

# Answers

## TOPIC 8 Algebra

### 8.2 Using variables

- 1 a  $x + 420$   
 b  $3x$   
 c  $x - 130$   
 d The nearby nest has 60 more ants.  
 e The nearby nest has 90 fewer ants.  
 f The nest is one quarter of the size of the original nest.
- 2 a  $x + y$  b  $x + y + 260$   
 c  $x + y + 90$  d  $x + y - 260$
- 3 Between 9.00 am and 9.15 am one Danish pastry was sold. In the next hour-and-a-half, a further 11 Danish pastries were sold. No more Danish pastries had been sold at 12.30 pm, but in the next half-hour 18 more were sold. No Danish pastries were sold after 1.00 pm.
- 4 a  $a + b$  b  $a + b + c$   
 c  $b + 4$  d  $a - 6$
- 5 a  $y + 7$  b  $y - 8$   
 c  $5y$  d  $14 - y$   
 e  $\frac{y}{3}$  f  $8y + 3$
- 6 a  $a + b$  b  $a - b$   
 c  $2b - 3a$  d  $ab$   
 e  $2ab$  f  $3a + 7b$   
 g  $a^2$  h  $\frac{a^2}{5}$
- 7 a  $\$27y$  b  $\$14d$  c  $\$(27r + 14h)$
- 8 a  $t + 2$  b  $t + g$   
 c  $t - 5$  d  $2t$
- 9 a Various answers are possible; an example is shown. The number of passengers doubled at the next stop and continued to increase, more than quadrupling in the first nine minutes. At 7.22 pm, 5 people alighted the train, and by 7.25 pm the same number of passengers were on the train as there were at the beginning. By 7.34 pm there were 12 fewer passengers than there were at the beginning.  
 b 7.22 pm c 7.19 pm d 7.34 pm
- 10 a The number of bacteria in each of these intervals is double the number of bacteria in the previous interval.  
 b The bacteria could be dividing in two.  
 c It is lower than expected, based on the previous pattern of growth.  
 d Some of the bacteria may have died, or failed to divide and reproduce.
- 11 a Odd b Even  
 c i  $n + 2$ ,  $n + 4$  and  $n + 6$   
 ii  $n - 2$
- 12  $TSA = 2x^2 + 4xh$ ;  $V = x^2h$
- 13  $TSA = 5xh + 3x^2$ ;  $V = \frac{3}{2}x^2h$
- 14  $TSA_{\text{square box}} = 2000 \text{ cm}^2$ ;  $V_{\text{square box}} = 6000 \text{ cm}^3$   
 $TSA_{\text{rectangular box}} = 2700 \text{ cm}^2$ ;  $V_{\text{rectangular box}} = 9000 \text{ cm}^3$

### Challenge 8.1

Number is 4; Bill's is age 15.

### 8.3 Substitution

- 1 a 6 b 14 c 30 d 1  
 e 9 f 1 g 7 h 3  
 i 6 j 15 k 19 l 4  
 m 5 n 10 o 20 p 5  
 q 22 r 2 s  $\frac{89}{10}(8\frac{9}{10})$  t  $\frac{39}{10}(3\frac{9}{10})$

- 2 a 42 b 3 c 54 d 1  
 e 9 f 15 g 15 h 21  
 i 24.3 j 21 k 1 l 4.8  
 m 18.3 n 8.1 o 16.2 p  $\frac{7}{4}(1\frac{3}{4})$
- 3 a 7 b 7 c -3 d 3  
 e 4 f 10 g 50 h 1  
 i -15 j -4 k 37 l 15  
 m 40 n  $2\frac{1}{3}$  o 19 p 1
- 4 150 m  
 5 10 cm  
 6  $C = \$64$
- 7 a  $48 \text{ cm}^2$  b  $8400 \text{ m}^2$  c  $4.472 \text{ m}^2$  or  $44\,720 \text{ cm}^2$
- 8 a  $F = 212^\circ\text{F}$  b  $28^\circ\text{C} = 82.4^\circ\text{F}$  c  $32^\circ\text{F}$
- 9 a  $100 \text{ km} \approx 60 \text{ miles}$  b  $248 \text{ km} \approx 148.8 \text{ miles}$   
 c  $12.5 \text{ km} \approx 7.5 \text{ miles}$
- 10 If  $x = 0$ , then the expression becomes  $\frac{0}{0}$ , which is indeterminate.
- 11 172.625
- 12 a  $V = x(x + 5)(x - 2)$   
 b 1200  
 c Because  $1.5 - 2 < 0$
- 13 a  $13p - 10$   
 b 2590

### 8.4 Working with brackets

- 1 a 36 b 4 c 84 d 18  
 e 56 f 35 g 90 h 350  
 i 55 j 20 k 133 l 147  
 m 784 n 250 o 9800 p 200
- 2 a 90 b 16 c 6 d 32  
 e 72 f 90 g 7 h 36  
 i 60 j 58 k 21 l 180  
 m 576 n 32 o -26 p 33
- 3 a 62 cm b 97.8 cm
- 4 B
- 5 a  $720^\circ$  b  $540^\circ$  c  $180^\circ$   
 d  $360^\circ$  e  $3240^\circ$
- 6 a  $CD = m + 4n$  b  $BC = 3m + n$   
 c Perimeter  $= 8m + 18n$
- 7 a  $A = \frac{1}{2}(m + n)(m - n)$   
 b  $A = \frac{1}{2}(21 \times 9) = 94.5 \text{ cm}^2$   
 c If  $m < n$ , then  $(m - n) < 0$ . Negative height is impossible.  
 d If  $m$  and  $n$  move closer in value, then the length of the base of the triangle gets closer to  $2m$  (or  $2n$ ) and the height gets closer to zero.
- 8 Check with your teacher.

### 8.5 Substituting positive and negative numbers

- 1 a 3 b 9 c -9 d 3  
 e -9 f -12 g -12 h 2  
 i 5 j -13 k 9 l 3  
 m -2 n -3 o -8 p -2  
 q 0 r -109 s 6 t 16
- 2 a 18 b 32 c -15 d 12  
 e 16 f 24 g 22 h 155  
 i 32 j 3 k 6 l 21
- 3 a -9 b 1 c 3 d 40  
 e -8 f 2 g -3 h 12  
 i  $-\frac{4}{5}$  j -2 k -1 l -6  
 m 125 n 175 o -5 p -12.5  
 q -74 r -117 s -104 t -15.5
- 4  $\frac{30}{7}$  or  $4\frac{2}{7}$

5 If  $x$  is negative then  $5x$  will also be a negative integer (less than or equal to  $-5$ ). Subtracting this number is equivalent to adding a positive integer. The result will be positive.

6 Check with your teacher.

7 a Negative

b Positive

8 a  $\approx 0.804x^d$

b  $\approx 3.216$

c  $\frac{x}{2}$

### 8.6 Number laws and variables

1 a i 11 ii 11, same

b i 25 ii 25, same

c i 31 ii 31, same

d i 32 ii 32, same

e i  $-5$  ii 5, different

f i  $-18$  ii 18, different

g i  $-28$  ii 28, different

h i 1 ii  $-1$ , different

2 a i  $-10$  ii  $-10$ , same

b i  $-180$  ii  $-180$ , same

c i  $-40$  ii  $-40$ , same

d i  $-350$  ii  $-350$ , same

e i  $-\frac{2}{5}$  ii  $-\frac{5}{2}$ , different

f i  $-1$  ii  $-1$ , same

g i  $-\frac{4}{5}$  ii  $-\frac{5}{4}$ , different

h i  $-\frac{14}{45}$  ii  $-\frac{45}{14}$ , different

3 a True b False c False

d True e True f True

g False h False i False

j True k False l True

4 a i 13 ii 13, same

b i 24 ii 24, same

c i 40 ii 40, same

d i  $-3$  ii  $-7$ , different

e i  $-35$  ii  $-71$ , different

f i  $-43$  ii  $-67$ , different

5 a i  $-64$  ii  $-64$ , same

b i 768 ii 768, same

c i  $-1536$  ii  $-1536$ , same

d i  $-4$  ii  $-1$ , different

e i  $-6$  ii  $-\frac{1}{6}$ , different

f i  $\frac{8}{5}$  ii  $\frac{1}{10}$ , different

6 a False b False c False

d True e True f False

7 C

8 D

9 a i 6 ii 6

b The answers are equal because of the use of the Associative Law.

10 a  $-10$  b  $-12$

11 a  $(-3p + 4q)$  b  $\frac{1}{(3p - 4q)}$

c  $15, -\frac{1}{15}$  d The result is the identity, which is 1.

### 8.7 Simplifying expressions

1 a  $6c$  b  $-3c$  c  $4a$

d  $q$  e  $-3h$  f  $2x$

g  $-6a$  h  $4f$  i  $-3p$

j  $h$  k  $18b$  l  $3t$

m  $13m$  n  $-x$  o  $20z$

p  $10p$  q  $17g$  r  $3b$

s  $14t$  t  $-7j$  u  $-15l$

v  $8m$  w  $0$  x  $10t$

2 a  $10x - 2y$  b  $7x - 12$  c  $11 - 2f$

d  $6 - u$  e  $7m + 3p$  f  $4r - 5h$

g  $17a - 5b$  h  $9t - 2$  i  $17 - 3g$

j  $10m - 2n$  k  $7k - 12$  l  $4n - 9$

m  $12 - 2b$  n  $20 - 12h$  o  $9y - 2g - 6$

p  $11h - 8$  q  $4s - 2t$  r  $14l - 5m$

s  $3k - 13h + 7$  t  $5 - 4t$  u  $7g - 2$

v  $19f - 10k$

3 a  $3x^2$  b  $5y^2$  c  $4a^3$

d  $7d^2$  e  $-g^2$  f  $10y^3$

g  $7b^2$  h  $a^2$  i  $-g^2$

j  $4a^2 + 9$  k  $23x^2$  l  $11s^2 + 4$

m  $8a^2 + 5a$  n  $b^2 + 23b$  o  $t^2 - 4g - 7$

p  $8g^3 - g^2 + 22$  q  $18ab + 3$  r  $11xy$

s  $3fg + 3s$  t  $12ab - 5$  u  $20ab^2 - 14ac$

4 Check with your teacher.

5 Check with your teacher.

6 a  $14t$  b  $12t$  c  $13t$  d  $39t$

7 a  $2x + 15$  b  $9x + 4$  c  $15x$  d  $7.5x - 2$

### Challenge 8.2

$x = 3$

### 8.8 Multiplying and dividing expressions with variables

1 a  $12g$  b  $21h$  c  $24d$  d  $15z$

e  $30r$  f  $35t$  g  $12u$  h  $42p$

i  $21gy$  j  $22ht$  k  $24gx$  l  $70ah$

m  $36dm$  n  $15ch$  o  $18gx$  p  $12.5bt$

q  $156mn$  r  $72ad$  s  $6abc$  t  $12fgh$

u  $48wx$  v  $231abd$  w  $24xy$  x  $10.5xy$

y  $132qs$  z  $24abc$

2 a  $4f$  b  $2h$  c  $5x$  d  $3g$

e  $2r$  f  $2$  g  $2$  h  $2$

i  $\frac{2}{3}$  j  $\frac{1}{2}$  k  $\frac{6}{7}$  l  $\frac{2}{3}$

m  $\frac{1}{3}$  n  $\frac{1}{2}$  o  $\frac{2}{3}$  p  $\frac{1}{34}$

q  $9$  r  $\frac{5}{12}$  s  $\frac{16}{11}(1\frac{5}{11})$  t  $3$

3 a  $5fg$  b  $3cd$  c  $\frac{2xy}{3}$  d  $cg$

e  $y$  f  $\frac{p}{2}$  g  $\frac{3a}{4}$  h  $\frac{3d}{4}$

i  $5$  j  $\frac{5r}{7}$  k  $\frac{2y}{7}$  l  $\frac{4b}{3}$

m  $13y$  n  $\frac{2}{5}$  o  $2a$  p  $\frac{g}{2}$

q  $\frac{11m}{5}$  r  $\frac{1}{6}$  s  $\frac{2d}{5}$  t  $\frac{b}{7}$

4 a  $-15f$  b  $12d$  c  $-33ag$

d  $27gt$  e  $20dht$  f  $-18st$

g  $42dw$  h  $24abce$  i  $-33abf$

j  $18absx$  k  $-75hqt$  l  $144pw$

m  $-21abg$  n  $-51abgh$  o  $-49gh$

p  $-40hjk$  q  $112.5xy$  r  $-144prt$

s  $30abc$  t  $-408wx$  u  $-90abcdf$

5 a  $-\frac{a}{2}$  b  $-\frac{a}{3}$  c  $-12j$  d  $\frac{1}{2d}$

e  $-\frac{4}{5l}$  f  $-\frac{x}{4}$  g  $-\frac{6}{7}$  h  $\frac{f}{5j}$

i  $-\frac{2x}{3}$  j  $-\frac{1}{6}$  k  $-\frac{m}{4}$  l  $\frac{st}{2}$

m  $-2$  n  $\frac{b}{3}$  o  $-\frac{d}{5h}$  p  $-\frac{12}{11p}$

q  $\frac{14ef}{9}$  r  $-\frac{18x}{7}$  s  $\frac{3}{2r}$  t  $-\frac{11}{12t}$

6 a  $2a^2$  b  $25p^2$  c  $-30x^2$   
d  $7a^2b$  e  $6b^2cd$  f  $-160x^2y$   
g  $42p^2q^2$  h  $-30mn^2t^2$  i  $-18xy^2z^2$   
j  $-42abc^2$  k 0 l  $-18w^2x^2y^2z^2$   
m  $6a^{11}$  n  $2x^5y^3$  o  $10m^9$   
p  $\frac{5p^{10}q^5}{6}$  q  $\frac{4x^2yz^2}{3}$  r  $\frac{3b}{5}$

7 a  $\frac{6}{a^2}$  b  $\frac{10b^2}{3}$  c  $\frac{5}{w}$  d  $\frac{9k}{5}$  e  $\frac{3g}{5a}$   
f  $\frac{16h^2}{9d^2}$  g  $\frac{5t}{n}$  h  $\frac{-27h^2}{2g^2}$  i  $\frac{8y}{w}$  j  $6fz$

8 The pronumeral must be a common factor of every term of the numerator and every term of the denominator before it can be cancelled.

9 -5

10  $\frac{7}{6}$

11 a  $V_{\text{small container}} = ab^2$ ,  $V_{\text{large container}} = 4ab^2$  b 4 times

### 8.9 Expanding brackets

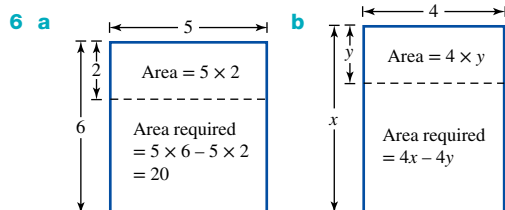
1 a  $3d + 12$  b  $2a + 10$   
c  $4x + 8$  d  $5r + 35$   
e  $6g + 36$  f  $2t - 6$   
g  $7d + 56$  h  $18x - 54$   
i  $48 + 12c$  j  $42 + 21x$   
k  $90g + 135$  l  $1.5t + 9$   
m  $11t - 22$  n  $6t - 18$   
o  $t^2 + 3t$  p  $x^2 + 4x$   
q  $g^2 + 7g$  r  $2g^2 + 10g$   
s  $3fg + 9f$  t  $6mn - 12m^2$

2 a  $9x - 6$  b  $3x^2 - 18xy$   
c  $15xy - 45y^2$  d  $100y - 250$   
e  $-3c - 9$  f  $-15x - 20$   
g  $-5x^2 - 30x$  h  $-12y - 2y^2$   
i  $-6t + 18$  j  $-20f + 8f^2$   
k  $27xy - 18x$  l  $-6bh + 18h^2$   
m  $20ab + 12ac$  n  $-6ag + 21a^2$   
o  $15ab + 30ac$  p  $-18w^2 + 10wz$   
q  $48m^2 + 120m$  r  $6k^2 - 15k$

3 a  $35x + 49$  b  $3c - 4$   
c  $22c - 2c^2$  d  $6v + 30$   
e  $5d^2 - 12d$  f  $11y + 12$   
g  $26r + r^2$  h  $9g - 37$   
i  $11f - 12g - 7$  j  $9x$   
k  $-5k - 10$  l  $18x + 6rx$   
m  $8r - 13$  n  $18gh - 24g$   
o  $11t + 1$  p  $24 + 11r - 11r^2$

4 a  $5x + 8$  b  $9x + 23$   
c  $6y + 26$  d  $d + 22$   
e  $14h$  f  $21m - 4$   
g  $28f - 1$  h  $4a - 3a^2 - 35$   
i  $6 - t^2 + 2t$  j  $7m$

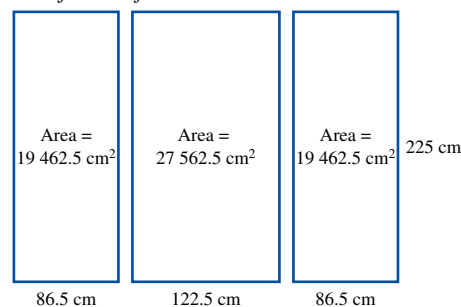
5 a  $10hk + 21h + 20k$  b  $15n - 6ny$   
c  $8gm + 24g - 18$  d  $18ab + 67b$   
e  $5a^2 - 35a - 35$  f  $11cf + 3c$   
g  $28x - 5xy - 29$  h  $37vw + 55v - 24$   
i  $6xy - 45x$  j  $77mn - 16m + 12n$



7 a  $x^2 + 3x + 2$  b  $a^2 + 7a + 12$   
c  $c^2 - c - 6$  d  $y^2 - 16$   
e  $u^2 - 5u + 6$  f  $k^2 - 7k + 10$

8 a  $\$50 - d$  b i Sale price ( $P$ ) =  $3(50 - d)$  ii  $P = 150 - 3d$   
c Amount of change ( $C$ ) =  $200 - (150 - 3d) = 50 + 3d$

9 a  $A_{\text{left}} = fm - 36f$ ;  $A_{\text{centre}} = fm$ ;  $A_{\text{right}} = fm - 36f$   
b  $A = 3fm - 72f$  c  $A = 3m^2 + 235.5m - 7380$



### 8.10 Factorising

1 a 2 b 3 c 6 d 13  
e 7 f 2 g 3 h 4

2 a  $2g$  b  $3m$  c 11 d  $2m$   
e  $2a$  f  $12g$  g  $2g$  h  $11l$   
i  $4mn$  j  $4c$  k  $4c$  l  $x$

3 a  $3(x + 2)$  b  $2(y + 2)$  c  $5(g + 2)$   
d  $4(2x + 3)$  e  $3(2f + 3)$  f  $4(3c + 5)$   
g  $2(d + 4)$  h  $2(x - 2)$  i  $6(2g - 3)$   
j  $11(h + 11)$  k  $4(s - 4)$  l  $4(2x - 5)$   
m  $12(g - 2)$  n  $2(7 - 2b)$  o  $16(a + 4)$   
p  $12(4 - q)$  q  $8(2 + f)$  r  $12(1 - d)$

4 a  $3(gh + 4)$  b  $2y(x + 3)$   
c  $4p(3q + 1)$  d  $7g(2 - h)$   
e  $2k(8j - 1)$  f  $2g(6e + 1)$   
g  $4(3k + 4)$  h  $m(7n + 6)$   
i  $7b(2a + 1)$  j  $5a(1 - 3bc)$   
k  $2r(4 + 7t)$  l  $12ab(2m + 1)$   
m  $2b(2 - 3a)$  n  $4g(3f - 4h)$   
o  $b(a - 2c)$  p  $7x(2 - 3y)$   
q  $k(11j + 3)$  r  $3p(1 + 9q)$   
s  $c(12a - 4 + 3d)$  t  $4(g + 2h - 4)$   
u  $14s(2 + t)$  v  $3v(5u + 9w)$

5  $2ab$   
6  $12a^3b^2c$   
7  $\frac{3}{5}$   
8  $\frac{3(x + 1)}{2(x - 1)}$   
9  $\frac{28}{3}$  or  $9\frac{1}{3}$   
10  $7y$   
11 a  $(3x - 6)(2x + 8)(ax - 5a)$  b  $\frac{4}{3a(x + 4)(x - 5)}$   
12 a  $6(x - 3)(x + 2)$  b  $x > 3$   
13 a  $\frac{3xy^3}{2z}$  b  $\frac{x(y - 1)^2}{18z(z - 1)}$  c  $\frac{xz^3}{15y^3}$  d  $\frac{4xz^3(x + 1)^2}{3y^2(y - 1)^3}$

### Investigation — Rich task

- Let  $l$  represent the number of long words and  $s$  represent the number of sentences. The Rix index =  $\frac{l}{s}$ .
- Six sentences and 19 long words
- 3.17
- Grade 8
- to 7 Check with your teacher.

### Code puzzle

Let's hope nothing develops.