^\circ)$	$x = 7 \cos 270^\circ = 7(0) = 0$	$y = 7 \sin 270^\circ = -7$	$(0, -7)$

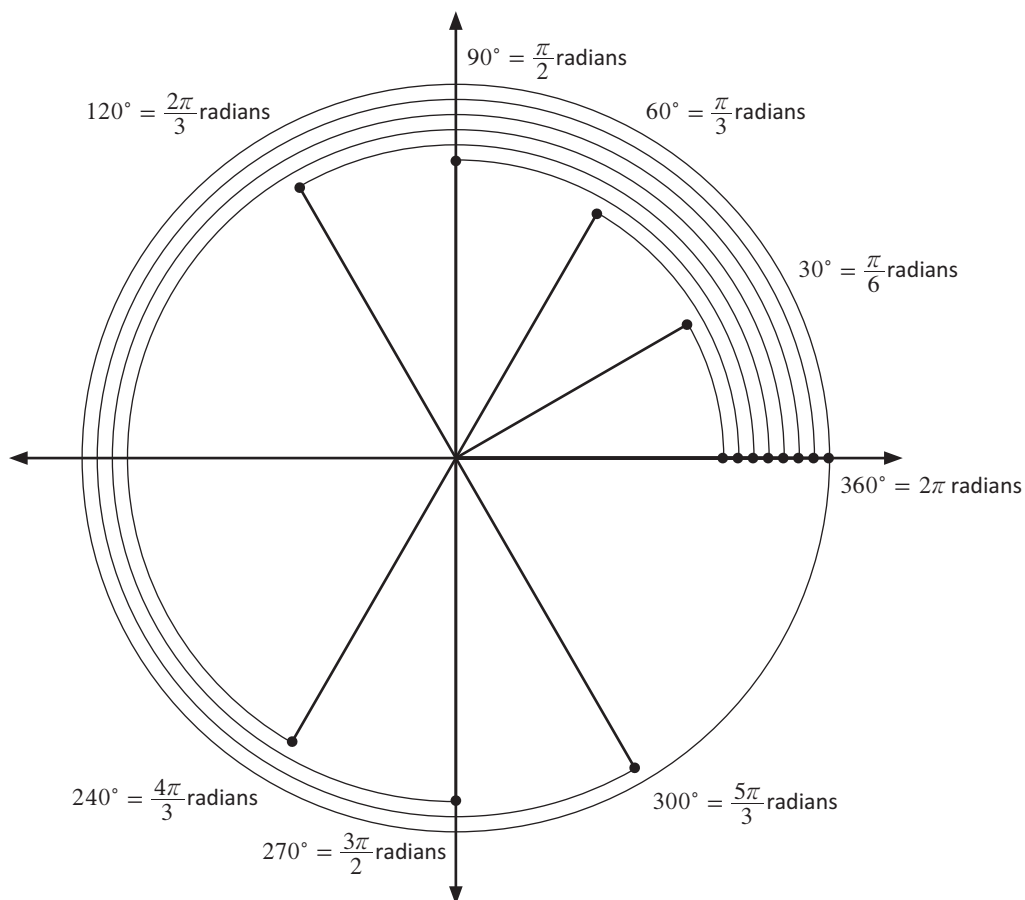
Using Our Knowledge

10.

θ	$\frac{\theta}{360^\circ}$	Arc length covered by θ	θ in Radians
30°	$\frac{30^\circ}{360^\circ} = \frac{1}{12}$	$\frac{1}{12} \times 2\pi r = \frac{\pi}{6}r$	$\frac{\pi}{6}r \div r = \frac{\pi}{6}$
60°	$\frac{60^\circ}{360^\circ} = \frac{1}{6}$	$\frac{1}{6} \times 2\pi r = \frac{\pi}{3}r$	$\frac{\pi}{3}r \div r = \frac{\pi}{3}$
90°	$\frac{90^\circ}{360^\circ} = \frac{1}{4}$	$\frac{1}{4} \times 2\pi r = \frac{\pi}{2}r$	$\frac{\pi}{2}r \div r = \frac{\pi}{2}$
120°	$\frac{120^\circ}{360^\circ} = \frac{1}{3}$	$\frac{1}{3} \times 2\pi r = \frac{2\pi}{3}r$	$\frac{2\pi}{3}r \div r = \frac{2\pi}{3}$
240°	$\frac{240^\circ}{360^\circ} = \frac{2}{3}$	$\frac{2}{3} \times 2\pi r = \frac{4\pi}{3}r$	$\frac{4\pi}{3}r \div r = \frac{4\pi}{3}$
270°	$\frac{270^\circ}{360^\circ} = \frac{3}{4}$	$\frac{3}{4} \times 2\pi r = \frac{3\pi}{2}r$	$\frac{3\pi}{2}r \div r = \frac{3\pi}{2}$
300°	$\frac{300^\circ}{360^\circ} = \frac{5}{6}$	$\frac{5}{6} \times 2\pi r = \frac{5\pi}{3}r$	$\frac{5\pi}{3}r \div r = \frac{5\pi}{3}$
360°	$\frac{360^\circ}{360^\circ} = 1$	$\frac{1}{1} \times 2\pi r = 2\pi r$	$2\pi r \div r = 2\pi$

Using Our Knowledge

11.



12. a $\frac{\pi}{3}$ rad

b $\frac{\pi}{2}$ rad

c $\frac{2\pi}{9}$ rad

d $\frac{5\pi}{9}$ rad

e $\frac{5\pi}{4}$ rad

f $\frac{7\pi}{4}$ rad

13. θ (degrees) = θ (radians) $\times \frac{180^\circ}{\pi}$

14. a 120°

b 45°

c 150°

d 270°

e 240°

f 315°

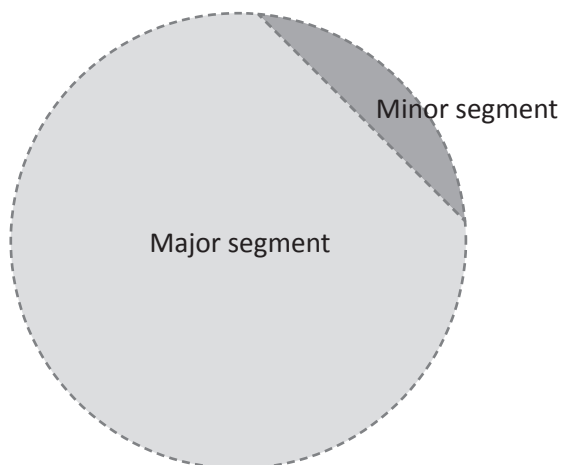
15. a π cm

b 4π cm

c 8π cm

d $2\pi r$ cm

16. a


b Area sector = 18.85 cm^2 (2 d.p.)

c Area triangle = $9\sqrt{3} \text{ cm}^2$ or 15.59 cm^2 (2 d.p.)

d Area minor segment = 3.26 cm^2 (2 d.p.)

17. $10\pi \text{ cm}$

18. $A = \frac{\pi}{2} \text{ rad}$

19. Area = $27\pi - \left(\frac{153}{4}\right)\sqrt{3} \text{ cm}^2$

20. Perimeter = $6\pi \text{ cm}$

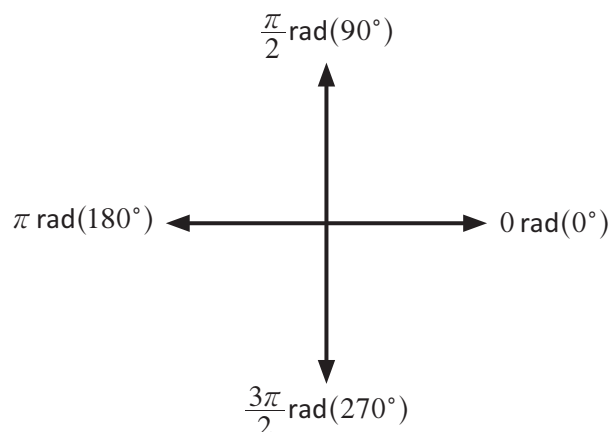
21. Total area = 202.79 cm^2 (2 d.p.)

Thinking More

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{2}$	1	0	Undefined
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1
$\frac{3\pi}{2}$	-1	0	Undefined

Thinking More

23.



24. a

Rule in degrees	Rule in radians
$\sin(90^\circ - \theta) = \cos \theta$	$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$
$\cos(90^\circ - \theta) = \sin \theta$	$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$
$\tan(90^\circ - \theta) = \cot \theta$	$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$

b

Rule in degrees	Rule in radians
$\sin(180^\circ - \theta) = +\sin \theta$	$\sin(\pi - \theta) = \sin \theta$
$\cos(180^\circ - \theta) = -\cos \theta$	$\cos(\pi - \theta) = -\cos \theta$
$\tan(180^\circ - \theta) = -\tan \theta$	$\tan(\pi - \theta) = -\tan \theta$

24. c

Rule in degrees	Rule in radians
$\sin(180^\circ + \theta) = -\sin \theta$	$\sin(\pi + \theta) = -\sin \theta$
$\cos(180^\circ + \theta) = -\cos \theta$	$\cos(\pi + \theta) = -\cos \theta$
$\tan(180^\circ + \theta) = +\tan \theta$	$\tan(\pi + \theta) = +\tan \theta$

d

Rule in degrees	Rule in radians
$\sin(360^\circ - \theta) = -\sin \theta$	$\sin(2\pi - \theta) = -\sin \theta$
$\cos(360^\circ - \theta) = +\cos \theta$	$\cos(2\pi - \theta) = +\cos \theta$
$\tan(360^\circ - \theta) = -\tan \theta$	$\tan(2\pi - \theta) = -\tan \theta$

25.

Expression of θ	θ (radians)	Quadrant of θ	Simplification Rule
$\sin \frac{\pi}{6}$	$\frac{\pi}{6}$	1 st	$\sin \frac{\pi}{6}$
$\cos \frac{\pi}{2}$	$\frac{\pi}{2}$	1 st	$\cos \frac{\pi}{2}$
$\sin \frac{3\pi}{4}$	$\frac{3\pi}{4}$	2 nd	$\sin \frac{3\pi}{4} = \sin\left(\pi - \frac{3\pi}{4}\right) = \sin \frac{\pi}{4}$
$\tan \frac{11\pi}{12}$	$\frac{11\pi}{12}$	2 nd	$\tan \frac{11\pi}{12} = \tan\left(\pi - \frac{\pi}{12}\right) = -\tan \frac{\pi}{12}$
$\cos \frac{5\pi}{4}$	$\frac{5\pi}{4}$	3 rd	$\cos \frac{5\pi}{4} = \cos\left(\pi + \frac{\pi}{4}\right) = -\cos \frac{\pi}{4}$
$\tan \frac{2\pi}{3}$	$\frac{2\pi}{3}$	2 nd	$\tan \frac{2\pi}{3} = \tan\left(\pi - \frac{\pi}{3}\right) = -\tan \frac{\pi}{3}$
$\cos \frac{9\pi}{10}$	$\frac{9\pi}{10}$	2 nd	$\cos \frac{9\pi}{10} = \cos\left(\pi - \frac{\pi}{10}\right) = -\cos \frac{\pi}{10}$
$\sin \frac{19\pi}{10}$	$\frac{19\pi}{10}$	4 th	$\sin \frac{19\pi}{10} = \sin\left(2\pi - \frac{\pi}{10}\right) = -\sin \frac{\pi}{10}$
$\tan \frac{7\pi}{4}$	$\frac{7\pi}{4}$	4 th	$\tan \frac{7\pi}{4} = \tan\left(2\pi - \frac{\pi}{4}\right) = -\tan \frac{\pi}{4}$
$\sin \frac{\pi}{10}$	$\frac{\pi}{10}$	1 st	$\sin \frac{\pi}{10}$
$\cos \frac{8\pi}{7}$	$\frac{8\pi}{7}$	3 rd	$\cos \frac{8\pi}{7} = \cos\left(\pi + \frac{\pi}{7}\right) = -\cos \frac{\pi}{7}$
$\tan \frac{15\pi}{8}$	$\frac{15\pi}{8}$	4 th	$\tan \frac{15\pi}{8} = \tan\left(2\pi - \frac{\pi}{8}\right) = -\tan \frac{\pi}{8}$

26.

θ as a fraction	θ as a decimal	Quadrant
$\frac{\pi}{2}$	1.5707...	1 st
$\frac{\pi}{6}$	0.5235...	1 st
$\frac{4\pi}{3}$	4.1887...	3 rd
$\frac{3\pi}{4}$	2.3561...	2 nd
$\frac{7\pi}{4}$	5.4977..	4 th

Thinking More

27.	Point	r	θ	Polar Coordinates (r, θ)	$x = r \cos \theta$	$y = r \sin \theta$	(x, y) Coordinates
	A	5	$\frac{\pi}{6}$	$(5, \frac{\pi}{6})$	$5 \cos \frac{\pi}{6} = \frac{5\sqrt{3}}{2}$	$5 \sin \frac{\pi}{6} = \frac{5}{2}$	$(\frac{5\sqrt{3}}{2}, \frac{5}{2})$
	B	4	$\frac{\pi}{2}$	$(4, \frac{\pi}{2})$	$4 \cos \frac{\pi}{2} = 0$	$4 \sin \frac{\pi}{2} = 4$	(0, 4)
	C	3	$\frac{7\pi}{6}$	$(3, \frac{7\pi}{6})$	$3 \cos \frac{7\pi}{6} = -\frac{3\sqrt{3}}{2}$	$3 \sin \frac{7\pi}{6} = -\frac{3}{2}$	$(-\frac{3\sqrt{3}}{2}, -\frac{3}{2})$
	D	5	$\frac{7\pi}{4}$	$(5, \frac{7\pi}{4})$	$5 \cos \frac{7\pi}{4} = \frac{5\sqrt{2}}{2}$	$5 \sin \frac{7\pi}{4} = -\frac{5\sqrt{2}}{2}$	$(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2})$
	E	2	π	$(2, \pi)$	$2 \cos \pi = -2$	$2 \sin \pi = 0$	(-2, 0)

28.	(x, y)	Quadrant	$r = \sqrt{x^2 + y^2}$	$A = \tan^{-1}(\frac{y}{x})$	θ	(r, θ)
	(3, -4)	4 th	$r = \sqrt{3^2 + (-4)^2} = 5$	$\tan^{-1}(\frac{-4}{3}) = -0.927...$	$2\pi + A = 5.36$	(5, 5.36)
	($\sqrt{3}, \sqrt{6}$)	1 st	$r = \sqrt{(\sqrt{3})^2 + (\sqrt{6})^2} = 3$	$\tan^{-1}(\frac{\sqrt{6}}{\sqrt{3}}) = \tan^{-1} \sqrt{2} = 0.955...$	$A = 0.96$	(3, 0.96)
	(-6, 8)	2 nd	$r = \sqrt{(-6)^2 + (8)^2} = 10$	$\tan^{-1}(\frac{8}{-6}) = -0.927...$	$\pi + A = 2.21$	(10, 2.21)
	(-12, -9)	3 rd	$r = \sqrt{(-12)^2 + (-9)^2} = 15$	$\tan^{-1}(\frac{-12}{-9}) = 0.927...$	$\pi + A = 4.07$	(15, 4.07)
	(-5, 0)	—	$r = \sqrt{(-5)^2 + (0)^2} = 5$	$\tan^{-1}(\frac{0}{-5}) = 0$	π	(5, π)
	(0, 5)	—	$r = \sqrt{(0)^2 + (5)^2} = 5$	$\tan^{-1}(\frac{5}{0}) = \text{undefined}$	$\frac{\pi}{2}$	(5, $\frac{\pi}{2}$)
	(-1, $-\sqrt{3}$)	3 rd	$r = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$	$\tan^{-1}(\frac{-\sqrt{3}}{-1}) = \frac{\pi}{3}$	$A + \pi = \frac{4\pi}{3}$	(2, $\frac{4\pi}{3}$)
	($\sqrt{2}, -\sqrt{7}$)	4 th	$r = \sqrt{(\sqrt{2})^2 + (-\sqrt{7})^2} = 3$	$\tan^{-1}(\frac{-\sqrt{7}}{\sqrt{2}}) = -1.0799...$	$A + 2\pi = 5.20$	(3, 5.20)

29.

(r, θ)	$x = r \cos \theta$	$y = r \sin \theta$	(x, y)
$(4, \frac{\pi}{3})$	$4 \cos \frac{\pi}{3} = 4(\frac{1}{2}) = 2$	$4 \sin \frac{\pi}{3} = 4(\frac{\sqrt{3}}{2}) = 2\sqrt{3}$	$(2, 2\sqrt{3})$
$(\sqrt{2}, \frac{\pi}{4})$	$\sqrt{2} \cos \frac{\pi}{4} = 1$	$\sqrt{2} \sin \frac{\pi}{4} = 1$	$(1, 1)$
$(3, \frac{5\pi}{4})$	$3 \cos \frac{5\pi}{4} = -\frac{3}{\sqrt{2}}$	$3 \sin \frac{5\pi}{4} = -\frac{3}{\sqrt{2}}$	$(-\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}})$
$(2, \frac{15\pi}{8})$	$2 \cos \frac{15\pi}{8} = 1.8477...$	$2 \sin \frac{15\pi}{8} = -0.7653...$	$(1.85, -0.77)$ (to 2 d.p.)
$(1, \frac{\pi}{2})$	$\cos \frac{\pi}{2} = 0$	$\sin \frac{\pi}{2} = 1$	$(0, 1)$
$(3, \pi)$	$3 \cos \pi = -3$	$3 \sin \pi = 0$	$(-3, 0)$
$(5, \frac{3\pi}{2})$	$5 \cos \frac{3\pi}{2} = 0$	$5 \sin \frac{3\pi}{2} = -5$	$(0, -5)$

