

ICE-EM MATHEMATICS

Australian Curriculum Edition

Year **8**
Book 1



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Preface

ICE-EM Mathematics is a series of textbooks for students in years 5 to 10 throughout Australia who study the Australian Mathematics Curriculum.

Background

The International Centre of Excellence for Education in Mathematics (ICE-EM) was established in 2004 with the assistance of the Australian Government and is managed by the Australian Mathematical Sciences Institute (AMSI). The Centre originally published the series as part of a program to improve mathematics teaching and learning in Australia. AMSI is now collaborating with Cambridge University Press to publish Australian Curriculum editions of the series.

ICE-EM developed the program and textbooks in recognition of the importance of mathematics in modern society and the need to enhance the mathematical capabilities of Australian students. Students who use the series will have a strong foundation for work or further study.

You can read more about AMSI at www.amsi.org.au, and also see the mathematics involved in a variety of careers at www.mathscareers.org.au.

Features

ICE-EM Mathematics provides a progressive development from upper primary to middle secondary school. The year 10 textbooks incorporate all material for the 10A course, and selected topics in earlier books carefully prepare students for this. *ICE-EM Mathematics* is an excellent preparation for all of the Australian Curriculum's year 11 and 12 mathematics courses.

The writers of the series are some of Australia's most outstanding mathematics teachers and subject experts. The textbooks are clearly and carefully written, and contain background information, examples and worked problems.

Each chapter addresses a specific Australian Curriculum content strand and set of sub-strands. The exercises within chapters take an integrated approach to the concept of proficiency strands, rather than separating them out. Students are encouraged to develop and apply understanding, fluency, problem-solving and reasoning skills in every exercise.

The series places a strong emphasis on understanding basic ideas, along with mastering essential technical skills. Mental arithmetic and other mental processes are major focuses, as is the development of spatial intuition, logical reasoning and understanding of the concepts.

Problem-solving lies at the heart of mathematics, so *ICE-EM Mathematics* gives students a variety of different types of problems to work on, which help them develop their reasoning skills. Challenge exercises at the end of each chapter contain problems and investigations of varying difficulty that should catch the imagination and interest of students. The final chapter in each 7–10 textbook contains additional problems that cover new concepts for students who wish to explore the subject even further.

The problems and examples in the *ICE-EM Mathematics* series are written in a way that deliberately does not require the use of a calculator, except in appropriate contexts, until year 9. During primary and early secondary years, students need to become confident with mental and written calculations, using a variety of techniques. These skills are essential to students' mathematical development, and lead to a feeling of confidence and mathematical self-reliance. Furthermore, as different states have varying requirements and expectations of calculator use, the series is designed to be as calculator neutral as possible.

Additional resources

HOTmaths

Cambridge HOTmaths provides an integrated program for users of the ICE-EM Mathematics series, combining the best of textbook and interactive online resources. The presence of a HOTmaths icon (shown at the right) in the header of a chapter topic shows that resources are available for that topic. Materials are accessible from a dropdown menu in HOTmaths, organised by textbook chapter and topic/lesson structure. For more information, see www.hotmaths.com.au.



TIMES and SAM modules

The TIMES and SAM web resources were developed by the ICE-EM Mathematics author team at AMSI and are written around the structure of the Australian Curriculum. These resources have been mapped against your ICE-EM Mathematics book and are available to teachers and students by clicking on the icons in the margins in the PDF or app version of the book.

The **TIMES** (The Improving Mathematics Education in Schools) modules are written for teachers. Each module contains an in-depth discussion of a component of the curriculum along with exercises, answers, screencasts of worked examples, and notes on historical context and links forward. The **SAM** (Supporting Australian Mathematics) modules are a collection of succinct, easy-to-navigate lessons written for students in Years 7 to 9 and their teachers. Each lesson is based on an Australian Curriculum topic and includes examples and short quizzes.

Guide to the icons

The blue and green icons in the margins of this book are based on the original AMSI logo:



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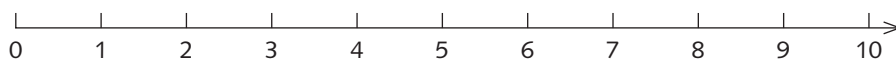
1

Number and Algebra

Whole numbers

This chapter is a brief review of whole numbers. Whole numbers, also known as the **counting numbers**, are the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ...

The whole numbers can be visualised as points equally spaced on the number line.



We can add, subtract, multiply, divide and take powers of numbers. Performing calculations using these operations can be done mentally using a variety of strategies, or by using written algorithms.

1A

Addition and subtraction of whole numbers



We begin this chapter with a review of addition and subtraction of whole numbers.

Addition

Any two numbers can be added. When two or more whole numbers are added, the result is called the **sum**.

The order in which we add two or more whole numbers does not matter. Adding them in any order will give the same answer.

Subtraction

Subtraction gives the difference between two numbers. For example, the difference of 8 and 5 is 3, and this is written $8 - 5 = 3$.

We can think of subtraction as:

- taking away one number from another. For example: I had eight tennis balls, but I gave five away, so now I only have three tennis balls.
- finding what we have to add to one number to get the other. For example: I have eight tennis balls and she has five. I have to add 3 to 5 to get 8, so the difference is 3.

Example 1

Calculate these sums mentally by adding the numbers in a more convenient order.

a $29 + 60 + 21$

b $33 + 42 + 67$

Solution

$$\begin{aligned}\mathbf{a} \quad 29 + 60 + 21 &= 29 + 21 + 60 \\ &= 50 + 60 \\ &= 110\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 33 + 42 + 67 &= 33 + 67 + 42 \\ &= 100 + 42 \\ &= 142\end{aligned}$$

Example 2

Calculate these differences mentally.

a $45 - 27$

b $136 - 77$

Solution

$$\begin{aligned} \text{a } 45 - 27 &= 25 - 7 \\ &= 18 \end{aligned}$$

or

$$\begin{aligned} 45 - 27 &= 48 - 30 \\ &= 18 \end{aligned}$$

or

$$\begin{aligned} 27 + 3 + 15 &= 45 \\ \text{Therefore } 45 - 27 &= 18. \end{aligned}$$

$$\begin{aligned} \text{b } 136 - 77 &= 139 - 80 \\ &= 59 \end{aligned}$$

or

$$\begin{aligned} 77 + 23 + 36 &= 136 \\ \text{Therefore } 136 - 77 &= 23 + 36 \\ &= 59. \end{aligned}$$

Note: Other strategies are possible.

The standard algorithms

Example 3

The numbers of students and staff who attended the local swimming carnival from five schools were 1023, 896, 704, 199 and 1189. How many people attended in total?

Solution

$$\begin{array}{r} 1023 \\ 896 \\ 704 \\ 199 \\ + 1189 \\ \hline 4011 \end{array}$$

4011 people attended the swimming carnival.

There are two standard algorithms for subtraction. It is easier to check your calculations if you use Method 1 in Example 4.

Example 4

Of the 4011 people who attended the swimming carnival, 2118 left by the end of lunchtime. How many were present for the start of the afternoon races?

(continued on next page)

Solution

Method 1

$$\begin{array}{r} 4 \overset{1}{0} \overset{1}{1} \overset{1}{1} \\ - 2 \overset{1}{1} \overset{1}{1} 8 \\ \hline 1 \ 8 \ 9 \ 3 \end{array}$$

Method 2

$$\begin{array}{r} \overset{9}{3} \overset{10}{4} \overset{1}{0} \overset{1}{1} \overset{1}{1} \\ - 2 \ 1 \ 1 \ 8 \\ \hline 1 \ 8 \ 9 \ 3 \end{array}$$

1893 people were at the swimming carnival for the start of the afternoon races.
A subtraction can always be checked by addition. In this case, $1893 + 2118 = 4011$.



Addition and subtraction

- The **any-order property for addition** states that a list of numbers can be added together in any order to give the sum of the numbers.
- The any-order property summarises the commutative and associative laws for addition.
- Numbers in addition and subtraction calculations can often be rearranged to make mental calculations simpler.
- Written algorithms for addition and subtraction are useful when the numbers are large.
- Subtraction is the reverse of addition. For example, $8 + 5 = 13$ and $13 - 5 = 8$.

The commutative and associative laws of addition say, for example, that $3 + 4 = 4 + 3$ and $(3 + 4) + 5 = 3 + (4 + 5)$.

Exercise 1A

Example 1

- 1 Calculate these sums mentally by adding the numbers in a more convenient order.

a $19 + 48 + 11$

b $28 + 53 + 22$

c $15 + 57 + 85$

d $37 + 88 + 13$

e $22 + 24 + 26 + 28$

f $31 + 33 + 37 + 39$

- 2 Calculate these sums mentally by adding the numbers in stages.

a $25 + 36$

b $37 + 89$

c $54 + 27$

d $189 + 34$

e $12 + 16 + 15 + 22$

f $23 + 24 + 28 + 21$

Example 2

- 3 Calculate these differences mentally, either by subtracting in stages or by adding in stages.

a $35 - 17$

b $53 - 27$

c $84 - 36$

d $96 - 28$

e $134 - 76$

f $252 - 177$

4 Calculate:

a $36 + 130 + 1644$

b $910 + 49 + 8371$

c $9221 + 839 + 65$

d $324 + 538 + 718$

e $229 + 1038 + 432$

f $251 + 489 + 12 + 37$

g $82 + 374 + 392 + 28$

h $125 + 561 + 351 + 1442$

i $255 + 195 + 582 + 55$

5 Carry out these subtractions.

a $762 - 387$

b $1921 - 428$

c $1405 - 386$

d $2764 - 1689$

e $918 - 579$

f $1297 - 399$

g $1089 - 137$

h $45\,678 - 34\,898$

Example 3

6 The numbers of people living in five apartment blocks in the city are 1098, 956, 423, 156 and 42. How many people live in these apartment blocks in total?

Example 4

7 In 1975, the population of Hobart was 217 135. In 1985, the population was 231 135. What was the population increase from 1975 to 1985?

8 There are 3047 cats living in Brownville and 6857 in Wugtown. What is the combined cat population of the two towns?

9 In 1860, the population of the Maitland shire was 1998. By 2001, the population had increased to 53 803. By how many people had the population increased in that time?

10 A furniture store ordered 234 tables, and 587 more chairs than tables. How many chairs were ordered? How many items were ordered in total?

11 In 2003, the number of ACT residents under the age of 15 was 63 900. If the total population was 322 800, how many people aged 15 and over lived in the ACT in that year?

12 The Mitchell Dam holds 496 709 000 litres of water. The Gaudry Weir holds 278 700 900 litres. If both water storage facilities are filled to capacity, how much water do they hold in total? What is the difference, in litres, between the capacities of the two facilities?

13 Doug is 1708 mm tall, Colin is 1639 mm tall and Peter is 1836 mm tall. How much taller is:

a Doug than Colin?

b Peter than Colin?

c Peter than Doug?

14 In 2004, there were 5938 students in state primary schools and 4133 students in state secondary schools in Toowoomba. What was the total number of students in state primary and secondary schools in that year?

15 In 1898, the population of the German Empire was 45 234 061. If 28 318 592 were Lutherans, 124 567 belonged to other protestant denominations, 561 612 were Jewish and the remainder Roman Catholic, how many Roman Catholics were there?

16 In 1891, the population of London was 4 211 743. The counties of Durham, Gloucestershire and Devonshire had populations of 1 024 369, 548 886 and 636 225, respectively. By how many did the population of London exceed the total population of these three counties?

17 A box contained 3487 coloured balls, of which 546 were white, 1364 were red, 987 were blue and the rest were black. How many black balls were there?

1B Multiplication and division of whole numbers



Multiplication and division are useful arithmetic operations. We use them all the time in everyday life. Repeated addition can be completed as a multiplication.

For example, $3 \times 5 = 5 + 5 + 5$.

Multiplication

Any two whole numbers can be multiplied together. The result is called the **product** of the numbers.

For example, the product of 15 and 13 is $15 \times 13 = 195$.

A list of whole numbers can be multiplied, two at a time, in any order, and the result will always be the same.

The following strategies are useful when doing multiplication mentally.

- Grouping numbers together in ways that will make the calculations easier.

Example 5

Find the product $8 \times 5 \times 7 \times 2$.

Solution

$$\begin{aligned} 8 \times 5 \times 7 \times 2 &= 8 \times 7 \times 10 && \text{(Multiply } 5 \times 2 = 10 \text{ first.)} \\ &= 560 \end{aligned}$$

- Using the distributive law. For example, $3 \times 23 = 3 \times (20 + 3) = 60 + 9 = 69$

Example 6

Calculate **a** 21×36 . **b** 28×36

Solution

$$\begin{aligned} \text{a } 21 \times 36 &= 20 \times 36 + 1 \times 36 && \text{b } 28 \times 36 = 30 \times 36 - 2 \times 36 \\ &= 720 + 36 && &= 1080 - 72 \\ &= 756 && &= 1008 \end{aligned}$$

There are other possible methods.

The **multiplication algorithm** is used for larger numbers.

Example 7

A brick wall is to have 37 rows, with 128 bricks in each row.
How many bricks are needed?

Solution

$$\begin{array}{r} 128 \\ \times 37 \\ \hline 896 \\ 3840 \\ \hline 4736 \end{array}$$

(Multiply 128 by 7.)
(Multiply 128 by 30.)

4736 bricks are required.



Multiplication of whole numbers

- The **any-order property for multiplication** states that a list of numbers can be multiplied together, in any order, to give the product of the numbers.
- The any-order property summarises the commutative and associative laws for multiplication.
- Multiplication is **distributive** over addition and subtraction:
 $(20 + 1) \times 36 = 20 \times 36 + 1 \times 36 = 720 + 36 = 756$
 $(30 - 2) \times 36 = 30 \times 36 - 2 \times 36 = 1080 - 72 = 1008$
- When multiplying mentally, it is often effective to:
 - take the factors in a different order
 - use the distributive law.
- Larger numbers can be multiplied using the multiplication algorithm.
- It is important to know your multiplication tables.

The commutative and associative laws of multiplication say, for example, that $3 \times 4 = 4 \times 3$ and $(3 \times 4) \times 5 = 3 \times (4 \times 5)$.

Division

Any whole number can be divided by any non-zero whole number, called the **divisor**, to give a **quotient** and a **remainder**.

Division without remainder is the reverse process of multiplication. For example, $30 \div 5 = 6$ is the reverse of $30 = 6 \times 5$. Six is the quotient and the remainder is 0.

Often, we do not have exact division. For example, 5 does not divide exactly into 32. We write $32 \div 5 = 6$ remainder 2. This is another way of writing $32 = 6 \times 5 + 2$.

There are two quite different ways to interpret a division such as ' $32 \div 5 = 6$ remainder 2'.

- 'Divide 32 people into groups of 5. There will be 6 groups, with 2 people left over.'
- 'Divide 32 people into 5 equal groups. Each group will have 6 people, and there will be 2 people left over.'

Example 8

53 chocolates are put into boxes of 12. How many boxes are filled, and how many chocolates are left over?

Solution

$$53 \div 12 = 4 \text{ remainder } 2$$

$$\text{That is, } 53 = 4 \times 12 + 5$$

We can fill 4 boxes with 5 chocolates left over.

There are several useful strategies for mental division.

Example 9

a Calculate $864 \div 8$ mentally.

b Calculate $752 \div 8$ mentally.

Solution

a Split 864 into $800 + 64$.

$$\begin{aligned} 864 \div 8 &= 800 \div 8 + 64 \div 8 \\ &= 100 + 8 \\ &= 108 \end{aligned}$$

Divide successively by 2

$$\begin{aligned} \text{or } 864 \div 8 &= 432 \div 4 \\ &= 216 \div 2 \\ &= 108 \end{aligned}$$

b Split 752 into $800 - 48$.

$$\begin{aligned} 752 \div 8 &= 800 \div 8 - 48 \div 8 \\ &= 100 - 6 \\ &= 94 \end{aligned}$$

$$\begin{aligned} \text{or } 752 \div 8 &= 376 \div 4 \\ &= 188 \div 2 \\ &= 94 \end{aligned}$$

The short division algorithm

The short division algorithm is effective when the divisor is small.

Example 10

- a** The humbug-making machine produced 2592 humbugs in an afternoon, packaged into bags of 8. How many bags were produced?
- b** The next morning, the humbug-making machine produced 2845 humbugs. Calculate the number of bags of 8 humbugs produced.

Solution

- a** In this case the divisor is 8, so the short division algorithm can be used.

$$\begin{array}{r} 324 \\ 8 \overline{) 2592} \end{array} \quad \text{324 bags of humbugs were produced that afternoon.}$$

- b** $\begin{array}{r} 355 \\ 8 \overline{) 2845} \end{array}$ remainder 5

355 bags of 8 humbugs were made, with 5 humbugs left over.

Fraction notation for division

We can also write a division as a fraction.

For example, $864 \div 8 = 108$ can be written as $\frac{864}{8} = 108$.

Other examples using this notation are

$$\frac{0}{3} = 0 \quad \text{and} \quad \frac{108}{12} = 9.$$

When there is a remainder, we can write the answer using a mixed numeral,

$$\frac{27}{8} = 3\frac{3}{8}.$$

In the next chapter we will study calculations such as $8 \times 3\frac{3}{8} = 27$.

The long division algorithm

When the divisor is large, we use the **long division algorithm**.

Example 11

On Fridays, the humbugs produced by the humbug-making machine are packed into mega-bags of 36 humbugs. One Friday, 8318 humbugs were produced. How many mega-bags were produced?

Solution

$$\begin{array}{r}
 231 \text{ remainder } 2 \\
 36 \overline{) 8318} \\
 \underline{72} \\
 111 \text{ (Bring down 1.)} \\
 \underline{108} \\
 38 \text{ (Bring down 8.)} \\
 \underline{36} \\
 2
 \end{array}$$

We use : $36 \times 1 = 36$
 $36 \times 2 = 72$
 $36 \times 3 = 108$

To start, consider $83 \div 36$:

$36 \times 2 = 72$ and $36 \times 3 = 108$

The 2 is placed above the line, and multiplied by 36 to give 72, which is subtracted from 83 to give the correct 'carry'.

Thus $8318 \div 36 = 231$ remainder 2,
 so 231 mega-bags were produced, with 2 humbugs left over.

Example 12

Find $9617 \div 27$.

Solution

It is useful to begin by writing down at least some of the multiples of 27 to the right of the calculation.

$$\begin{array}{r}
 356 \\
 27 \overline{) 9617} \\
 \underline{81} \\
 151 \\
 \underline{135} \\
 167 \\
 \underline{162} \\
 5
 \end{array}$$

$27 \times 1 = 27$
 $27 \times 2 = 54$
 $27 \times 3 = 81$
 $27 \times 4 = 108$
 $27 \times 5 = 135$
 $27 \times 6 = 162$
 $27 \times 7 = 189$
 $27 \times 8 = 216$
 $27 \times 9 = 243$

$9617 \div 27 = 356$ remainder 5.



Division of whole numbers

- Any whole number can be divided by any non-zero whole number to give a quotient and remainder.
- A division such as $30 \div 5 = 6$ can be interpreted in two ways:
 - ‘dividing 30 people into 5 equal groups’
 - ‘dividing 30 people into groups of 5’
- Division is **distributive** over addition and subtraction.

For example:

$$(800 + 64) \div 8 = 800 \div 8 + 64 \div 8 = 100 + 8 = 108$$

$$(800 - 48) \div 8 = 800 \div 8 - 48 \div 8 = 100 - 6 = 94$$

- A division statement such as $30 \div 7 = 4$ remainder 2 is equivalent to the multiplication and addition statement $30 = 7 \times 4 + 2$.
- Every multiplication statement is equivalent to a division statement.
- To perform divisions, you must know your multiplication tables.
- When performing division mentally, the distributive law is often useful.

For example:

$$\begin{aligned} 330 \div 15 &= (300 + 30) \div 15 \\ &= 20 + 2 \\ &= 22 \end{aligned}$$

- Larger numbers can be divided using the long division algorithm.



Exercise 1B

Example 5

- 1 Calculate these products mentally by multiplying the numbers in a more convenient order.

a $7 \times 5 \times 9 \times 2$

b $12 \times 25 \times 11 \times 4$

c $15 \times 9 \times 4$

d $35 \times 11 \times 2$

Example 6

- 2 Calculate these products mentally by using the fact that one factor is close to a multiple of 10.

a 21×33

b 18×28

c 42×16

d 49×34

Example 9

- 3 Calculate these divisions mentally.

a $749 \div 7$

b $352 \div 4$

c $1272 \div 12$

d $2964 \div 6$

Example 10

- 4 Calculate the quotient and remainder in each division.

a $165 \div 7$

b $441 \div 9$

c $800 \div 12$

d $1244 \div 11$

- 5 Carry out each calculation using the multiplication algorithm.

a 353×14

b 572×29

c 79×186

d 612×49

e 432×88

f 895×35

g 135×36

h 936×24

i 506×56

j 506×66

- 6 Use short division to calculate:

a $556 \div 2$

b $540 \div 4$

c $8624 \div 8$

d $498 \div 6$

e $4050 \div 6$

f $1496 \div 11$

g $78\,093 \div 9$

h $39\,240 \div 9$

i $6005 \div 7$

j $9756 \div 12$

- 7 Use the long division algorithm to calculate:

a $828 \div 13$

b $1050 \div 16$

c $2344 \div 16$

d $8554 \div 17$

e $2806 \div 23$

f $4042 \div 19$

g $1498 \div 18$

h $4708 \div 19$

- 8 The district sports carnival allows teams of no more than 87. If 128 schools each send a team of the maximum size, how many students take part?
- 9 A school hall has 30 rows of seats. Each row has 28 seats. How many seats are there?
- 10 An apartment block has 44 floors. Each floor has 18 apartments. How many apartments are there in the block?
- 11 A car park has 34 rows and each row has 42 parking spaces. How many cars can be parked?
- 12 Pete planted 80 rows of 30 tomato plants. If each plant produced 43 tomatoes, what was the total crop?
- 13 An office building has 15 floors, with 23 rooms and 4 corridors on each floor. If each room has 7 lights and each corridor has 15 lights, what is the total number of lights in the building?
- 14 In three hours, Rory and Sarah counted all the cars that passed by their house. In the first hour they counted 192 cars, in the second they counted 234 cars, and in the third they counted 203 cars. If each car carried five people, how many people passed by Rory and Sarah's house in a car during that three-hour period?
- 15 Tara is comparing mobile phone plans. She uses her phone for SMS messages only. Plan A costs \$33.00 per month and includes a maximum of 150 SMS messages. Plan B has no monthly charge but charges 24c for each SMS. Which plan would have been cheaper last month, when she sent 149 SMSs?
- 16 The average distance between Mercury and Venus is 51 078 090 km. What is the average distance between Mars and Jupiter if it is 11 times the average distance between Mercury and Venus?

- 17** Tony is offered two options for buying a particular car. He can pay the marked price of \$52 800 in cash. Alternatively, he can pay a \$10 000 deposit and then pay instalments of \$1805 a month for 24 months. How much more would he pay for the car on the instalment plan?
- 18** A greengrocer bought a sack of potatoes weighing 51 kg. He divided the potatoes into bags so that each bag held 3 kg of potatoes. How many bags of potatoes did he get from his sack?
- 19** A humbug-making machine produced 12 429 humbugs in three days.
- a** How many bags of 8 is this? How many are left over?
- b** How many mega-bags of 36 is this? How many are left over?
- 20** A calendar is to be invented for a newly settled planet, which has 2304 days in its year. The designer has to split the days up into weeks. How many weeks will there be if he uses weeks of:
- a** 6 days **b** 8 days **c** 12 days
- d** If he makes each month have 144 days, how many weeks will there be in a month for each of parts **a–c**?
- 21** A family were stuck in their house after a serious storm. During the storm, the clock was damaged so that only the hour hand worked. The family stayed inside the house for 636 hours after the storm. How many times did the hour hand go around the clock before they left?



Review exercise

1 Calculate these sums mentally.

a $38 + 22$

b $32 + 25$

c $35 + 27$

d $42 + 19$

e $13 + 17$

f $8 + 89$

g $14 + 76$

h $29 + 46$

i $84 + 6$

j $32 + 9$

k $11 + 9 + 33$

l $2 + 15 + 38$

m $61 + 24 + 9$

n $4 + 42 + 36$

o $27 + 6 + 3$

p $16 + 24 + 5$

q $16 + 55 + 24 + 45$ **r** $72 + 19 + 28 + 21$ **s** $22 + 17 + 18 + 23$ **t** $23 + 37 + 64 + 6$

u $15 + 64 + 26 + 45$

2 Carry out these subtractions mentally.

a $31 - 9$

b $32 - 28$

c $94 - 66$

d $56 - 48$

e $521 - 428$

f $405 - 386$

g $764 - 689$

h $618 - 509$

i $813 - 619$

j $754 - 749$

k $328 - 68$

l $462 - 387$

3 Carry out these multiplications mentally.

a $25 \times 4 \times 9$

b $50 \times 78 \times 2$

c $1 \times 35 \times 20$

d $3 \times 7 \times 4 \times 5$

e $5 \times 76 \times 2$

f $8 \times 5 \times 40$

g $6 \times 5 \times 10 \times 2$

h $5 \times 47 \times 20$

i $13 \times 6 \times 0$

4 Lake Superior, the largest freshwater lake in the world, has an area of $82\,732 \text{ km}^2$. The second largest, Lake Victoria, has an area of $69\,753 \text{ km}^2$. How much larger is Lake Superior?

5 Copy and complete the calculations by finding a digit for each *.

$$\begin{array}{r} \text{a} \quad 2 \ 3 \ 1 \\ \quad 2 \ 6 \ * \\ + \ 1 \ * \ 6 \\ \hline \quad * \ 3 \ 1 \end{array}$$

$$\begin{array}{r} \text{b} \quad \quad * \ * \ * \\ \quad \quad 6 \ 3 \ 7 \\ + \quad 5 \ 6 \ 7 \\ \hline \quad * \ 0 \ 4 \ 7 \end{array}$$

$$\begin{array}{r} \text{c} \quad \quad * \ 7 \ * \\ \quad \times \quad \quad 9 \\ \hline \quad 3 \ 3 \ * \ 4 \end{array}$$

- 6 There are 3257 dogs in Snugsville and 2673 in Cudstown. What is the combined dog population of the two towns?
- 7 A school has 2137 students. The number of boys at the school is 1586. How many girls are there at the school?
- 8 Five hundred and seventy-six chocolates are to be divided among 24 people. How many chocolates will each person receive?
- 9 A youth club has 80 members. There are 10 more boys than girls. How many boys are there?
- 10 4000 apples are packed into boxes, each box holding 75 apples. How many boxes are required?
- 11 A club started the year with 82 members. During the year, 36 people left and 28 people joined. How many people belonged to the club at the end of the year?
- 12 A fruit grower sold 780 boxes of oranges. On average, the boxes weigh 21 kg each. What was the total weight (in kg) of the boxes of oranges?
- 13 An orchard of 60 hectares (ha) has 2100 trees. How many trees are there on each hectare (on average)?

Challenge exercise



- 1 The towns Thomas, Callaghan, Barker, Dixon and Evans are located on a straight highway in that order. The distance from Thomas to Evans is 20 km. The distance from Thomas to Dixon is 15 km. The distance from Callaghan to Evans is 10 km. Barker is halfway between Callaghan and Dixon. What is the distance from Callaghan to Barker?
- 2 Using each of the digits 1 to 9 only once, write them in such a way that you produce two numbers that add up to 99 999.
- 3 A family had a collection of animals. When the council asked them how many birds and how many beasts they had, they answered, 'Well, we have 36 heads and 100 feet in total.' How many birds and how many beasts did the family have?
- 4 Take a number that does not contain zero and in which all the digits are different.
 - Double the number and then add 4.
 - Multiply by 5 and then add 12.
 - Multiply by 10 and then subtract 320.
 - Cross out the zeroes in your answer.

What number do you get? Explain your answer.

- 5
 - a What is the smallest number that must be subtracted from 5762, so that the result is exactly divisible by 19?
 - b What number divided by 367 will give 59 as the quotient and 126 as the remainder?
 - c Divide 931 into two parts such that one part is greater than the other by 127.
- 6
 - a The average of 7 numbers is 36. An eighth number is chosen so that the average is now 45. What is the eighth number?
 - b The average of 7 numbers is 17 and the average of four of them is 14. What is the average of the other three numbers?
 - c The average of 6 numbers is 29 and the average of four other numbers is 44. What is the average of the 10 numbers?
 - d A batsman had an average of 38 runs for 5 innings. If he scored 62 in his next innings, what was then his average?

Division by factors

- 7
 - a Divide 263 052 by 36 by first dividing by 9 and then by 4.
 - b Divide 56 133 by 231 by first dividing by 3, then by 7 and finally by 11 ($231 = 3 \times 7 \times 11$).



8 Consider the following division using factors: $69\,237 \div 45$:

$$\begin{array}{r|l} 5 & 6\,9\,2\,3\,7 \\ \hline 9 & 1\,3\,8\,4\,7 \quad \text{remainder } 2 \\ & 1\,5\,3\,8 \quad \text{remainder } 5 \end{array}$$

Quotient = 1538

$$\begin{aligned} \text{Remainder} &= 5 \times 5 + 2 \\ &= 27 \end{aligned}$$

Use this method to divide 69 237 by 231.



CHAPTER

2

Number and Algebra

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Fractions and decimals

In Chapter 1 we reviewed the whole numbers. This chapter is a review of fractions and decimals.

Fractions have been used since ancient times. For instance, dividing a harvest into equal parts and distributing different amounts to different families would often have involved fractions.

Fractions were used by the ancient Egyptians and Babylonians. Systematic use of decimal fractions did not appear until the sixteenth century. Decimals are now a central part of day-to-day calculations in almost every walk of life.

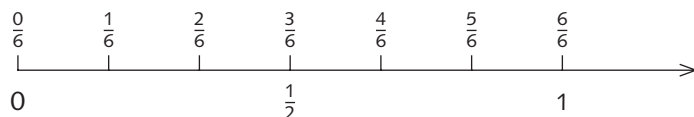
2A Equivalent fractions and simplest form



Equivalent fractions

Two fractions are **equivalent** if they are represented by the same point on the number line. We regard equivalent fractions as being equal.

The markers $\frac{3}{6}$ and $\frac{1}{2}$ are represented by the same point on the number line below, so they are equivalent.



This is simply the statement that, starting at 0, taking one step of length $\frac{1}{2}$ and taking 3 steps of length $\frac{1}{6}$ both get us to the same point.

$$\begin{aligned}\frac{1}{2} &= \frac{1 \times 3}{2 \times 3} \\ &= \frac{3}{6}\end{aligned}$$

To form equivalent fractions, start with a fraction and either:

- multiply its numerator and denominator by the same non-zero whole number.

For example:

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

- divide the numerator and denominator by the same common factor.

For example:

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

Simplest form and cancelling

A fraction is in **simplest form** or **lowest terms** if the numerator and denominator have no common factor other than 1.

For example, the fraction $\frac{5}{12}$ is in simplest form because the highest common factor of 5 and 12 is 1.

However, $\frac{9}{12}$ is not in simplest form since 9 and 12 have 3 as a common factor.

Indeed $\frac{9}{12} = \frac{3}{4}$.

Example 1

Simplify:

a $\frac{36}{45}$

b $\frac{98}{21}$

Solution

$$\begin{aligned}\text{a } \frac{36}{45} &= \frac{\cancel{36}^4}{\cancel{45}_5} \text{ (dividing each by 9)} \\ &= \frac{4}{5}\end{aligned}$$

$$\begin{aligned}\text{b } \frac{98}{21} &= \frac{\cancel{98}^{14}}{\cancel{21}_3} \text{ (dividing each by 7)} \\ &= \frac{14}{3} \\ &= 4\frac{2}{3}\end{aligned}$$

Note: Two fractions are equivalent if they have the same simplest form.

Example 2

Write 'yes' or 'no' to indicate whether the fractions in each pair are equivalent.

a Is $\frac{3}{4}$ equivalent to $\frac{45}{60}$?

b Is $\frac{5}{9}$ equivalent to $\frac{70}{108}$?

c Is $\frac{99}{100}$ equivalent to $\frac{9}{10}$?

d Is $\frac{9}{27}$ equivalent to $\frac{33}{99}$?

e Is $\frac{18}{99}$ equivalent to $\frac{60}{330}$?

Solution

a Yes. (Multiply numerator and denominator of $\frac{3}{4}$ by 15.)

b No. ($\frac{5}{9}$ is in simplest form. The simplest form of $\frac{70}{108}$ is $\frac{35}{54}$.)

c No. (Both fractions are in simplest form and they are different.)

d Yes. (The simplest form of both fractions is $\frac{1}{3}$.)

e Yes. (The simplest form of both fractions is $\frac{2}{11}$.)



Equivalent fractions and simplest form

- Two fractions are said to be **equivalent** if they are represented by the same point on a number line. Equivalent fractions are equal.
- We say that a fraction is in **simplest form** if the numerator and denominator have no common factor other than 1.
- We can test when two fractions are **equivalent** by seeing if they have the same simplest form.
- Starting with a given fraction, the fractions obtained by multiplying its numerator and its denominator by the same non-zero whole number are equivalent to it.
- Starting with a given fraction, the fractions obtained by dividing its numerator and its denominator by the same common factor are equivalent to it.

Comparison of fractions: which is larger?

If two or more fractions have the same denominator, then the one with the larger numerator is the larger fraction.

Equivalent fractions are used to compare fractions when the denominators of the fractions given are different. First find equivalent fractions with the same denominator for the given fractions, and then compare the numerators.

Example 3

Order the following fractions from smallest to largest:

$$\frac{3}{10}, \frac{3}{4}, \frac{1}{2}, \frac{7}{20} \text{ and } \frac{3}{5}.$$

Solution

The LCM of the denominators 2, 4, 5, 10 and 20 is 20.

$$\frac{3}{10} = \frac{6}{20}, \frac{3}{4} = \frac{15}{20}, \frac{1}{2} = \frac{10}{20}, \frac{7}{20} = \frac{7}{20}, \frac{3}{5} = \frac{12}{20}$$

$$\text{Writing these in order: } \frac{6}{20}, \frac{7}{20}, \frac{10}{20}, \frac{12}{20}, \frac{15}{20}$$

$$\text{The order is } \frac{3}{10} < \frac{7}{20} < \frac{1}{2} < \frac{3}{5} < \frac{3}{4}.$$



Exercise 2A

Example 1

1 Write each fraction in simplest form.

a $\frac{10}{15}$

b $\frac{17}{34}$

c $\frac{18}{21}$

d $\frac{20}{24}$

e $\frac{45}{35}$

f $\frac{56}{14}$

g $\frac{105}{147}$

h $\frac{84}{224}$

i $\frac{20}{25}$

j $\frac{24}{36}$

k $\frac{14}{21}$

l $\frac{24}{32}$

m $\frac{45}{36}$

n $\frac{56}{42}$

o $\frac{105}{84}$

p $\frac{112}{48}$

Example 2

2 Test whether the fractions in each pair are equivalent.

a $\frac{2}{3}, \frac{10}{15}$

b $\frac{2}{3}, \frac{12}{21}$

c $\frac{2}{3}, \frac{22}{33}$

d $\frac{7}{8}, \frac{56}{64}$

e $\frac{7}{8}, \frac{49}{64}$

f $\frac{3}{5}, \frac{18}{30}$

Example 3

3 Write the six fractions with a common denominator, and hence order them from smallest to largest.

a $\frac{11}{10}, \frac{2}{5}, \frac{5}{4}, \frac{9}{10}, \frac{3}{4}, \frac{4}{5}$

b $\frac{5}{6}, \frac{4}{5}, \frac{7}{10}, \frac{3}{4}, \frac{11}{15}, \frac{23}{30}$

4 Rewrite these fractions with denominator 63.

$\frac{1}{3}, \frac{2}{3}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{1}{9}, \frac{2}{9}, \frac{5}{9}, \frac{8}{21}$

5 Arrange the fractions from smallest to largest.

a $\frac{7}{8}, \frac{2}{3}, \frac{5}{6}$

b $\frac{11}{16}, \frac{2}{3}, \frac{3}{4}$

c $\frac{11}{12}, \frac{31}{45}, \frac{2}{3}$

d $\frac{7}{12}, \frac{5}{8}, \frac{2}{3}$

e $\frac{25}{54}, \frac{1}{2}, \frac{5}{9}$

f $\frac{31}{48}, \frac{2}{3}, \frac{11}{16}$



A **proper fraction** is greater than or equal to 0 and less than 1. For example, $\frac{2}{7}$ and $\frac{3}{8}$.

A **mixed numeral** is a whole number plus a proper fraction. For example, $1\frac{1}{3}$ and $2\frac{2}{5}$ are mixed numerals.

An **improper fraction** is a fraction whose numerator is greater than or equal to the denominator, so that it is greater than or equal to 1. For example, $\frac{9}{8}$ and $\frac{13}{7}$.

An improper fraction can be written as a mixed numeral or a whole number. In general, improper fractions should be written as mixed numerals with the fractional parts in simplest form. For example:

$$\frac{20}{6} = 3\frac{1}{3}$$

To add and subtract fractions with the same denominator, simply add or subtract the numerators.

Example 4

Evaluate:

a $\frac{18}{19} + \frac{13}{19}$

b $1\frac{1}{6} + 2\frac{5}{6}$

c $\frac{5}{16} + \frac{11}{16} + \frac{7}{16}$

Solution

a $\frac{18}{19} + \frac{13}{19} = \frac{31}{19}$
 $= 1\frac{12}{19}$

b $1\frac{1}{6} + 2\frac{5}{6} = 1 + 2 + \frac{1}{6} + \frac{5}{6}$
 $= 3 + \frac{6}{6}$
 $= 4$

c $\frac{5}{16} + \frac{11}{16} + \frac{7}{16} = \frac{23}{16}$
 $= 1\frac{7}{16}$

Example 5

Evaluate:

a $\frac{12}{13} - \frac{5}{13}$

b $1\frac{8}{19} - \frac{9}{19}$

Solution

a $\frac{12}{13} - \frac{5}{13} = \frac{7}{13}$

b $1\frac{8}{19} - \frac{9}{19} = \frac{27}{19} - \frac{9}{19}$
 $= \frac{18}{19}$

If the denominators are different, use the lowest common multiple of the denominators to find equivalent fractions.

Example 6

Evaluate $\frac{2}{3} + \frac{5}{8}$.

Solution

$$\begin{aligned}\frac{2}{3} + \frac{5}{8} &= \frac{16}{24} + \frac{15}{24} \quad (\text{The LCM of 3 and 8 is 24.}) \\ &= \frac{31}{24} \\ &= 1\frac{7}{24}\end{aligned}$$

Example 7

Evaluate $\frac{5}{9} - \frac{1}{6}$.

Solution

$$\begin{aligned}\frac{5}{9} - \frac{1}{6} &= \frac{10}{18} - \frac{3}{18} \quad (\text{The LCM of 9 and 6 is 18.}) \\ &= \frac{7}{18}\end{aligned}$$

Example 8

Evaluate:

a $2\frac{3}{8} + 1\frac{5}{6}$

b $21\frac{1}{6} - 19\frac{5}{8}$

(continued on next page)

Solution

$$\begin{aligned}\mathbf{a} \quad 2\frac{3}{8} + 1\frac{5}{6} &= 2\frac{9}{24} + 1\frac{20}{24} \\ &= 3 + \frac{9}{24} + \frac{20}{24} \\ &= 3 + \frac{29}{24} \\ &= 3 + 1 + \frac{5}{24} \\ &= 4\frac{5}{24}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 21\frac{1}{6} - 19\frac{5}{8} &= 21\frac{4}{24} - 19\frac{15}{24} && \left(\text{Notice that } \frac{1}{6} < \frac{5}{8}. \right) \\ &= 20 + 1\frac{4}{24} - \left(19 + \frac{15}{24} \right) \\ &= (20 - 19) + \left(\frac{28}{24} - \frac{15}{24} \right) \\ &= 1\frac{13}{24}\end{aligned}$$

or

$$\begin{aligned}21\frac{1}{6} - 19\frac{5}{8} &= 21 + \frac{1}{6} - 19 - \frac{5}{8} \\ &= 21 - 19 + \frac{4}{24} - \frac{15}{24} \\ &= 2 - \frac{11}{24} \\ &= 1\frac{13}{24}\end{aligned}$$

Many applications involve addition and subtraction of fractions.

Example 9

Five identical trucks, carrying $\frac{5}{6}$ load, $\frac{3}{8}$ load, $\frac{3}{4}$ load, $\frac{1}{2}$ load and $\frac{2}{3}$ load of potatoes, arrived at the chip factory. How many truckloads of potatoes were delivered in total?

Solution

$$\begin{aligned}\frac{5}{6} + \frac{3}{8} + \frac{3}{4} + \frac{1}{2} + \frac{2}{3} &= \frac{20}{24} + \frac{9}{24} + \frac{18}{24} + \frac{12}{24} + \frac{16}{24} \\ &= \frac{75}{24} \\ &= \frac{25}{8} \\ &= 3\frac{1}{8}\end{aligned}$$

$3\frac{1}{8}$ truckloads of potatoes were delivered.

Example 10

Brad has a roll of electrical cable with $3\frac{3}{10}$ m left on it. How much is left if he uses $2\frac{7}{8}$ m?

Solution

$$\begin{aligned}3\frac{3}{10} - 2\frac{7}{8} &= \frac{33}{10} - \frac{23}{8} \\ &= \frac{132}{40} - \frac{115}{40} \quad (\text{LCM is 40.}) \\ &= \frac{17}{40}\end{aligned}$$

There is $\frac{17}{40}$ m of cable left.

Exercise 2B

Example 4

1 Find the value of:

a $\frac{3}{4} + \frac{5}{4}$

b $\frac{1}{5} + \frac{3}{5} + \frac{4}{5}$

c $\frac{3}{10} + \frac{2}{10}$

d $\frac{3}{5} + \frac{2}{5} + \frac{7}{5}$

e $\frac{2}{3} + \frac{5}{3}$

f $\frac{2}{7} + \frac{8}{7}$

Example 5

2 Evaluate:

a $\frac{3}{4} - \frac{1}{4}$

b $\frac{7}{12} - \frac{5}{12}$

c $2\frac{2}{5} - 1\frac{1}{5}$

d $1\frac{7}{19} - \frac{9}{19}$

e $2\frac{2}{5} - 1\frac{4}{5}$

f $3\frac{5}{12} - 2\frac{7}{12}$

Example 6

3 Evaluate:

a $\frac{2}{3} + \frac{3}{4}$

b $\frac{9}{10} + \frac{10}{11}$

c $\frac{5}{6} + \frac{3}{10}$

d $\frac{1}{9} + \frac{1}{10}$

e $\frac{8}{9} + \frac{7}{8}$

f $\frac{5}{7} + \frac{3}{5}$

g $\frac{1}{3} + \frac{5}{8}$

h $\frac{7}{8} + \frac{3}{7}$

i $\frac{5}{10} + \frac{3}{5}$

j $\frac{3}{4} + \frac{11}{12}$

k $\frac{5}{8} + \frac{5}{6}$

Example 7

4 Evaluate:

a $\frac{5}{6} - \frac{2}{3}$

b $\frac{1}{3} - \frac{1}{4}$

c $\frac{20}{33} - \frac{35}{77}$

d $\frac{5}{7} - \frac{3}{5}$

e $\frac{7}{8} - \frac{5}{6}$

f $\frac{8}{9} - \frac{7}{8}$

g $\frac{1}{9} - \frac{1}{10}$

h $\frac{2}{7} - \frac{1}{5}$

i $\frac{11}{12} - \frac{5}{6}$

j $\frac{7}{20} - \frac{2}{15}$

k $\frac{3}{11} - \frac{1}{5}$

Example 8

5 Evaluate:

a $2\frac{1}{5} + 3\frac{2}{3}$

b $1\frac{2}{3} + 1\frac{1}{4}$

c $2\frac{4}{5} + 1\frac{1}{4}$

d $3\frac{1}{2} + 2\frac{2}{3}$

e $1\frac{1}{5} + 2\frac{2}{3}$

f $5\frac{1}{10} + 3\frac{7}{8}$

g $1\frac{1}{11} + 3\frac{4}{5}$

h $7\frac{3}{8} + 2\frac{1}{5}$

i $6\frac{4}{5} + 7\frac{5}{8}$

Example 8

6 Evaluate:

a $3\frac{3}{4} - 1\frac{1}{2}$

b $13\frac{3}{4} - 2\frac{7}{8}$

c $27\frac{5}{8} - 14\frac{1}{3}$

d $52\frac{3}{4} - 26\frac{7}{8}$

e $22\frac{1}{4} - 11\frac{1}{3}$

f $52\frac{7}{10} - 3\frac{5}{11}$

- 7 A man walks $2\frac{7}{8}$ km and then runs for $1\frac{3}{4}$ km. How far has he travelled in total?
- 8 The distance from Davidson to Clare is $5\frac{3}{10}$ km. A boy rides $2\frac{3}{4}$ km from Davidson along the road to Clare. How much further does he have to ride to Clare?
- 9 Jacinta has $\frac{2}{3}$ of a litre of water in a jug and then pours in $\frac{1}{4}$ of a litre. How much water is in the jug now?
- 10 A box of chocolates has 24 chocolates in it. Marie ate $\frac{1}{2}$ of the box of chocolates and then ate $\frac{3}{8}$ of the same box of chocolates the next day. What fraction of the box of chocolates did she eat in total?
- 11 I have a length of wire and cut off $\frac{3}{7}$ of it. What fraction of the wire do I have left?
- 12 Dimitri devotes $\frac{1}{3}$ of the day to schoolwork and he spends $\frac{1}{10}$ of the day watching television. As a fraction of the day, how much more time is spent on schoolwork than on television?
- 13 A family travelling to Albury cover one-third of the journey before 1 p.m. and a further one-quarter of the journey between 1 p.m. and 2 p.m. What fraction of the journey have they travelled by 2 p.m.?
- 14 A unit square is subdivided into 36 squares of equal area. Seven of these squares are painted black and the remainder are painted white. What fraction of the unit square is black?
- 15 A rectangular chocolate block is broken into 24 equal pieces. Samantha takes 5 of these pieces and Jacob takes 7 of them. What fraction of the original chocolate block is left?
- 16 In a packet of jellybeans, $\frac{1}{3}$ of the jellybeans are purple, $\frac{1}{8}$ are black and $\frac{1}{4}$ are red.
- a What fraction of the jellybeans are either purple or black?
 - b What fraction of the jellybeans are either purple or red?
 - c What fraction of the jellybeans are purple or red or black?
 - d What fraction of the jellybeans are not purple or red or black?



Multiplication of fractions

When multiplying fractions, we first multiply the two numerators, then multiply the two denominators, and then simplify if we can.

Always look out for cancelling. In most cases, cancel common factors first, then multiply.

Example 11

Evaluate $\frac{10}{21} \times \frac{9}{16}$.

Solution

$$\begin{aligned}
 \frac{10}{21} \times \frac{9}{16} &= \frac{\overset{5}{\cancel{10}^7}}{\underset{7}{\cancel{21}^3}} \times \frac{\overset{3}{\cancel{9}^1}}{\underset{8}{\cancel{16}^2}} && \text{(Divide 10 and 16 by 2; divide 9 and 21 by 3.)} \\
 &= \frac{5}{7} \times \frac{3}{8} \\
 &= \frac{15}{56}
 \end{aligned}$$

When multiplying mixed numerals, first convert to improper fractions, cancel common factors if possible and then multiply.

Example 12

Evaluate $5\frac{1}{4} \times 2\frac{1}{3}$.

Solution

$$\begin{aligned}
 5\frac{1}{4} \times 2\frac{1}{3} &= \frac{21}{4} \times \frac{7}{3} && \text{(Convert to improper fractions.)} \\
 &= \frac{\overset{7}{\cancel{21}^3}}{4} \times \frac{7}{\underset{1}{\cancel{3}^1}} && \text{(Divide 21 and 3 by 3.)} \\
 &= \frac{7}{4} \times \frac{7}{1} \\
 &= \frac{49}{4} \\
 &= 12\frac{1}{4}
 \end{aligned}$$

Remember that multiplication of fractions is required when the word ‘of’ is used.

An everyday use of multiplication of fractions arises when we take part of a quantity or measurement. The key word here is ‘of’.

Example 13

Approval has been given for $\frac{4}{5}$ of a class of 30 students to be immunised. How many injections will the nurse give to students in that class?

Solution

$$\begin{aligned}\frac{4}{5} \text{ of } 30 &= \frac{4}{5} \times 30 \\ &= \frac{4}{5} \times \frac{30}{1} \\ &= \frac{4}{\cancel{5}^1} \times \frac{\cancel{30}^6}{1} \\ &= \frac{4}{1} \times \frac{6}{1} \\ &= 24\end{aligned}$$

Hence 24 injections will be given.

Example 14

There is $\frac{5}{8}$ of a packet of cereal at the start of the week, and Jack eats $\frac{2}{3}$ of that amount by the end of the week.

- a** What fraction of a whole packet of cereal has Jack eaten by the end of the week?
- b** Assuming Jack will eat the same amount of cereal each week, how much does he eat in 5 weeks?

(continued on next page)

Solution

- a** We need to work out $\frac{2}{3}$ of $\frac{5}{8}$.

$$\begin{aligned}\frac{2}{3} \times \frac{5}{8} &= \frac{2 \times 5}{3 \times 8} \\ &= \frac{10}{24} \\ &= \frac{5}{12}\end{aligned}$$

- b** Jack has eaten $\frac{5}{12}$ of a packet of cereal over the week.

For 5 weeks, Jack needs

$$\begin{aligned}\frac{5}{12} \times 5 &= \frac{5}{12} \times \frac{5}{1} \\ &= \frac{25}{12} \\ &= 2\frac{1}{12} \text{ packets.}\end{aligned}$$

Jack will eat $2\frac{1}{12}$ packets of cereal over 5 weeks.

Division of fractions

The **reciprocal** of a non-zero fraction is obtained by interchanging the numerator and the denominator. For example, the reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$ and the reciprocal of 5 is $\frac{1}{5}$.

The product of a fraction and its reciprocal is always 1.

$$\frac{3}{7} \times \frac{7}{3} = 1 \quad \text{and} \quad 5 \times \frac{1}{5} = 1$$

Division of fractions is the reverse process of multiplication, so to divide by a non-zero fraction we multiply it by its reciprocal.

Example 15

Evaluate:

a $\frac{2}{3} \div \frac{4}{27}$

b $16 \div \frac{2}{3}$

c $\frac{2}{3} \div 16$

Solution

$$\begin{aligned} \text{a } \frac{2}{3} \div \frac{4}{27} &= \frac{2}{3} \times \frac{27}{4} && \text{(The reciprocal of } \frac{4}{27} \text{ is } \frac{27}{4} \text{.)} \\ &= \frac{\cancel{2}^1}{\cancel{3}^1} \times \frac{\cancel{27}^9}{\cancel{4}^2} \\ &= \frac{9}{2} \\ &= 4\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{b } 16 \div \frac{2}{3} &= \frac{16}{1} \times \frac{3}{2} && \text{(The reciprocal of } \frac{2}{3} \text{ is } \frac{3}{2} \text{.)} \\ &= \frac{\cancel{16}^8}{1} \times \frac{3}{\cancel{2}^1} \\ &= \frac{24}{1} \\ &= 24 \end{aligned}$$

$$\begin{aligned} \text{c } \frac{2}{3} \div 16 &= \frac{2}{3} \times \frac{1}{16} && \text{(The reciprocal of 16 is } \frac{1}{16} \text{.)} \\ &= \frac{\cancel{2}^1}{3} \times \frac{1}{\cancel{16}^8} \\ &= \frac{1}{24} \end{aligned}$$

As with multiplication, mixed numerals must first be converted to improper fractions before dividing.

Example 16

Evaluate $2\frac{3}{4} \div 1\frac{5}{8}$.

(continued on next page)

Solution

$$\begin{aligned}2\frac{3}{4} \div 1\frac{5}{8} &= \frac{11}{4} \div \frac{13}{8} \\&= \frac{11}{\cancel{4}^1} \times \frac{\cancel{8}^2}{13} \\&= \frac{22}{13} \\&= 1\frac{9}{13}\end{aligned}$$

Example 17

Robin has $6\frac{3}{4}$ m of timber and wants to cut from it pieces that are $2\frac{1}{4}$ m long.
How many such lengths can he cut?

Solution

$$\begin{aligned}6\frac{3}{4} \div 2\frac{1}{4} &= \frac{27}{4} \div \frac{9}{4} && \text{(Convert each mixed numeral to an improper fraction.)} \\&= \frac{27}{4} \times \frac{4}{9} && \text{(Multiply by the reciprocal of } \frac{9}{4}, \text{ that is, } \frac{4}{9} \text{.)} \\&= \frac{\cancel{27}^3}{\cancel{4}^1} \times \frac{\cancel{4}^1}{\cancel{9}^1} && \text{(Cancel and then multiply.)} \\&= 3\end{aligned}$$

Thus cutting $6\frac{3}{4}$ m of timber into pieces of length $2\frac{1}{4}$ m gives exactly 3 pieces.



Multiplying and dividing fractions

- To multiply fractions:
 - first cancel factors common to a numerator and a denominator
 - then multiply the numerators and multiply the denominators.
- The **reciprocal** of a non-zero fraction is formed by interchanging the numerator and the denominator.
- To divide by a non-zero fraction, multiply by its reciprocal.
- To multiply or divide mixed numerals, first change them to improper fractions.

Exercise 2C

1 Evaluate:

a $\frac{2}{3}$ of $\frac{5}{16}$

b $\frac{3}{4}$ of $\frac{11}{16}$

c $\frac{5}{11}$ of 22

d $\frac{3}{5}$ of $\frac{11}{20}$

e $\frac{5}{12}$ of 32

f $\frac{6}{11}$ of 25

Example 11

2 Evaluate:

a $\frac{1}{2} \times \frac{1}{3}$

b $\frac{1}{7} \times \frac{1}{11}$

c $\frac{3}{4} \times \frac{1}{3}$

d $\frac{7}{8} \times \frac{4}{5}$

e $\frac{3}{28} \times \frac{2}{3}$

f $\frac{5}{18} \times \frac{3}{5}$

g $\frac{16}{21} \times \frac{7}{8}$

h $\frac{14}{15} \times \frac{7}{8}$

i $\frac{14}{3} \times \frac{3}{7}$

j $\frac{7}{11} \times \frac{22}{35}$

k $\frac{7}{5} \times \frac{5}{13}$

l $\frac{3}{4} \times \frac{8}{15}$

m $\frac{5}{8} \times \frac{7}{12}$

n $\frac{15}{22} \times \frac{11}{30}$

o $\frac{7}{4} \times \frac{12}{35}$

Example 12

3 Evaluate:

a $1\frac{1}{5} \times 2\frac{1}{3}$

b $2\frac{2}{3} \times 3\frac{1}{5}$

c $4\frac{1}{10} \times 5\frac{2}{5}$

d $2\frac{4}{11} \times 1\frac{1}{3}$

e $1\frac{3}{5} \times 1\frac{2}{3}$

f $2\frac{1}{4} \times 1\frac{1}{5}$

g $2\frac{1}{5} \times \frac{3}{11}$

h $2\frac{1}{4} \times 1\frac{1}{8}$

i $3\frac{1}{3} \times 1\frac{2}{3}$

Example 13

4 $\frac{7}{8}$ of a group of 968 students have brown eyes. How many students is this?

5 Find the value of:

a $\frac{3}{5}$ of 20 L

b $\frac{5}{8}$ of 10 km

c $\frac{5}{8}$ of 10 000 m

d $\frac{3}{4}$ of 50 m

e $\frac{4}{5}$ of 60 kg

f $\frac{3}{5}$ of 800 mm

g $\frac{3}{7}$ of 140 m

h $\frac{7}{8}$ of 100 km

Example 14

6 Evaluate:

a $\frac{5}{8} \div \frac{4}{11}$

b $\frac{7}{15} \div \frac{2}{11}$

c $\frac{2}{3} \div \frac{4}{7}$

d $15 \div \frac{18}{5}$

e $\frac{3}{4} \div 10$

f $\frac{5}{11} \div 7$

g $\frac{5}{8} \div \frac{11}{3}$

h $\frac{5}{12} \div 15$

i $\frac{2}{3} \div \frac{5}{8}$

j $16 \div \frac{2}{3}$

k $10 \div \frac{2}{5}$

l $\frac{15}{8} \div \frac{3}{4}$

7 Evaluate:

a $6\frac{3}{4} \div 1\frac{1}{5}$ **b** $2\frac{3}{5} \div 1\frac{1}{2}$ **c** $5\frac{1}{4} \div 1\frac{1}{2}$ **d** $10 \div 1\frac{1}{5}$ **e** $5\frac{1}{3} \div 10$

f $2\frac{1}{5} \div 1\frac{1}{3}$ **g** $7\frac{1}{5} \div 9$ **h** $2\frac{1}{5} \div 3\frac{3}{4}$ **i** $5\frac{1}{3} \div 6\frac{1}{4}$

- 8 It takes a cabinet maker $2\frac{1}{4}$ days to make a polished table top. How many can he make in 9 weeks?
- 9 How many seconds does it take for a man running at $9\frac{1}{2}$ m/s to run 100 m?
- 10 **a** What number multiplied by $2\frac{1}{4}$ gives $1\frac{1}{8}$?
b What number divided by $1\frac{1}{3}$ gives $2\frac{1}{4}$?
- 11 At Castlefield College, $\frac{4}{7}$ of the students are boys.
a What fraction of students are girls?
b If there are 360 boys, how many girls are there?
- 12 Four-fifths of the jellybeans in a jar are black. There are 288 black jellybeans in the jar. How many jellybeans are there in total?
- 13 Three-quarters of the trees in a forest are acacias. It is known that there are 6000 trees in the forest.
a How many acacias are there in the forest?
b What fraction of the trees are not acacias?
- 14 Three-sevenths of a sum of money is \$36. How much is $\frac{3}{4}$ of the sum of money?
- 15 Five-twelfths of a farm covers 325 hectares (ha). What is the area of the whole farm?
- 16 The difference between $\frac{2}{3}$ and $\frac{3}{4}$ of a sum of money is \$200. What is the sum of money?
- 17 A tank that is $\frac{2}{3}$ full contains 1350 L of water. How many litres does it hold when it is full?
- 18 After cycling $\frac{1}{3}$ of a journey, a cyclist has 28 km further to go. What is the length of the journey?
- 19 A rope $10\frac{7}{8}$ m is cut into five equal length pieces of rope. How long is each piece?

2D The unitary method



The idea of the **unitary method** is to base calculations on one part of the whole. Often, the ‘part’ is a fraction of the whole. Here we take another look at ‘of’ and multiplication from this point of view.

Fraction of a quantity: the unitary method

In the **unitary method** of solving problems, the most important step is the very first line, where the ‘parts’ and the ‘whole’ are identified.

Example 18

The class was told that each pupil must read $\frac{3}{8}$ of a 496-page book before the first day of term. How many pages must each pupil read?

Solution

The ‘whole’ is the 496-page book.

The ‘part’ is $\frac{1}{8}$ of the pages in the book,

so 8 parts is 496 pages.

(This line is the key step.)

$\div 8$ 1 part is 62 pages.

(The operation is in the box to the left.)

$\times 3$ 3 parts is 186 pages.

(We have now got to the ‘3’ in $\frac{3}{8}$.)

Hence each pupil must read 186 pages.

Going from a fraction to the whole

The unitary method can be used to solve problems where we are given a fraction of the whole.

Example 19

There are 51 people at Sacha’s barbecue. This is $\frac{3}{5}$ of those invited. How many people were invited?

Solution

The ‘part’ to use is $\frac{1}{5}$ of those invited,

so 3 parts is 51 people.

(This line is the key step.)

$\div 3$ 1 part is 17 people.

$\times 5$ 5 parts is 85 people.

(This is the ‘whole’.)

Hence 85 people were invited to the party.



Exercise 2D

Example 18

- 1 Use the unitary method to solve these problems.

a $\frac{2}{3}$ of 540 b $\frac{5}{8}$ of 968 c $\frac{5}{12}$ of 1440

- 2 Use the unitary method to solve these problems.

a Find $\frac{4}{5}$ of \$160. b Find $\frac{3}{7}$ of 210 kg.

c Amina's father agrees to pay $\frac{7}{12}$ of the price of a new saxophone. If the saxophone costs \$3780, how much will Amina's father pay?

d Binh's farm has produced 4600 cubic metres of hay. If he keeps $\frac{3}{8}$ of this to feed his stock, how much will he have left to sell at the market?

Example 19

- 3 a $\frac{3}{8}$ of an amount of money is \$543. What is the amount of money?

b $\frac{3}{4}$ of the students in a school are boys. There are 726 boys in the school. How many students are there in the school?

- 4 Twenty-one students have signed up for the school music camp. This is $\frac{3}{7}$ of the maximum number of people allowed to attend the camp. How many more students can still sign up?

- 5 Use the unitary method in reverse to solve these problems.

a If $\frac{3}{5}$ of a container used for storing milk is 150 litres, what is the capacity of the container?

b Massima has saved \$495, which is $\frac{11}{12}$ of the cost of a stereo. What is the price of the stereo?

c Jehan saves \$105 every week, which is $\frac{3}{5}$ of his weekly wage. What is his weekly wage?

d Conrad is recovering from an operation and has been told to walk 10 000 steps per day. In 45 minutes, he has walked 3000 steps. If he walks at the same rate, how long will it take him to walk the required number of steps?

- 6 In the basketball grand final, Simon scored 16 goals. If Simon scored $\frac{1}{6}$ of his team's goals, what was their final number of goals?

- 7 One goat needs two-fifths of a hectare if it is to produce high-quality milk. How many hectares are needed for a herd of 72 goats?

- 8 Sam wants to make five banana pancakes. He has a recipe for 20 pancakes, and the recipe requires two-thirds of a cup of milk. How much milk is needed for 5 pancakes?

2E Decimal notation



Decimals are an extension of the base-ten number system. The whole-number place-value notation is extended to include tenths, hundredths, thousandths, and so on. The **decimal point** separates the whole-number part from the fractional part.

Here is a reminder of what a decimal means. The decimal

$$123.456 = 100 + 20 + 3 + \frac{4}{10} + \frac{5}{100} + \frac{6}{1000}.$$

This process is called writing 123.456 in **expanded form**. We can also write $123.456 = 123 \frac{456}{1000}$.

Using expanded form, we can write a decimal as a fraction or mixed numeral.

Example 20

Write 2.1255 as a mixed numeral with the fractional part in simplest form.

Solution

$$\begin{aligned} 2.1255 &= 2 \frac{1255}{10000} \\ &= 2 \frac{251}{2000} \end{aligned}$$

Converting fractions to decimals

Converting fractions with denominators that are powers of 10 to decimals is straightforward.



Example 21

Convert $8 \frac{123}{1000}$ to a decimal.

Solution

$$8 \frac{123}{1000} = 8.123$$



Some fractions do not have denominators that are powers of 10 but they can also be converted into decimals.

Example 22

Write each fraction as a decimal.

a $\frac{3}{25}$

b $\frac{7}{40}$

c $\frac{19}{125}$

Solution

$$\begin{aligned}\mathbf{a} \quad \frac{3}{25} &= \frac{3 \times 4}{25 \times 4} \\ &= \frac{12}{100} \\ &= 0.12\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad \frac{7}{40} &= \frac{7 \times 25}{40 \times 25} \\ &= \frac{175}{1000} \\ &= 0.175\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad \frac{19}{125} &= \frac{19 \times 8}{125 \times 8} \\ &= \frac{152}{1000} \\ &= 0.152\end{aligned}$$

Comparing and ordering decimals

Selecting the larger of two decimals is done by comparing the leftmost digit and then comparing from left to right until the digits being compared are different. You can then decide which is larger.

Example 23

Five children measure their arm spans. Their results are 1.64 m, 1.5 m, 1.595 m, 1.328 m and 1.593 m.

Put these measurements in order from smallest to largest.

Solution

First compare the units. They are all the same, so compare the tenths digit. The largest tenths digit is the 6 in 1.64 and the smallest is the 3 in 1.328. Now compare the hundredths digits and finally the thousandths to see that the order is:

$$1.328 < 1.5 < 1.593 < 1.595 < 1.64.$$

Example 24

Order these fractions from smallest to largest by first converting them to decimals:

$$\frac{3}{4}, 7\frac{5}{8}, \frac{6265}{700} \text{ and } \frac{228}{400}.$$

Solution

$$\frac{3}{4} = \frac{75}{100} \\ = 0.75$$

$$7\frac{5}{8} = 7\frac{625}{1000} \quad (\text{Multiply numerator and denominator by 125.}) \\ = 7.625$$

$$\frac{6265}{700} = \frac{895}{100} \quad (\text{Divide numerator and denominator by 7.}) \\ = 8.95$$

$$\frac{228}{400} = \frac{57}{100} \quad (\text{Divide numerator and denominator by 4.}) \\ = 0.57$$

So the order is $\frac{280}{400} < \frac{3}{4} < 7\frac{5}{8} < \frac{6265}{700}$ or $0.57 < 0.75 < 7.625 < 8.95$.

Exercise 2E

Example 20

- 1 Convert these numbers to a proper fraction or mixed numeral.

a 2.1 **b** 5.023 **c** 6.71 **d** 2.006 **e** 0.076
f 5.68 **g** 0.0085 **h** 2.008 **i** 16.875 **j** 23.625

Example 21, 22

- 2 Convert these fractions and mixed numerals to decimals.

a $51\frac{3}{4}$ **b** $\frac{7}{8}$ **c** $36\frac{3}{25}$ **d** $112\frac{17}{50}$ **e** $87\frac{39}{200}$

Example 23

- 3 Arrange these numbers in order from smallest to largest.

a 2.5834, 2.35, 2.83, 2.435, 2.5
b 18.0099573, 18.1, 18.02, 18.1002, 18.21
c 6.6, 6.66, 66.06, 60.66, 60.006
d 55.2, 47.682, 55.24, 55.16, 56.001

Example 24

- 4 Convert these mixed numerals to decimals, then put them in increasing order.

$4\frac{3}{4}$, $4\frac{3}{5}$, $4\frac{31}{100}$, $4\frac{5}{8}$



Addition and subtraction

Addition and subtraction of decimals follow the same ideas as for whole numbers and use the standard algorithms. It is important that we only add like to like: we can only add tens to tens, ones to ones, tenths to tenths, and so on. For this reason, we must line up the decimal points of the numbers when we use the algorithms for addition and subtraction.

Example 25

Calculate the sum of 3.4518 and 4.09.

Solution

Inserting two extra zeroes helps to line up the digits and the decimal point.

$$\begin{array}{r} 3.4518 \\ + 4.0900 \\ \hline 7.5418 \end{array}$$

Example 26

The wiring plan for an electronic robot calls for 0.82 m, 1.5 m and 13.285 m of cable. How much cable is needed in total?

Solution

$$\begin{array}{r} 0.820 \\ 1.500 \\ + 13.285 \\ \hline 15.605 \end{array}$$

15.605 m of cable is needed in total.

Example 27

Coffee comes in bags that weigh 4.2 kg. In one week, 1.83 kg of coffee was used. How much was left?

Solution

$$\begin{array}{r} 4.210 \\ - 1.813 \\ \hline 2.37 \end{array}$$

2.37 kg of coffee was left.

Multiplication and division by powers of 10

Multiplying and dividing decimals by 10, 100, 1000, 10000 and so on is easy.



Multiplication and division of decimals by powers of 10

- When any number is multiplied by 10, each digit is multiplied by 10. This corresponds to moving the decimal point one place to the right and inserting a zero if necessary.
- Multiplying by $100 = 10^2$ corresponds to moving the decimal point 2 places to the right and inserting zeroes if necessary.

Multiplying by $1000 = 10^3$ corresponds to moving the decimal point 3 places to the right and inserting zeroes if necessary.

Multiplying by $10000 = 10^4$ corresponds to moving the decimal point 4 places to the right and inserting zeroes if necessary.

- When any number is divided by 10, each digit is divided by 10. This corresponds to moving the decimal point one place to the left and inserting a zero if necessary.
- Dividing by $100 = 10^2$ corresponds to moving the decimal point 2 places to the left and inserting zeroes if necessary.

Dividing by $1000 = 10^3$ corresponds to moving the decimal point 3 places to the left and inserting zeroes if necessary.

Dividing by $10000 = 10^4$ corresponds to moving the decimal point 4 places to the left and inserting zeroes if necessary.

Multiplication of decimals

Example 28

Evaluate 2.451×100 .

Solution

$$2.451 \times 100 = 245.1 \quad (\text{The decimal point is moved two places to the right.})$$

Example 29

Evaluate $2.451 \div 1000$.

Solution

$2.451 \div 1000 = 0.002451$ (The decimal point is moved 3 places to the left and the 2 zeroes are inserted.)

Multiplying a decimal by a whole number

The following example presents a suitable setting out.

Example 30

Six children collected lengths of timber for recycling. They collected 3.87 m of timber each. How much timber did they collect in total?

Solution

Multiply the length collected by one child by 6.

$$\begin{array}{r} 3.87 \\ \times \quad 6 \\ \hline 23.22 \end{array}$$

The children collected 23.22 m of timber in total.

Multiplying one decimal by another

We can multiply decimals by converting each decimal to a fraction, multiplying the fractions (without cancelling) and then converting the result back to a decimal.

Example 31

Calculate 0.03×0.18 .

Solution

$$\begin{aligned} 0.03 \times 0.18 &= \frac{3}{100} \times \frac{18}{100} \\ &= \frac{54}{10000} \\ &= 0.0054 \end{aligned}$$

or

$0.03 \times 0.18 = 0.0054$.
Multiply 18 by 3 and place the decimal point so that the total number of places after the decimal point is the same on both sides of the equation.

Division of decimals



Dividing a decimal by a whole number

We can use the division algorithm to do this.

$$\begin{array}{r} 0.9 \\ 4 \overline{) 3.6} \end{array}$$

The procedure is the same as for whole numbers. The decimal point in the quotient is aligned directly above the decimal point in the dividend.

Example 32

Divide 3.6 by 5.

Solution

$$\begin{array}{r} 0.72 \\ 5 \overline{) 3.60} \end{array}$$

Dividing one decimal by another

There are two methods for dividing one decimal by another.

Example 33

Evaluate $3.6 \div 0.05$.

Solution

Method 1

Multiply numerator and denominator by the same factor to obtain a whole-number divisor.

$$\begin{aligned} 3.6 \div 0.05 &= \frac{3.6 \times 100}{0.05 \times 100} \\ &= \frac{360}{5} \\ &= 72 \end{aligned}$$

Method 2

Convert each number to a fraction.

$$\begin{aligned} 3.6 \div 0.05 &= \frac{36}{10} \div \frac{5}{100} \\ &= \frac{36}{10} \times \frac{100}{5} \\ &= \frac{36}{\cancel{10}^1} \times \frac{\cancel{100}^{10}}{5} \\ &= \frac{360}{5} \\ &= 72 \end{aligned}$$

Example 34

Divide 31.4 by 0.05.

Solution

Method 1

$$\begin{aligned}\frac{31.4}{0.05} &= \frac{31.4 \times 100}{0.05 \times 100} \\ &= \frac{3140}{5} \\ &= 628\end{aligned}$$

Method 2

$$\begin{aligned}31.4 \div 0.05 &= 31\frac{4}{10} \div \frac{5}{100} \\ &= \frac{314}{10} \times \frac{100}{5} \\ &= \frac{314}{\cancel{10}^1} \times \frac{\cancel{100}^{10^2}}{\cancel{5}^1} \\ &= 628\end{aligned}$$

Exercise 2F

Example 25, 26, 27

- 1 Calculate these sums and differences using the standard algorithms.

a $9.77 + 37.8$

b $78.9 + 0.89 + 45 + 4.664$

c $562.6 - 43.18$

d $307.05 - 89.77$

Example 28, 29

- 2 Evaluate:

a 2.3×10

b 0.003×100

c $2.6 \div 10$

d $260 \div 1000$

e $260 \div 10000$

f 0.0075×100

g $56.1 \div 100$

h 2.63×10000

i 2.5×100

j $526.7 \div 1000$

k 32.3×10000

l 4.6×1000

Example 30

- 3 Evaluate:

a 5.63×7

b 22.867×5

c 426.8×9

d 456.23×7

e 56.23×24

f 69.56×32

Example 31

- 4 Calculate:

a 0.3×0.7

b 0.8×0.06

c 0.9×0.07

d 0.006×0.7

e 0.06×0.004

f 0.05×0.08

g 31.504×1.2

h 0.061×0.002

Example 32

- 5 Evaluate:

a $2.376 \div 2$

b $9.2 \div 5$

c $5.48 \div 4$

d $42.7 \div 5$

e $9.7 \div 2$

f $43.2 \div 5$

- 6** Calculate:
- a** $36 \div 0.9$ **b** $2800 \div 0.0007$ **c** $0.378 \div 0.03$
d $63.147 \div 0.07$ **e** $84 \div 2.4$ **f** $560 \div 0.008$
g $7200 \div 0.09$ **h** $0.144 \div 0.012$ **i** $450.56 \div 0.08$
- 7** The population of Sydney was 3.9 million in 1996 and increased to 4.2 million in 2001. By what number of people did the population increase over the 5 years? What was the average annual increase?
- 8** What is the perimeter of a table with side lengths 1.8 m, 0.87 m, 0.87 m and 1.43 m?
- 9** Between June 1996 and June 2001, the population of Brisbane increased by 133 400 people to reach 1.7 million. What was the population in June 1996?
- 10** What is the mass, in kilograms, of each of the following amounts of olive oil if one litre of olive oil has a mass of 0.878 kg?
- a** 2 L **b** 3.5 L **c** 0.8 L
d 1.7 L **e** 423 mL **f** 400 mL
- 11** On average, Melburnians ate 18.34 kg of cheese each in 2001. If the population of Melbourne in 2001 was 3.5 million, what was the total amount of cheese eaten in Melbourne in that year?
- 12** If the 321 700 people living in Darwin produce an average of 1.87 kg of garbage per person per day, how much garbage is produced by Darwin residents in one non-leap year?
- 13** A rectangular block of land measures 15.78 m by 27.93 m. Calculate the perimeter and area of the block.
- 14** Find, as a decimal, the missing number in each statement.
- a** $10 \times \square = 0.4$ **b** $109.9 = \square \div 0.7$ **c** $\square \times 2.3 = 9.2$
d $\square = 5.3 \times 96.4$ **e** $\square \div 1.098 = 25.87$ **f** $45.2 \div 0.8 = \square$
- 15** Gold is measured in troy ounces. One troy ounce is equal to 31.103 g. What is the weight, in grams, of each of the following gold nuggets?
- a** The Welcome Nugget, found at Ballarat in 1858, weighing 2217 ounces
b The Welcome Stranger Nugget, found at Moliagul in 1869, weighing 2284 ounces
c The Beyers and Holtermann specimen, which was reported to have contained 3000 ounces of gold



We have already seen how to convert certain fractions to decimals.

For example:

$$\frac{3}{10} = 0.3, \quad \frac{3}{4} = 0.75.$$

The method used was to multiply the denominator by some number to produce a power of 10 and multiply the numerator by that same number. Thus,

$$\frac{3}{8} = \frac{3 \times 125}{8 \times 125} = \frac{375}{1000} = 0.375$$

This method will only work if the only prime factors of the denominator are 2 or 5, and will produce a decimal with a finite number of digits. Such decimals are called **terminating decimals**. We will see shortly that a fraction whose denominator has a prime factor other than 2 or 5 does not produce a terminating decimal.



A better method for converting fractions to decimals is to use the division algorithm.

Example 35

Evaluate $\frac{3}{8}$ as a decimal by using the division algorithm.

Solution

$$\begin{array}{r} 0.375 \\ 8 \overline{)3.0^6 0^4 0} \end{array}$$

$$\text{So } \frac{3}{8} = 0.375.$$



Recurring decimals

If we apply the division algorithm to a fraction whose denominator has a prime factor other than 2 or 5, we see that the process does not terminate.



Example 36

Use the division algorithm to write $\frac{4}{11}$ as a repeating decimal.

Solution

$$\begin{array}{r} 0.3636363\ldots \\ 11 \overline{) 4.070707070707\ldots} \end{array}$$

Hence $\frac{4}{11} = 0.3636363636\ldots$

We write $\frac{4}{11} = 0.\dot{3}6$

We write $0.363636\ldots$ as $0.\dot{3}6$ to indicate that the digits 36 repeat indefinitely. This is an example of a recurring decimal. We place dots on the first and last digit of the recurring cycle.

Example 37

Use the division algorithm to convert $\frac{5}{12}$ to a decimal.

Solution

$$\begin{array}{r} 0.4166666\ldots \\ 12 \overline{) 5.020808080808\ldots} \end{array}$$

Hence $\frac{5}{12} = 0.4166666666\ldots$

We write $\frac{5}{12} = 0.41\dot{6}$

We write $0.41666666\ldots$ as $0.41\dot{6}$ to show that only the digit 6 repeats indefinitely. This is an example of an eventually recurring decimal.



Note the conventions for writing the repeating cycles in the final answers:

- In $0.41\dot{6} = 0.4166666666\ldots$, the dot goes over the repeating digit 6.
- In $0.\dot{3}6 = 0.3636363636\ldots$, the dots go over the first and last digits of the repeating cycle.

Other fractions give decimals with longer repeating strings of digits.

For example, $\frac{2}{7} = 0.285714285714\ldots$. This can be written as $0.\dot{2}8571\dot{4}$.



Example 38

Write each of these numbers as a recurring decimal.

a $\frac{5}{11}$

b $4\frac{6}{7}$

c $5\frac{7}{12}$

Solution

$$\begin{array}{r} 0.4\ 5\ 4\ 5\dots \\ 11 \overline{) 5.0\overset{6}{0}\overset{5}{0}\overset{6}{0}\dots} \\ \hline \end{array}$$

$$\frac{5}{11} = 0.4\dot{5}$$

$$\begin{array}{r} 0.8\ 5\ 7\ 1\ 4\ 2\ 8\ 5\dots \\ 7 \overline{) 6.0\overset{4}{0}\overset{5}{0}\overset{1}{0}\overset{3}{0}\overset{2}{0}\overset{6}{0}\overset{4}{0}\dots} \\ \hline \end{array}$$

$$4\frac{6}{7} = 4.8\dot{5}714\dot{2}$$

$$\begin{array}{r} 0.5\ 8\ 3\ 3\dots \\ 12 \overline{) 7.0\overset{10}{0}\overset{4}{0}\overset{4}{0}\dots} \\ \hline \end{array}$$

$$5\frac{7}{12} = 5.58\dot{3}$$

The infinite string of digits in the statement $\frac{5}{12} = 0.41666666666\dots$ means that the sequence 0.4, 0.41, 0.416, 0.4166, 0.41666, 0.416666, 0.4166666, 0.41666666... gets closer and closer to $\frac{5}{12}$. No term can ever quite get to $\frac{5}{12}$, but we can find terms ‘as close as we like’ to $\frac{5}{12}$ by taking more and more digits in the string.

Similarly, the statement $\frac{4}{11} = 0.3636363636\dots$ means that the sequence 0.3, 0.36, 0.363, 0.3636, 0.36363, 0.363636, 0.3636363, 0.36363636 gets ‘as close as we like’ to $\frac{4}{11}$.

Rounding decimals

The last few decimal places of a decimal such as 3.1415927 may have no practical value. For example, if 3.1415927 represents the number of kilometres between two farmyard gates, then the final digit 7 represents $\frac{7}{10}$ of a millimetre, which is completely irrelevant. When we round numbers, we write them correct to a certain number of decimal places.

Writing a decimal correct to a number of decimal places

The rules for rounding are as follows.

Suppose that we want to round 10.12589 correct to two decimal places.

- Identify the rounding digit in the second decimal place – in this case it is 2.
- Look at the next digit to the right of the rounding digit – in this case it is 5.
 - If this next digit is 0, 1, 2, 3 or 4, leave the rounding digit alone.
 - If this next digit is 5, 6, 7, 8 or 9, increase the rounding digit by 1.

In this case, the next digit is 5, so the rounding digit increases from 2 to 3.

- Now discard all the digits after the rounding digit.

Hence 10.12589 is approximately 10.13, correct to two decimal places. We write this as $10.12589 \approx 10.13$ correct to two decimal places.

Example 39

Write 3.16493 correct to:

- a** 2 decimal places **b** 3 decimal places **c** 4 decimal places

Solution

- a** $3.16493 \approx 3.16$ to 2 decimal places: the rounding digit is 6, and the digit to the right is smaller than 5.
- b** $3.16493 \approx 3.165$ to 3 decimal places: the rounding digit is 4, and the digit to the right is larger than 5.
- c** $3.16493 \approx 3.1649$ to 4 decimal places: the rounding digit is 9, and the digit to the right is smaller than 5.

The rounding procedure can also be used with repeating decimals.

Example 40

Write $0.\dot{3}\dot{6}$ correct to:

- a** 2 decimal places **b** 3 decimal places **c** 4 decimal places

Solution

$$0.\dot{3}\dot{6} = 0.363636 \dots$$

- a** $0.\dot{3}\dot{6} = 0.36$ (correct to 2 decimal places)
- b** $0.\dot{3}\dot{6} = 0.364$ (correct to 3 decimal places)
- c** $0.\dot{3}\dot{6} = 0.3636$ (correct to 4 decimal places)



Exercise 2G

1 Convert these fractions and mixed numerals to decimals.

a $\frac{2}{5}$

b $\frac{4}{5}$

c $\frac{3}{25}$

d $\frac{7}{8}$

e $4\frac{3}{20}$

f $7\frac{3}{8}$

g $45\frac{3}{4}$

h $7\frac{17}{20}$

i $\frac{17}{5}$

j $\frac{28}{25}$

2 Convert these fractions to decimals.

a $\frac{2}{7}$

b $\frac{3}{7}$

c $\frac{2}{3}$

d $\frac{2}{9}$

e $\frac{3}{11}$

f $\frac{4}{9}$

g $\frac{7}{11}$

h $\frac{8}{9}$

i $\frac{2}{13}$

3 Convert these fractions to decimals.

a $\frac{1}{6}$

b $\frac{5}{6}$

c $\frac{5}{12}$

d $\frac{7}{12}$

e $\frac{11}{12}$

f $\frac{4}{9}$

4 Convert these fractions and mixed numerals to decimals. (They are all recurring decimals.)

a $64\frac{5}{9}$

b $78\frac{5}{6}$

c $45\frac{2}{11}$

d $\frac{11}{12}$

e $3\frac{2}{7}$

5 Round these decimals correct to the given numbers of decimal places.

a 463.1529 (2 places) **b** 7.2811 (1 place) **c** 79.497 (2 places)

d 0.0649 (3 places) **e** 7.99 (1 place) **f** 85.6 (nearest whole number)

6 Express each fraction as a decimal correct to 2 decimal places.

a $\frac{2}{7}$

b $\frac{5}{6}$

c $\frac{4}{7}$

d $\frac{7}{12}$

e $\frac{4}{9}$

7 Write $0.\dot{7}4\dot{6}$ correct to:

a 2 decimal places

b 4 decimal places

c 8 decimal places

Review exercise



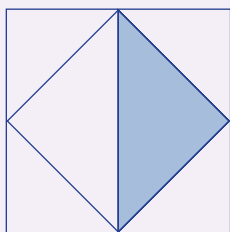
- 1 From this list of fractions: $\frac{35}{42}, \frac{22}{33}, \frac{10}{15}, \frac{17}{34}, \frac{112}{224}, \frac{20}{24}, \frac{18}{21}, \frac{6}{7}$
write two fractions that are equivalent to each of the following.
 - a $\frac{12}{18}$
 - b $\frac{5}{6}$
 - c $\frac{42}{49}$
 - d $\frac{1}{2}$
- 2 Order each set of fractions from largest to smallest.
 - a $\frac{3}{2}, \frac{2}{3}, \frac{10}{12}, \frac{8}{6}, \frac{5}{4}, \frac{2}{4}$
 - b $2\frac{3}{7}, 3\frac{1}{8}, 2\frac{2}{9}, 2\frac{5}{6}, 3\frac{1}{9}, 2\frac{6}{7}$
- 3 Evaluate:
 - a $\frac{5}{12} \div \frac{1}{6}$
 - b $\frac{7}{15} \div \frac{14}{25}$
 - c $\frac{19}{13} \times \frac{52}{81} \div \frac{16}{45}$
 - d $\frac{5}{16} \times \frac{15}{16} \div \frac{125}{32}$
- 4 Convert mixed numerals to fractions before carrying out these calculations.
 - a $5\frac{1}{4} \times 2\frac{1}{7}$
 - b $3\frac{1}{5} \div 1\frac{1}{25}$
 - c $5\frac{1}{2} \times 6\frac{1}{2} \div 3\frac{1}{4}$
 - d $7\frac{6}{7} \times 3\frac{15}{16} \div 4\frac{2}{5}$
 - e $1\frac{1}{3} \times 2\frac{1}{4}$
 - f $\frac{5}{12} \times \frac{3}{14}$
 - g $\frac{5}{12} \div \frac{3}{4}$
 - h $3\frac{7}{8} \div \frac{1}{4}$
 - i $1\frac{1}{4} \times 2\frac{1}{8}$
- 5 Calculate:
 - a $\frac{5}{12} + \frac{1}{3}$
 - b $\frac{2}{3} + \frac{5}{12}$
 - c $\frac{5}{6} - \frac{3}{4}$
 - d $1\frac{1}{3} - 2\frac{1}{5}$
 - e $3\frac{1}{4} + 2\frac{3}{5}$
 - f $3\frac{3}{4} + 2\frac{1}{5}$
 - g $7\frac{1}{2} + 2\frac{1}{7}$
 - h $4\frac{1}{3} + 1\frac{3}{4}$
 - i $4\frac{2}{5} + 3\frac{3}{4}$
 - j $2\frac{1}{2} - \frac{3}{8}$
 - k $1\frac{1}{2} - \frac{1}{5}$
 - l $1\frac{1}{5} + \frac{1}{2}$
- 6 Evaluate:
 - a $1\frac{3}{4} - \frac{5}{6} + 2\frac{1}{2}$
 - b $5 - 1\frac{1}{2} + \frac{5}{8}$
 - c $3\frac{1}{4} - 2\frac{3}{4} + 2\frac{1}{2}$
 - d $6\frac{1}{2} + 2\frac{3}{5} - 1\frac{4}{5}$
 - e $2\frac{3}{4} + 9\frac{1}{2} + 1\frac{3}{4}$
 - f $2\frac{1}{3} - \frac{3}{4} + 1\frac{2}{3}$
- 7 Nina is training hard to become a better swimmer. She spends 17 hours per week doing freestyle, 9 hours per week doing backstroke, and 4 hours per week in the gym. What fraction of her training time is spent out of the pool?
- 8 Alejandro allocates $\frac{1}{4}$ of his pocket money to transport costs, $\frac{1}{10}$ to telephone calls and $\frac{3}{5}$ to clothes, and he saves the rest. If he receives \$18 per week, what are his expenses for transport, telephone calls and clothes? What fraction of his pocket money does Alejandro save each week?



- 9 In a garden, $\frac{1}{3}$ of the area is used for vegetables and $\frac{2}{5}$ for flowers and plants. If the rest of the garden consists of lawns, what fraction of the garden is occupied by lawns?
- 10 There are 1236 boys at a school. This is $\frac{3}{5}$ of the total number of students at the school. How many students are there at the school?
- 11 A particular area of a plantation contains 5325 trees. This is $\frac{5}{8}$ of the total number of trees in the plantation. How many trees are there in the plantation?
- 12 A football squad consists of 24 players. Only $\frac{1}{8}$ of the squad are injured. How many players are fit to play?
- 13 Round these decimals correct to the given number of decimal places.
- | | |
|-----------------|-------------------|
| a 0.089734 | 4 decimal places |
| b 654.0786 | 3 decimal places |
| c 3.14159265 | 5 decimal places |
| d 3.14159265 | 3 decimal places |
| e 2.71828182846 | 10 decimal places |
| f 2.71828182846 | 4 decimal places |
- 14 Calculate:
- | | | |
|----------------------------|-----------------------|----------------------------|
| a 4.56×10 | b 0.0028×100 | c $1.03052 \times 10\,000$ |
| d 0.00000043×1000 | | e $24.87 \div 10$ |
| f $971.2 \div 100$ | g $0.003 \div 1000$ | h $340 \div 10\,000$ |
| i 1000×1.00836 | j $2.894 \div 100$ | |
- 15 Calculate:
- | | | |
|-------------------|---------------------|------------------------|
| a $3.045 + 34.98$ | b $34.7605 + 0.089$ | c $452.906 + 8.006$ |
| d $45 - 39.06$ | e $56.089 - 7.308$ | f $451.23 - 356.8$ |
| g 67.9×3 | h 0.41×0.6 | i 0.345×0.002 |
| j $4 \div 0.02$ | k $67 \div 0.04$ | l $0.08 \div 0.6$ |
- 16 Calculate the cost of:
- a 3.75 kilograms of butter at \$1.50 a kilogram
- b $3\frac{1}{2}$ dozen eggs at \$2.80 per dozen
- c 4.35 kilograms of cheese at \$7.50 a kilogram



- 17 Kyle has \$8.55 to buy his mum's favourite marmalade. If the marmalade is selling at \$2.99 per jar, how many jars can Kyle buy?
- 18 Inouk wanted to edge her garden beds with timber. She measured the lengths of the edges as 1.48 metres, 9.2 metres, 0.93 metres, 4.02 metres and 5.45 metres. How many 4.2-metre lengths of timber garden edging did she have to buy? If the timber garden edging costs \$3.44 per metre, how much did Inouk's garden edging project cost her?
- 19 Calculate how much you save on each item when you buy in bulk.
- a 3 cans of beetroot for \$2.85 or 98 cents each
 - b 6 jars of tomato paste for \$8.40 or \$1.50 each
 - c 4 bottles of soft drink for \$3.70 or 93 cents each
- 20 One-quarter of a class arrived at 7 p.m. for the school concert, half of them arrived at 7.15 p.m., three children told the teacher that they were not able to attend, and the rest are running late. If there are 60 children in the class:
- a how many children arrived at 7 p.m.?
 - b what fraction of the class are running late? How many children is this?
 - c what fraction told the teacher they were not able to attend the concert?
- 21 Belgian cheese costs \$1.70 for $\frac{1}{2}$ a kilogram. How much will $2\frac{1}{2}$ kilograms of Belgian cheese cost?
- 22 The midpoints of the sides of a square are joined as shown.



What fraction of the large square is shaded?

- 23 Linda's doctor tells her she should get 10 hours of sleep each night. She knows that she sleeps $\frac{3}{8}$ of a day already. How much more sleep should she be getting each day?
- 24 Catherine is $\frac{5}{6}$ of her father's height and her brother Peter is $\frac{7}{9}$ of their father's height. If Peter is 140 cm tall, how tall is Catherine?
- 25 Zubin's large dog eats $3\frac{1}{3}$ tins of dog food every day. Each tin contains $1\frac{1}{7}$ kg of food.
- a How much dog food does the dog eat in a seven-day week?
 - b How much dog food does the dog eat in a 52-week year?



26 Evaluate:

$$\text{a } \frac{\frac{3}{5} - \frac{1}{2}}{\frac{4}{5}}$$

$$\text{b } \frac{\frac{6}{11} - \frac{1}{3}}{\frac{7}{9}}$$

$$\text{c } \frac{\frac{5}{8}}{3\frac{1}{2} - 1\frac{1}{4}}$$

$$\text{d } \frac{\frac{7}{11}}{2\frac{2}{3} - 1\frac{1}{2}}$$

$$\text{e } \frac{\frac{5}{8} - \frac{1}{3}}{\frac{5}{8} + \frac{1}{3}}$$

$$\text{f } \frac{5\frac{4}{9}}{2\frac{1}{3} + 5\frac{1}{3}}$$

$$\text{g } \frac{\frac{1}{2} + \frac{3}{4} + \frac{5}{6}}{\frac{2}{3} + \frac{1}{4} + \frac{7}{6}}$$

$$\text{h } \frac{3\frac{1}{4} - 2\frac{3}{5}}{3\frac{1}{4} - 2\frac{3}{5}}$$

27 Evaluate:

$$\text{a } \frac{5}{2 + \frac{1}{3}}$$

$$\text{b } \frac{2}{3 - \frac{1}{4}}$$

$$\text{c } \frac{2}{4 + \frac{5}{6\frac{1}{4}}}$$

$$\text{d } \frac{9}{6 - \frac{3}{1 - \frac{1}{4}}}$$

- 28 The label on Ben's dog's vitamin bottle says he should give the dog 0.09 millilitres per kilogram of body weight. Ben works out that the dog's dosage should be 5.9976 millilitres. What whole number of millilitres should Ben round this to? How much does Ben's dog weigh?

Challenge exercise



- 1 Rebecca has a collection of 45 books. The collection consists of novels and textbooks written in either Chinese or English. $\frac{4}{5}$ of the novels are in English and $\frac{3}{4}$ of the textbooks are in English. The total number of books written in English is 35. How many of her books are textbooks written in English?
- 2 Ava thought she had bought $3\frac{1}{2}$ m of cloth from the market. When she got home, she found that the stall holder had used a ruler that was 4 cm short of 1 m. What was the real length, in metres, of Ava's cloth?
- 3 Angel spent $\frac{7}{24}$ of her weekly salary on Monday, $\frac{1}{4}$ on Tuesday and $\frac{1}{3}$ of it on Wednesday. What fraction of her salary was left on Thursday?
- 4 The fractions $\frac{1}{2}$ and $\frac{1}{3}$ are said to be **consecutive unit fractions**.
 - a Complete each of these subtractions.
 - i $\frac{1}{2} - \frac{1}{3}$
 - ii $\frac{1}{4} - \frac{1}{5}$
 - iii $\frac{1}{7} - \frac{1}{8}$
 - iv $\frac{1}{9} - \frac{1}{10}$
 - b State a rule for subtracting consecutive unit fractions.
 - c Write $\frac{1}{12}$ as the difference of two consecutive unit fractions.
- 5 Which of the fractions $\frac{1}{4}$, $\frac{1}{2}$, $\frac{5}{8}$ and $1\frac{4}{5}$ is the closest to each of the fractions below?
 - a $\frac{4}{6}$
 - b $\frac{4}{12}$
 - c $\frac{4}{9}$
 - d $\frac{9}{17}$
 - e $\frac{47}{96}$
 - f $\frac{31}{120}$
 - g $\frac{199}{100}$
 - h $\frac{299}{402}$
- 6
 - a Show that $27 \times 37 = 999$. Then use the fact that $0.\dot{9} = 1$ to show that $0.\dot{0}2\dot{7} \times 37 = 1$.
 - b Hence show that $\frac{1}{37} = 0.\dot{0}2\dot{7}$.
 - c By multiplying $\frac{1}{37} = 0.\dot{0}2\dot{7}$ by 10 and 100, show that $\frac{10}{37} = 0.\dot{2}7\dot{0}$ and $\frac{100}{37} = 2.\dot{7}0\dot{2}$, and hence show that $\frac{26}{37} = 0.\dot{7}0\dot{2}$.
 - d By multiplying $\frac{1}{37} = 0.\dot{0}2\dot{7}$ by 2, 3, 4 and 26, show that $\frac{2}{37} = 0.\dot{0}5\dot{4}$, $\frac{3}{37} = 0.\dot{0}8\dot{1}$, $\frac{4}{37} = 0.\dot{1}0\dot{8}$ and $\frac{26}{37} = 0.\dot{7}0\dot{2}$.



- e Using similar methods, find $\frac{1}{27}$, $\frac{10}{27}$, $\frac{19}{27}$, and then $\frac{2}{27}$, $\frac{3}{27}$ and $\frac{4}{27}$ as recurring decimals.
- 7 a Show that $101 \times 99 = 9999$.
- b Hence find $\frac{1}{101}$ as a recurring decimal.
- c Use the methods of the previous question to find $\frac{1}{101}$, $\frac{100}{101}$ and $\frac{1000}{101}$ as recurring decimals, and hence find $\frac{91}{101}$ as a recurring decimal.
- d Find $\frac{2}{101}$, $\frac{3}{101}$ and $\frac{4}{101}$ as recurring decimals.
- 8 Four bells commence tolling together and ring every 1 , $1\frac{1}{4}$, $1\frac{1}{5}$ and $1\frac{1}{6}$ seconds respectively. After what time will they first ring together again?



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CHAPTER

3

Number and Algebra

Review of factors and indices

This chapter is about the whole numbers 0, 1, 2, 3, ... and multiplication. The whole number 6 divides exactly into the whole number 24. The number 1001 can be written as $7 \times 13 \times 11$, which is a product of prime numbers. Each whole number can be factorised into prime numbers. In this chapter, we will learn how to factor numbers. The numbers 72 and 81 share some common factors. The number 9 is called the **highest common factor** of 72 and 81. Factoring numbers into products of primes gives a simple way to find their highest common factor. It will also assist us in finding the square roots and cube roots of numbers. Primes and factoring have important applications in telecommunications technology.



All numbers in this chapter are whole numbers.

Factors

The numbers 1, 2, 3, 6, 9 and 18 are the **factors** of 18, because each of them divides into 18 without remainder.

Common factors and the HCF

On the next page, we list the factors of 18 and 24 and compare the lists. The factors that are common to both lists have been boxed.

Factors of 18: $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{6}$, 9, 18

Factors of 24: $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, 4, $\boxed{6}$, 8, 12, 24

The **common factors** of 18 and 24 are 1, 2, 3 and 6.

The largest of these common factors is 6. This is called the **HCF** – the **highest common factor** – of 18 and 24.

Example 1

Find the highest common factor of 18, 72 and 54.

Solution

The factors of 18 are 1, 2, 3, 6, 9 and 18.

The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72.

The factors of 54 are 1, 2, 3, 6, 9, 18, 27 and 54.

So the highest factor common to 18, 72 and 54 is 18.

Multiples

To write down the non-zero multiples of 4, count in 4s until you have as many as you want:

4, 8, 12, 16, 20, 24, 28, 32, ...

Common multiples and the LCM

Look at the lists of the first few non-zero multiples of 18 and 24 below and compare them. The multiples that are common to both lists have been boxed.

Multiples of 18: 18, 36, 54, $\boxed{72}$, 90, 108, 126, $\boxed{144}$, 162, 180, 198, $\boxed{216}$, ...

Multiples of 24: 24, 48, $\boxed{72}$, 96, 120, $\boxed{144}$, 168, 192, $\boxed{216}$, ...

So the **common multiples** of 18 and 24 are 72, 144, 216, . . . The least of these common multiples is 72. This is called the **LCM** – the **lowest common multiple** – of 18 and 24.

Example 2

Find the lowest common multiple of 6, 8 and 12.

Solution

Non-zero multiples of 6 are 6, 12, 18, 24, 30, 36, . . .

Non-zero multiples of 8 are 8, 16, 24, 32, 40, . . .

Non-zero multiples of 12 are 12, 24, 36, . . .

So the lowest common multiple of 6, 8 and 12 is 24.



Factors and the HCF

- A **factor** of a number is a number that divides into it without remainder. For example, 6 is a factor of 24 because $24 = 6 \times 4$ or, equivalently, $24 \div 6 = 4$.
- There is natural pairing of the factors of any number. For example, the factors of 24 are 1 and 24, 2 and 12, 3 and 8, 4 and 6.
- The **highest common factor**, or **HCF**, of two or more numbers is the largest number that is a factor of all of them.



Multiples and the LCM

- The non-zero **multiples** of a number are found by counting in that number. For example, the non-zero multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, . . .
- The **lowest common multiple**, or **LCM**, of two or more numbers is the smallest number that is a multiple of all of them.

Prime numbers and composite numbers

A **prime number** is a number greater than 1 whose only factors are 1 and itself.

The list of prime numbers begins

2, 3, 5, 7, 11, . . .

The whole numbers greater than 1 that are not prime are called **composite numbers**. Such numbers have at least three factors, including 1 and itself. The list of composite numbers begins

4, 6, 8, 9, 10, . . .

The numbers **0** and **1** are special, because they are neither prime nor composite.



Prime numbers and composite numbers

- A **prime number** is a number greater than 1 whose only factors are 1 and itself.
- A **composite number** is a whole number greater than 1 that is not prime. It will therefore have at least 3 factors.
- The numbers **0** and **1** are neither prime nor composite.



Exercise 3A

Example 1

- 1 List all the factors of both numbers in each pair, then write down their HCF.

- | | | |
|--------------------|-------------------|-------------------|
| a 8 and 12 | b 6 and 20 | c 14 and 9 |
| d 20 and 22 | e 1 and 8 | f 5 and 15 |

- 2 Find the HCF of:

- | | | |
|------------------------|-----------------------|------------------------|
| a 30 and 24 | b 15 and 21 | c 12 and 72 |
| d 25 and 16 | e 36 and 16 | f 26 and 65 |
| g 12, 18 and 30 | h 15, 6 and 14 | i 60, 20 and 10 |

Example 2

- 3 List the first few non-zero multiples of both numbers in each pair, then write down their LCM.

- | | | |
|-------------------|--------------------|-------------------|
| a 4 and 6 | b 10 and 12 | c 3 and 7 |
| d 5 and 10 | e 1 and 5 | f 6 and 15 |

- 4 Find the LCM of:

- | | | |
|---------------------|----------------------|---------------------------|
| a 8 and 12 | b 8 and 9 | c 17 and 1 |
| d 12 and 15 | e 7 and 49 | f 20 and 90 |
| g 3, 4 and 6 | h 8, 9 and 12 | i 12, 8, 10 and 30 |

- 5 **a i** Count to 30 in 2s.

ii Count to 30 in 3s.

iii Count to 30 in 5s.

- b** Write down all the numbers from 2 to 30 that you did not list in part **a**. This list, together with 2, 3 and 5, is a list of all the 10 prime numbers less than 30.

- c** Write down all the composite numbers between 30 and 50.

- 6 Find all the prime divisors of:
- a 8 b 12 c 15 d 30
- e 44 f 38 g 125
- 7 a Write 10 and 14 as products of two prime numbers.
 b Write 30 and 42 as products of three prime numbers.
 c Write each of 8 and 9 as a power of a prime number.
- 8 Write each of these prime numbers as the sum of two squares.
 a 5 b 13 c 17 d 29 e 37 f 41
- 9 The Guavalo Palace has three long corridors, whose lengths are 24 m, 48 m and 60 m, respectively. The decorators are designing a standard piece of carpeting that will fit an exact number of times into each corridor. What is the longest piece of carpet that they can choose?
- 10 The baker's delivery truck visits the caravan park every 4 days. The fishmonger visits every 5 days. If they both visit today, how many days will pass before they visit on the same day again?
- 11 Three different bugs living in a puddle take 4, 8 and 3 minutes respectively to swim around the edge of the puddle. If they start at the same point at 12 p.m., what time will it be when they are all together again at that point?
- 12 1001 is the product of which three prime numbers?
- 13 **Goldbach's conjecture** states that every even number greater than 2 can be expressed as the sum of two prime numbers. Show that this is true for the even numbers between 140 and 160.
- 14 The prime factors of 210 are 2, 3, 5 and 7. Find six other factors of 210 without using division.
- 15 Explain why a square number has an odd number of factors, and a non-square number has an even number of factors. (*Hint*: Think about the pairing procedure that you used to find all the factors of a number.)

3B Indices and the index laws



In this section we discuss some properties of products of repeated factors. We begin with a review of powers.

Powers



Powers provide a useful way of writing a product of repeated factors. For example, 16 can be written as the product of four 2s:

$$\begin{aligned} 16 &= 2 \times 2 \times 2 \times 2 \\ &= 2^4 \end{aligned} \quad (\text{We read this as '2 to the power 4'.})$$

- The small number 4 at the top right is called the **index** or **exponent**. (Note: The plural of *index* is *indices*.)
- The whole expression 2^4 is called a **power**. It is the fourth power of 2. We say 'two to the fourth'.
- The number 2 in this expression is called the **base**.

Arrays provide a useful way to visualise powers. Here are some examples with powers of 3.

$$3^1 = 3$$

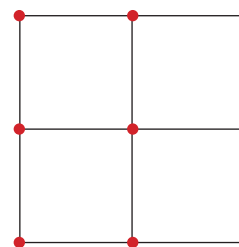
Read 3^1 as 'three to the power of one'.



$$3^1 = 3$$

$$3^2 = 3 \times 3 = 9$$

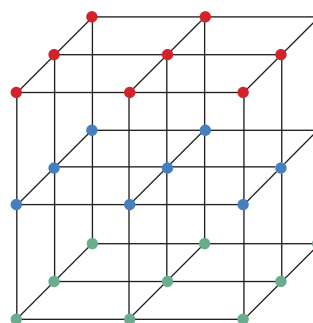
Read 3^2 as 'three squared' or 'three to the power of two'.



$$3^2 = 9$$

$$3^3 = 3 \times 3 \times 3 = 27$$

Read 3^3 as 'three cubed' or 'three to the power of three'.



$$3^3 = 27$$

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

Read 3^4 as 'three to the power of four'.

$$3^n = 3 \times 3 \times 3 \times \dots \times 3 \times 3, \text{ with } n \text{ factors of } 3 \text{ in the product.}$$

$$a^m = a \times a \times a \times \dots \times a \times a, \text{ with } m \text{ factors of } a \text{ in the product.}$$



Index notation

- A **power** is the product of a certain number of factors, all of which are the same. For example: $2^4 = 2 \times 2 \times 2 \times 2$.
- The number 2 in the example above is called the **base** of the power.
- The number 4 in the example above is called the **index** or **exponent**.
- For any number a , $a^1 = a$.

Index laws

There are rules, called the **index laws**, for handling indices with the same base.



Index law 1: A rule for multiplying powers of the same base

There is a very simple way to multiply two or more powers of the same base. Here is an example that explains the rule.

$$\begin{aligned} 7^2 \times 7^3 &= (7 \times 7) \times (7 \times 7 \times 7) && \text{(Write out each power.)} \\ &= 7^5 && \text{(There are five 7s multiplied together.)} \end{aligned}$$

Notice that we add the two indices to get the result:

$$\begin{aligned} 7^2 \times 7^3 &= 7^{2+3} \\ &= 7^5 \end{aligned}$$



Index law 1

When multiplying numbers written using powers, if the base number is the same, add the indices.

$$a^m \times a^n = a^{m+n}$$

Example 3

Write $5^2 \times 5^7$ as a single power of 5.

Solution

$$\begin{aligned} 5^2 \times 5^7 &= 5^{2+7} \\ &= 5^9 \end{aligned}$$

Index law 2: A rule for dividing powers of the same base

There is also a simple rule for dividing powers of the same base. Again, an example shows what the rule must be.

$$\begin{aligned}5^7 \div 5^3 &= \frac{5^7}{5^3} \\&= \frac{\cancel{5}^1 \times \cancel{5}^1 \times \cancel{5}^1 \times 5 \times 5 \times 5 \times 5}{\cancel{5}^1 \times \cancel{5}^1 \times \cancel{5}^1} \quad (\text{Three factors cancel out.}) \\&= 5 \times 5 \times 5 \times 5 \quad (\text{This leaves } 7 - 3 = 4 \text{ factors behind.}) \\&= 5^{7-3} \quad (\text{Notice that we subtract the indices to get the result.}) \\&= 5^4\end{aligned}$$



Index law 2

When dividing a number a^m by another number a^n , subtract the indices.

$$a^m \div a^n = a^{m-n} \quad \text{or} \quad \frac{a^m}{a^n} = a^{m-n}$$

Example 4

Write **a** $3^6 \div 3^2$ as a single power of 3. **b** $3^7 \div 3^6$ as a single power of 3.

Solution

$$\begin{aligned}\text{a } 3^6 \div 3^2 &= \frac{3^6}{3^2} \\&= 3^4\end{aligned}$$

$$\begin{aligned}\text{b } 3^7 \div 3^6 &= \frac{3^7}{3^6} \\&= 3\end{aligned}$$

Index law 3: A rule for raising a power to a power

Once again, an example makes the rule for raising a power to a power clear.

$$\begin{aligned}(8^3)^4 &= 8^3 \times 8^3 \times 8^3 \times 8^3 \quad (\text{This is what '8}^3 \text{ to the power of 4' means.}) \\&= 8^{3+3+3+3} \quad (\text{index law 1}) \\&= 8^3 \times 4 \\&= 8^{12}\end{aligned}$$



Index law 3

When a power is raised to another power, multiply the indices.

$$(a^m)^n = a^{m \times n}$$

Example 5

Write $(5^3)^2$ as a single power of 5.

Solution

$$\begin{aligned}(5^3)^2 &= 5^{3 \times 2} \\ &= 5^6\end{aligned}$$

Example 6

Write $(2^4)^6$ as a single power of 2.

Solution

$$\begin{aligned}(2^4)^6 &= 2^{4 \times 6} \\ &= 2^{24}\end{aligned}$$

Index law 4: Powers of products

It is often useful to expand a power of a product.

For example, we can expand:

$$\begin{aligned}(3 \times 4)^5 &= (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \\ &= (3 \times 3 \times 3 \times 3 \times 3) \times (4 \times 4 \times 4 \times 4 \times 4) \\ &= 3^5 \times 4^5\end{aligned}$$



Index law 4

A power of a product is the product of the powers.

$$(a \times b)^m = a^m \times b^m$$

Example 7

- a** Expand the brackets in $(5 \times 4)^2$ and write the answer as a product of powers. Then calculate the answer.
- b** Use the above index law to evaluate $5^4 \times 2^4$.

(continued on next page)

Solution

$$\begin{aligned}\mathbf{a} \quad (5 \times 4)^2 &= 5^2 \times 4^2 \\ &= 25 \times 16 \\ &= 400\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 5^4 \times 2^4 &= (5 \times 2)^4 \\ &= 10^4 \\ &= 10\,000\end{aligned}$$

Index law 5: Powers of quotients

The brackets in a power of a quotient can also be expanded.

$$\begin{aligned}\left(\frac{2}{3}\right)^5 &= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \\ &= \frac{2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3} \\ &= \frac{2^5}{3^5}\end{aligned}$$



Index law 5

A power of a quotient is the quotient of the powers.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Example 8

Evaluate $\frac{20^3}{5^3}$.

Solution

$$\begin{aligned}\frac{20^3}{5^3} &= \left(\frac{20}{5}\right)^3 \\ &= 4^3 \\ &= 64\end{aligned}$$



Indices



Index law 1

To multiply powers of the same base, add the indices.

$$a^m \times a^n = a^{m+n}$$

Index law 2

To divide powers of the same base, subtract the indices.

$$a^m \div a^n = a^{m-n}$$

Index law 3

To raise a power to a power, multiply the indices.

$$(a^m)^n = a^{m \times n}$$

Index law 4

A power of a product is the product of the powers.

$$(a \times b)^m = a^m \times b^m$$

Index law 5

A power of a quotient is the quotient of the powers.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Zero index

Clearly $\frac{4^3}{4^3} = 1$. If the index laws are to apply then $4^{3-3} = 4^0 = 1$. Hence we define $a^0 = 1$, for all non-zero a .

For example: $2^0 = 1$, $5^0 = 1$, $2 \times 6^0 = 2$ and $(2 \times 6)^0 = 1$.



Exercise 3B

- Copy and complete this sentence.
'The expression 5^3 is a _____, in which 5 is the _____ and 3 is the _____.'
- Evaluate:
a 4^2 b 9^2 c 12^2 d 100^2 e 2000^2 f 25^2
- a Write down three powers of base 7.
b Write down three powers with index 7.

c Write 49, 144 and 8 as powers with indices greater than 1.

d Write 64 as a power with:

i index 1 ii index 2 iii index 3 iv index 6

4 a Evaluate all the powers of 2 up to 2^{10} .

b Evaluate all the powers of 5 up to 5^4 .

5 Simplify each of the following, using the index law given in brackets.

a $2^2 \times 2^3$ (index law 1)

b $3^2 \times 4^2 \times 3^2$ (index law 1)

c $3^3 \div 3^2$ (index law 2)

Example 3

6 Write each expression as a single power.

a $7^3 \times 7^2$

b $11^2 \times 11^4$

c $11^5 \times 11$

d $3^2 \times 3^7$

e $13^2 \times 13^5$

f $10^2 \times 10^3$

Example 4

7 Write each expression as a single power.

a $7^3 \div 7^2$

b $11^5 \div 11^2$

c $5^5 \div 5^2$

d $3^3 \div 3$

e $6^{12} \div 6^{11}$

f $6^{13} \div 6^{12}$

g $\frac{5^7}{5^2}$

h $\frac{7^{11}}{7^2}$

i $\frac{5^{12}}{5^7}$

j $\frac{2^{11}}{2^2}$

k $\frac{2^{15}}{2^3}$

l $\frac{5^{17}}{5^{13}}$

Example 5, 6

8 Write each expression as a single power.

a $(2^3)^2$

b $(3^5)^2$

c $(2^5)^2$

d $(11^2)^3$

e $(5^2)^3$

f $(7^2)^4$

9 Write each expression as a single power.

a $3^6 \times 3^7$

b $8^5 \times 8^2$

c $5^3 \times 5 \times 5^4$

d $10^{12} \times 10^{13} \times 10^{20}$

e $6^7 \div 6^3$

f $2^{12} \div 2^4$

g $7^{10} \div 7$

h $4^6 \times 4^5 \div 4^9$

i $(12^4)^3$

j $(11^7)^9$

k $(4^5)^2 \times 4^6$

l $(10^4)^6 \div 10^{20}$

10 Evaluate:

a $2^4 \times 5^4$

b $4^3 \times 5^3$

c $2^5 \times 5^3$

11 Evaluate:

a 5^0

b 7^0

c 2×6^0

d $(2 \times 6)^0$

e 7×11^0

f $6^2 \times 6^0$

g $\frac{7^{11}}{7^{10} \times 7^0}$

h $(6^7)^0$

i $(5^0)^6$



Brackets are used to control the order in which operations are done. For example:

$$\begin{array}{lll} \text{a} & (3 + 4) \times 5 = 7 \times 5 & \text{b} \quad (5 \times 4)^3 = 20^3 \\ & = 35 & = 8000 \\ & & \text{c} \quad 20 - (8 + 3) = 20 - 11 \\ & & = 9 \end{array}$$

However, when there are no brackets, we need a set of conventions, known as the **order of operations**, to control the order in which operations are done.



Order of operations

- Evaluate expressions inside brackets first.
- In the absence of brackets, and within brackets, carry out operations in the following order:
 - indices
 - multiplication and division from left to right
 - addition and subtraction from left to right.

In the following examples, the brackets have been removed from the expressions **a** to **c** above.

The order of operations conventions mean that the operations are done in different order, and the answers are quite different. We can now see the effect of removing the brackets from **a**, **b** and **c**.

$$\begin{array}{ll} \text{a} & 3 + 4 \times 5 = 3 + 20 \\ & = 23 \quad \text{(Do multiplication before addition.)} \\ \text{b} & 5 \times 4^3 = 5 \times 64 \\ & = 320 \quad \text{(Calculate the power first.)} \\ \text{c} & 20 - 8 + 3 = 12 + 3 \\ & = 15 \quad \text{(Do addition and subtraction from left to right.)} \end{array}$$

Example 9

Evaluate:

$$\begin{array}{lll} \text{a} & 3 \times (6 + 5) & \text{b} \quad 17 + 3 \times 2 \\ \text{d} & (50 + 10)^2 & \text{e} \quad (16 - 4)^2 \\ & & \text{f} \quad 20 - 4 \times 3 \end{array}$$

Solution

$$\begin{array}{ll} \text{a} & 3 \times (6 + 5) = 3 \times 11 \\ & = 33 \\ \text{b} & 17 + 3 \times 2 = 17 + 6 \\ & = 23 \\ \text{c} & 7 \times 3^2 = 7 \times 9 \\ & = 63 \\ \text{d} & (50 + 10)^2 = 60^2 \\ & = 3600 \\ \text{e} & (16 - 4)^2 = 12^2 \\ & = 144 \\ \text{f} & 20 - 4 \times 3 = 20 - 12 \\ & = 8 \end{array}$$

Example 10

Evaluate:

a $3^3 \times (2^2 + 12) \div 6 - 4^2$

b $3^3 \times 2^2 + 12 \div (6 - 4)^2$

c $(3^3 \times 2^2 + 12 \div 6 - 4)^2$

Solution

a $3^3 \times (2^2 + 12) \div 6 - 4^2 = 27 \times (4 + 12) \div 6 - 16$
 $= 27 \times 16 \div 6 - 16$
 $= 432 \div 6 - 16$
 $= 72 - 16$
 $= 56$

b $3^3 \times 2^2 + 12 \div (6 - 4)^2 = 27 \times 4 + 12 \div 4$
 $= 108 + 3$
 $= 111$

c $(3^3 \times 2^2 + 12 \div 6 - 4)^2 = (27 \times 4 + 12 \div 6 - 4)^2$
 $= (108 + 2 - 4)^2$
 $= 106^2$
 $= 11\,236$



Exercise 3C

Example 9

1 Use the order of operations conventions to evaluate these expressions.

a $3 \times (5 + 11)$

b $3 \times 5 + 11$

c $(20 - 8) \times 2$

d $20 - 8 \times 2$

e $(9 \times 2)^3$

f 9×2^3

g $(20 \div 2)^2$

h $20 \div 2^2$

i $(11 - 1)^4$

j $11 - 1^4$

k $(90 + 10)^2$

l $90 + 10^2$

2 Evaluate these expressions. You will need to use the 'work from left to right' convention.

a $40 - (17 - 12)$

b $40 - 17 - 12$

c $52 - (16 + 12)$

d $52 - 16 + 12$

e $(44 - 20) - (12 - 8)$

f $44 - 20 - 12 - 8$

g $(60 + 25) - (20 + 15)$

h $60 + 25 - 20 + 15$

i $48 \div (12 \times 2)$

j $48 \div 12 \times 2$

k $90 \div (6 \div 3)$

l $90 \div 6 \div 3$

3 Evaluate:

a $6^2 + 20 \div 2^2 + 15$

b $4^3 + 6 \times 8 + 5^2$

c $2^3 + (2^2 - 2) \times 2^4$

d $2^2 \times 2^2 + 2 \div 2 - 2^2$

Example 10

e $2^3 \times 2^3 + 2 \div 2 - 2^3$

f $2^3 \times (2^3 + 2) \div 2 - 2^3$

g $18 \div 6 - 5 + 3^2 \times 4^3$

h $7^3 \times 2^2 + 12$

i $\frac{3^3 \times 4^2}{2 \times 3^2}$

4 Copy these statements, inserting brackets if necessary to make them true.

a $4 \times 3 + 7 = 40$

b $70 - 20 \div 5 = 10$

c $2^2 + 6 \times 4 \div 2^2 = 7$

d $4 + 3^2 - 2 = 11$

e $3 + 2^2 \times 2^3 - 7^2 = 7$

f $4 + 4^4 \div 2^3 + 2 = 26$

5 Copy these statements, inserting brackets if necessary to make them true.

a $4 \times 2 + 7 = 36$

b $4 \times 2 + 7 = 15$

c $20 - 2 + 2 - 2 + 2 = 12$

d $20 - 2 + 2 - 2 + 2 = 16$

e $70 - 20 \div 5 = 10$

f $70 - 20 \div 5 = 66$

g $2^2 + 2 \times 8 \div 2^2 = 12$

h $2^2 + 2 \times 8 \div 2^2 = 8$

i $2^2 + 2 \times 8 \div 2^2 = 144$

j $11 - 2 + 3^2 = 144$

k $11 - 2 + 3^2 = 18$

l $11 - 2 + 3^2 = 0$



We have already seen that a number is divisible by another number if the second number divides it exactly, with no remainder.

Sometimes it is easy to see that one number divides another exactly.

For example, 2 divides 12 because $12 = 2 \times 6$.

As another example, 10 divides 10000 exactly, because $10000 = 10 \times 1000$.

But what about situations where it is not so easy to see if one number is divisible by another? Sometimes it is helpful to know about divisibility without having to use the division algorithm.

You may already be aware of the following divisibility properties of numbers and their relationship to counting patterns.

Divisibility tests for 2, 10 and 5

A number that ends in a zero is divisible by 10. For example, 20, 450 and 23 890 are all divisible by 10. This can be seen in the counting pattern:

10, 20, 30, 40, . . . , 670, 680, 690, 700, . . .

Numbers that do not end in 0 are not divisible by 10.

The numbers that are divisible by 2 – the even numbers – can be listed by counting by 2s:

0, 2, 4, 6, 8, 10, . . . , 452, 454, 456, . . .

It is easy to see why this test for divisibility by 2 works.

For example, we write 352 as a multiple of 10 plus its last digit: $35 \times 10 + 2$. Since we know that any multiple of 10 is divisible by 2, then we only need to look at the last digit. If this last digit is divisible by 2, then the entire number is divisible by 2. Conversely, if the last digit of a number is not even, then the number is not divisible by 2.

If the last digit of a number is 5, then it is divisible by 5. For example, 15, 225 and 439 785 are all divisible by 5. Also, it follows that a number divisible by 10 is also divisible by 5.

So any number that ends in 0 or 5 is also divisible by 5. Conversely, if the last digit of a number is not 0 or 5, then the number is not divisible by 5. The counting pattern shows the numbers that are divisible by 5:

5, 10, 15, 20, 25, . . . , 985, 990, 995, 1000, . . .

Divisibility tests for 3 and 9

A number is divisible by 3 if the sum of its digits is divisible by 3. Any number whose digit sum is not divisible by 3 is not itself divisible by 3. For example, 39 is divisible by 3 because the sum of its digits is equal to $3 + 9 = 12$, and 12 is divisible by 3.

A number is divisible by 9 if the sum of its digits is divisible by 9. Any number whose digit sum is not divisible by 9 is not itself divisible by 9. For example, 13 968 is divisible by 9 because the sum of its digits is equal to $1 + 3 + 9 + 6 + 8 = 27$ and 27 is divisible by 9.

The proof of why the test for divisibility by 9 works is illustrated for 684 in the example below.

In expanded form, 684 can be written as:

$$684 = 100 \times 6 + 10 \times 8 + 4.$$

We know that $100 = 99 + 1$ and $10 = 9 + 1$, so we can rewrite the above expression as:

$$\begin{aligned} 684 &= (99 \times 6 + 1 \times 6) + (9 \times 8 + 1 \times 8) + 4 \\ &= 99 \times 6 + 6 + 9 \times 8 + 8 + 4 \\ &= (99 \times 6 + 9 \times 8) + 6 + 8 + 4. \end{aligned}$$

Since 9 divides 99, 9 divides $(99 \times 6 + 9 \times 8)$. We are left with $6 + 8 + 4$ (the sum of the digits), which is equal to $6 + 8 + 4 = 18$. This is divisible by 9, so the entire number is divisible by 9.

Note also that any number that is divisible by 9 is also divisible by 3, so 684 is divisible by both 9 and 3. The proof of the test for divisibility by 3 is essentially the same as that for divisibility by 9.

Divisibility test for 6

A number is divisible by 6 if it is both even and divisible by 3. This is because we know that $2 \times 3 = 6$. It follows that if a number is divisible by 3 and it is even, then it is divisible by 6.

Divisibility tests for 4 and 8

A number is divisible by 4 if the number formed by the last two digits is divisible by 4. Conversely, if the number formed by the last two digits of a number is not divisible by 4, then the number itself is not divisible by 4.

For example, 124 is divisible by 4 because the number formed by its last two digits is 24, which is divisible by 4.

The reason why the test for divisibility by 4 works is illustrated in the following example.

$$\begin{aligned} 324 &= 300 + 24 \\ &= (3 \times 100) + 24 \end{aligned}$$

Since we know that 100 is divisible by 4, we are left with 24. Since 24 is divisible by 4, then 324 is divisible by 4.

A number is divisible by 8 if the number formed by the last three digits is divisible by 8. Conversely, if the number formed by the last 3 digits of a number is not divisible by 8, then the number itself is not divisible by 8.

For example, 5472 is divisible by 8 because 472 is divisible by 8. Since 8 divides 1000, 8 divides any multiple of 1000, so we only need to deal with the last three digits.

Divisibility test for 11

To test a number for divisibility by 11, we follow this procedure. Add up the digits in the odd positions, then add up the digits in the even positions. If the difference of these two numbers is divisible by 11, then the given number is divisible by 11.

For example, 82786 is divisible by 11 because:

$$(8 + 7 + 6) - (2 + 8) = 21 - 10 \\ = 11.$$

Example 11

Is 1848 divisible by 2, 3, 4, 5, 9 and 11?

Solution

1848 is divisible by 2 as it ends in 8.

The sum of the digits is $1 + 8 + 4 + 8 = 21$. Since 21 is divisible by 3, 1848 is divisible by 3.

Since the number formed from the last two digits (48) is divisible by 4, the given number is divisible by 4.

1848 does not end in 5 so it is not divisible by 5.

The sum of the digits is $1 + 8 + 4 + 8 = 21$. Since 21 is not divisible by 9, 1848 is not divisible by 9.

The sum of the odd-position digits minus the sum of the even-position digits is $(8 + 8) - (1 + 4) = 16 - 5 = 11$. This is divisible by 11, so 1848 is also divisible by 11.

Summary: 1848 is divisible by 2, 3, 4 and 11, but is not divisible by 5 or 9.

By the way, $1848 = 2^3 \times 3 \times 7 \times 11$ as a product of its primes.

Exercise 3D

Example 11

- 1 Test each number for divisibility by 2, 4 and 8.

a 2896 b 56374 c 1858732 d 280082

Example 11

- 2 Test each number for divisibility by 5, 25 and 125.

(A number is divisible by 25 if its last two digits are 00, 25, 50 or 75. A number is divisible by 125 if its last three digits are 000, 125, 250, 375, 500, 625, 750 or 875, since 125 divides 1000.)

a 3765 b 68275 c 444300 d 556875

3 Test each number for divisibility by 3, 9 and 11.

a 5679 **b** 7425 **c** 71 643 **d** 1727

4 Test each number for divisibility by 6 and 12.

(A number is divisible by 12 if it is divisible by both 4 and 3, because $12 = 4 \times 3$.)

a 3798 **b** 5772 **c** 9909 **d** 48 882

5 Test each number for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, 11 and 12.

a 672 **b** 49 395
c 136 290 **d** 242 010 000 437 000 361

6 Fill in the boxes to make a six-digit number 3__1__3__ divisible by:

a 3 **b** 5 **c** 3 and 5
d 2, 4 and 9 **e** 3, but not 9 **f** 11
g 4 and 11 **h** 6 and 8, but not 9 **i** 3, but not 6

7 List the other numbers by which a number is divisible if it is divisible by:

a 9 **b** 2 and 3 **c** 2 and 5 **d** 8

8 Test each number for divisibility by 12, 15 and 18.

(A number is divisible by 15 if it is divisible by both 3 and 5, because $15 = 3 \times 5$. A number is divisible by 12 if it is divisible by both 4 and 3, because $12 = 4 \times 3$. A number is divisible by 18 if it is divisible by both 2 and 9, because $18 = 2 \times 9$.)

a 36 450 **b** 21 942 **c** 2041 200 **d** 2007 000 000



Any whole number can be broken down, by factorisation, into a product of prime numbers. This means that prime numbers are the building blocks from which all whole numbers greater than 1 are built up by multiplication. For example:

$$\begin{array}{lcl} 12 = 3 \times 4 & & 12 = 6 \times 2 \\ = 3 \times 2 \times 2 & \text{or} & = 2 \times 3 \times 2 \\ = 2^2 \times 3 & & = 2^2 \times 3 \end{array}$$

We have just found the **prime factorisation** of 12 in two different ways. The prime factorisation of 12 is $2^2 \times 3$.

Factorisation into primes always gives the same answer, apart from the order of the factors. The final answer is usually written using index notation, with the primes placed in increasing order. This fact is called the **fundamental theorem of arithmetic**.

Here are two of the many ways to carry out the factorisation of 420 into primes.

$$\begin{array}{lcl} 420 = 2 \times 210 & & 420 = 60 \times 7 \\ = 2 \times 21 \times 10 & \text{or} & = 6 \times 10 \times 7 \\ = 2 \times 3 \times 7 \times 2 \times 5 & & = 2 \times 3 \times 2 \times 5 \times 7 \\ = 2^2 \times 3 \times 5 \times 7 & & = 2^2 \times 3 \times 5 \times 7 \end{array}$$

The example below shows another standard way to set out these calculations.

Example 12

Express 1260 as the product of its prime factors.

Solution

2	1 2 6 0
3	6 3 0
5	2 1 0
7	4 2
3	6
	2

Hence $1260 = 2 \times 3 \times 5 \times 7 \times 3 \times 2$
 $= 2 \times 2 \times 3 \times 3 \times 5 \times 7$
 $= 2^2 \times 3^2 \times 5 \times 7.$

Using prime factorisation to find the LCM

At the beginning of this chapter, we found the lowest common multiple (LCM) of two numbers by writing down lists of the multiples of each number. For larger numbers this is not very practical – prime factorisation provides a better method.

To find the LCM of 24 and 18, we first factorise each number as a product of primes:

$$\begin{aligned} 24 &= 2 \times 2 \times 2 \times 3 \\ &= 2^3 \times 3 \end{aligned}$$

$$\begin{aligned} 18 &= 2 \times 3 \times 3 \\ &= 2 \times 3^2 \end{aligned}$$

Both numbers have powers of 2 as a factor. The larger, 2^3 , must be taken as a factor of the LCM, otherwise the LCM could not be a multiple of 24. Similarly, the larger power of 3 is 3^2 . It must be taken as a factor of the LCM because the LCM must be a multiple of 18. So the LCM of 24 and 18 is:

$$\begin{aligned} 2^3 \times 3^2 &= 8 \times 9 \\ &= 72. \end{aligned}$$

Example 13

What is the LCM of 180 and 144?

Solution

Factorise each number into a product of primes.

$$\begin{aligned} 180 &= 2 \times 2 \times 3 \times 3 \times 5 \\ &= 2^2 \times 3^2 \times 5 \end{aligned}$$

$$\begin{aligned} 144 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ &= 2^4 \times 3^2 \end{aligned}$$

Take the greatest index of each prime.

$$\begin{aligned} \text{The LCM of 180 and 144 is } 2^4 \times 3^2 \times 5^1 &= 16 \times 9 \times 5 \\ &= 720. \end{aligned}$$

Using prime factorisation to find the HCF

Prime factorisation provides an effective method for finding the highest common factor (HCF) of two or more numbers.

To find the HCF of 24 and 18, we again factorise each number into primes:

$$\begin{aligned} 24 &= 2 \times 2 \times 2 \times 3 \\ &= 2^3 \times 3^1 \end{aligned}$$

$$\begin{aligned} 18 &= 2 \times 3 \times 3 \\ &= 2^1 \times 3^2 \end{aligned}$$

Each number contains a power of 2 and a power of 3 in its prime factorisation.

To get the highest common factor of 24 and 18, we take the lower power of 2, which is $2 = 2^1$, and multiply it by the lower power of 3, which is $3 = 3^1$. No higher power than 3^1 can divide the HCF. No higher power than 2^1 can divide the HCF. The HCF of 24 and 18 is therefore $2 \times 3 = 6$.

The reason this works is that we cannot take any higher power of 2 and still have a factor of 18, nor can we take a higher power of 3 and still have a factor of 24.

Example 14

What is the HCF of 180 and 144?

Solution

Factorise both numbers into primes.

$$\begin{aligned}180 &= 2 \times 2 \times 3 \times 3 \times 5 \\ &= 2^2 \times 3^2 \times 5^1\end{aligned}$$

$$\begin{aligned}144 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ &= 2^4 \times 3^2\end{aligned}$$

Take the smallest index of each prime common factor.

The HCF of 180 and 144 is $2^2 \times 3^2 = 36$.



Factors and divisibility

- The **fundamental theorem of arithmetic** states that every whole number greater than 1 can be expressed as a product of prime numbers. This product for each number is unique apart from the order of the prime factors.
- Prime factorisation can be used to find the lowest common multiple. Take the *greatest* index for each prime.
- Prime factorisation can be used to find the highest common factor. Take the *smallest* index for each prime.

Squares and square roots

The **square** of a number is the product of that number with itself. For example, 8 squared is the number $8 \times 8 = 64$. The number 64 is called a **perfect square**, because it is the square of a whole number.

The **square root** of a number is the number that when multiplied by itself gives the first number. For example, $8^2 = 64$, so 8 is the square root of 64.

We write $\sqrt{64} = 8$.

Cubes and cube roots

The **cube** of a number is defined similarly. So the cube of 4 is $4 \times 4 \times 4 = 4^3 = 64$.

The **cube root** of 64 is 4 because $4^3 = 64$, and we write $\sqrt[3]{64} = 4$.

The cube root of 125 is 5, because the cube of 5 is 125. We write $\sqrt[3]{125} = 5$, because $5^3 = 125$.

We can see that taking the cube root is the reverse operation to cubing.

Note that most numbers do not have a square or cube root that is a whole number. For example:

- Neither $\sqrt{2}$ nor $\sqrt[3]{2}$ is a whole number.
- $\sqrt{9} = 3$ because $3^2 = 9$, but $\sqrt[3]{9}$ is not a whole number.
- $\sqrt{8}$ is not a whole number, but $\sqrt[3]{8} = 2$ because $2^3 = 8$.

Example 15

What is the cube root of 1000?

Solution

$$\begin{aligned} 1000 &= 10 \times 10 \times 10 \\ &= 10^3 \\ \sqrt[3]{1000} &= 10 \end{aligned}$$

Finding square roots and cube roots by prime factorisation

Prime factorisation can help us find the square root of a perfect square. For larger numbers, prime factorisation is the best way of finding their square roots and cube roots, if these are whole numbers.

For example, here is the prime factorisation of 324.

$$\begin{aligned} 324 &= 4 \times 81 \\ &= 2 \times 2 \times 9 \times 9 \\ &= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \\ &= 2^2 \times 3^4 \end{aligned}$$

Since both indices are even, we can use the index laws to write 324 as the square of a whole number.

$$\begin{aligned} 324 &= 2^2 \times 3^4 \\ &= (2^1 \times 3^2)^2 \quad (\text{index laws 3 and 4}) \end{aligned}$$

Since the square of $2^1 \times 3^2$ is 324, $\sqrt{324} = 2 \times 3^2 = 18$.

However, the indices are not multiples of 3, so $\sqrt[3]{324}$ is not a whole number.

Example 16

Find the square root of 7056, using the prime factorisation of 7056.

Solution

7	7	0	5	6
7	1	0	0	8
3		1	4	4
3			4	8
2			1	6
2				8
2				4
2				2
				1

$$\begin{aligned}\text{Hence } 7056 &= 2^4 \times 3^2 \times 7^2 \\ &= (2^2 \times 3 \times 7)^2\end{aligned}$$

Therefore, 7056 is a square number and $\sqrt{7056} = 2^2 \times 3 \times 7 = 84$.

It is not necessary to factor one prime at a time, as the following example shows.

Example 17

Find $\sqrt[3]{1728}$.

Solution

The prime factorisation of 1728 is

$$\begin{aligned}1728 &= 4 \times 432 \\ &= 4 \times 4 \times 108 \\ &= 4 \times 4 \times 4 \times 27 \\ &= (2^2)^3 \times 3^3 \\ &= (2^2 \times 3)^3, \\ \text{so } \sqrt[3]{1728} &= 2^2 \times 3 = 12.\end{aligned}$$

Note that we could not calculate $\sqrt{1728}$ in this way because not all the indices are even, so $\sqrt{1728}$ is not a whole number.



Perfect squares and square roots

- A number multiplied by itself gives the **square** of the original number.
- The **square root** of a square number is the number that when multiplied by itself gives the original number.
- The square of a whole number is called a **perfect square**.
- We can use prime factorisation to determine whether a whole number is a square and to find the square root of a number.



Cubes and cube roots

- The **cube** of a number a is the product $a \times a \times a = a^3$.
- Taking the **cube root** is the inverse operation of cubing.
- We can use prime factorisation to determine whether a whole number is a cube and to find the cube root of a number.



Exercise 3E

Example 12

1 Find the prime factorisation of:

a 8 **b** 24 **c** 45 **d** 891 **e** 735 **f** 2904

Example 13, 14

2 Given the prime factorisations $144 = 2^4 \times 3^2$, $108 = 2^2 \times 3^3$ and $405 = 3^4 \times 5$:

- a** find the HCF of 144 and 108.
b find the LCM of 144 and 108.
c find the HCF of 108 and 405.
d find the LCM of 108 and 405.
e find the HCF of 108, 144 and 405.
f find the LCM of 108, 144 and 405.

3 Use prime factorisation to find:

- | | |
|-------------------------------------|------------------------------------|
| a the HCF of 36 and 45 | b the LCM of 36 and 45 |
| c the HCF of 112 and 21 | d the LCM of 112 and 21 |
| e the LCM of 12, 15 and 40 | f the LCM of 24, 72 and 108 |
| g the HCF of 72 and 126 | h the HCF of 24, 60 and 112 |
| i the HCF of 70, 105 and 280 | |

- 4 Given the prime factorisation, find the square root and the cube root of each number below, if they are whole numbers.
- a $3375 = 3^3 \times 5^3$ b $729 = 3^6$ c $4356 = 2^2 \times 3^2 \times 11^2$
- 5 Find the prime factorisation of each number below. Then find its square root and cube root, if they are whole numbers.
- a 225 b 324 c 512 d 1728
- e 360 f 1936 g 216 h 10648
- 6 Mr and Mrs Cinco have quintuplets, three boys and two girls. When buying toys, to avoid squabbles, they always make sure that the toy collection can be shared equally whether the five children play together, the girls play alone or the boys play alone.
- At the beginning of the year, the number of toys in the collection is 180. What is the smallest number of toys they can buy for the children's birthday so that the family can continue sharing the toys in the same way as before?
- 7 Find the prime factorisation of 360, and use it to write down all 24 factors of 360.

Review exercise



- 1 The product of the factors of 12 is:
A 12 **B** 28 **C** 1728 **D** 144 **E** 3556
- 2 The sum of the factors of 18 is:
A 9 **B** 36 **C** 5832 **D** 39 **E** 2916
- 3 Write each of these numbers as the sum of two multiples of 8.
a 96 **b** 40 **c** 184 **d** 480 **e** 296
- 4 Find the LCMs of the following sets of numbers.
a 8 and 6 **b** 10 and 18 **c** 16 and 24
d 60 and 72 **e** 6, 10 and 15 **f** 10, 12 and 18
g 36, 45 and 27 **h** 12, 60 and 102 **i** 30, 40, 50 and 75
- 5 Find the HCFs of the following sets of numbers.
a 6 and 9 **b** 45 and 75 **c** 12 and 35
d 60 and 70 **e** 21 and 35 **f** 132 and 680
g 12, 18 and 30 **h** 18, 30 and 90 **i** 42, 189 and 231
- 6 Evaluate:
a 6^2 **b** 11^2 **c** 50^2 **d** 31^2 **e** 87^2 **f** 143^2
- 7 Write each expression as a single power.
a $8^2 \times 8^4$ **b** $2^{13} \div 2^9$ **c** $3^5 \times 3^5 \times 3^5$ **d** $4^9 \times 4^{19}$
e $10^3 \times 10^2 \times 10^3 \times 10^4$ **f** $7^{11} \div 7^2 \times 7^8$ **g** $(5^2)^3$
h $(8^2)^3 \div 8^2$ **i** $12^4 \times (12^5)^2$
- 8 **a** The LCM of $2^2 \times 3^1 \times 5^4$ and $2^3 \times 3^2 \times 7^2$ is:
A $2 \times 5^4 \times 7^2$ **B** $2^3 \times 3^2 \times 5^4 \times 7^2$ **C** $2^2 \times 3^1 \times 5^1 \times 7^1$
b The HCF of $2^2 \times 3^1 \times 5^4$ and $2^3 \times 3^2 \times 7^2$ is:
A 75 **B** 3528 **C** 12 **D** 210 **E** $2^3 \times 3^2$
- 9 Standing on the seashore, Stephanie can see two ships' lights flashing. One light flashes at intervals of 12 seconds and the other at intervals of 18 seconds.
a If they flash together once, how long is it before they flash together again?
b If they flash together at the start of a 5-minute interval, how many times will they flash together in those 5 minutes?
- 10 Show that each of the whole numbers between 28 and 37 can be written as the product of prime numbers.



- 11** Express each of these numbers as a product of its prime factors and so find its square root.
a 256 **b** 441 **c** 1156 **d** 7569 **e** 18225 **f** 20736
- 12** Write each prime number as the sum of two square numbers.
a 73 **b** 137 **c** 229
- 13** Test each of the following numbers for divisibility by 3, by 4 and by 12.
a 64 **b** 1336 **c** 3972
- 14** Write a five-digit number that has one of its digits repeated three times and is:
a divisible by 2
b divisible by 5 and 2
c divisible by 3 and 4
d divisible by 11
e divisible by 8 and 3
f divisible by 2 and 11 but not 4
- 15** Evaluate:
a $3 \times 5^2 + 6^2 - 2^3$ **b** $(3 \times 5^2 + 6)^2 - 2^3$ **c** $(3 \times 5)^2 + (6^2 - 2)^3$
d $[(3 \times 5)^2 + 6]^2 - 2^3$ **e** $3 \times (5^2 + 6^2 - 2)^3$ **f** $(3 \times 5^2 - 2 \times 6^2 - 2)^3$
- 16** A yellow bulb flashes every 4 minutes and a blue bulb flashes every 7 minutes. If both bulbs flash together at 9 a.m., what is the first time after 10 a.m. that both bulbs will flash together?
- 17** How many factors do each of the following have?
a 2^3 **b** 5^3 **c** 7^3 **d** 11^3
- 18** How many factors do each of the following have?
a 2^5 **b** 3^5 **c** 7^5 **d** 11^5
- 19** Write down the factors of:
a 111 **b** 1001 **c** 1111
- 20** Find the highest common factor of each of the following pairs.
a $2^3 \times 3^4, 2^2 \times 3^6 \times 5$ **b** $2^4 \times 3 \times 5, 2^3 \times 3^2 \times 5^2$

Challenge exercise



- 1 A number is greater than 48 squared and less than 49 squared. If 6 squared is one of its factors, and it is a multiple of 13, what is the number?
- 2 Given a six-digit number $328xyz$, find the values of x , y and z such that the number is divisible by 3, 4 and 5, and is the smallest six-digit number starting with 328 that has this property. How many other numbers have this property?
- 3 When multiplying a number by 11, we can use the following technique. First, write down the last digit of the number to be multiplied. Then, *working from right to left*, write down the sum of every pair of adjacent numbers. Remember to carry correctly. Finally, write down the first digit of the original number. For example:

$$\begin{array}{ccccccc}
 & & 5 & 3 & 8 & 9 & \\
 & \nearrow & & \nearrow & & \nearrow & \\
 5389 \times 11 = & 5 & 9 & 2 & 1 & 7 & 9 \\
 \text{first digit} & & & & & & \text{last digit} \\
 & & 5+3 & & 3+8 & & 8+9
 \end{array}$$

- a Write down, using the above method:
 - i 17089×11
 - ii 2223×11
 - iii 1654998×11
 - iv 1654998×111
 - b Briefly explain why this method works.
- 4 What three-digit number leaves a remainder of 1 when divided by 2, 3, 4, 5 or 6, and no remainder when divided by 7?
 - 5 The local takeaway chicken shop has all of its employees working the same number of hours each day. The total number of hours for Thursday is 133. How many employees work on Thursdays?
 - 6 Hoc knew that before he closed his novel, the product of the page numbers was 1190. What were the two open pages?
 - 7 It is 7 p.m. now. What will the time be 23 999 999 996 hours later?
 - 8 What is the smallest whole number that is divisible by 11 and leaves a remainder of 1 when divided by any of the numbers from 2 to 10?
 - 9 What is the largest prime factor of 93 093?
 - 10 The product of two whole numbers is 10 000 000. Neither number is a multiple of 10. What are the two numbers?
 - 11 A teacher multiplied the ages of all of the students (all teenagers) in her class and came up with the number 15 231 236 267 520.
 - a What is the prime factorisation of this number?
 - b Use the prime factorisation to help you work out the number of students in the class, their ages and the number of students of each age.



- 12** Twelve is the smallest whole number that has exactly 6 different factors. What is the smallest whole number that has exactly 24 different factors?
- 13 a** What is the smallest whole number that satisfies all of the following conditions?
- Dividing by 7 gives a remainder of 4.
 - Dividing by 8 gives a remainder of 5.
 - Dividing by 9 gives a remainder of 6.
- b** What is the smallest whole number that satisfies the conditions in part **a** and also the following ones?
- Dividing by 6 gives a remainder of 3.
 - Dividing by 11 gives a remainder of 8.
- 14 a** If the 4-digit number $8mn9$ is a perfect square, find the values of m and n .
- b** The digits 1, 2, 3, 4 and 5 are each used once in the 5-digit number $abcde$ such that the 3-digit number abc is divisible by 4, bcd is divisible by 5, and cde is divisible by 3. Find the values of a , b , c , d and e .
- c** The digits 1, 2, 3, 4, 5 and 6 are each used in the 6-digit number $abcdef$. The 3-digit number abc is divisible by 4, bcd is divisible by 5, cde is divisible by 3, and def is divisible by 11. Find the values of a , b , c , d , e and f .
- 15** Helena has a basket of chocolates. There are fewer than 500 chocolates in the basket. Helena knows that she can divide all of the chocolates into bags holding 2 each, 3 each or 6 each. She also knows that it is not possible to put them into bags of 4, 5, 9 or 10. If she tries to put 7 in each bag, there are 3 left over. How many chocolates are in Helena's basket?
- 16** What is the next number after 64 whose square roots and cube roots are both whole numbers?

CHAPTER

4

Number and Algebra

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Negative numbers

You have probably come across examples of negative numbers already. They are the numbers that are less than zero. Negative numbers are used, for example, in the measurement of temperature.

The temperature 0°C is the temperature at which water freezes, known as *freezing point*. The temperature that is 5 degrees colder than freezing point is written as -5°C .

In some Australian cities, the temperature drops below zero. Canberra has a lowest recorded temperature of -10°C . The lowest recorded temperature in Australia was -23°C , recorded at Charlotte's Pass in NSW. Here are some other lowest temperatures:

Alice Springs	-7°C	Paris	-24°C	London	-16°C
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Negative numbers are also used to record heights below sea level.

For example, the surface of the Dead Sea in Israel is at 417 metres below sea level. This can also be written as -417 metres above sea level. This is the lowest point on land anywhere on Earth. The lowest point on land in Australia is at Lake Eyre, which is 15 metres below sea level. This can also be written as -15 metres above sea level.



The whole numbers together with the negative whole numbers are called the **integers**.

..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

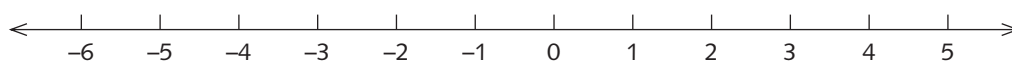
The numbers 1, 2, 3, 4, 5, ... are called the **positive integers**.

The numbers ..., -5, -4, -3, -2, -1 are called the **negative integers**.

The number 0 is neither positive nor negative.

The number line

The integers can be represented by points on a number line. The line is infinite in both directions, with the positive integers to the right of zero and the negative integers to the left of zero. The numbers are equally spaced.



An integer a is **less than** another integer b if a lies to the left of b on the number line.

The symbol $<$ is used for **less than**. For example, $-3 < -1$.



An integer b is **greater than** another integer a if b lies to the right of a on the number line.

The symbol $>$ is used for **greater than**. For example, $1 > -5$.

A practical illustration of this is that a temperature of -10°C is colder than a temperature of -5°C , and $-10 < -5$. Similarly, $-5 < 0$.

Example 1

- a** List all the integers less than 5 and greater than -2.
- b** List all the integers less than 1 and greater than -4.

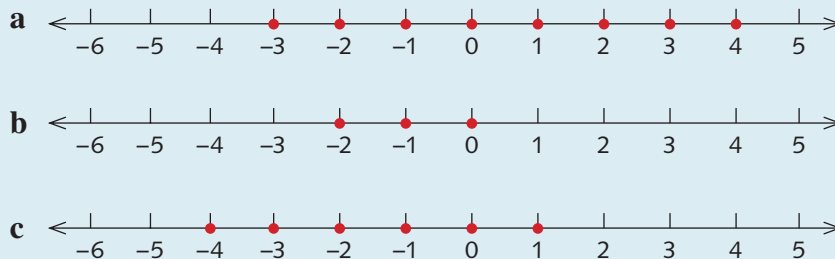
Solution

- a** -1, 0, 1, 2, 3, 4
- b** -3, -2, -1, 0

Example 2

- a** Draw a number line and indicate on it with dots all the integers less than 5 and greater than -4.
- b** Draw a number line and indicate on it with dots all the integers less than 1 and greater than -3.
- c** Draw a number line and indicate on it with dots all the integers greater than -5 and less than 2.

Solution

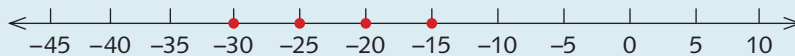


Example 3

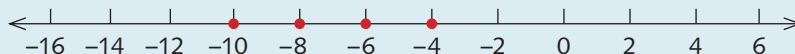
- a** The sequence 10, 5, 0, -5, -10, ... is 'going down by fives'.
Write down the next four terms.
- b** The sequence -16, -14, -12, ... is 'going up by twos'.
Write down the next four terms.

Solution

- a** The next four terms are -15, -20, -25, -30.



- b** The next four terms are -10, -8, -6, -4.



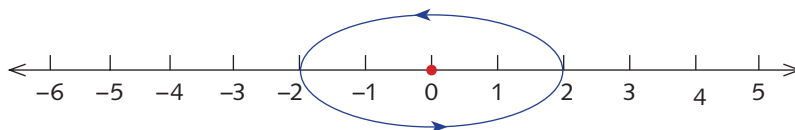
The opposite of an integer

We call -2 the number **opposite** to 2.

The integer -2 has opposite $-(-2) = 2$.

The operation of forming the opposite can be visualised by putting a pin in the number line at 0 and rotating the number line by 180° .

The opposite of 2 is -2.



The opposite of -2 is 2, so $-(-2) = 2$.

The number 0 is different from all of the other numbers. The opposite of 0 is 0.

The operation of taking the opposite can be visualised by swapping the number on one side of zero with the matching number the same distance from zero on the other side.



Exercise 4A

Example 1

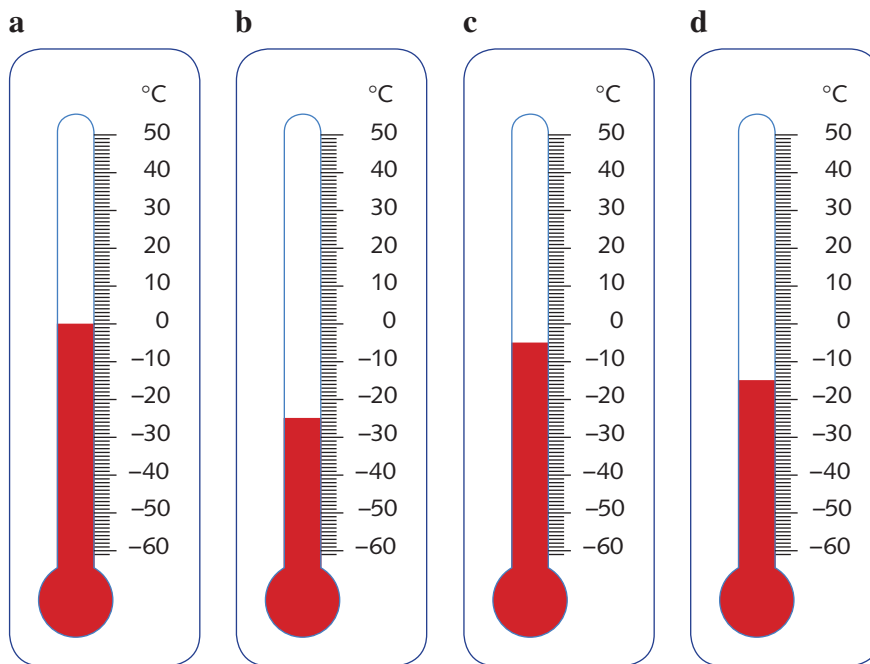
- 1 a List the integers less than -16 and greater than -22 .
b List the integers greater than -114 and less than -105 .
c List the integers less than 3 and greater than -8 .
d List the integers greater than -124 and less than -117 .

Example 2

- 2 a Draw a number line and mark the numbers -12 , -2 , 2 and -8 on it.
b Draw a number line and mark the numbers -8 , -3 , -5 and 4 on it.
c Draw a number line and mark the integers less than 5 and greater than -6 on it.
d Draw a number line and mark the integers less than 1 and greater than -9 on it.

Example 3

- 3 The sequence $-16, -13, -10, \dots$ is 'going up by threes'. Find the next three terms.
- 4 The sequence $15, 11, 7, \dots$ is 'going down by fours'. Find the next three terms.
- 5 The sequence $-50, -43, -36, \dots$ is 'going up by sevens'. Find the next four terms.
- 6 Determine the readings for each of the thermometers shown below.



- 7 Determine the opposites of these integers.

- | | | | |
|-----------|--------|--------------|----------|
| a 6 | b -3 | c 34 | d -5 |
| e -72 | f 67 | g -456 | h 10 000 |
| i -3500 | j 7800 | k $-45\,000$ | l 76 000 |

4B Addition and subtraction of a positive integer



If a submarine is at a depth of -250 m and then rises by 20 m, its final position is -230 m. This can be written as $-250 + 20 = -230$.

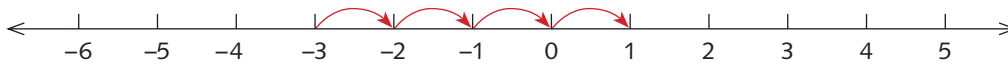
Joseph has \$3000 and he spends \$5000. He now has a debt of \$2000, so it is natural to interpret this as $3000 - 5000 = -2000$.

The number line and addition

The number line provides a useful picture for addition and subtraction of integers.

Addition of a positive integer

When you add a positive integer, move to the right along the number line.



For example, to calculate $-3 + 4$, start at -3 and move 4 steps to the right. We see that $-3 + 4 = 1$.

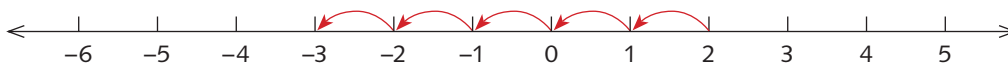
A practical example of this is: 'I started with a debt of \$3 and I earned \$4. I now have \$1'.

Subtraction of a positive integer

We can think of subtraction as **taking away**.

When you subtract a positive integer, move to the left along the number line.

For example, to calculate $2 - 5$, start at 2 and move 5 steps to the left. We see that $2 - 5 = -3$.



The same question arises in a practical way: 'I had \$2 and I spent \$5. I now have a debt of \$3'.

Example 4

Calculate:

a $-6 + 9$

b $-1 + 7$

c $-2 + 9$

d $-9 + 2$

Solution

a $-6 + 9 = 3$ (Start at -6 on the number line and move 9 steps to the right.)

b $-1 + 7 = 6$ (Start at -1 on the number line and move 7 steps to the right.)

c $-2 + 9 = 7$ (Start at -2 on the number line and move 9 steps to the right.)

d $-9 + 2 = -7$ (Start at -9 on the number line and move 2 steps to the right.)

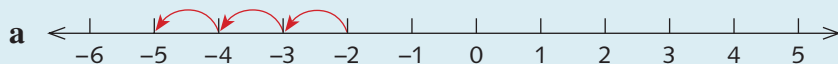
Example 5

Calculate:

a $-2 - 3$

b $2 - 8$

Solution



Start at -2 and move 3 steps to the left. We see that $-2 - 3 = -5$.

b $2 - 8 = -6$ (Start at 2 on the number line and move 8 steps to the left)

Exercise 4B

Example 4

1 Write the answers to these additions.

a $-5 + 4$

b $-7 + 11$

c $-6 + 2$

d $-11 + 15$

e $-4 + 9$

f $-7 + 11$

g $-1 + 10$

h $-3 + 10$

i $-15 + 7$

j $-11 + 11$

k $-17 + 10$

l $-13 + 4$

m $-16 + 16$

n $-70 + 33$

o $-60 + 99$

p $-21 + 15$

q $-75 + 12$

r $-96 + 6$

s $-345 + 600$

t $-23 + 89$

Example 5

2 Write the answers to these subtractions.

a $-5 - 6$

b $6 - 8$

c $1 - 9$

d $3 - 11$

e $7 - 12$

f $-2 - 8$

g $4 - 10$

h $2 - 5$

i $-3 - 4$

j $4 - 6$

k $-10 - 6$

l $-26 - 12$

m $-17 - 10$

n $-13 - 6$

o $14 - 20$

p $3 - 96$

q $54 - 59$

r $-66 - 30$

s $28 - 64$

t $12 - 21$

u $-100 - 28$

v $-136 - 144$

3 Write the answers to these additions and subtractions.

a $-450 + 50$

b $-500 + 80$

c $-100 - 70$

d $-315 + 430$

e $-420 - 45$

f $-300 + 500$

g $-500 - 700$

h $-1500 + 500$

i $-4000 - 600$

j $8000 - 10\,000$

k $4500 - 5000$

l $-5000 - 2500$

4 Work from left to right to calculate:

a $2 - 4 + 8$

b $4 - 10 + 20$

c $4 - 12 + 8$

d $23 - 30 + 17$

e $2 + 3 - 5$

f $3 - 7 + 11$

g $-3 - 7 + 12$

h $4 - 11 - 21$

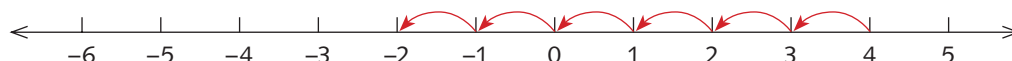


In the previous section, we considered addition and subtraction of a positive integer. In this section, we will add and subtract negative integers.

Addition of a negative integer

Adding a negative integer to another integer means that you take a certain number of steps to the *left* on a number line.

The result of the addition $4 + (-6)$ is the number you get by moving 6 steps to the left, starting at 4.



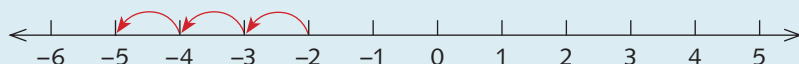
$$4 + (-6) = -2$$

Note that $4 + (-6)$ is the same as $4 - 6$.

Example 6

Calculate $-2 + (-3)$.

Solution



$-2 + (-3)$ is the number you get by moving 3 steps to the left, starting at -2 . That is, -5 .

Notice that $-2 - 3$ is also equal to -5 .

All additions of this form can be completed in a similar way. For example:

$$4 + (-7) = -3 \quad \text{and} \quad 4 - 7 = -3$$

$$-11 + (-3) = -14 \quad \text{and} \quad -11 - 3 = -14$$

This suggests the following rule.



To add a negative integer, subtract its opposite

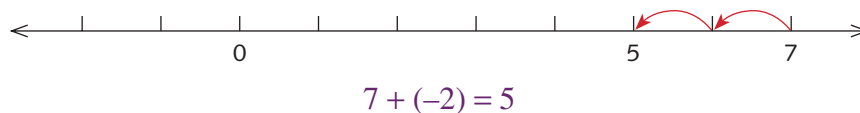
For example:

$$\begin{aligned} 4 + (-10) &= 4 - 10 \\ &= -6 \end{aligned}$$

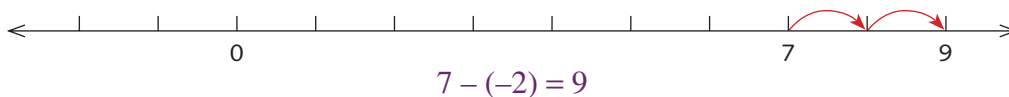
$$\begin{aligned} -7 + (-12) &= -7 - 12 \\ &= -19 \end{aligned}$$

Subtraction of a negative integer

We have already seen that adding -2 means taking 2 steps to the left. For example:



We want subtracting -2 to be the reverse process of adding -2 , so to subtract -2 , we will take 2 steps to the *right*. For example:



There is a very simple way to state this rule:



To subtract a negative number, add its opposite

For example:

$$\begin{aligned} 7 - (-2) &= 7 + 2 \\ &= 9 \end{aligned}$$

Example 7

Evaluate:

a $20 + (-3)$ **b** $-3 + (-8)$ **c** $6 - (-20)$ **d** $-13 - (-8)$ **e** $-20 - (-3)$

Solution

a $20 + (-3) = 20 - 3$
 $= 17$

b $-3 + (-8) = -3 - 8$
 $= -11$

c $6 - (-20) = 6 + 20$
 $= 26$

d $-13 - (-8) = -13 + 8$
 $= -5$

e $-20 - (-3) = -20 + 3$
 $= -17$

Example 8

Calculate $6 - (-17)$.

Solution

$$\begin{aligned} 6 - (-17) &= 6 + 17 \\ &= 23 \end{aligned}$$

Example 9

Calculate $-17 + (-6) - (-22)$.

Solution

$$\begin{aligned} -17 + (-6) - (-22) &= -17 - 6 + 22 \\ &= -23 + 22 \\ &= -1 \end{aligned}$$

Example 10

The minimum temperature on Saturday is -12°C and the maximum temperature is -1°C . Calculate the difference (maximum temperature – minimum temperature).

Solution

$$\begin{aligned} \text{Maximum temperature} - \text{minimum temperature} &= -1 - (-12) \\ &= -1 + 12 \\ &= 11^{\circ}\text{C} \end{aligned}$$

Example 11

Evaluate:

a $347 + (-625)$

b $456 - (-356)$

c $-234 + 568$

d $-120 - (-105)$

Solution

$$\begin{aligned} \text{a } 347 + (-625) &= 347 - 625 \\ &= -(625 - 347) \\ &= -278 \end{aligned}$$

$$\begin{aligned} \text{b } 456 - (-356) &= 456 + 356 \\ &= 812 \end{aligned}$$

$$\begin{aligned} \text{c } -234 + 568 &= 568 + (-234) \\ &= 568 - 234 \\ &= 334 \end{aligned}$$

$$\begin{aligned} \text{d } -120 - (-105) &= -120 + 105 \\ &= -(120 - 105) \\ &= -15 \end{aligned}$$



Exercise 4C

Example 6

1 Write the answers to these additions.

- | | | | |
|----------------------|----------------------|-----------------------|-----------------------|
| a $7 + (-2)$ | b $-7 + 2$ | c $-5 + 12$ | d $-15 + (-3)$ |
| e $-13 + 17$ | f $-7 + (-3)$ | g $-7 + (-2)$ | h $-6 + (-11)$ |
| i $12 + (-5)$ | j $-13 + 5$ | k $-13 + (-4)$ | l $-24 + 8$ |

Example 7,8

2 Write the answers to these subtractions.

- | | | | |
|--------------------|-----------------------|------------------------|-----------------------|
| a $3 - 6$ | b $5 - 11$ | c $-4 - 7$ | d $13 - (-3)$ |
| e $-12 - 8$ | f $-5 - (-3)$ | g $-7 - (-5)$ | h $6 - (-11)$ |
| i $-11 - 5$ | j $-14 - (-5)$ | k $-23 - (-15)$ | l $31 - (-10)$ |

3 Write the answer to these additions and subtractions.

- | | | | |
|----------------------|---------------------|----------------------|----------------------|
| a $2 + (-3)$ | b $4 + (-6)$ | c $5 + (-8)$ | d $4 + (-7)$ |
| e $6 + (-11)$ | f $6 - (-3)$ | g $-4 - (-1)$ | h $-6 - (-2)$ |
| i $-6 - 3$ | | | |

Example 9

4 Evaluate:

- | | | |
|------------------------------|------------------------------|-------------------------------|
| a $13 - 27 + (-26)$ | b $-12 - 14 + 8$ | c $-41 + 56 - 2$ |
| d $32 - 42 - (-8)$ | e $7 + 13 - 16$ | f $-32 - (-42) - (-3)$ |
| g $-37 - 17 + 25$ | h $6 - (-27) + 45$ | i $16 + (-3) - (-7)$ |
| j $-100 + 48 - (-80)$ | k $-50 - 70 - (-100)$ | l $20 - (-10) - (-30)$ |

Example 11

5 Evaluate:

- | | |
|--------------------------|----------------------------|
| a $256 - (-100)$ | b $-850 - (-246)$ |
| c $-658 - (-790)$ | d $-9860 - (-3755)$ |

Example 10

- 6 The temperature in Warsaw on a winter's day went from a minimum of -17°C to a maximum of -1°C . By how much did the temperature rise?
- 7 The temperature in Daylesford on a very cold winter's day went from -2°C to 9°C . What was the change in temperature?

- 8 The following table shows a list of minimum and maximum temperatures for a number of cities. Complete the table, showing the increase in each case.

Minimum	Maximum	Increase
-11°C	6°C	
-16°C	7°C	
-35°C	-4°C	
-25°C	-15°C	
-7°C	-2°C	
-13°C	2°C	
-11°C	5°C	

- 9 The temperature in Canberra on a very cold day went from 8°C to -2°C . What was the change in temperature?
- 10 A group of students have devised their own form of indoor cricket. A point is given for each run scored, but 5 points are deducted each time a player is given out. (It is possible to end up with a negative score.)
Give the total point score for each of these situations.
- a 45 runs and 2 players given out
 - b 50 runs and 10 players given out
 - c 45 runs and 10 players given out

4D

Multiplication and division with negative integers



Multiplication with negative integers

$5 \times (-3)$ means 5 lots of -3 added together. That is,

$$5 \times (-3) = (-3) + (-3) + (-3) + (-3) + (-3) \\ = -15.$$

Just as $8 \times 6 = 6 \times 8$, we will take -3×5 to be the same as $5 \times (-3)$.

All products such as $5 \times (-3)$ and -3×5 are treated in the same way.

For example:

$$-6 \times 3 = 3 \times (-6) \\ = -18$$

$$-15 \times 4 = 4 \times (-15) \\ = -60$$

$$-5 \times 10 = 10 \times (-5) \\ = -50$$

The question remains as to what we might mean by multiplying two negative integers together. We first investigate this by looking at a multiplication table.

In the left-hand column below, we are taking multiples of 5. The products go down by 5 each time.

In the right-hand column, we are taking multiples of -5 . The products go up by 5 each time.

$3 \times 5 = 15$	$3 \times (-5) = -15$
$2 \times 5 = 10$	$2 \times (-5) = -10$
$1 \times 5 = 5$	$1 \times (-5) = -5$
$0 \times 5 = 0$	$0 \times (-5) = 0$
$-1 \times 5 = -5$	$-1 \times (-5) = ?$
$-2 \times 5 = -10$	$-2 \times (-5) = ?$

The pattern suggests that it would be natural to take $-1 \times (-5)$ equal to 5 and $-2 \times (-5)$ equal to 10 so that the pattern continues in a natural way.

All products such as $-5 \times (-2)$ and $-5 \times (-1)$ are treated in the same way.

For example:

$$-6 \times (-2) = 12$$

$$-3 \times (-8) = 24$$

$$-20 \times (-5) = 100$$

We have the following rules.



The sign of the product of two integers

- The product of two positive numbers is positive.
For example, $4 \times 7 = 28$.
- The product of a negative number and a positive number is a negative number.
For example, $-4 \times 7 = -28$.
- The product of two negative numbers is a positive number.
For example, $-4 \times (-7) = 28$.

Example 12

Evaluate each product.

a $4 \times (-20)$

b -7×10

c $-25 \times (-40)$

Solution

a $4 \times (-20) = -80$

b $-7 \times 10 = -70$

c $-25 \times (-40) = 25 \times 40$
 $= 1000$

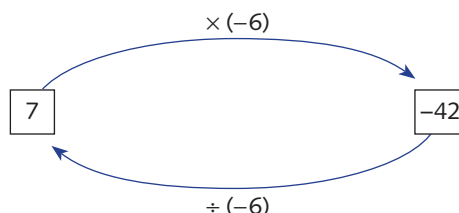
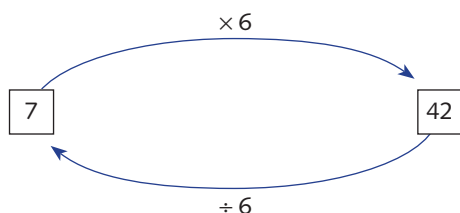
Division with negative integers

Every multiplication statement, for non-zero numbers, has an equivalent division statement. For example, $7 \times 3 = 21$ is equivalent to $21 \div 3 = 7$ and to $21 \div 7 = 3$. We will use this fact to establish the rules for division involving the integers.

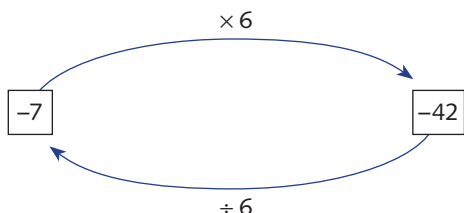
Here are some more examples:

$7 \times 6 = 42$ is equivalent to
 $42 \div 6 = 7$.

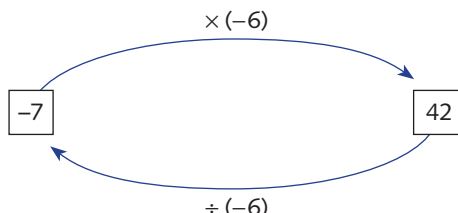
$7 \times (-6) = -42$ is equivalent to
 $-42 \div (-6) = 7$.



$-7 \times 6 = -42$ is equivalent to
 $-42 \div 6 = -7$.



$-7 \times (-6) = 42$ is equivalent to
 $42 \div (-6) = -7$.



The sign of the quotient of two integers

- The quotient of two positive numbers is positive.
 For example, $28 \div 7 = 4$.
- The quotient of a positive number and a negative number is a negative number.
 For example, $28 \div (-7) = -4$.
- The quotient of a negative number and a positive number is a negative number.
 For example, $-28 \div 7 = -4$.
- The quotient of two negative numbers is a positive number.
 For example, $-28 \div (-7) = 4$.

Example 13

Evaluate:

a $-54 \div 9$

b $-28 \div (-4)$

c $72 \div (-9)$

Solution

a $-54 \div 9 = -6$

b $-28 \div (-4) = 7$

c $72 \div (-9) = -8$

Example 14

Evaluate:

a $\frac{-63}{7}$

b $\frac{-72}{-4}$

c $\frac{108}{-12}$

Solution

a $\frac{-63}{7} = -9$

b $\frac{-72}{-4} = 18$

c $\frac{108}{-12} = -9$



Exercise 4D

Example 12

1 Carry out these multiplications.

a $6 \times (-2)$

b $7 \times (-2)$

c $7 \times (-10)$

d $12 \times (-4)$

e $12 \times (-18)$

f -5×3

g -6×4

h -5×12

i -11×7

j -12×5

k $-20 \times (-7)$

l $16 \times (-4)$

m $-7 \times (-20)$

n $-14 \times (-14)$

o -19×9

p $25 \times (-4)$

q $-20 \times (-9)$

r $-6 \times (-18)$

s -15×20

t $-12 \times (-25)$

Example 13

2 Carry out these divisions.

a $-18 \div 3$

b $-28 \div 2$

c $-35 \div 5$

d $-27 \div 3$

e $-120 \div 4$

f $18 \div (-3)$

g $36 \div (-2)$

h $45 \div (-9)$

i $21 \div (-3)$

j $120 \div (-3)$

k $-52 \div (-4)$

l $-72 \div (-8)$

m $-100 \div (-20)$

n $-121 \div (-11)$

o $-64 \div (-8)$

p $-144 \div (-8)$

q $-52 \div (-13)$

r $-600 \div (-10)$

s $-168 \div 6$

t $-385 \div 11$

Example 14

3 Evaluate:

a $\frac{4}{-1}$

b $\frac{-6}{-1}$

c $\frac{8}{-2}$

d $\frac{12}{-4}$

4 Evaluate:

a $\frac{-60}{-12}$

b $\frac{-65}{13}$

c $\frac{-72}{12}$

d $\frac{-120}{-8}$

e $\frac{136}{-4}$

f $\frac{-610}{5}$

g $\frac{-240}{15}$

h $\frac{396}{-4}$

i $\frac{-70}{10}$

j $\frac{720}{-24}$

k $\frac{-360}{-90}$

l $\frac{-4323}{3}$

5 Evaluate:

a $3 \times (-4) \times (-7)$

b $5 \times (-7) \times (-8)$

c $60 \times (-3) \times (-12)$

d $-45 \times (-8) \times 10$

e $48 \times (-10) \div 3$

f $8 \times (-10) \div 5$

g $55 \times (-3) \times (-30)$

h $-40 \times (-3) \times (-5)$

6 Copy and complete these statements.

a $2 \times \dots = -50$

b $5 \times \dots = -75$

c $-7 \times \dots = 63$

d $\dots \times (-8) = 72$

e $80 \div \dots = -10$

f $-45 \div \dots = 5$

g $-321 \div \dots = 3$

h $5664 \div \dots = -708$

4E Indices and order of operations



You need to be particularly careful with the order of operations conventions when working with indices.

For example, $-4^2 = -16$ and $(-4)^2 = 16$. In the first case, 4 is first squared and then the opposite is taken. In the second case, -4 is squared. Notice how different the two answers are.

Also:

$$\begin{array}{lcl} -3 \times 5 + 2 = -15 + 2 & \text{and} & -3 \times (5 + 2) = -3 \times 7 \\ = -13 & & = -21. \end{array}$$



Order of operations

- Evaluate expressions inside brackets first.
- In the absence of brackets, carry out operations in the following order:
 - powers
 - multiplication and division from left to right
 - addition and subtraction from left to right.

Example 15

Evaluate:

a $(-5)^2$

b -5^2

c $-6 - 5 + 4$

d $6 \times (-3) + 8$

e $-2 \times (-6) + 11 - 16$

Solution

a $(-5)^2 = -5 \times (-5)$
 $= 25$

b $-5^2 = -(5 \times 5)$
 $= -25$

c $-6 - 5 + 4 = -11 + 4$
 $= -7$

d $6 \times (-3) + 8 = -18 + 8$
 $= -10$

e $-2 \times (-6) + 11 - 16 = 12 + 11 - 16$
 $= 7$

Example 16

Evaluate:

a $7 \times (-6 + 8)$

b $-3 + 8 \times (7 - 12)$

c $6 - (5 + 3)$

d $11 \times (-2 + 8)$

e $-7 \times (-6 + 10) - 15$

f $4 \times (-6) + 3 \times 8$

g $3 \times (-6)^2 + 3 \times 21$

Solution

a $7 \times (-6 + 8) = 7 \times 2$
 $= 14$

b $-3 + 8 \times (7 - 12) = -3 + 8 \times (-5)$
 $= -3 + (-40)$
 $= -43$

c $6 - (5 + 3) = 6 - 8$
 $= -2$

d $11 \times (-2 + 8) = 11 \times 6$
 $= 66$

e $-7 \times (-6 + 10) - 15 = -7 \times 4 - 15$
 $= -28 - 15$
 $= -43$

f $4 \times (-6) + 3 \times 8 = -24 + 24$
 $= 0$

g $3 \times (-6)^2 + 3 \times 21 = 3 \times 36 + 63$
 $= 108 + 63$
 $= 171$

Exercise 4E

Example 15

1 Evaluate:

a $(-9)^2$

b $-(7)^2$

c $3 \times (-5)^2$

d $-9 \times (-4)^2$

e $(-20)^2 \times -(4)^2$

f $(-13)^2$

g $(-6)^3$

h $(-3)^4$

i $(-5)^5$

j $(-7)^6$

k $(-1)^7$

2 Evaluate:

a $-6 + 30 - 15$

b $-5 - (-10) + 30$

c $-8 + 12 - 18$

d $-6 + 12 - (-15)$

e $-15 + 7 - 10$

f $75 - (-40) + 60$

g $-12 + 20 - 70$

h $-60 - 33 - 57$

3 Evaluate:

$$\begin{array}{lll} \mathbf{a} & -3 \times (-16 + 8) & \mathbf{b} \quad -4 + 6 \times (11 - 14) \quad \mathbf{c} \quad -6 - 18 + 4 \\ \mathbf{d} & -6 \times (-3 + 12) & \mathbf{e} \quad -2 \times (-6 + 16) - 25 \quad \mathbf{f} \quad -15 + 5 \times (-3 + 12) \\ \mathbf{g} & (-11 + 5) \times 12 + (-15) & \mathbf{h} \quad (-18 - 4) \times 26 - (-12) \end{array}$$

4 Evaluate:

$$\begin{array}{lll} \mathbf{a} & -(3 - 27) & \mathbf{b} \quad -(37 - 64) \quad \mathbf{c} \quad 15 + (8 - 16) \\ \mathbf{d} & -53 + (5 - 11) & \mathbf{e} \quad 16 - 21 + 5 \times (-3) \quad \mathbf{f} \quad -3 \times (86 - 97) \\ \mathbf{g} & -15 \times (3 - 11) & \mathbf{h} \quad 6 \times (14 - 41) \quad \mathbf{i} \quad -8 \times (11 - 28) \\ \mathbf{j} & (34 + 34) - (-6) \times (-120) & \mathbf{k} \quad (56 + 44) + (-10) \times 20 \end{array}$$

5 Evaluate:

$$\begin{array}{lll} \mathbf{a} & -5 \times (-26 + 8) & \mathbf{b} \quad -4 + 6 \times (15 - 12) \quad \mathbf{c} \quad -6 - (25 + 4) \\ \mathbf{d} & -6 \times (-4 + 12) & \mathbf{e} \quad -2 \times (-8 + 16) - 40 \quad \mathbf{f} \quad -99 + 5 \times (-62 + 12) \\ \mathbf{g} & -81 + 5 \times (51 + (-45)) & \mathbf{h} \quad -28 - 4 \times (26 - (-14)) \end{array}$$

6 Evaluate:

$$\begin{array}{ll} \mathbf{a} & 60 \div (-5) \div 12 \\ \mathbf{b} & 90 \times (-4) \div 10 \\ \mathbf{c} & 60 \div 10 \times 3 \\ \mathbf{d} & 80 \times (-5) \div 25 \end{array}$$

7 Evaluate:

$$\begin{array}{ll} \mathbf{a} & (-20)^2 + 3 \times (-10) \\ \mathbf{b} & (-20)^2 \times (-10)^3 \\ \mathbf{c} & 3 \times (-10)^3 + 10^2 \\ \mathbf{d} & (-4) \times (-10)^2 \times (-10) \end{array}$$

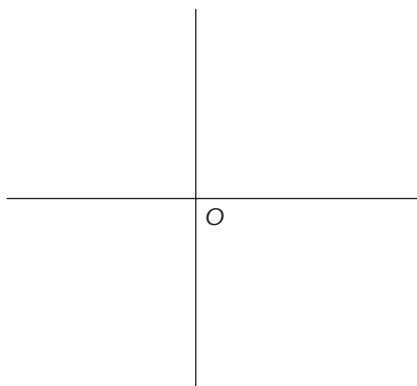
8 A shop manager buys 200 shirts at \$26 each and sells them for a total of \$4000. Calculate the total purchase price, and subtract this from the total amount gained from sales.**9** A man puts \$2000 into a bank account every month for 12 months.**a** What is his bank balance at the end of 12 months, given that he does not withdraw any money?**b** He writes a cheque for \$40 000. What is his new bank balance?

4F The Cartesian plane



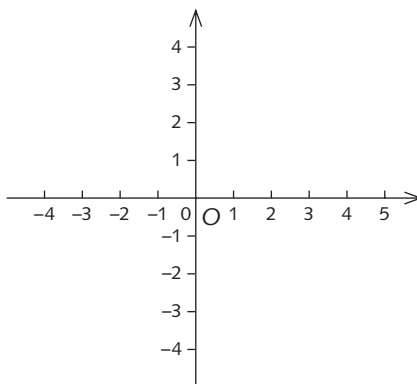
We have previously represented numbers as points on the number line. This idea can be extended by taking pairs of numbers to represent points in a plane. This is called the **Cartesian plane**.

We start with two perpendicular straight lines. They intersect at a point O called the **origin**. We leave the right angle sign out for clarity.



Each of the lines is called an **axis**. The plural of *axis* is *axes*.

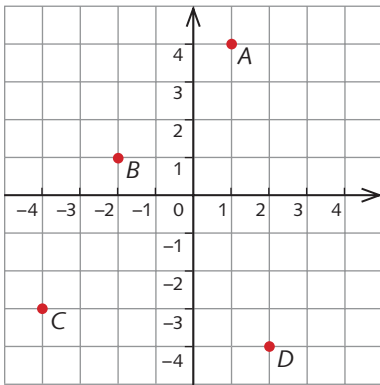
Next we mark off intervals of unit length along each axis, and mark each axis as a number line with 0 at the point O . The arrows are drawn to show that the axes extend infinitely in both directions and to indicate which way is the positive direction.



The axes are called the **coordinate axes** or sometimes the **Cartesian coordinate axes**. They are named after the French mathematician and philosopher René Descartes 1596–1650. He introduced coordinate axes to show how algebra could be used to solve geometric problems. Although the idea is simple, it revolutionised mathematics.

Now we imagine adding vertical and horizontal lines to the diagram through the integer points on the axes. We can describe each point where the lines meet by a pair of integers. This pair of integers is called the **coordinates** of the point. The first number is the **horizontal coordinate** and the second number is the **vertical coordinate**.

For example, the coordinates of the point labelled *A* below are (1, 4). This is where the line through the point 1 on the horizontal axis and the line through the point 4 on the vertical axis meet. We move 1 unit to the right of the origin and 4 units up to reach *A*.



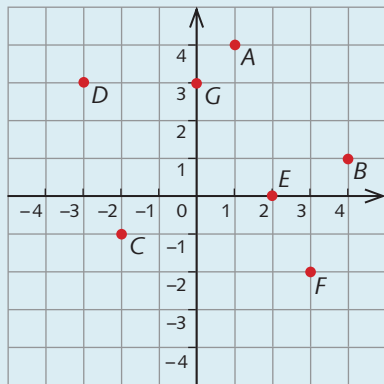
The point *D* has coordinates (2, -4).
We move 2 units to the right of the origin
and 4 units down to get *D*.

The point *B* has coordinates (-2, 1).
We move 2 units to the left of the origin
and 1 unit up to get *B*.

The point *C* has coordinates (-4, -3).
We move 4 units to the left of the origin
and 3 units down.

Example 17

Give the coordinates of each of the points marked on the Cartesian plane below.

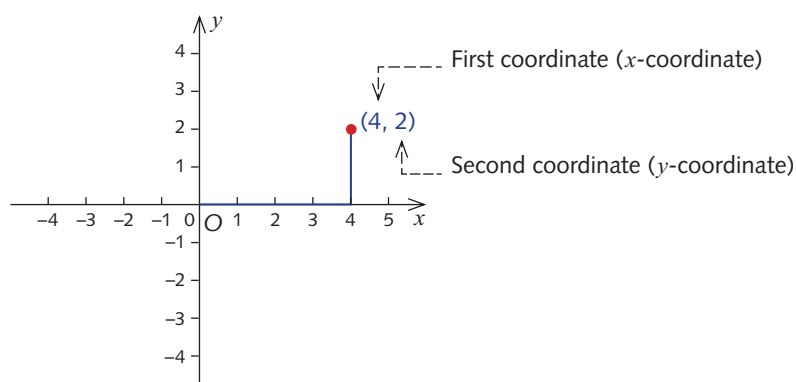


Solution

$A(1, 4)$	$B(4, 1)$	$C(-2, -1)$	$D(-3, 3)$
$E(2, 0)$	$F(3, -2)$	$G(0, 3)$	

Remember, the first coordinate tells us where to go from the origin in the horizontal direction. If it is negative, we go to the left of the origin; if it is positive, we go to the right of the origin.

The second coordinate tells us where to go from the origin in the vertical direction. If it is negative, we go below the origin; if it is positive, we go above the origin.



The first coordinate is usually called the x -coordinate and the second coordinate is usually called the y -coordinate.

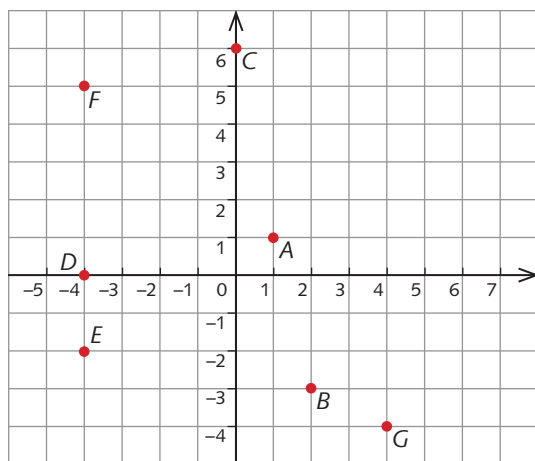
Exercise 4F

- 1 On a Cartesian plane, plot each of the following points. The coordinates of the points are given.

a $A(4, 1)$ **b** $B(-2, 3)$ **c** $C(-2, -2)$ **d** $D(4, -1)$
e $E(1, 3)$ **f** $F(0, -2)$ **g** $G(-3, 0)$ **h** $H(0, 5)$

Example 17

- 2 Give the coordinates of each of the points A to G below.



- 3** Do this exercise on graph paper. Set out the coordinate axes by marking intervals of length 1 unit along the horizontal and vertical axes.
- a** Plot the points $O(0, 0)$, $A(3, 0)$, $C(3, 3)$ and $D(0, 3)$, and join the points to form the intervals OA , AC , CD and DO . Describe the shape formed and evaluate its area.
 - b** Plot the points $A(-3, 0)$, $B(6, 0)$ and $C(0, 4)$, and join the points to form the intervals AB , BC and CA . Describe the shape formed and evaluate its area.
 - c** Plot the points $A(-4, -3)$, $B(7, -3)$, $C(7, 2)$ and $D(-4, 5)$, and join the points to form the intervals AB , BC , CD and DA . Describe the shape formed and evaluate its area.
 - d** Plot the points $A(-4, 4)$, $B(1, 4)$, and $C(-4, 1)$, and join the points to form the intervals AC , CB , BA . Describe the shape formed and evaluate its area.
 - e** Plot the points $O(0, 0)$ and $A(1, 2)$, and draw the line passing through these points. Plot the points $B(-2, 3)$ and $C(0, 2)$, and draw the line passing through these points. Describe the relationship between the lines.
 - f** Plot the points $A(1, 1)$, $B(3, 3)$, $C(7, 3)$ and $D(5, 1)$, and join the points to form the intervals AB , BC , CD and DA . Describe the shape formed and evaluate its area.

4G Negative fractions



Fractions were reviewed in Chapter 2.

We can now introduce negative fractions as well. They lie to the left of 0 on the number line.

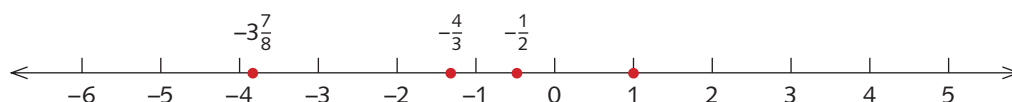
Each positive fraction has an opposite fraction.

For example, $\frac{5}{4}$ has the opposite $-\frac{5}{4} = -1\frac{1}{4} = -1.25$.

A situation where we see negative fractions is in temperatures.

A temperature of -4.5°C ($-\frac{9}{2}^\circ\text{C}$) is 4.5°C below zero.

Several negative fractions are marked on the number line below.



The arithmetic for integers, which we have considered in the previous sections, also extends to fractions.

Addition and subtraction of negative fractions

Example 18

Write the answer to each of these additions and subtractions.

a $-\frac{1}{2} + 2$

b $-1\frac{1}{2} + 2$

c $-\frac{1}{3} - \frac{1}{2}$

d $-\frac{2}{3} + \frac{1}{5}$

Solution

a $-\frac{1}{2} + 2 = 1\frac{1}{2}$

b $-1\frac{1}{2} + 2 = \frac{1}{2}$

c $-\frac{1}{3} - \frac{1}{2} = -\frac{2}{6} - \frac{3}{6}$
 $= -\frac{5}{6}$

d $-\frac{2}{3} + \frac{1}{5} = -\frac{10}{15} + \frac{3}{15}$
 $= -\frac{7}{15}$

Division by negative integers giving fractions

Example 19

Divide:

a 265 by -2

b -3765 by -7

Solution

a First divide 265 by 2. The result is $132\frac{1}{2}$.

For $265 \div (-2)$, the signs are different and the result is $-132\frac{1}{2}$.

b First divide 3765 by 7. The result is $537\frac{6}{7}$.

For $-3765 \div (-7)$, the signs are the same and $-3765 \div (-7) = 537\frac{6}{7}$.

Recall the discussion of how signs behave under multiplication and division on pages 99–101.

Multiplication and division of negative fractions

The methods we have been using with the positive fractions also work with the negative fractions.

Example 20

Give the answer to each of these multiplications and divisions as a fraction.

a $-\frac{1}{2} \times 2$

b $-1\frac{1}{2} \div 2$

c $-\frac{1}{3} \div \frac{1}{2}$

d $-\frac{2}{3} \times \left(-\frac{1}{5}\right)$

Solution

a $-\frac{1}{2} \times 2 = -1$

b $-1\frac{1}{2} \div 2 = -\frac{3}{2} \times \frac{1}{2}$
 $= -\frac{3}{4}$

c $-\frac{1}{3} \div \frac{1}{2} = -\frac{1}{3} \times \frac{2}{1}$
 $= -\frac{2}{3}$

d $-\frac{2}{3} \times \left(-\frac{1}{5}\right) = \frac{2}{3} \times \frac{1}{5}$
 $= \frac{2}{15}$



Exercise 4G

- 1 Draw a number line from -5 to 5 and mark on it the positions of

$$-3\frac{1}{2}, -4\frac{3}{4}, -3\frac{1}{4}, -1\frac{1}{2}, 1\frac{1}{2}.$$

- 2 Arrange each of these sets of numbers from smallest to largest.

a $-2, \frac{1}{4}, -1, \frac{-5}{-3}, -\frac{7}{5}, \frac{1}{2}, 1$

b $-\frac{7}{4}, \frac{-11}{-3}, -\frac{11}{5}, \frac{14}{5}, \frac{7}{2}$

c $-\frac{2}{3}, -\frac{4}{5}, -\frac{11}{12}, \frac{12}{13}$

d $-1\frac{11}{13}, -\frac{15}{13}, -2, -1$

Example 18

- 3 Evaluate:

a $-\frac{1}{2} + 3$

b $-1\frac{1}{2} + 4$

c $-\frac{2}{3} - \frac{1}{3}$

d $-\frac{2}{3} + \frac{1}{3}$

e $-\frac{3}{4} - \frac{3}{4}$

f $-\frac{2}{3} + \frac{3}{5}$

g $-\frac{3}{4} + \frac{1}{4}$

h $-\frac{1}{5} - \frac{2}{5}$

i $-\frac{3}{4} + 3$

j $-\frac{3}{5} - 3$

k $-1\frac{1}{2} + \frac{-7}{9}$

l $\frac{3}{7} - 2\frac{1}{2}$

m $-3\frac{1}{2} + 3$

n $-2\frac{1}{2} + 4$

o $-\frac{2}{5} - \frac{1}{3}$

p $-\frac{3}{4} + \frac{1}{3}$

q $-\frac{3}{5} - \frac{3}{4}$

r $-\frac{2}{3} + \frac{1}{4}$

s $-\frac{3}{4} + \frac{1}{5}$

t $-\frac{1}{5} - \frac{2}{7}$

u $-\frac{3}{4} + 3\frac{2}{9}$

v $-\frac{3}{5} - 3\frac{2}{7}$

w $-\frac{5}{13} + \left(-3\frac{5}{7}\right)$

x $\frac{3}{8} - \left(-\frac{2}{7}\right)$

- 4 Divide:

a 2735 by -2

b -6322 by 3

c -5524 by -7

Example 20 a,d

- 5 Evaluate:

a $2 \times \left(-\frac{3}{8}\right)$

b $-\frac{3}{8} \times \left(-\frac{5}{9}\right)$

c $-\frac{11}{12} \times \left(-\frac{5}{9}\right)$

d $-\frac{9}{5} \times \left(-\frac{3}{4}\right)$

e $-\frac{11}{5} \times \frac{5}{12}$

f $4 \times \left(-\frac{3}{8}\right) \times \frac{5}{9}$

g $5 \times \left(-\frac{3}{10}\right)$

h $\frac{11}{12} \times \left(-\frac{4}{3}\right)$

i $-\frac{2}{3} \times \frac{5}{6}$

j $-\frac{5}{12} \times \frac{7}{5}$

k $-\frac{5}{8} \times \frac{7}{10}$

l $-\frac{3}{5} \times \frac{25}{18}$

6 Evaluate:

$$\begin{array}{llll}
 \mathbf{a} \ 2 \div \left(-\frac{1}{3}\right) & \mathbf{b} \ \frac{3}{4} \div \left(-\frac{2}{3}\right) & \mathbf{c} \ -3 \div 5 & \mathbf{d} \ -7 \div 12 \\
 \mathbf{e} \ \frac{3}{4} \div \left(-\frac{11}{12}\right) & \mathbf{f} \ -\frac{2}{3} \div \left(-\frac{4}{9}\right) & \mathbf{g} \ -\frac{3}{8} \div \frac{5}{12} & \mathbf{h} \ 5 \div \frac{7}{6} \\
 \mathbf{i} \ -\frac{2}{3} \div \left(-\frac{1}{6}\right) & \mathbf{j} \ \frac{5}{7} \div 10 & \mathbf{k} \ \frac{3}{7} \div \frac{4}{9} & \mathbf{l} \ -5 \div \frac{7}{10}
 \end{array}$$

7 Rewrite each expression in simplest form.

$$\begin{array}{lll}
 \mathbf{a} \ \frac{1}{5} - \left(\frac{1}{3} + \frac{1}{2}\right) & \mathbf{b} \ -2\frac{7}{8} + \left(-\frac{1}{3}\right) \times \frac{4}{3} & \mathbf{c} \ \left(-1\frac{1}{2} + 1\frac{3}{4}\right) \times 3\frac{1}{2} \\
 \mathbf{d} \ -\frac{1}{4} - \frac{1}{3} \times 4 & \mathbf{e} \ \frac{1}{2} - 2 \times \frac{3}{8} & \mathbf{f} \ -\frac{3}{4} - \left(2 - \frac{1}{2}\right) \\
 \mathbf{g} \ 4 - \left(-3\frac{1}{2}\right) - 2 & \mathbf{h} \ \frac{1}{6} - \frac{1}{3} + \frac{1}{6} & \mathbf{i} \ 2 \times \left(-\frac{1}{3}\right) - 4\frac{1}{3}
 \end{array}$$

8 Evaluate:

$$\begin{array}{ll}
 \mathbf{a} \ -\frac{1}{2} + 2\frac{1}{2} - \frac{1}{4} + \frac{3}{4} & \mathbf{b} \ -3\frac{1}{4} - \frac{1}{4} + 6 - 2\frac{1}{2} \\
 \mathbf{c} \ 2 \times \left(-\frac{3}{2}\right) \times \left(-\frac{4}{7}\right) + \frac{2}{7} & \mathbf{d} \ 3\frac{1}{3} \times (-3) + 10\frac{1}{2} \div 3 \\
 \mathbf{e} \ 5\frac{1}{4} \div (-7) + \frac{3}{4} & \mathbf{f} \ -\frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} - \frac{1}{6}
 \end{array}$$

9 Evaluate:

$$\begin{array}{llll}
 \mathbf{a} \ \left(-\frac{1}{2}\right)^2 & \mathbf{b} \ \left(-\frac{1}{2}\right)^3 & \mathbf{c} \ \left(-\frac{1}{2}\right)^4 & \mathbf{d} \ \left(-\frac{1}{2}\right)^5 \\
 \mathbf{e} \ -16 \times \left(-\frac{1}{2}\right)^5 & \mathbf{f} \ -32 \times \left(-\frac{1}{4}\right)^2 & \mathbf{g} \ 24 \times \left(-\frac{1}{2}\right)^3 & \mathbf{h} \ \left(-\frac{1}{3}\right)^3 \times -81
 \end{array}$$



Review exercise

1 Evaluate:

- a** $25 + (-2)$ **b** $-36 + 22$ **c** $-35 + 50$ **d** $-51 + (-44)$
e $-32 + 16$ **f** $-45 + (-23)$ **g** $-160 + (-20)$ **h** $-50 + (-10)$
i $110 + (-40)$ **j** $-120 + 40$ **k** $-135 + (-25)$ **l** $60 - (-25)$

2 In an indoor cricket match, a team has made 25 runs and lost 7 wickets. What is the score of the team? (A run adds 1 to the score and a wicket subtracts 5.)

3 The temperature in June at a base in Antarctica varied from a minimum of -60°C to a maximum of -35°C . What was the value of:

- a** maximum temperature – minimum temperature?
b minimum temperature – maximum temperature?

4 The temperature in Canberra had gone down to -5°C . The temperature in a heated house was a cosy 22°C . What was the value of:

- a** inside temperature – outside temperature?
b outside temperature – inside temperature?

5 Evaluate:

- a** $125 \times (-2)$ **b** -36×11 **c** -35×50 **d** $-51 \times (-40)$
e -3×16 **f** $-50 \times (-23)$ **g** $-160 \times (-20)$ **h** $-50 \times (-10)$
i $11 \times (-40)$ **j** -120×20 **k** $-32 \times (-4)$ **l** -25×8

6 Evaluate:

- a** $125 \div (-5)$ **b** $-36 \div 9$ **c** $-35 \div 5$ **d** $-51 \div (-3)$
e $-16 \div (-4)$ **f** $-50 \div (-10)$ **g** $-160 \div (-20)$ **h** $-1500 \div (-10)$
i $120 \div (-40)$ **j** $-120 \div 20$ **k** $-40 \div (-5)$ **l** $624 \div (-6)$

7 Evaluate:

- a** $-4 \times (6 - 7)$ **b** $7 \times (11 - 20)$ **c** $-3 \times (5 + 15)$
d $-6 \times (-4 - 6)$ **e** $-12 \times (-6 + 20)$ **f** $-(-4)^2$
g $(3 - 7) \times (11 - 15)$ **h** $(10 - 3) \times (-3 + 10)$ **i** $(-5 - 10) \times (10 - 4)$

8 Start with the number -5 , add 11 and then subtract 20. Multiply the last number you obtained by 4. What is the final result?

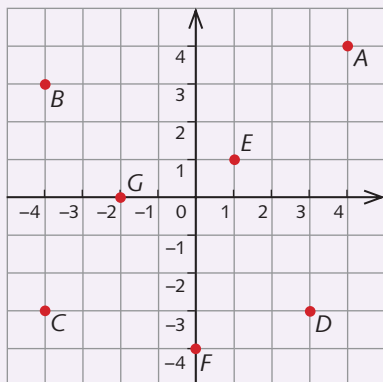
9 Start with -100 , subtract 200 and then add -300 . Divide the result by 100. What is the final result?

10 On a Cartesian plane, plot each of the following points. The coordinates of the points are given.

- a** $A(1, 1)$ **b** $B(2, -3)$ **c** $C(0, 6)$
d $D(-4, 0)$ **e** $E(-4, -2)$ **f** $F(-4, 5)$



- 11** Find the coordinates of each of the points A to G below.



- 12** Evaluate:

a $-\frac{1}{2} + 3$

b $-1\frac{1}{2} + 6$

c $-\frac{9}{13} - \frac{1}{13}$

d $-\frac{7}{8} + \frac{5}{8}$

e $-\frac{5}{12} + \left(-\frac{5}{6}\right)$

f $-\frac{5}{11} + 6$

- 13** Evaluate:

a $\frac{2}{3} \times \left(-\frac{1}{4}\right)$

b $3\frac{1}{3} \times (-9)$

c $-\frac{4}{3} \times \frac{3}{4} \div \frac{7}{8}$

d $-\frac{2}{3} \times \frac{3}{4} \times (-2)$

e $\frac{3}{4} \times \left(-\frac{4}{5}\right) \times \frac{1}{2}$

f $\frac{1}{2} - \frac{1}{3} \times \frac{1}{2}$

g $-\frac{2}{3} \times \frac{1}{4} \times \frac{3}{5}$

h $-\frac{5}{8} \times \frac{1}{2} \times \frac{2}{3}$

i $\frac{1}{4} - \frac{2}{3} \div 3$

- 14** AC electricity, which is what you get from a power point, alternates (hence the name alternating current) between positive and negative voltages. The highest positive voltage it reaches is 1.4 times the rated voltage, and the lowest voltage is -1.4 times the rated voltage. If the power point is rated as 230 V AC, what is the difference between the highest and lowest voltages? ('V' stands for the unit of measure 'volts')

- 15** Evaluate:

a $(-1)^2$

b $(-1)^3$

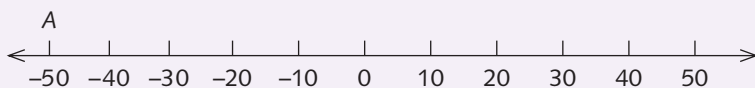
c $(-1)^4$

d $(-1)^5$

e $(-1)^6$

Comment on the results.

- 16** Johann is asked to walk along a straight pathway that is marked out as a number line, as shown in the diagram. Distances are in metres.



Johann starts at the point A, which is at -50 on the number line.



He is asked to walk along the path and follow these directions:

- Walk from A to the point -10 .
- Turn through 180° .
- Walk to the point -20 .
- Turn through 180° .
- Walk to the point 50 .
- Turn through 180° .
- Walk back to A.

How far has he walked?

- 17** The average of a list of numbers is obtained by adding the numbers, and then dividing by the number of numbers. For example, the average of 20, 30, 50 and 60 is given by

$$(20 + 30 + 50 + 60) \div 4 = 160 \div 4 \\ = 40$$

$$\text{or } \frac{20 + 30 + 50 + 60}{4} = \frac{160}{4} \\ = 40.$$

Find the average of each of these lists of numbers.

- a** $-20, -30, 40, 60$
b $-6, -1, -10, -20, 22$
c $-200, -300, -500, -1000$
d $-2, -3, -4, 0, 8, 1$

- 18** Find the average of each of these lists of numbers.

a $-1\frac{1}{4}, 2\frac{1}{2}$ **b** $-1, -1\frac{1}{2}, 0, 6$ **c** $-\frac{3}{4}, -\frac{1}{2}, -\frac{1}{8}, \frac{1}{2}, \frac{1}{4}$

- 19** If $a = -1\frac{1}{4}$ and $b = 3\frac{1}{2}$, evaluate:

a $a + b$ **b** $a - b$ **c** $b - a$ **d** ab
e $\frac{a}{b}$ **f** $\frac{b}{a}$ **g** $\frac{a+2}{b}$ **h** $\frac{b-a}{b}$

- 20** Evaluate:

a $(-3)^2 + 2 \times (-3) + 8$ **b** $(-3)^3 - 2 \times (-3) + 10$
c $6 - (-2)^2 - (-2)^3$ **d** $\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 + \frac{1}{2}$

- 21** Evaluate:

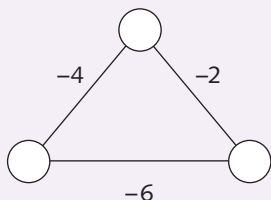
a $1 \div \frac{1}{2}$ **b** $1 \div \left(-\frac{1}{2}\right)$ **c** $1 \div \left(-\frac{3}{2}\right)$ **d** $1 \div \left(-\frac{3}{2}\right)^2$

Challenge exercise

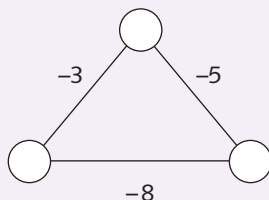


- 1 The integers on the edges of each triangle below are the sums of the integers in the adjoining two circles. Find the numbers in the circles.

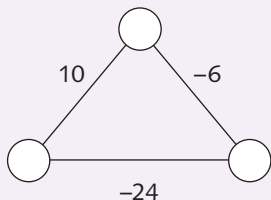
a



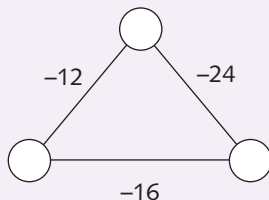
b



c



d



- 2 Put the three numbers 4, -2 and -5 into the boxes below

$$\square + \square - \square = \underline{\hspace{2cm}}$$

so that the answer is:

a 1

b 7

c -11

- 3 Put the three numbers 3, -3 and -4 into the boxes below

$$\square + \square - \square = \underline{\hspace{2cm}}$$

so that the answer is:

a 4

b -10

c 2

- 4 Which number needs to be placed in the box below to make the following a true statement?

$$3 - \square + (-5) = 0$$

- 5 Place brackets in each statement to make it true.

a $2 + -3 \times 3 + 4 = -7$

b $2 + -3 \times 3 + 4 \times 2 = 1$

c $2 - 5 \times 6 + 7 \times 6 - 5 = -71$

- 6 This is a magic square. All rows, columns and diagonals have the same sum. Complete the magic square.

8	-6	4
		-4



- 7 a** Find the value of $1 - 2 + 3 - 4 + 5 - 6$ by:
- i** working from left to right
 - ii** pairing $(1 - 2) + (3 - 4) + (5 - 6)$
- b** Evaluate $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + \dots + 99 - 100$.
- 8** Evaluate $100 + 99 - 98 - 97 + 96 + 95 - 94 - 93 + \dots + 4 + 3 - 2 - 1$.
- 9** The average of five given numbers is 2. If the smallest number is deleted, the average is 4. What is the smallest number?
- 10** Find the value of:
- a** $(2 - 4) + (6 - 8) + (10 - 12) + (14 - 16) + (18 - 20)$
 - b** $2 - 4 + 6 - 8 + 10 - 12 + \dots + 98 - 100$
- 11** What is the least product you could obtain by multiplying any two of the following numbers: $-7, -5, -1, 1$ and 3 ?
- 12** Evaluate:
- a** $\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6}$
 - b** $\frac{1}{2 \times 3} - \frac{1}{3 \times 4} + \frac{1}{4 \times 5} - \frac{1}{5 \times 6}$
 - c** $-\frac{1}{2 \times 3} + \frac{1}{3 \times 4} - \frac{1}{4 \times 5} + \frac{1}{5 \times 6}$
 - d** $-\frac{1}{5 \times 6} - \frac{1}{6 \times 7} - \frac{1}{7 \times 8} - \frac{1}{8 \times 9}$



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CHAPTER

5

Measurement and Geometry

Review of geometry

The word *geometry* comes from two Greek words meaning 'earth measurement'. Its name explains the origin of the subject. In ancient Egypt, the river Nile regularly overflowed its banks and washed away boundary markers between neighbouring properties. The boundaries had to be found again and this led to the study of the different shapes and sizes of triangles.

Geometry deals with points, lines, planes, angles, circles, triangles, and so on. The ancient Greeks valued its study greatly, and their work established the geometry you will meet in this book.

5A Angles at a point



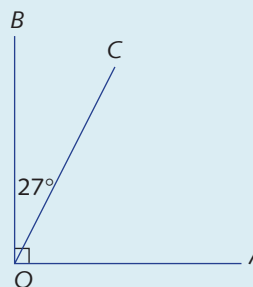
Clear, logical reasoning has always been an important part of geometry. It is very important in geometry to draw diagrams. This makes it easier to construct convincing arguments.

Angles at a point – geometric arguments

- The following can be used in arguments:
 - Adjacent angles can be added.
 - Angles in a revolution add to 360° .
 - Angles in a straight angle add to 180° .
 - Vertically opposite angles are equal.
- Two lines are called **perpendicular** if they meet at right angles.
- The Greek letters α (alpha), β (beta), γ (gamma) and θ (theta) are often used as pronumerals to represent angle size in geometry.

Example 1

Find $\angle AOC$ in the diagram opposite.

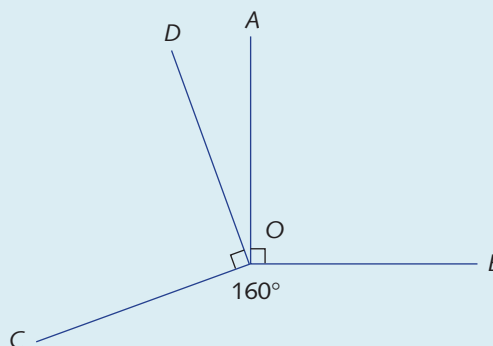


Solution

$\angle AOC + 27^\circ = 90^\circ$ (adjacent angles at O)
so $\angle AOC = 63^\circ$.

Example 2

Find $\angle AOD$ in the diagram below.

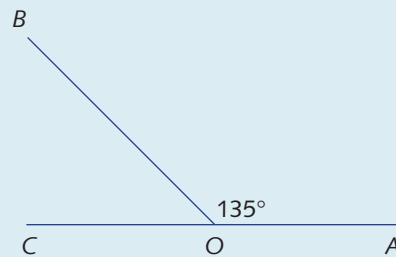


Solution

$\angle AOD + 90^\circ + 160^\circ + 90^\circ = 360^\circ$ (revolution at O)
so $\angle AOD = 20^\circ$.

Example 3

Find $\angle BOC$ in the diagram below.

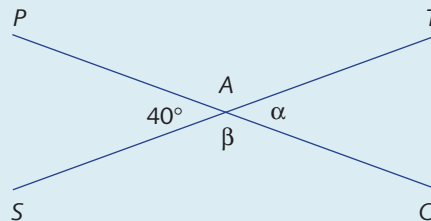


Solution

$\angle BOC + 135^\circ = 180^\circ$ (straight angle $\angle AOC$)
so $\angle BOC = 45^\circ$.

Example 4

Find α and β in the diagram opposite.



Solution

$\alpha = 40^\circ$ (vertically opposite angles at A).

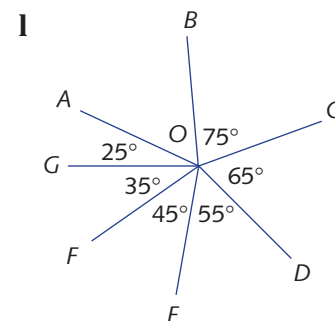
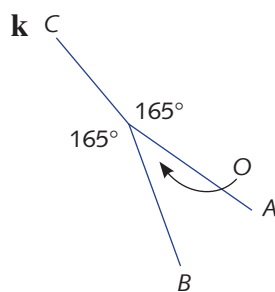
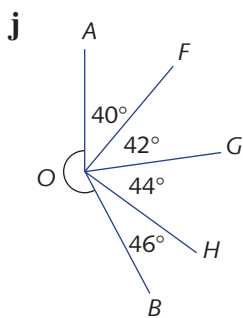
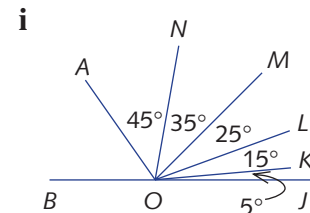
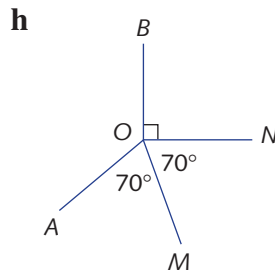
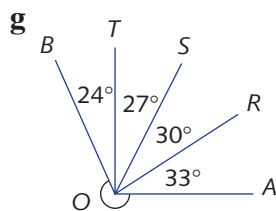
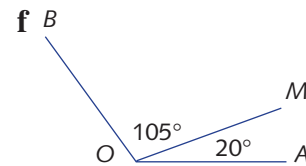
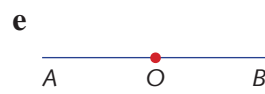
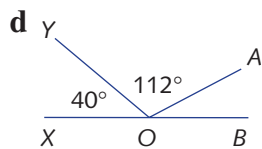
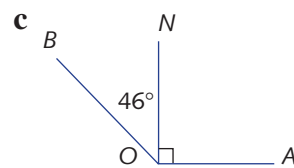
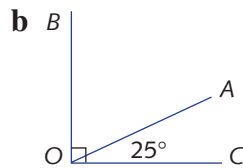
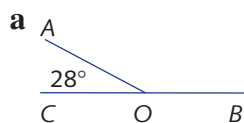
Also, $\beta + 40^\circ = 180^\circ$ (straight angle $\angle PAQ$),
so $\beta = 140^\circ$.



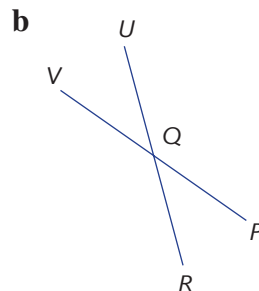
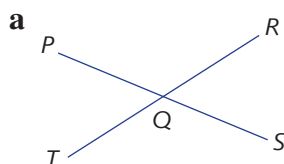
Exercise 5A

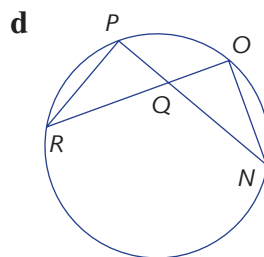
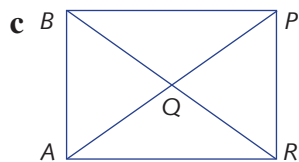
Example 1, 2, 3

- 1 Find the value of $\angle AOB$ in each diagram below, giving reasons for your answer. In parts g and j, find the reflex angle $\angle AOB$.



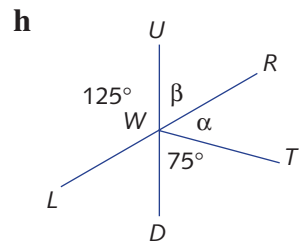
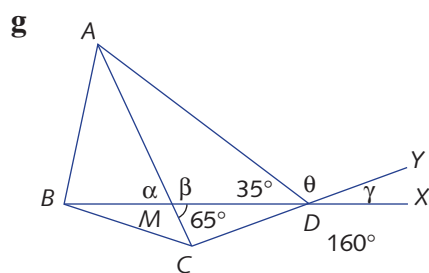
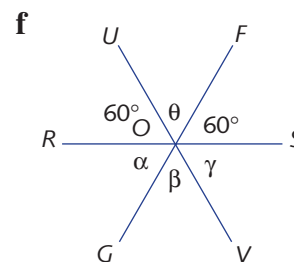
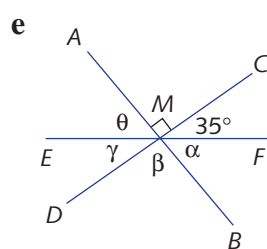
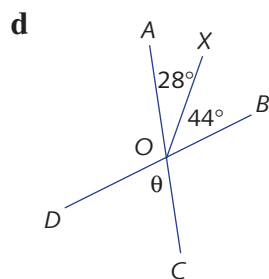
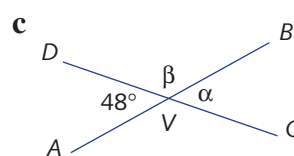
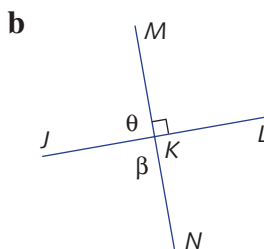
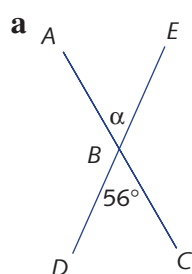
- 2 Name the angle that is vertically opposite to $\angle PQR$ in each diagram below.





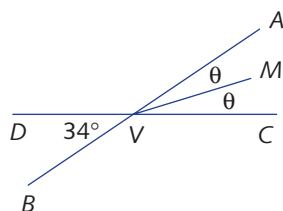
Example 4

3 Find the sizes of the angles denoted by α , β , γ and θ in each diagram below, giving reasons.

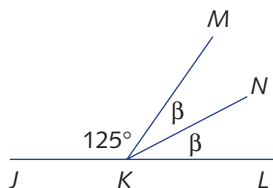


- 4 In each diagram below, angles marked with the same Greek letter are equal in size. Find the value of each, giving reasons.

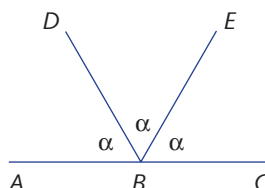
a



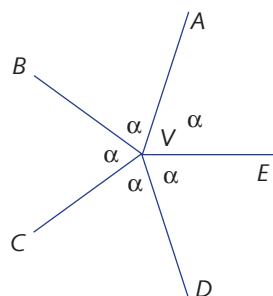
b



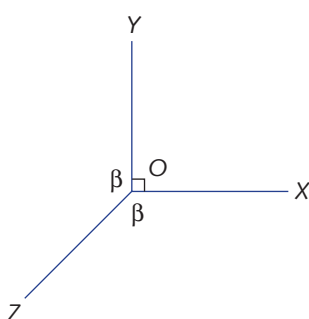
c



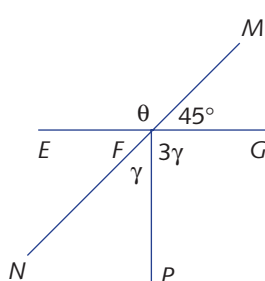
d



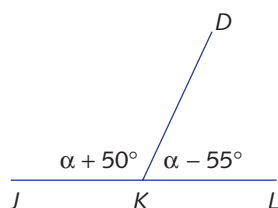
e



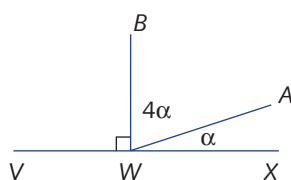
f



g



h



5B Angles associated with transversals

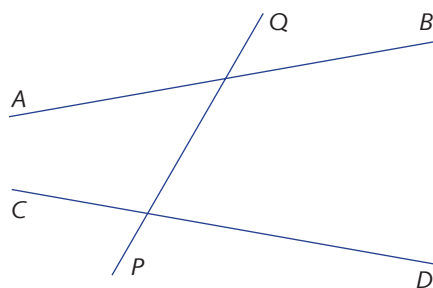
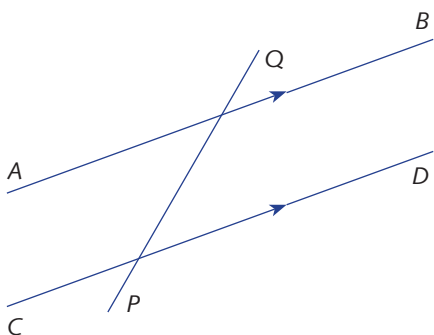


This section and the next one involve the relationships between angles and parallel lines.

As always in geometry, reasons should be as specific as possible. Make sure that you name any parallel lines that you are using in an argument.

Transversal

A **transversal** is a line that crosses two other lines. In both diagrams below, the line PQ is a transversal to the lines AB and CD .

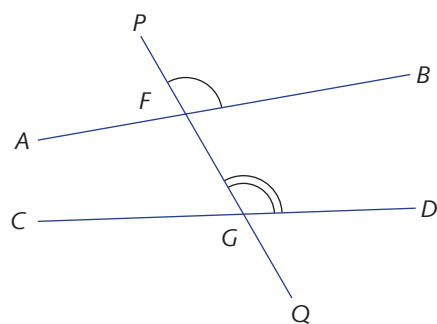


Notice that PQ is a transversal whether or not the two lines that it crosses are parallel.

Corresponding angles

The two marked angles in the diagram are called **corresponding angles**, because they are in *corresponding* positions around the two vertices F and G .

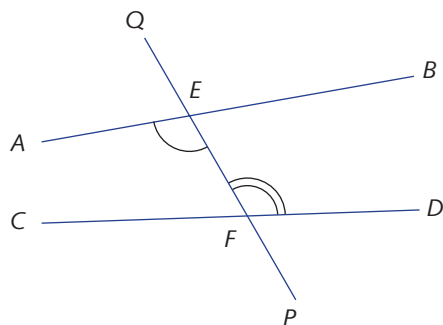
There are actually four pairs of corresponding angles in the diagram. Can you name the other three pairs?



Alternate angles

In the diagram opposite, the two marked angles are called **alternate angles**, because they are on *alternate* sides of the transversal PQ . The two angles must also be inside the two parallel lines.

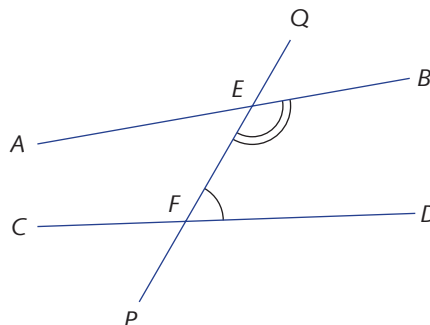
There is a second pair of alternate angles in the diagram. Can you name the angles of this pair?



Co-interior angles

In the diagram opposite, the two marked angles are called **co-interior angles**, because they are between the two lines and on the same side of the transversal PQ .

There is a second pair of co-interior angles in the diagram. Name the angles of this pair.



A transversal lying across parallel lines

We learnt in Chapter 6 at *ICE-EM Mathematics Year 7 Book 1* that when a transversal lies across two parallel lines, the alternate angles, the corresponding angles and the co-interior angles are closely related.



Angles associated with transversals

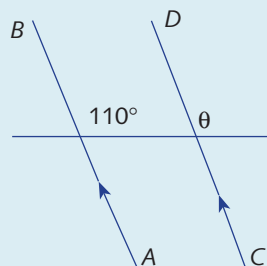
A transversal is a line that crosses two other lines. If the lines crossed by the transversal are parallel, then:

- **corresponding** angles are equal
- **alternate** angles are equal
- **co-interior** angles are supplementary.

Here are three examples that show how to set out your work with corresponding angles, alternate angles and co-interior angles. Your reason should first name the type of angles involved, and then name the relevant pair of parallel lines.

Example 5

Find θ in the diagram opposite.

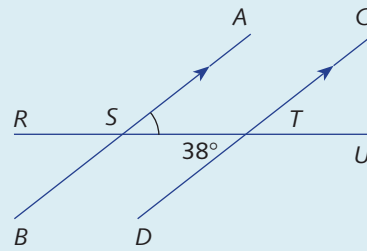


Solution

$$\theta = 110^\circ \text{ (corresponding angles, } AB \parallel CD\text{)}$$

Example 6

Find $\angle AST$ in the diagram opposite.

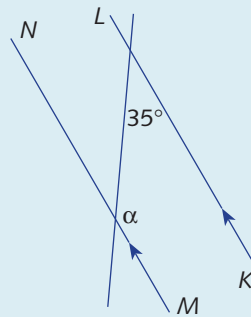


Solution

$$\angle AST = 38^\circ \text{ (alternate angles, } AB \parallel CD\text{)}$$

Example 7

Find α in the diagram opposite.



Solution

$$\alpha + 35^\circ = 180^\circ \text{ (co-interior angles, } KL \parallel MN\text{)}$$

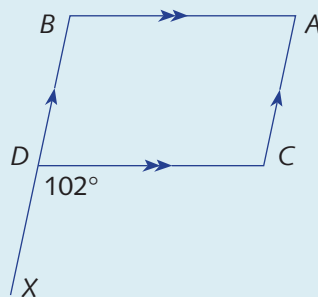
$$\alpha = 145^\circ$$

Two-step solutions

The solution of the problem below needs two steps, with a reason for each step. Notice that the pairs of parallel lines used are different in the two steps.

Example 8

Find $\angle BAC$ in the diagram opposite.



Solution

First, $\angle DCA = 102^\circ$ (alternate angles, $AC \parallel BD$).

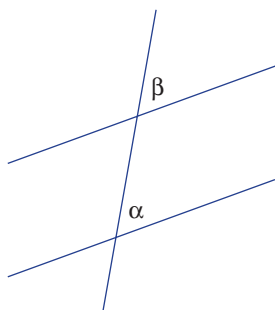
Hence $\angle BAC = 78^\circ$ (co-interior angles, $AB \parallel CD$).



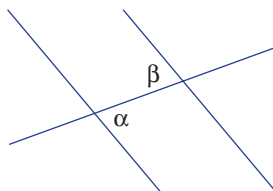
Exercise 5B

- 1 In each diagram below, identify each pair of angles marked with α and β as corresponding angles, alternate angles or co-interior angles.

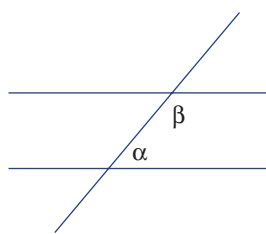
a



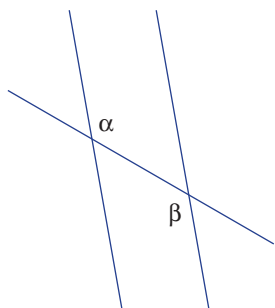
b



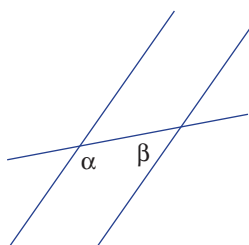
c



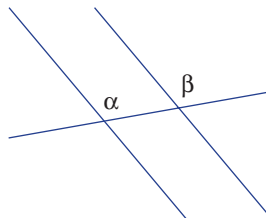
d

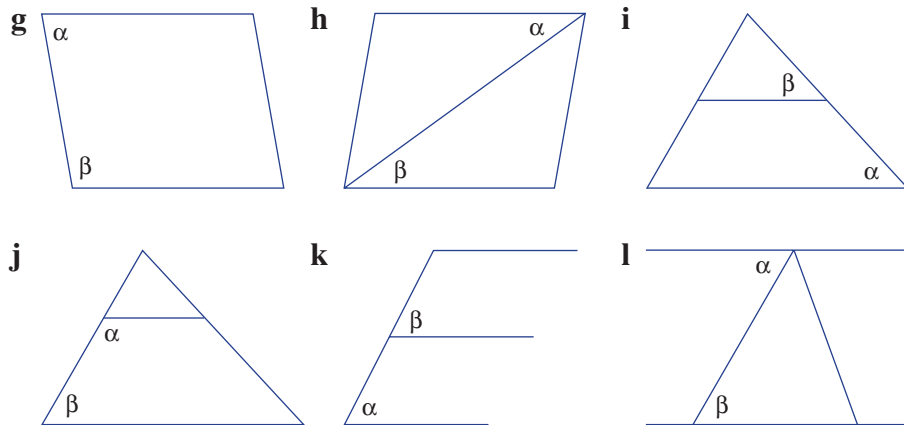


e



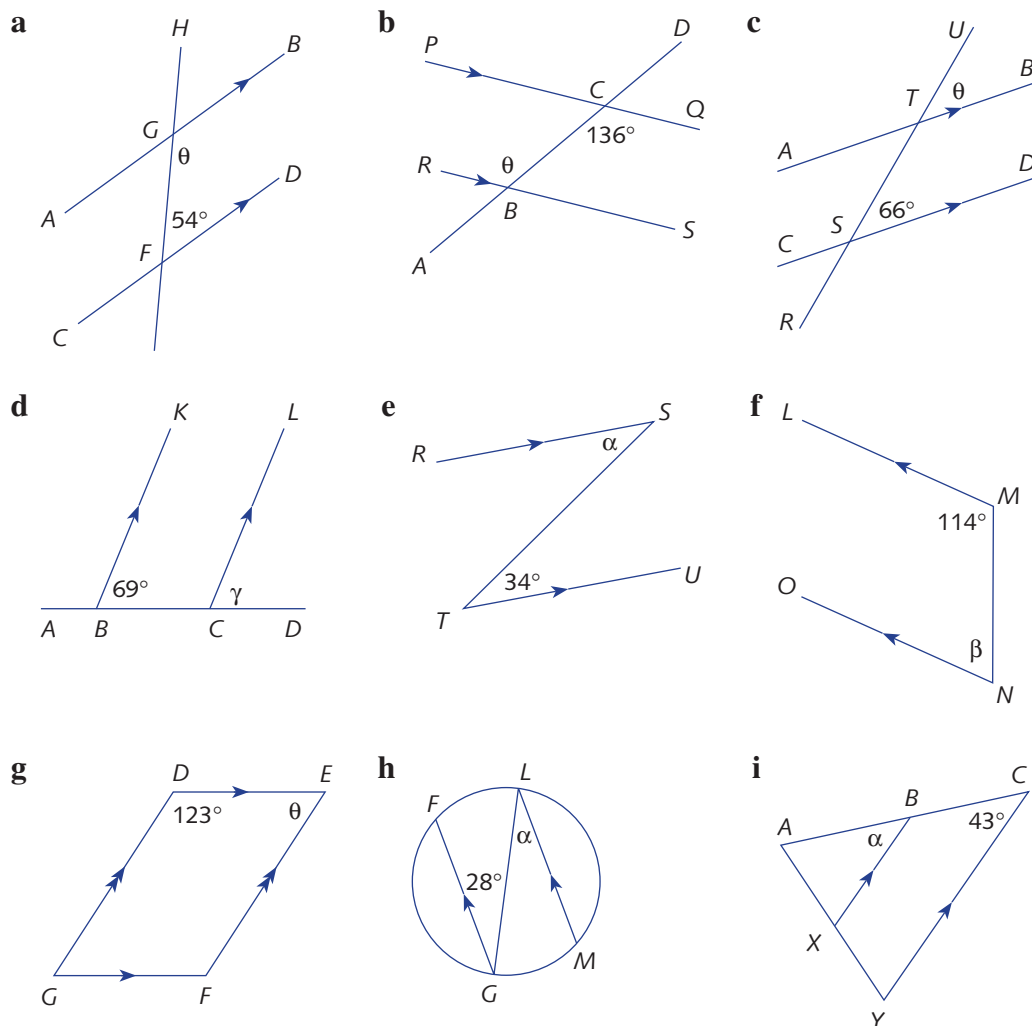
f

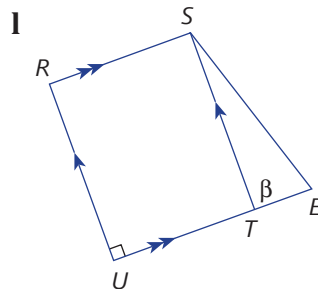
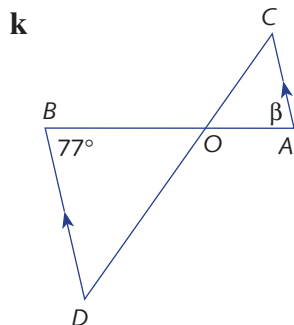
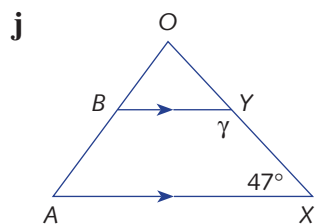




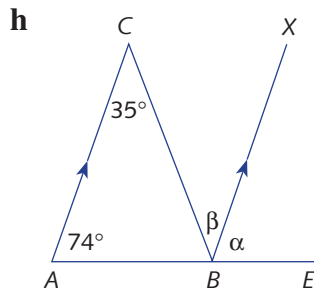
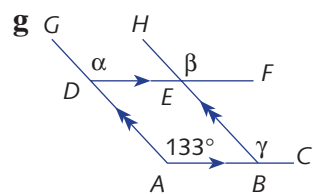
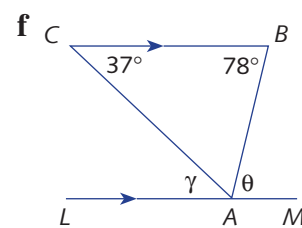
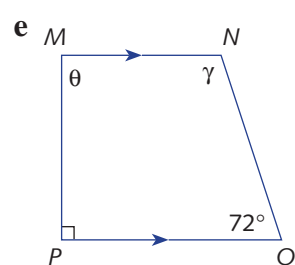
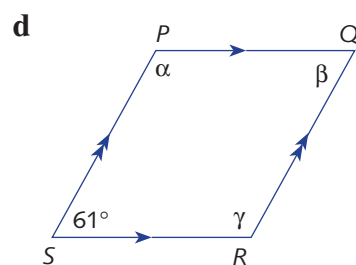
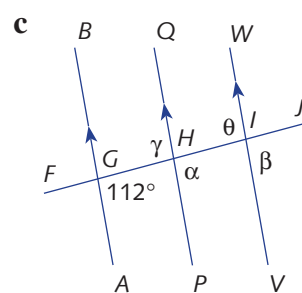
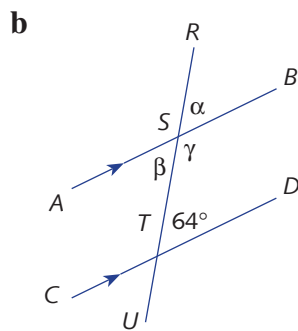
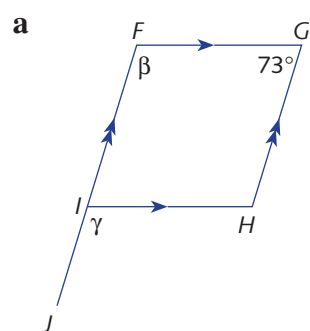
Example 5, 6, 7

- 2** Find the values of the pronumerals α , β , γ and θ in the diagrams below. Give careful reasons for all your statements, mentioning the relevant parallel lines.

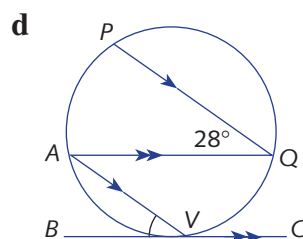
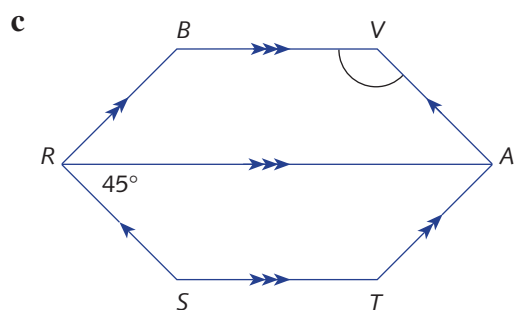
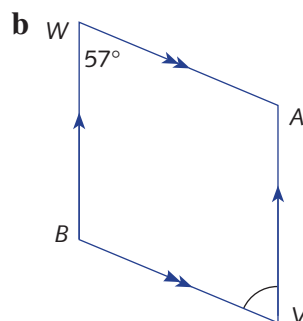
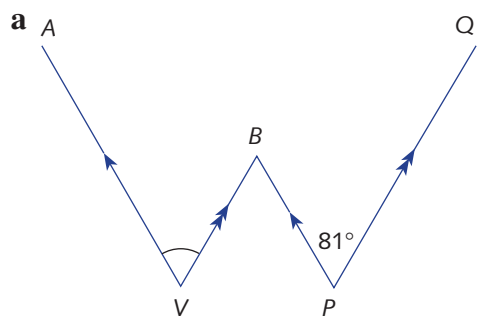




3 Find the values of α , β , γ and θ in the diagrams below, giving reasons.



- 4 Find the size of the marked angle $\angle AVB$ in each diagram below. The solution to each part will require two steps, each with its own reason.



5C Further problems involving parallel lines

This section deals with more complicated problems involving parallel lines:

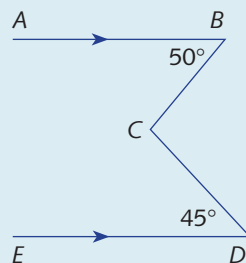
- problems where construction lines need to be added
- problems involving algebra.

Adding construction lines to solve a problem

Some problems cannot be solved until one or more extra lines, called **construction lines**, have been added to the diagram, as in the example below.

Example 9

Find $\angle BCD$ in the diagram opposite.



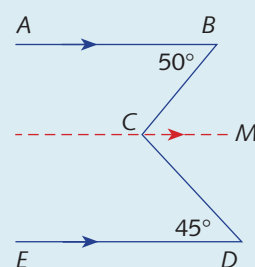
Solution

Construct the line CM through C parallel to AB and ED , as shown in the diagram opposite.

Then $\angle BCM = 50^\circ$ (alternate angles, $AB \parallel CM$),

and $\angle DCM = 45^\circ$ (alternate angles, $ED \parallel CM$).

Hence $\angle BCD = 95^\circ$ (adjacent angles at C).

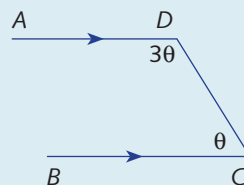


Problems involving algebra

In the example below, the value of θ is found using geometric arguments and algebra.

Example 10

Find θ in the diagram opposite.



Solution

$$\theta + 3\theta = 180^\circ \text{ (co-interior angles, } AD \parallel BC \text{)}$$

$$4\theta = 180^\circ$$

$$\theta = 45^\circ$$



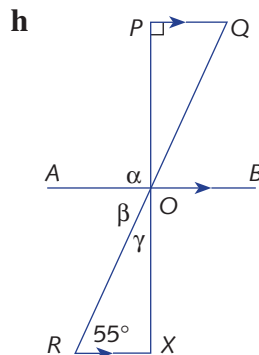
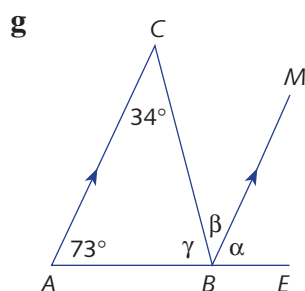
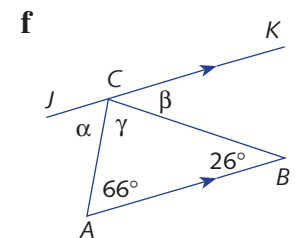
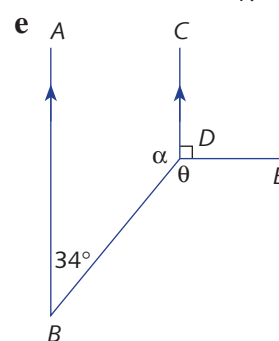
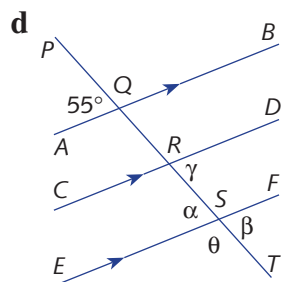
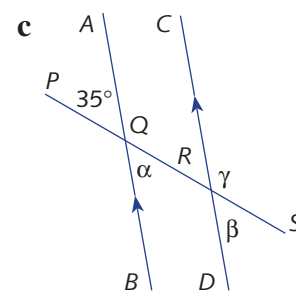
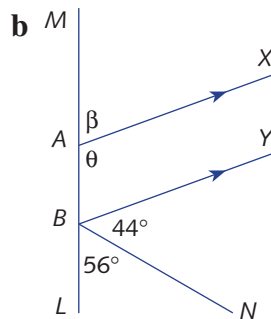
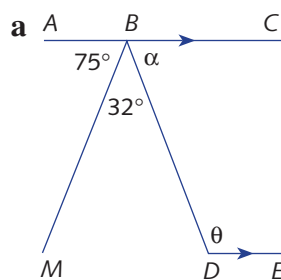
Further problems involving parallel lines

- Extra **construction lines** may need to be added to the diagram.
- Algebra may be required.

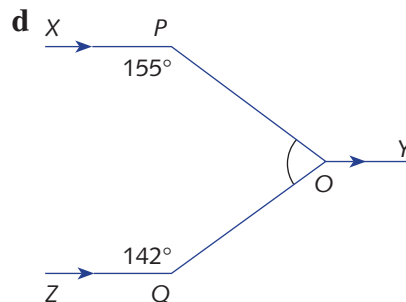
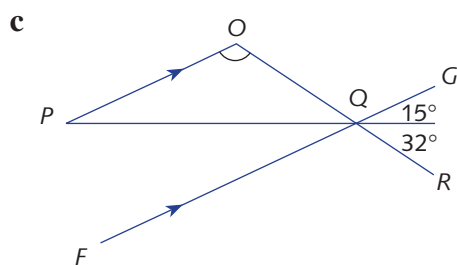
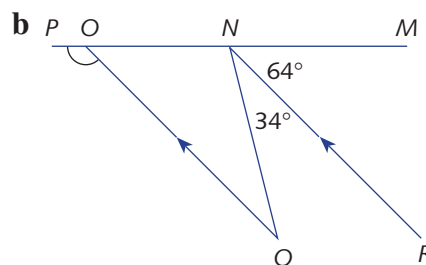
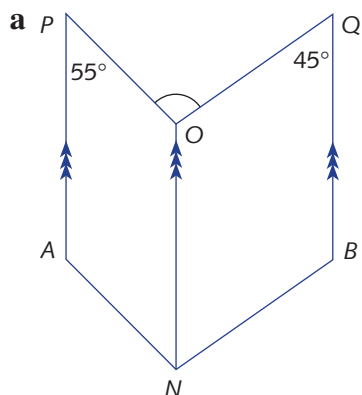


Exercise 5C

1 Find the values of α , β , γ and θ in the diagrams below, giving reasons.

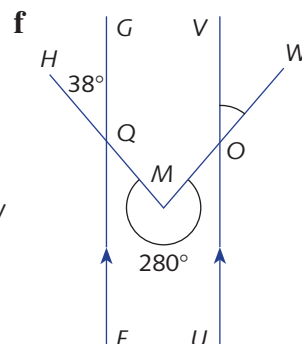
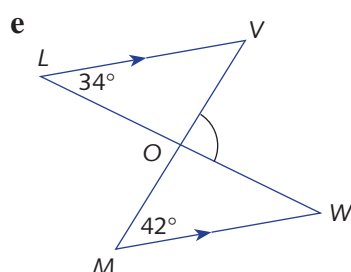
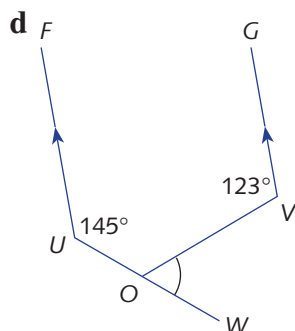
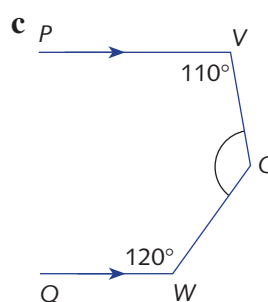
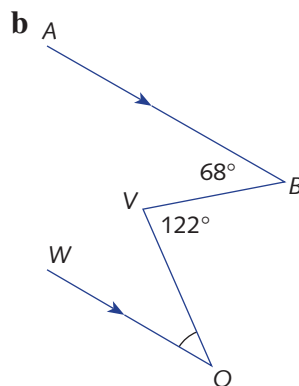
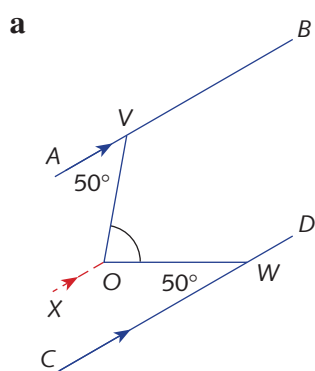


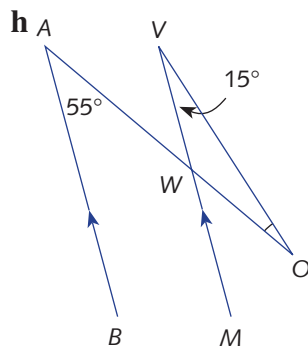
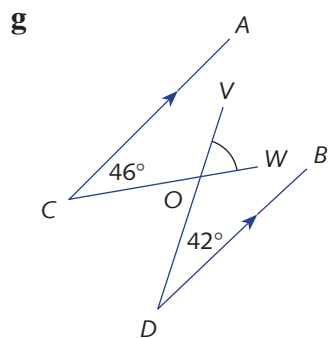
- 2 Find the size of the marked angle $\angle POQ$ in each diagram below. Several steps may be required.



Example 9

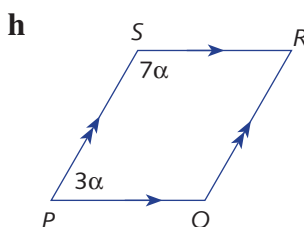
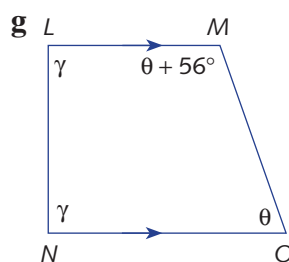
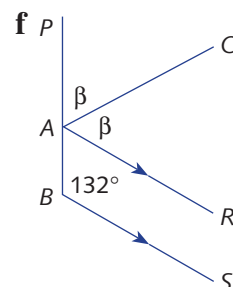
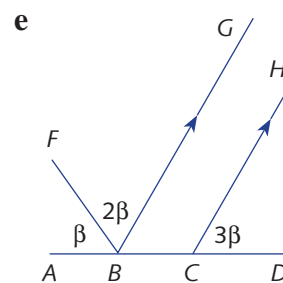
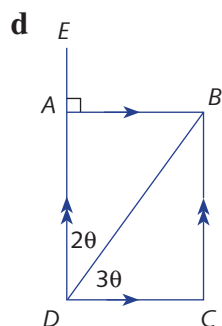
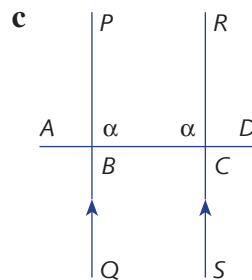
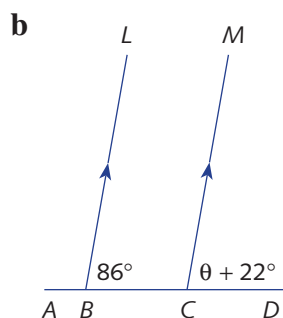
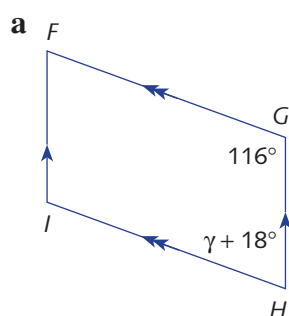
- 3 Copy each diagram, then add a suitable construction line in order to find the size of the marked angle $\angle VOW$ – the first has been done for you. Give reasons.





Example 10

4 Find the values of α , β , γ and θ , giving reasons.



5D Proving that two lines are parallel



Corresponding, alternate and co-interior angles can be used to prove that two lines are parallel.

Proving that two lines are parallel

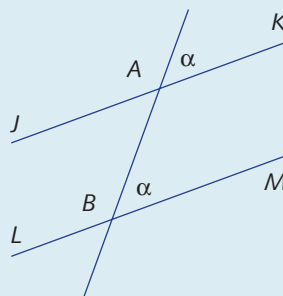
Suppose that a transversal crosses two other lines.

- If the corresponding angles are equal, then the lines are parallel.
- If the alternate angles are equal, then the lines are parallel.
- If the co-interior angles are supplementary, then the lines are parallel.

These three results are called the **converses** of the previous three results, because the logic works in reverse. Here are three examples that show how to set out arguments using them.

Example 11

Find any parallel lines in the diagram opposite.

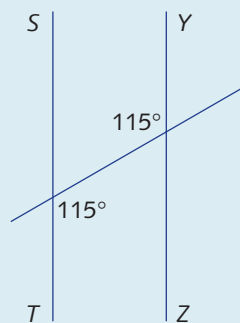


Solution

$JK \parallel LM$ (corresponding angles are equal)

Example 12

Find any parallel lines in the diagram opposite.

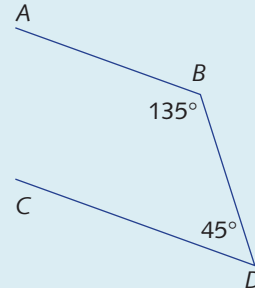


Solution

$ST \parallel YZ$ (alternate angles are equal)

Example 13

Find any parallel lines in the diagram opposite.



Solution

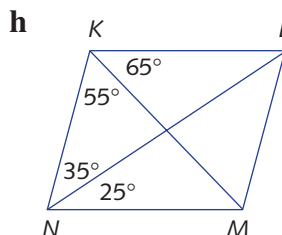
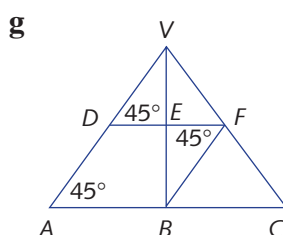
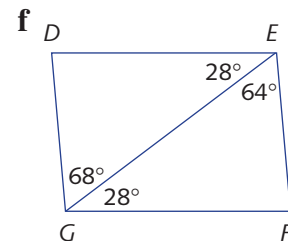
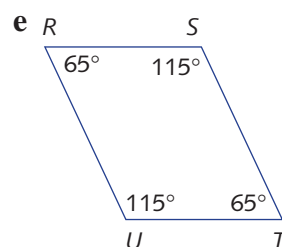
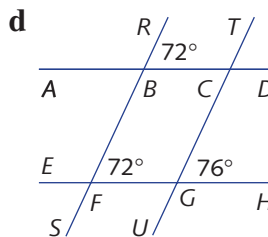
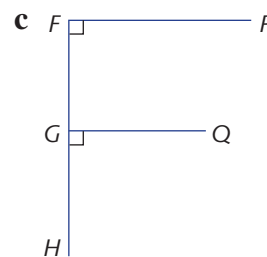
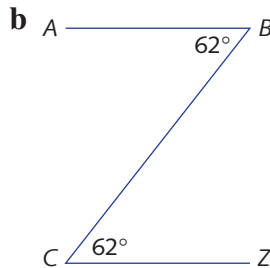
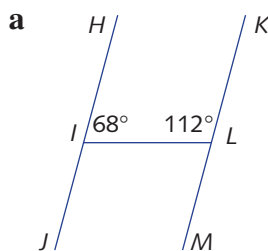
$135^\circ + 45^\circ = 180^\circ$.
Hence $AB \parallel CD$.

(co-interior angles are supplementary)

Exercise 5D

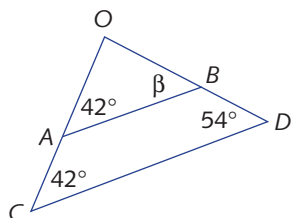
Example 11, 12, 13

1 In each diagram below, name all pairs of parallel lines, giving reasons.

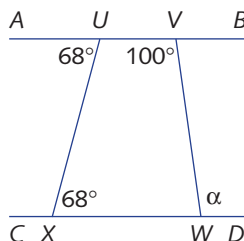


2 In each diagram below, give a reason why $AB \parallel CD$. Hence find the values of α , β , γ and θ .

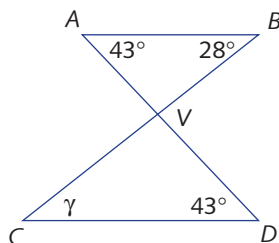
a



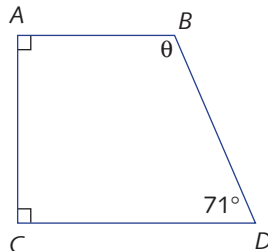
b



c

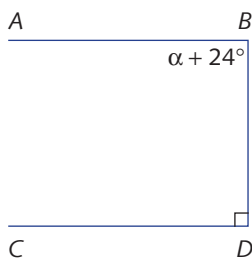


d

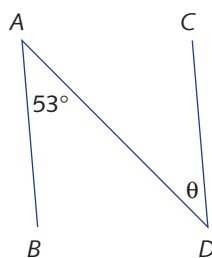


3 In each diagram below, write down the values of α , β , γ and θ that will make AB parallel to CD .

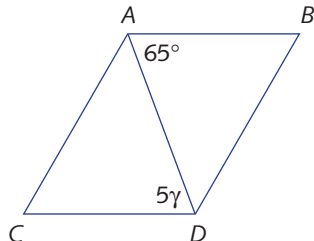
a



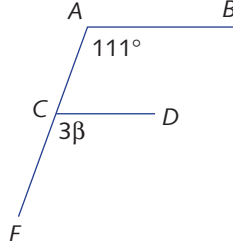
b



c



d



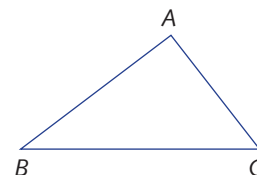
5E Angles in triangles



This section proves and applies two theorems about the angles of any triangle. In mathematics, a true statement and its proof are called a *theorem* – a Greek word meaning ‘a thing to be gazed upon’ or ‘a thing contemplated by the mind’. You may have seen these theorems already, but proving them will probably be new to you.

Triangles

A **triangle** is formed by taking three non-collinear points A , B and C and joining the three intervals AB , BC and CA .



These intervals are called the **sides** of the triangle, and the three points are called its **vertices**. (The singular of *vertices* is *vertex*.)

Proving the theorem about the angle sum of a triangle

We proved last year that the sum of the interior angles of a triangle is 180° . As we discussed then, checking a theorem many times doesn’t prove it – however many triangles you check, there are always more to check. Here is a totally convincing proof that applies to every triangle, whatever its shape.

The theorem and its proof are set out very formally in the manner traditional for geometry. The proof uses only the methods we have already developed about angles at a point and across transversals.

Theorem: The sum of the interior angles of a triangle is 180° .

Proof: Let ABC be a triangle.

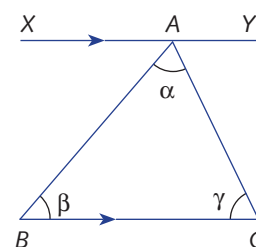
Let $\angle BAC = \alpha$, $\angle ABC = \beta$ and $\angle ACB = \gamma$.

We must prove that $\alpha + \beta + \gamma = 180^\circ$.

Draw the line XY parallel to BC and passing through the vertex A .

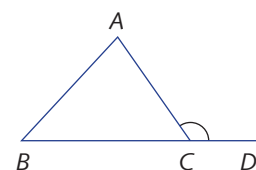
Then $\angle XAB = \beta$ (alternate angles, $XY \parallel BC$),
and $\angle YAC = \gamma$ (alternate angles, $XY \parallel BC$).

Hence $\alpha + \beta + \gamma = 180^\circ$ (straight angle $\angle XAY$).



The exterior angles of a triangle

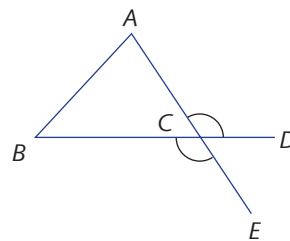
Let ABC be a triangle, with the side BC produced to D . (The word ‘produced’ means ‘extended’.) Then the angle $\angle ACD$ between the side AC and the extension CD is called an **exterior angle** of the triangle.



The angles $\angle CAB$ and $\angle ABC$ are called the **opposite interior angles**, because they are *opposite* the exterior angle at the vertex C .

There are two exterior angles at each vertex, as shown in the lower diagram opposite, and because they are vertically opposite, they are equal in size:

$\angle ACD = \angle BCE$ (vertically opposite at C).



Proving the exterior angle theorem

We proved last year that an exterior angle of a triangle is the sum of the opposite interior angles. The following is valid in every situation, whatever shape the triangle may have.

Theorem: An exterior angle of a triangle equals the sum of the two interior opposite angles.

Proof: Let ABC be a triangle, with BC produced to D .

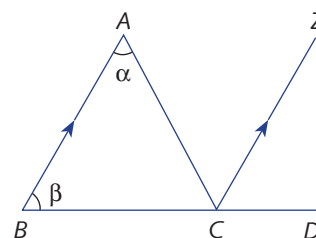
Let $\angle BAC = \alpha$ and $\angle CBA = \beta$.

We must prove that $\angle ACD = \alpha + \beta$.

Draw the line CZ through C parallel to BA .

$\angle ZCD = \beta$ (corresponding angles, $BA \parallel CZ$), and $\angle ACZ = \alpha$ (alternate angles, $BA \parallel CZ$).

Hence $\angle ACD = \alpha + \beta$ (adjacent angles at C).

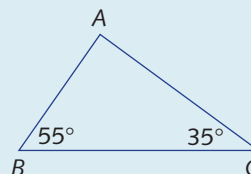


Angles in triangles

- The sum of the interior angles of a triangle is 180° .
- An exterior angle of a triangle equals the sum of the opposite interior angles.

Example 14

Find $\angle A$ in the triangle opposite.

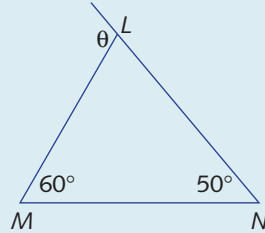


Solution

$\angle A + 55^\circ + 35^\circ = 180^\circ$ (angle sum of $\triangle ABC$),
so $\angle A = 90^\circ$.

Example 15

Find θ in the triangle opposite.



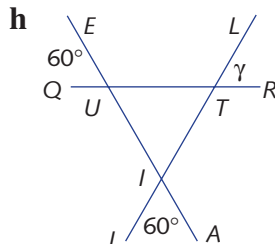
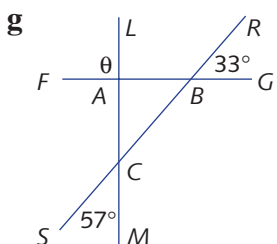
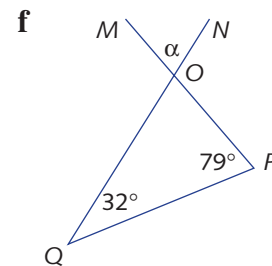
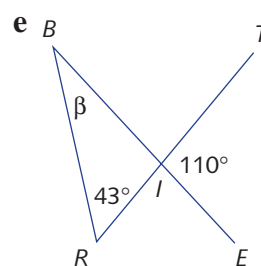
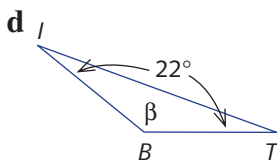
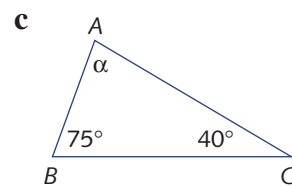
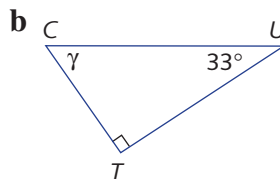
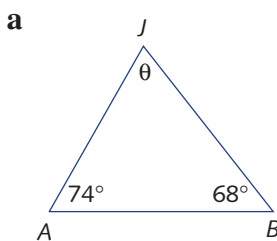
Solution

$\theta = 60^\circ + 50^\circ$ (exterior angle of $\triangle LMN$),
so $\theta = 110^\circ$.

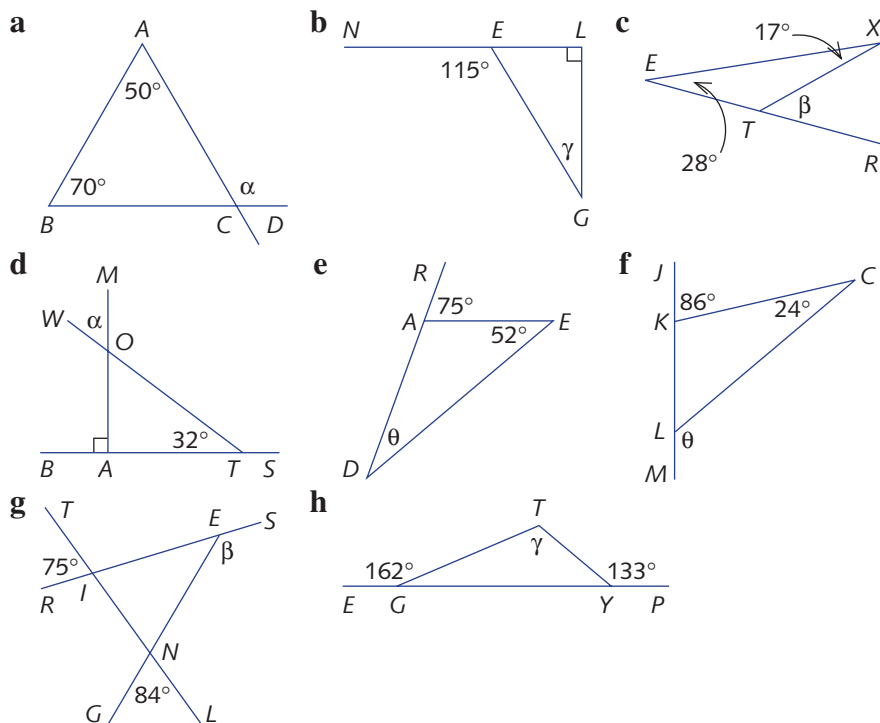
Exercise 5E

- Find the third angle of a triangle, two of whose angles are given.
a $60^\circ, 30^\circ$ **b** $120^\circ, 30^\circ$ **c** $111^\circ, 11^\circ$ **d** $62^\circ, 90^\circ$ **e** $28^\circ, 58^\circ$
- Find the values of α, β, γ and θ in the diagrams below, giving reasons.

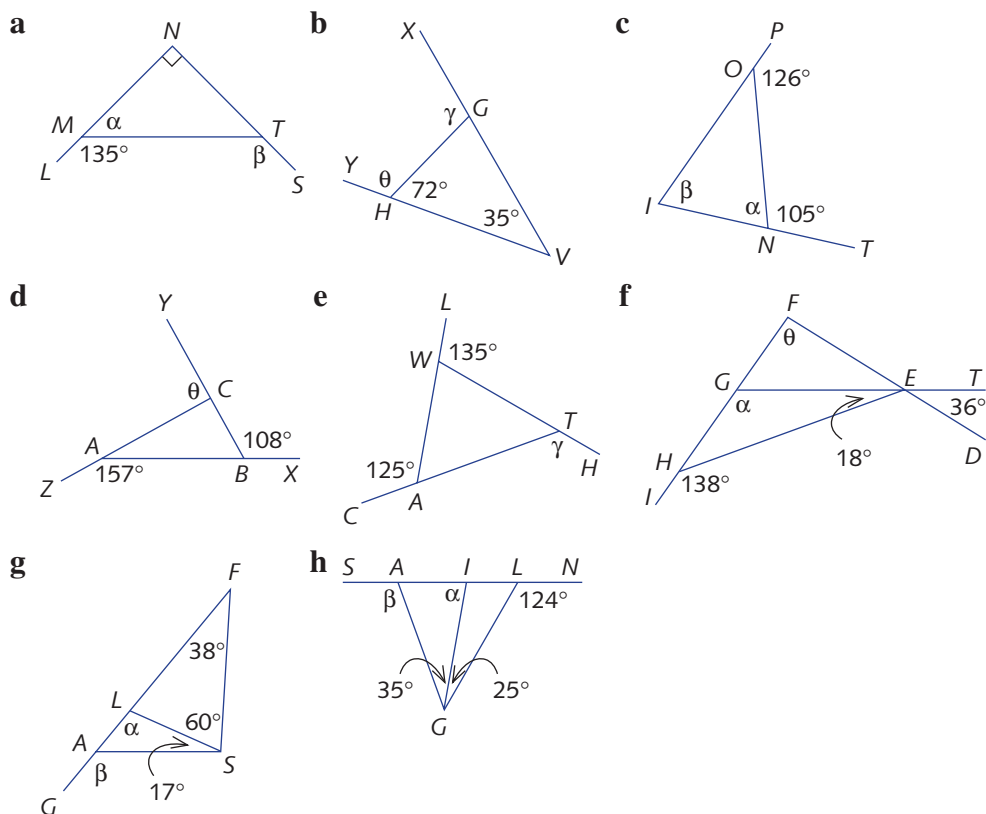
Example 14, 15



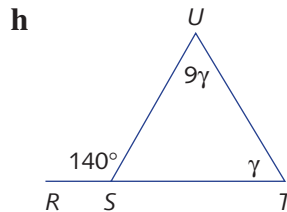
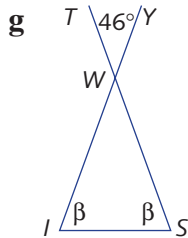
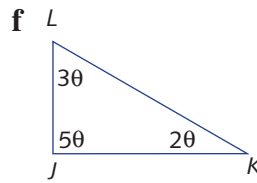
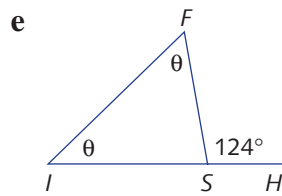
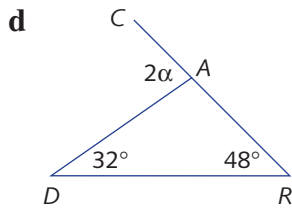
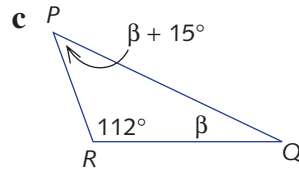
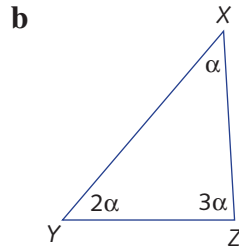
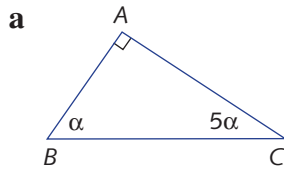
3 Use exterior angles – *not* interior angles – to find α , β , γ and θ in each diagram below. Give reasons.



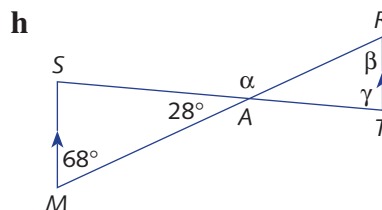
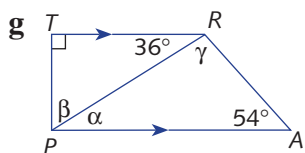
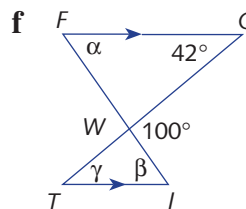
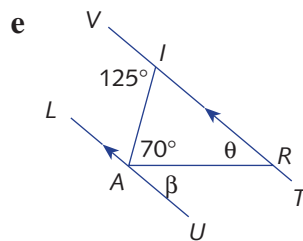
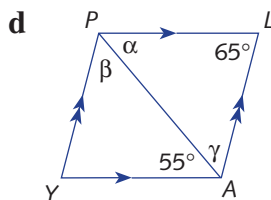
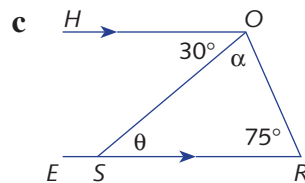
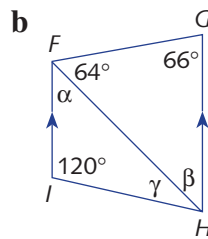
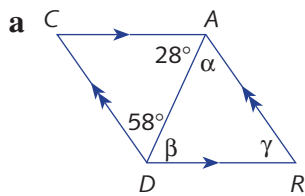
4 Find the values of the pronumerals in each diagram, giving reasons.



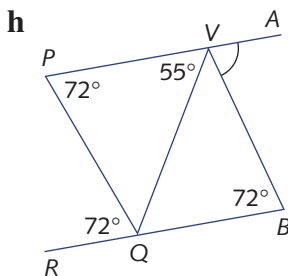
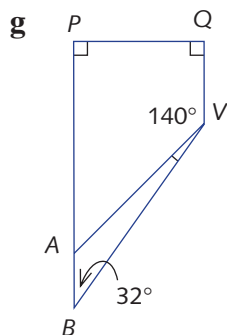
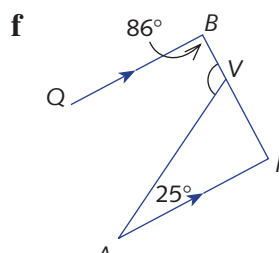
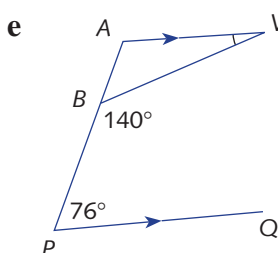
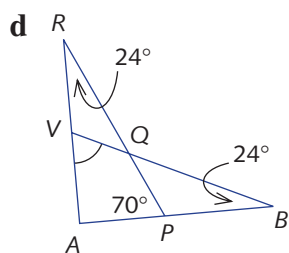
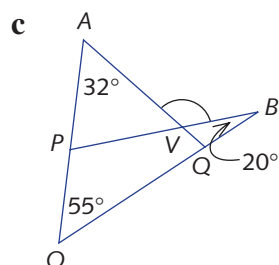
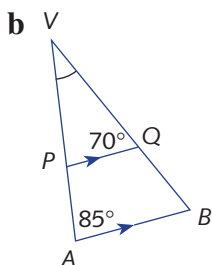
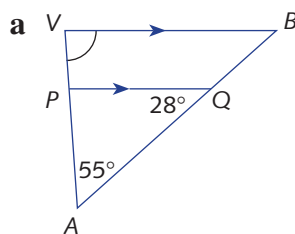
5 Find α , β , γ and θ in these diagrams, giving reasons.



6 Find α , β , γ and θ in these diagrams, giving reasons.



7 Find the size of the marked angle $\angle AVB$ in each diagram, giving reasons.

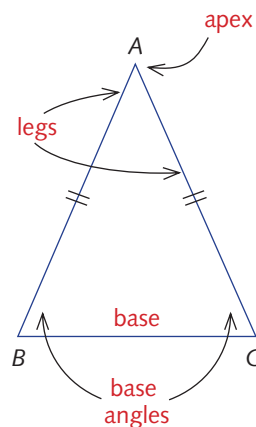




Isosceles triangles

An **isosceles triangle** is a triangle in which two (or more) sides have equal length.

- The equal sides AB and AC in the isosceles triangle ABC to the right are called the **legs**. They have been marked with double dashes to indicate that they are equal in length.
- The vertex A where the legs meet is called the **apex**.
- The third side BC is called the **base**.
- The angles $\angle ABC$ and $\angle ACB$ at the base are called **base angles**.

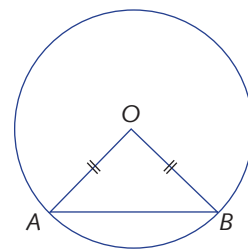


The word *isosceles* is a Greek word meaning ‘equal legs’ – *iso* means ‘equal’, and *sceles* means ‘legs’.

Constructing an isosceles triangle using a circle

In the diagram to the right, the equal radii AO and BO of the circle form the equal legs of the isosceles triangle OAB . The interval AB is the base and the centre O is the apex.

Using two radii of a circle is usually the simplest way to construct an isosceles triangle.



Isosceles triangles and their base angles

Last year, in Chapter 13 of *ICE-EM Mathematics Year 7 Book 2*, we developed in an informal way two results about isosceles triangles. The first is a property that all isosceles triangles have.

- The base angles of an isosceles triangle are equal.
(This result can also be stated as ‘If two sides of a triangle are equal, then the angles opposite those sides are equal.’)

The second is a test for a triangle to be isosceles, and is the converse of the first result.

- If two angles of a triangle are equal, then the sides opposite those angles are equal.

Later this year, we will have the techniques necessary to prove these theorems properly. In Chapter 14 of *Book 2* we will introduce *congruence*, which will allow us to prove these theorems properly.

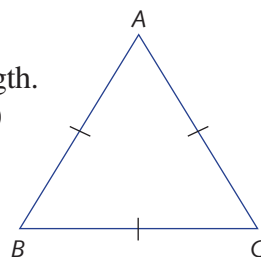
Isosceles triangles

- An **isosceles triangle** is a triangle with two (or more) sides equal.
 - The equal sides are called the **legs**; the legs meet at the **apex**; the third side is the **base**; and the angles opposite the legs are called **base angles**.
 - Two radii of a circle and the interval joining them form an isosceles triangle.
- The base angles of an isosceles triangle are equal.
- Conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal.

Equilateral triangles

An **equilateral triangle** is a triangle in which all three sides have equal length. (The name comes from Latin – *equi* means ‘equal’, and *latus* means ‘side’.)

The diagram to the right shows an equilateral triangle ABC . Notice how it is an isosceles triangle in three different ways, because the base could be taken as AB , BC or CA .



The interior angles of an equilateral triangle are all 60°

It is not hard to prove that all the angles of an equilateral triangle are 60° provided that we use the earlier theorem that the base angles of an isosceles triangle are equal.

Theorem: The interior angles of an equilateral triangle are all 60° .

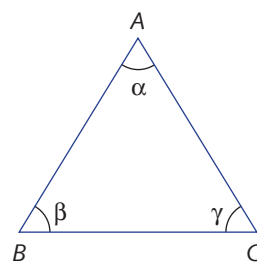
Proof: Let ABC be an equilateral triangle.

Let $\angle BAC = \alpha$, $\angle ABC = \beta$ and $\angle ACB = \gamma$.

We must prove that $\alpha = \beta = \gamma = 60^\circ$.

$\beta = \gamma$ (opposite sides AC and AB are equal), and $\beta = \alpha$ (opposite sides AC and BC are equal), so $\alpha = \beta = \gamma$.

But $\alpha + \beta + \gamma = 180^\circ$ (angle sum of $\triangle ABC$), so $\alpha = \beta = \gamma = 60^\circ$.



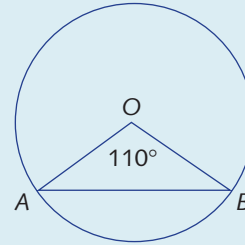
Equilateral triangles

- An equilateral triangle is a triangle with all three sides equal.
- The angles in an equilateral triangle are all 60° .

Here are examples of arguments using each of the three theorems in this section, and the fact that all the radii of any circle are equal.

Example 16

Find $\angle OBA$ in the diagram opposite.



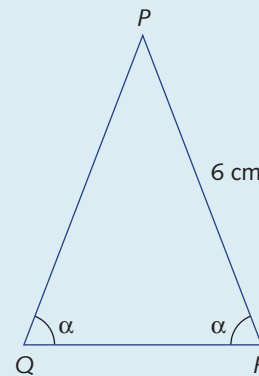
Solution

$AO = BO$ (radii),

Hence $\angle OBA = 35^\circ$ (base angles of isosceles $\triangle ABO$).

Example 17

Find the length of PQ in the diagram opposite.

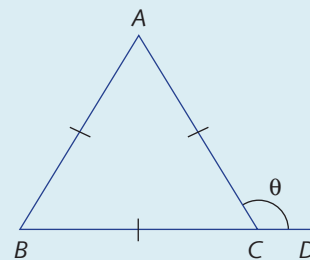


Solution

$PQ = 6$ cm (opposite angles $\angle PQR$ and $\angle PRQ$ are equal).

Example 18

Find θ in the diagram opposite.



(continued on next page)

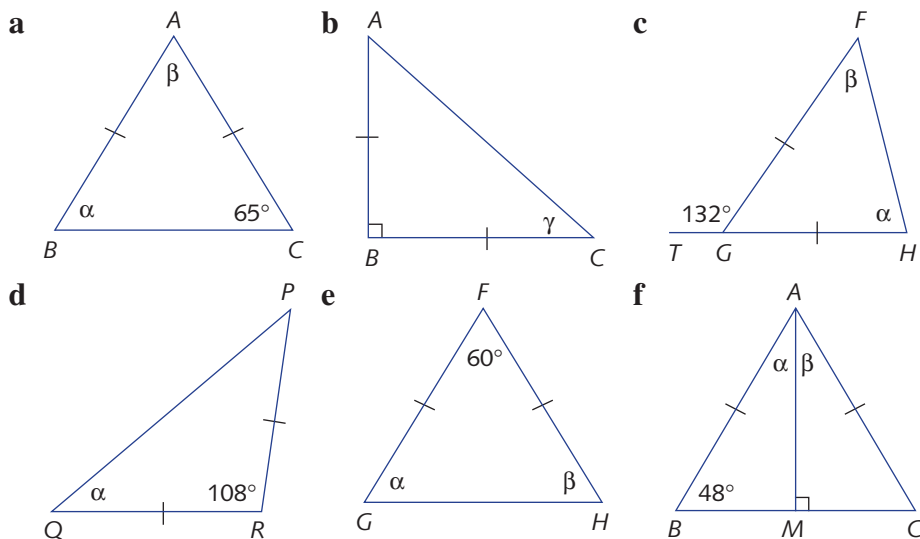
Solution

$\angle ABC = 60^\circ$ (equilateral $\triangle ABC$).
 Hence $\theta + 60^\circ = 180^\circ$ (straight angle $\angle BCD$),
 so $\theta = 120^\circ$.

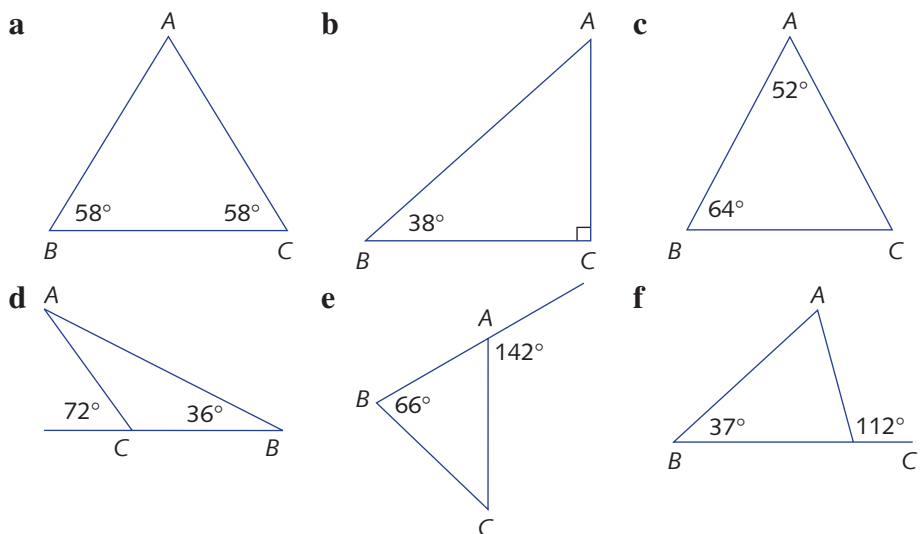
Exercise 5F

Example 18

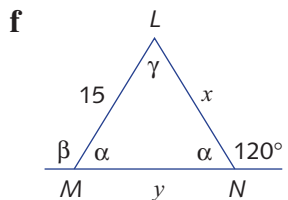
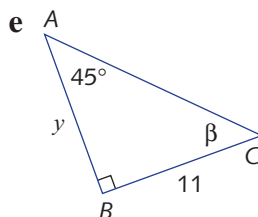
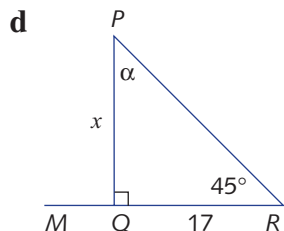
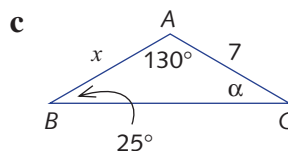
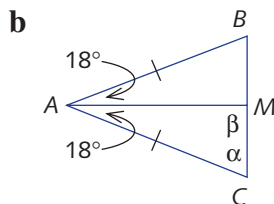
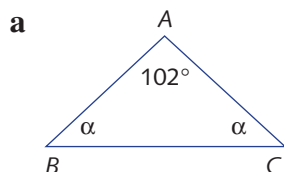
1 Find the value of each pronumeral, giving reasons.



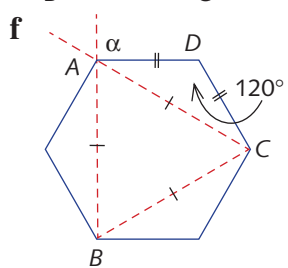
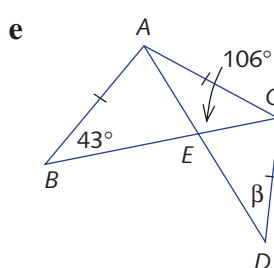
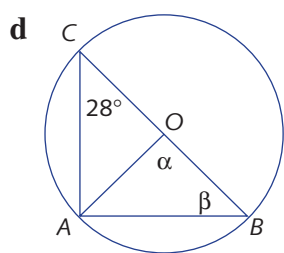
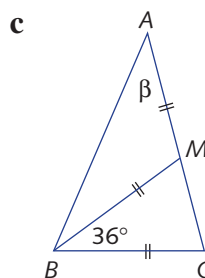
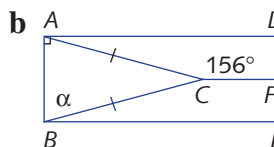
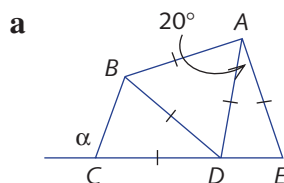
2 Determine whether $\triangle ABC$ in each part is an isosceles triangle, naming the equal sides. Give reasons.



- 3 a Let $\triangle ABC$ be isosceles, with $AB = AC$. Produce BC to D , and let $\angle ACD = 124^\circ$. Draw a diagram and find $\angle ABC$, giving reasons.
- b P and Q are points on a circle with centre O such that $\angle POQ = 48^\circ$. Draw a diagram and calculate the size of $\angle OPQ$.
- 4 Find the values of the pronumerals in these diagrams, giving reasons.

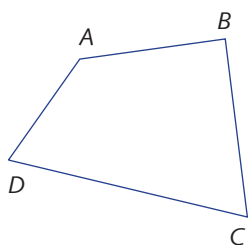


- 5 Find the values of the pronumerals in these diagrams, giving reasons. In part d, O is the centre of the circle.

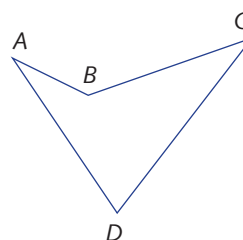




Take any four points A , B , C and D in the plane, no three of which are collinear, and join the intervals AB , BC , CD and DA . Provided that no two of these intervals cross over each other, the figure $ABCD$ is called a *quadrilateral* (from Latin – *quadri* means ‘four’, and *latus* means ‘side’). The four points are its **vertices** and the four intervals are its **sides**.

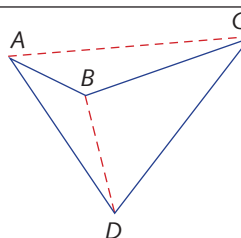
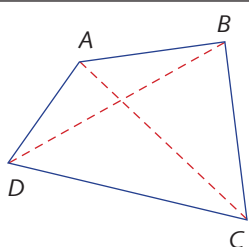


The quadrilateral above is called a **convex quadrilateral** because none of its four interior angles is a reflex angle.



The quadrilateral above is called a **non-convex quadrilateral** because one of its interior angles ($\angle ABC$) is a reflex angle.

(Note: No interior angle can be exactly 180° because the definition requires that no three of the vertices be collinear.)



The intervals AC and BD joining non-adjacent vertices are called the **diagonals** of the quadrilateral. The left-hand quadrilateral above is convex and so both diagonals are inside the quadrilateral. The right-hand quadrilateral above is non-convex and one diagonal is inside the quadrilateral and the other is outside.

The interior angles of a quadrilateral add to 360°

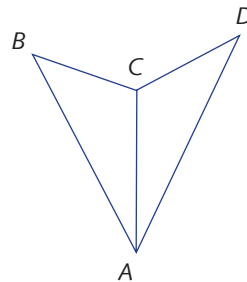
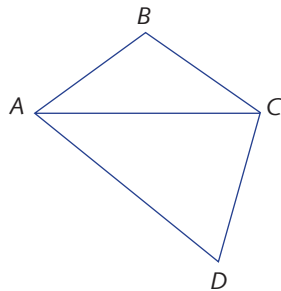
The interior angles of a quadrilateral always add to 360° .

The general proof divides the quadrilateral into two triangles and then applies the earlier theorem about the angle sum of a triangle.

Theorem: The sum of the interior angles of a quadrilateral is 360° .

Proof: Let $ABCD$ be a quadrilateral, labelled so that the diagonal AC is inside the quadrilateral.

(At least one diagonal lies inside the quadrilateral.)



We must prove that $\angle ABC + \angle BCD + \angle CDA + \angle DAB = 360^\circ$.

Join the diagonal AC .

The interior angles of $\triangle ABC$ add to 180° .

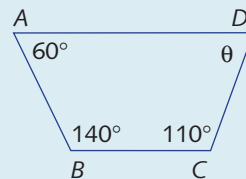
Similarly, the interior angles of $\triangle ADC$ add to 180° .

But the sum of the interior angles of $ABCD$ is the sum of the angles of $\triangle ABC$ and $\triangle ADC$.

Hence the interior angles of $ABCD$ add to 360° .

Example 19

Find θ in the diagram opposite.



Solution

$$\theta + 60^\circ + 110^\circ + 140^\circ = 360^\circ \quad (\text{angle sum of quadrilateral } ABCD)$$

$$\theta + 310^\circ = 360^\circ$$

$$\theta = 50^\circ$$

Quadrilaterals

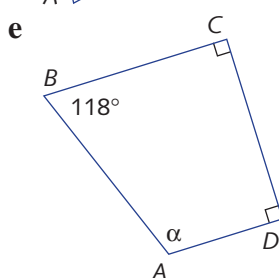
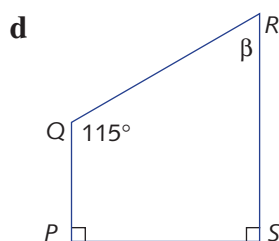
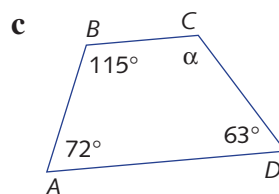
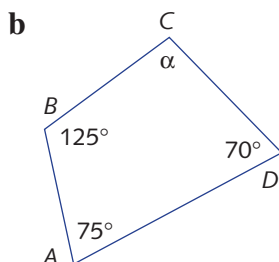
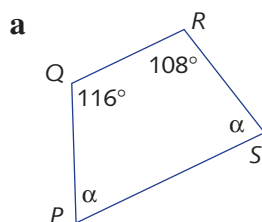
- A **quadrilateral** $ABCD$ has four **vertices**, no three of which are collinear, and four **sides**, no two crossing each other.
- A **convex quadrilateral** has no reflex interior angle. Both **diagonals** of a convex quadrilateral lie inside the figure.
- A **non-convex quadrilateral** has one reflex interior angle. One diagonal of a non-convex quadrilateral lies outside the figure, the other inside.
- The sum of the interior angles of a quadrilateral is 360° .



Exercise 5G

Example 19

- 1 Find the values of the pronumerals in these diagrams, giving reasons.



- 2 In each part, three angles of a quadrilateral are given. Calculate the size of the fourth angle of the quadrilateral. Sketch a quadrilateral with these angle sizes.

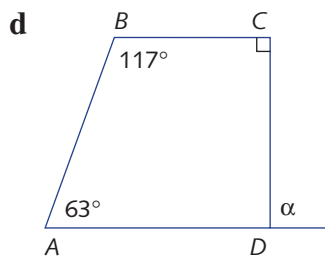
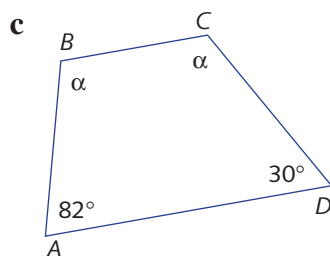
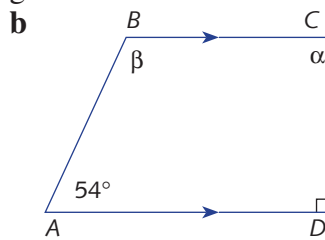
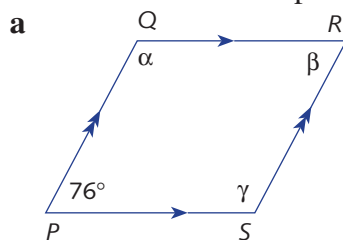
a $90^\circ, 90^\circ, 150^\circ$

b $45^\circ, 90^\circ, 135^\circ$

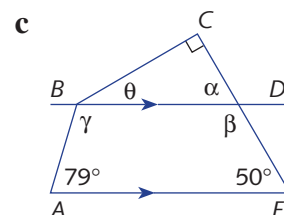
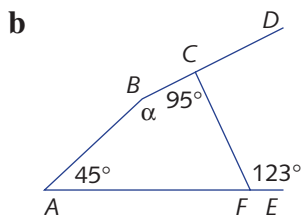
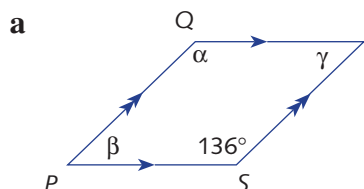
c $70^\circ, 160^\circ, 20^\circ$

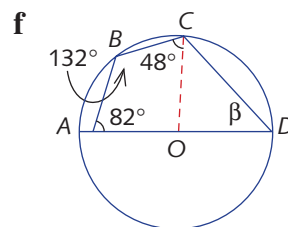
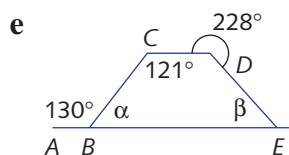
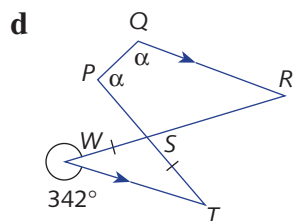
d $40^\circ, 40^\circ, 40^\circ$

- 3 Find the values of the pronumerals, giving reasons.

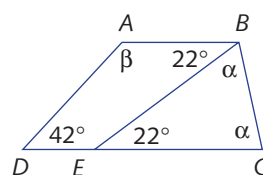
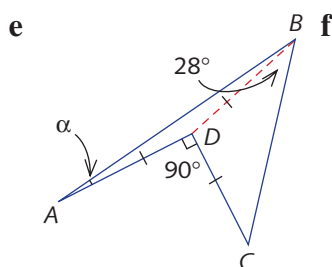
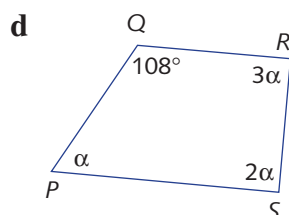
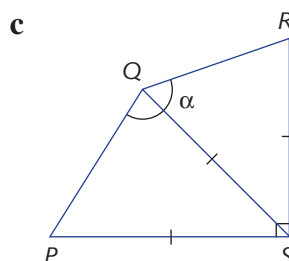
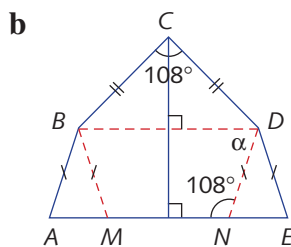
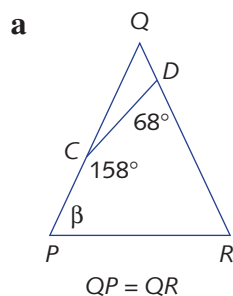


- 4 Find the values of the pronumerals, giving reasons. In part **f**, O is the centre of the circle.

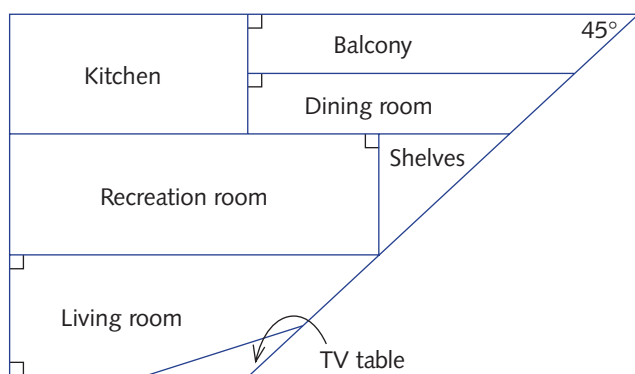




5 Find the values of the pronumerals, giving reasons.



6 The following diagram is a map of the first floor of Tony's house.

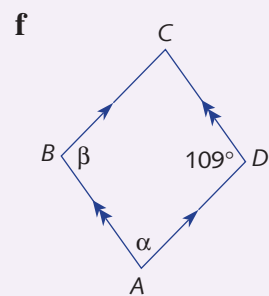
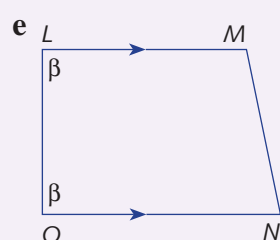
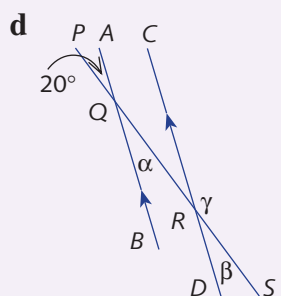
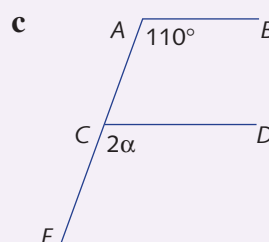
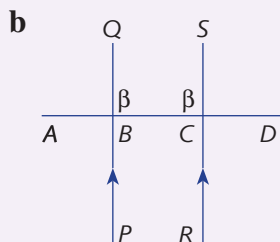
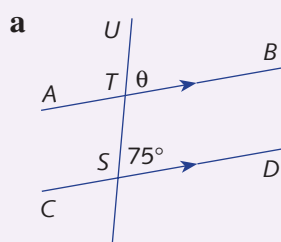


- Tony would like to build some wooden shelves at the end of the recreation room. Each shelf would be an isosceles triangle. Evaluate the internal angles of these shelves.
- Tony would also like to construct a TV table in the shape of an isosceles triangle. Evaluate the internal angles of the TV table.



Review exercise

1 Find the value of the pronumeral in each diagram. Give reasons in each case.



2 State the complement of each angle.

a 30°

b 63°

c 74°

d 84°

3 State the supplement of each angle.

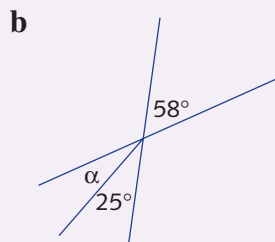
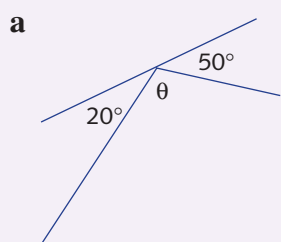
a 127°

b 76°

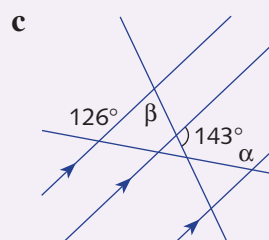
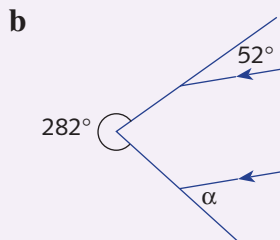
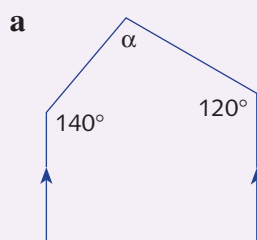
c 134°

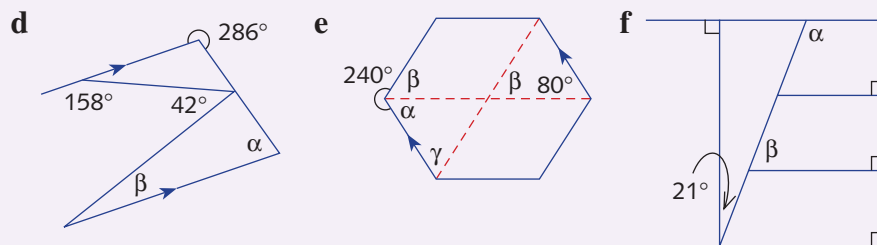
d 15°

4 Find the value of each pronumeral.

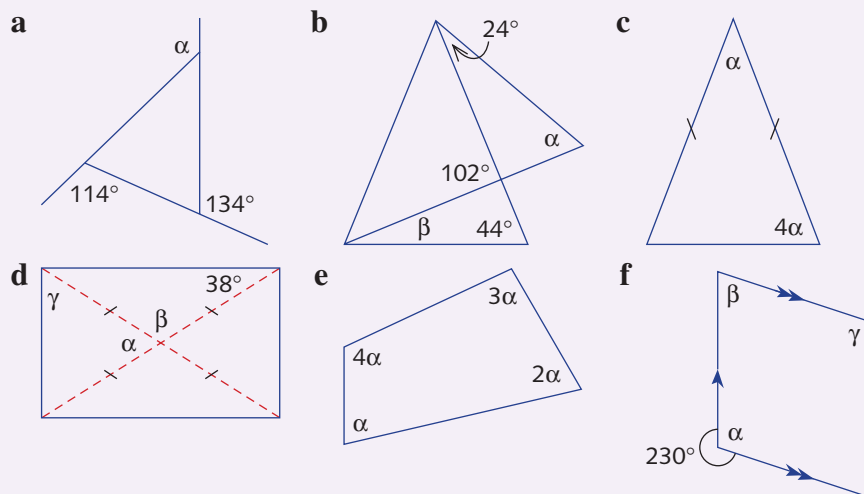


5 Find the values of the pronumerals in these diagrams.

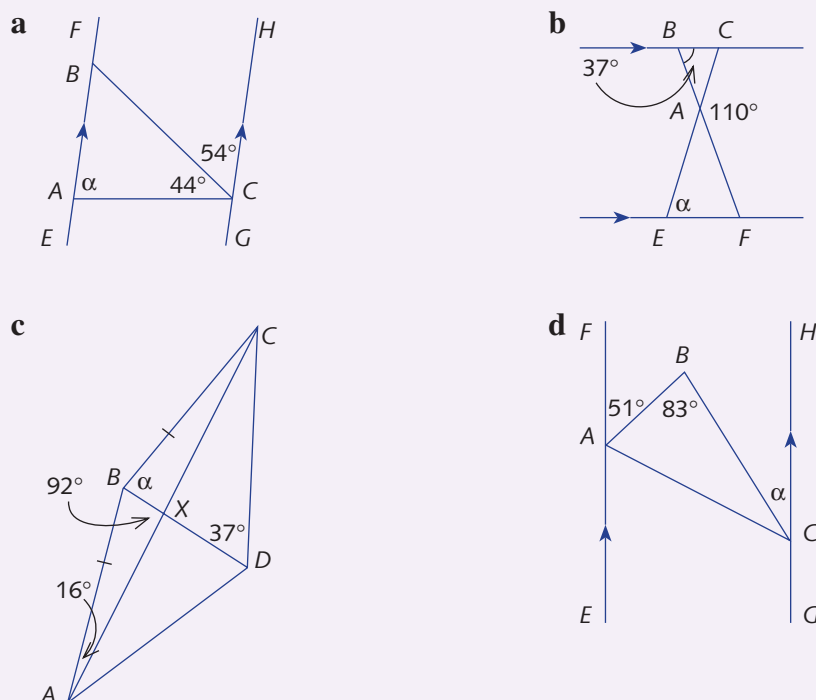




6 Find the values of the pronumerals in these diagrams.



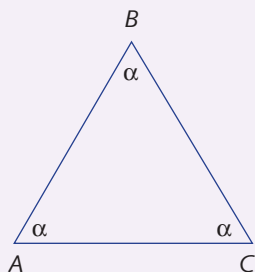
7 Find the value of α in each of these diagrams.



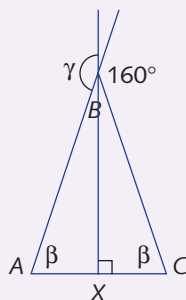


- 8 Explain why a triangle cannot have a reflex angle as one of its internal angles.
- 9 Find the values of α , β and γ in the diagrams below, and explain your reasoning.

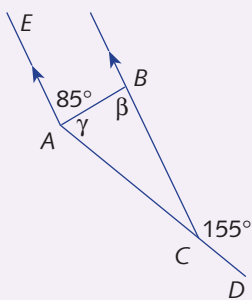
a



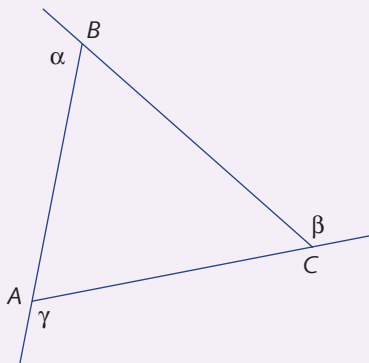
b



c



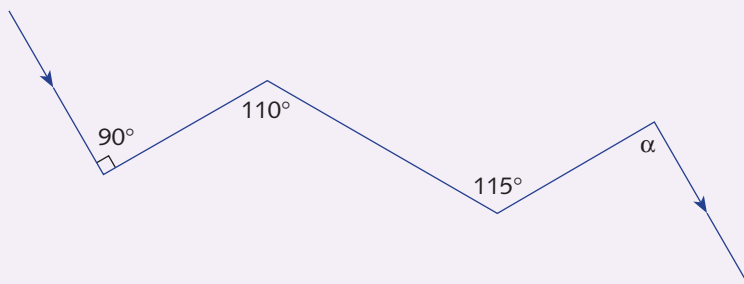
- 10 Find the value of $\alpha + \beta + \gamma$ in the diagram below.



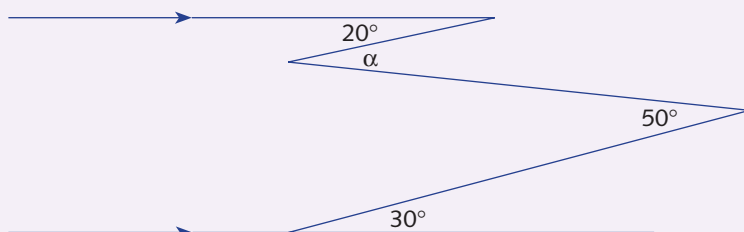
Challenge exercise



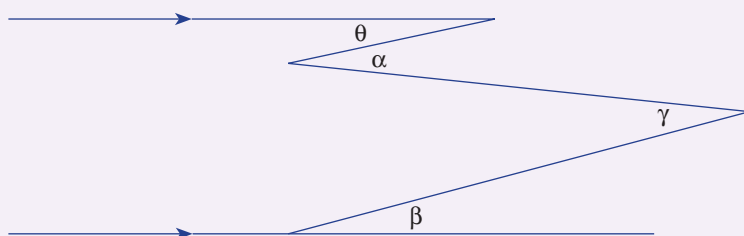
- 1 Calculate the unknown angle.



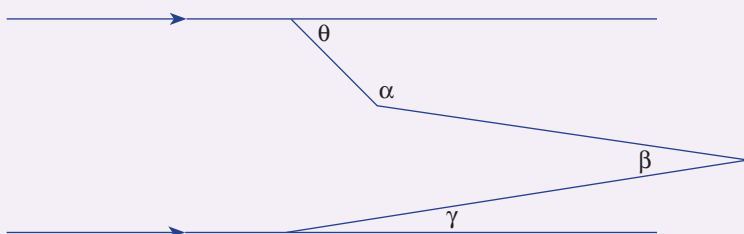
- 2 a Find the value of α .



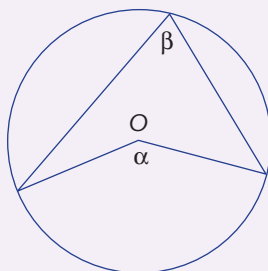
- b Find α in terms of β , γ and θ .



- c Find α in terms of β , γ and θ .

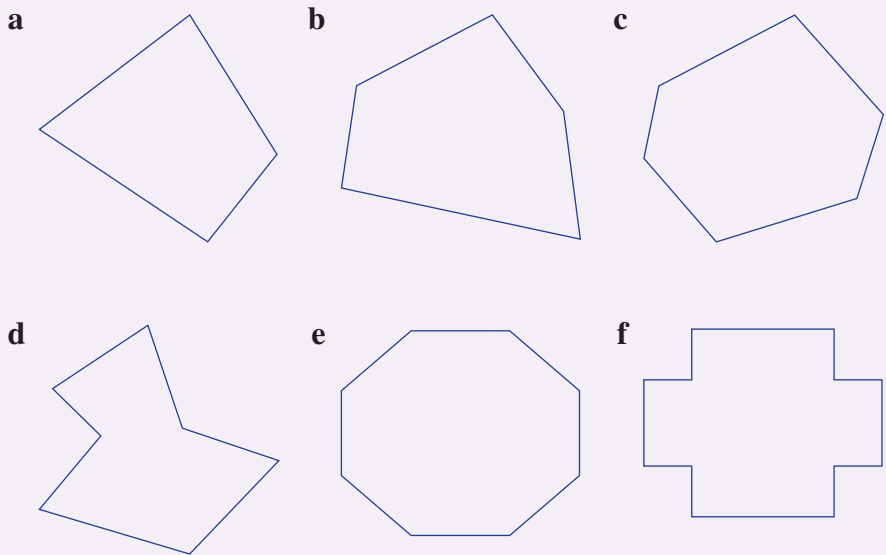


- 3 The point O is the centre of the circle. Find α in terms of β .

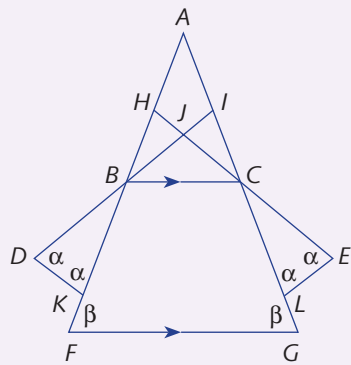




4 The sum of the internal angles of a triangle is 180° . Use this property of a triangle to find the sum of the internal angles in each of the diagrams below. What relationship do you notice?



5 Name all the isosceles triangles in the diagram below.



6 Consider the isosceles triangle ABC , with $AB = AC$. Suppose D is on AB , and E is on AC , so that $AD = DE$. If $\angle CED = 142^\circ$, calculate $\angle ABC$.

7 $ABCD$ is a square and AEB is an equilateral triangle. Calculate the two possible values of the size of $\angle DEC$.



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CHAPTER

6

Number and Algebra

Algebra – part 1

Algebra is an important part of mathematical language. It helps us to state ideas more simply. It also enables us to make general statements about mathematics and solve problems that are difficult to do otherwise.

Algebra was introduced to the Arabs in about 830 CE by Abu Abdullah Muhammad bin Musa al-Khwarizmi. The last part of his name gives us the word algorithm. His work was also influential in the introduction of algebra into Europe in the early thirteenth century.



In algebra, letters are used to stand for numbers. For example, if a box contains x stones and you put five more in, then there are $x + 5$ stones in the box. You may or may not know what the value of x is. In algebra, we call the letter a **pronumeral**.

Substitution involves giving a value to the letter.

For example, if $x = 20$, then $x + 5 = 25$.

We first recall some conventions and notation in algebra.

Multiplication: $5 \times x = 5x$ $a \times b = ab$

Division: $x \div 5 = \frac{x}{5}$ $a \div b = \frac{a}{b}$

Powers: $x \times x = x^2$ $a \times a \times a = a^3$ $z \times z \times z \times z = z^4$

The following table reminds us of the meanings of some commonly occurring algebraic expressions.

$2x + 3$	The number x is multiplied by 2, and 3 is added to the result.
$5x - 3$	The number x is multiplied by 5, and 3 is subtracted from the result.
$3(x - 1)$	One is subtracted from the number x , and the result is multiplied by 3.
$x^2 + 4$	The number x is multiplied by itself, and 4 is added to the result.
$\frac{x}{5} + 6$	The number x is divided by 5, and 6 is added to the result.
$\frac{x + 5}{6}$	5 is added to the number x , and the result is divided by 6.

In algebra we write:

$$x^1 = x$$

$$0 + x = x$$

$$1x = x$$

Example 1

Find the value of each expression below for:

a $x = 2$

$$x + 3$$

b $x = -3$

$$x - 3$$

$$2x + 3$$

$$\frac{x + 3}{2}$$

Solution

a When $x = 2$:

$$\begin{aligned}x + 3 &= 2 + 3 \\ &= 5\end{aligned}$$

$$\begin{aligned}2x + 3 &= 2 \times 2 + 3 \\ &= 7\end{aligned}$$

$$\begin{aligned}x - 3 &= 2 - 3 \\ &= -1\end{aligned}$$

$$\begin{aligned}\frac{x+3}{2} &= \frac{2+3}{2} \\ &= \frac{5}{2} \\ &= 2\frac{1}{2}\end{aligned}$$

b When $x = -3$:

$$\begin{aligned}x + 3 &= -3 + 3 \\ &= 0\end{aligned}$$

$$\begin{aligned}2x + 3 &= 2 \times (-3) + 3 \\ &= -6 + 3 \\ &= -3\end{aligned}$$

$$\begin{aligned}x - 3 &= -3 - 3 \\ &= -6\end{aligned}$$

$$\begin{aligned}\frac{x+3}{2} &= \frac{-3+3}{2} \\ &= 0\end{aligned}$$

Example 2

For $m = -1$ and $n = \frac{1}{2}$, evaluate:

a $m + n$

b $m - n$

c $2m - n$

d $m^2 + n^2$

e $4 - 2n$

Solution

$$\begin{aligned}\mathbf{a} \quad m + n &= -1 + \frac{1}{2} \\ &= -\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad m - n &= -1 - \frac{1}{2} \\ &= -1\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad 2m - n &= 2 \times (-1) - \frac{1}{2} \\ &= -2 - \frac{1}{2} \\ &= -2\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\mathbf{d} \quad m^2 + n^2 &= (-1)^2 + \left(\frac{1}{2}\right)^2 \\ &= 1 + \frac{1}{4} \\ &= 1\frac{1}{4}\end{aligned}$$

$$\begin{aligned}\mathbf{e} \quad 4 - 2n &= 4 - 2 \times \frac{1}{2} \\ &= 3\end{aligned}$$

Example 3

For $m = 2$, evaluate:

a m^2

b $(-m)^2$

c $-m^2$

d m^3

e $(-m)^3$

f $(3m)^2$

g $3m^2$

Solution

a $m^2 = 2^2$
 $= 4$

b $(-m)^2 = (-2)^2$
 $= -2 \times -2$
 $= 4$

c $-m^2 = -2^2$
 $= -4$

d $m^3 = 2^3$
 $= 8$

e $(-m)^3 = (-2)^3$
 $= (-2) \times (-2) \times (-2)$
 $= -8$

f $(3m)^2 = (3 \times 2)^2$
 $= 6^2$
 $= 36$

g $3m^2 = 3 \times 2^2$
 $= 12$

Example 4

For $x = -3$, evaluate:

a x^2

b $(-x)^2$

c $-x^2$

d x^3

e $(-x)^3$

f $(2x)^2$

g $2x^2$

Solution

a $x^2 = (-3)^2$
 $= 9$

b $(-x)^2 = 3^2$
 $= 9$

c $-x^2 = -(-3)^2$
 $= -1 \times (-3) \times (-3)$
 $= -9$

d $x^3 = (-3)^3$
 $= (-3) \times (-3) \times (-3)$
 $= -27$

e $(-x)^3 = (-(-3))^3$
 $= 3^3$
 $= 27$

f $(2x)^2 = (2 \times -3)^2$
 $= (-6)^2$
 $= 36$

g $2x^2 = 2 \times (-3)^2$
 $= 2 \times 9$
 $= 18$



Exercise 6A

Example 1

1 Substitute 2 for x in each expression and evaluate.

a $x + 5$

b $4x$

c $\frac{x}{2}$

d $\frac{7x}{2}$

e $3(x + 4)$

f $2(x - 1)$

g $\frac{x+8}{2}$

h $\frac{9-x}{4}$

2 Substitute -6 for x in each expression and evaluate.

a $x + 3$

b $-2x$

c $\frac{x}{6}$

d $\frac{x}{2}$

e $2(x + 5)$

f $2(x - 5)$

g $\frac{x+8}{2}$

h $5(x - 4)$

3 Evaluate each expression when $n = -3$.

a n^2

b $(-n)^2$

c $-n^2$

d n^3

e $(-n)^3$

f $(3n)^2$

g $3n^2$

Example 2,3

4 Evaluate each expression when $a = -2$, $b = 5$ and $c = -1$.

a $a + b$

b $a^2 + c$

c $a - c$

d c^5

e $a^2 + b^2$

f $a - b$

g $a^2 - c^2$

h $-c$

i $\frac{b}{c}$

j $\frac{b}{c^3}$

k $(2c)^2$

l $(2c)^3$

5 Evaluate each expression when $a = -2$, $b = 5$ and $c = -\frac{1}{3}$.

a c^2

b $a^2 - c^2$

c $a - b$

d $b - a$

6 Evaluate each expression when $x = -2$ and $y = -1$.

a $-(2x)$

b $-xy$

c $2x + 3y$

d $\frac{x}{y}$

e $\frac{6x}{y}$

f $x^2 + y^2$

g $x + y$

h $-5x + 6y$

i $-3y$

j x^2y

k $x^3 + y^3$

7 Evaluate each expression when $a = 3$, $b = 5$ and $c = -1$.

a $a + b + c$

b $a - c + b$

c $a^2 + b^2 + c^2$

d $a^2 - b^2 - c^2$

e $(-a)^2 + b^3 + c$

f $c - a - b$

g $c - b + a^2$

h $-(2a)^2 + b^3 + c$

i $(-3a)^3 + 2a - c$

6B Solving equations



Joe has a pencil case that contains a number of pencils. He has three other pencils.



We do not know how many pencils there are in the pencil case but we do know he has a total of 11 pencils. Let x be the number of pencils in the pencil case. Then the facts we are given tell us that

$$x + 3 = 11.$$

This statement is called an **equation** and we need to **solve** the equation for x . The solution is $x = 8$.

Reading equations

The equation $2x + 4 = 10$ can be read as ‘two times a number x plus 4 is equal to 10’.

The instruction ‘Solve the equation $2x + 4 = 10$ ’ can also be read as ‘A number x is multiplied by 2, and then 4 is added. The result is 10. Find the number.’

The number in this case is 3. This can be checked by $2 \times 3 + 4 = 10$.

Equivalent equations

Consider these equations

$$2x + 3 = 9 \quad \textcircled{1}$$

$$2x + 5 = 11 \quad \textcircled{2}$$

Equation $\textcircled{2}$ is obtained from equation $\textcircled{1}$ by adding 2 to each side of the equation.

Equation $\textcircled{1}$ is obtained by subtracting 2 from both sides of equation $\textcircled{2}$. Equations $\textcircled{1}$ and $\textcircled{2}$ are said to be **equivalent** equations.

Equation $\textcircled{3}$ below is obtained from equation $\textcircled{2}$ by subtracting 5 from each side of the equation.

$$2x = 6 \quad \textcircled{3}$$

$$x = 3 \quad \textcircled{4}$$

Equation $\textcircled{4}$ is obtained from equation $\textcircled{3}$ by dividing each side of the equation by 2. You can obtain equation $\textcircled{3}$ from equation $\textcircled{4}$ by multiplying each side by 2. All of the above equations are **equivalent**.



Equivalent equations

- If we add the same number to, or subtract the same number from, both sides of an equation, the new equation is **equivalent** to the original equation.
- If we multiply or divide both sides of an equation by the same non-zero number, the new equation is equivalent to the original equation.
- Equivalent equations have exactly the same solutions.

Example 5

Solve each equation for x and check that LHS = RHS.

a $x + 3 = 5$

b $3x = 23$

c $2x - 3 = 11$

d $4x + 3 = 10$

e $\frac{3x}{4} = 10$

Solution

a $x + 3 = 5$

$$\boxed{-3} \quad x + 3 - 3 = 5 - 3 \quad (\text{Subtract 3 from both sides.})$$
$$x = 2$$

Check: Left-hand side (LHS) = $2 + 3$
 $= 5$
 $= \text{right-hand side (RHS)}$

b $3x = 23$

$$\boxed{\div 3} \quad \frac{3x}{3} = \frac{23}{3} \quad (\text{Divide both sides by 3.})$$
$$= 7\frac{2}{3}$$

Check: LHS = $3 \times 7\frac{2}{3}$
 $= 3 \times \frac{23}{3}$
 $= 23$
 $= \text{RHS}$

c $2x - 3 = 11$

$$\boxed{+3} \quad 2x - 3 + 3 = 11 + 3 \quad (\text{Add 3 to both sides.})$$
$$2x = 14$$

$$\boxed{\div 2} \quad x = 7 \quad (\text{Divide both sides by 2.})$$

Check: LHS = $2 \times 7 - 3$
 $= 11$
 $= \text{RHS}$

(continued on next page)

d $4x + 3 = 10$

$\boxed{-3}$ $4x + 3 - 3 = 10 - 3$ (Subtract 3 from both sides.)

$$4x = 7$$

$\boxed{\div 4}$ $x = \frac{7}{4}$ (Divide both sides by 4.)

$$= 1\frac{3}{4}$$

Check: $\text{LHS} = 4 \times 1\frac{3}{4} + 3$

$$= 4 \times \frac{7}{4} + 3$$

$$= 10$$

$$= \text{RHS}$$

e $\frac{3x}{4} = 10$

$\boxed{\times 4}$ $3x = 40$ (Multiply both sides by 4.)

$\boxed{\div 3}$ $x = \frac{40}{3}$ (Divide both sides by 3.)

$$= 13\frac{1}{3}$$

Check: $\text{LHS} = \frac{3 \times 13\frac{1}{3}}{4}$

$$= \frac{40}{4}$$

$$= 10$$

$$= \text{RHS}$$

From now on we will not include the box to the left in the solutions.

Example 6

Solve each equations for x .

a $2x - 11 = -7$ **b** $6 - 4x = 22$ **c** $\frac{x}{5} + 7 = 22$

Solution

a $2x - 11 = -7$

$$2x = -7 + 11 \quad (\text{Add 11 to both sides.})$$

$$2x = 4$$

$$x = 2 \quad (\text{Divide both sides by 2.})$$

b $6 - 4x = 22$

$$-4x = 16 \quad (\text{Subtract 6 from both sides.})$$

$$x = -4 \quad (\text{Divide both sides by } -4.)$$

c $\frac{x}{5} + 7 = 22$

$$\frac{x}{5} = 15 \quad (\text{Subtract 7 from both sides.})$$

$$x = 15 \times 5 \quad (\text{Multiply both sides by 5.})$$
$$= 75$$

Example 7

In each case below, write an equation and solve it.

a A number has 7 added to it and the result is 35.

b A number is multiplied by 15 and the result is 165.

c A number is divided by 11 and 6 is added to it. The result is 13.

Solution

a Let x be the number.

$$x + 7 = 35$$

$$x = 28 \quad (\text{Subtract 7 from both sides.})$$

The number is 28.

b Let x be the number.

$$15x = 165$$

$$x = \frac{165}{15} \quad (\text{Divide both sides by 15.})$$

$$x = 11$$

The number is 11.

(continued on next page)

c Let x be the number.

$$\frac{x}{11} + 6 = 13$$

$$\frac{x}{11} = 13 - 6 \quad (\text{Subtract 6 from both sides.})$$

$$\frac{x}{11} = 7$$

$$x = 77 \quad (\text{Multiply both sides by 11.})$$

The number is 77.

Example 8

Match the equivalent equations in columns 1 and 2 of the table.

	Column 1	Column 2
a	$x + 1 = 6$	$2x = 14$
b	$2x = 4$	$6x - 3 = 15$
c	$x - 2 = 5$	$2x + 3 = 13$
d	$3x = 9$	$x + 5 = 7$

Solution

a $x + 1 = 6$

$$x = 5 \quad (\text{Subtract 1 from both sides.})$$

$$2x = 10 \quad (\text{Multiply both sides by 2.})$$

$$2x + 3 = 13 \quad (\text{Add 3 to both sides.})$$

b $2x = 4$

$$x = 2 \quad (\text{Divide both sides by 2.})$$

$$x + 5 = 7 \quad (\text{Add 5 to both sides.})$$

c $x - 2 = 5$

$$x = 7 \quad (\text{Add 2 to both sides.})$$

$$2x = 14 \quad (\text{Multiply both sides by 2.})$$

d $3x = 9$

$$x = 3 \quad (\text{Divide both sides by 3.})$$

$$6x = 18 \quad (\text{Multiply both sides by 6.})$$

$$6x - 3 = 15 \quad (\text{Subtract 3 from both sides.})$$



Exercise 6B

Example 5

1 Solve these equations. Check your solutions.

a $m + 4 = 11$

b $2a = 6$

c $\frac{x}{5} = 10$

d $4x + 5 = 16$

e $5x - 3 = 12$

f $2m + 8 = 12$

g $\frac{2x}{3} = 12$

h $\frac{3x}{4} = 15$

i $2n + 12 = 20$

Example 6

2 Solve these equations. Check your solutions.

a $2x - 6 = -12$

b $2x + 6 = -15$

c $2x - 3 = -8$

d $6 - 4x = 26$

e $5 - 2x = 12$

f $6 - 5x = 10$

g $\frac{x}{3} + 4 = 22$

h $\frac{x}{4} - 5 = 10$

i $\frac{x}{5} + 7 = 15$

3 Solve each equation for m .

a $m + 3 = 6$

b $m - 11 = 7$

c $m + 8 = -10$

d $m - 5 = -11$

e $6 - m = 10$

f $-8 + m = 6$

g $m + 6 = 3$

h $8 - m = -6$

i $m - 11 = -5$

4 Solve each equation for n .

a $2n = -6$

b $-3n = 9$

c $-5n = 25$

d $-3n = 16$

e $12n = 100$

f $18n = 46$

g $5n = 17$

h $-6n = -50$

i $3n = 17$

5 Solve each equation for x . Check your solutions.

a $\frac{x}{3} = 12$

b $\frac{x}{2} = 15$

c $\frac{x}{5} = -16$

d $\frac{x}{10} = -2$

e $\frac{x}{3} = -6$

f $\frac{x}{4} = -8$

6 Solve each equation for x . Check your solutions.

a $2x + 1 = 7$

b $5x - 1 = 11$

c $7x + 3 = 17$

d $4x + 2 = 18$

e $1 - 5x = 21$

f $5 - 20x = 100$

g $2 - 10x = 44$

h $5x - 11 = 30$

i $10x + 23 = 100$

7 Solve each equation for x .

a $x + 3 = 7$

b $5x = 27$

c $3x - 4 = 11$

d $5x + 6 = 14$

e $\frac{5x}{3} = 20$

f $\frac{x}{3} - 4 = 11$

8 Solve these equations.

a $x - 4 = 5$

b $3a = 36$

c $3z - 7 = 17$

d $11b + 4 = 121$

e $\frac{5x}{4} = 30$

f $\frac{x}{7} - 2 = 8$

- 9 In each case, write an equation, and solve it.
- a A number a has 5 added to it, and the result is 21.
 - b A number x is multiplied by 7, and the result is 35.
 - c A number z is multiplied by 5, and the result is 37.
 - d A number m is multiplied by 5, and then 3 is added. The result is 50.
 - e A number n is divided by 6, and the result is 10.
 - f A number p is divided by 3, and 5 is subtracted from it. The result is 23.

10 Solve each equation for x .

a $x + 5 = 6$ b $5x - 2 = 3$ c $2x + 7 = 4$
 d $\frac{4x}{7} = 11$ e $\frac{5x}{11} = 7$

11 In each case, write an equation and solve it.

- a A number x is subtracted from 20, and the result is 10.
- b A number m is multiplied by 2, and the result is subtracted from 6. The final result is 20.
- c A number n is divided by 8, and 6 is added to the result. The final result is 20.
- d A number p is multiplied by 7, and then 10 is added. The result is 60.
- e A number x is subtracted from 6, and the result is -10 .
- f A number y is multiplied by 7, and the result is subtracted from 15. The final result is -6 .
- g A number k is divided by 10, and 7 is subtracted from the result. The final result is -1 .

12 Match the equivalent equations in columns 1 and 2 of the table.

	Column 1	Column 2
a	$x + 6 = 11$	$7x = 4$
b	$2x - 3 = 5$	$2x + 2 = 10$
c	$7x + 6 = 10$	$x + 2 = 14$
d	$\frac{x}{3} = 4$	$x + 10 = 15$

13 Match the equivalent equations in columns 1 and 2 of the table.

	Column 1	Column 2
a	$\frac{x}{3} - 5 = 11$	$2x = 10$
b	$2x - 11 = 3$	$2x + 9 = 23$
c	$5 - 2x = 7$	$-2x = 2$
d	$x + 6 = 11$	$\frac{x}{3} = 16$



The **distributive law for multiplication over addition** implies:

$$\begin{aligned} 3 \times (x + 4) &= 3 \times x + 3 \times 4 \\ &= 3x + 12. \end{aligned}$$

The **distributive law for multiplication over subtraction** implies:

$$\begin{aligned} 3 \times (x - 4) &= 3 \times x - 3 \times 4 \\ &= 3x - 12. \end{aligned}$$

We can describe these results in general by using algebra.



The distributive law

- The distributive law for multiplication over addition:
 $a(b + c) = ab + ac$
- The distributive law for multiplication over subtraction:
 $a(b - c) = ab - ac$

This process of rewriting an expression to remove brackets is usually referred to as **expanding brackets**.



Example 9

Use the distributive law to expand the brackets.

a $5(x - 4)$

b $4(3x + 2)$

c $-2(4 + x)$

d $-4(5 - x)$

Solution

a $5(x - 4) = 5 \times x - 5 \times 4$
 $= 5x - 20$

b $4(3x + 2) = 4 \times 3x + 4 \times 2$
 $= 12x + 8$

c $-2(4 + x) = -8 - 2x$

d $-4(5 - x) = -20 + 4x$





Exercise 6C



Example 9

1 Expand brackets.

- a** $2(3a - 1)$ **b** $5(6p + 7)$ **c** $4(3 - x)$ **d** $3(a - 2)$
e $7(6 - 2x)$ **f** $3(7 - x)$ **g** $-4(5 - x)$ **h** $-6(7 - 2p)$
i $-3(5x - 2)$ **j** $-4(6x - 3)$ **k** $-3(2 - x)$ **l** $-2(3x + 1)$
m $-5(4p - 7)$ **n** $-4(3 - x)$ **o** $-2(a - 2)$ **p** $-7(-6 - 2x)$

2 First, substitute $x = 5$ in each expression and evaluate it. Next, expand the brackets and again substitute $x = 5$.

- a** $2(x + 3)$ **b** $5(2x + 4)$ **c** $2(x - 2)$ **d** $6(x - 3)$

3 Substitute $a = 2$ in each expression and evaluate.

- a** $3 + 2(a - 2)$ **b** $6(a - 2)$
c $3 + 2(5a - 2)$ **d** $4 + 5(a - 1)$

4 Evaluate each expression when $z = -2$.

- a** $5z - 2(z - 4)$ **b** $2 - 3(2 - 4z)$
c $7z + 5(2z - 4)$ **d** $11z + 2(3 - 4z)$

5 Evaluate each expression when $a = -4$.

- a** $6a - (4 - a)$ **b** $10 - 2(5 - 2a)$ **c** $15 - (6 - a)$
d $10 - 2(12 - 2a)$ **e** $-(6 - a)$ **f** $4a - (10 - a)$



We will now employ the methods introduced in the last section to solve equations that contain brackets.

Example 10

Solve these equations by first expanding the brackets.

a $2(3x - 1) = 15$

b $3(1 - 2x) = -13$

Solution

a $2(3x - 1) = 15$

$$6x - 2 = 15 \quad (\text{Expand the brackets.})$$

$$6x = 17 \quad (\text{Add 2 to both sides of the equation.})$$

$$x = \frac{17}{6} \quad (\text{Divide both sides of the equation by 6.})$$

$$= 2\frac{5}{6}$$

b $3(1 - 2x) = -13$

$$3 - 6x = -13 \quad (\text{Expand the brackets.})$$

$$-6x = -13 - 3 \quad (\text{Subtract 3 from both sides of the equation.})$$

$$-6x = -16$$

$$x = \frac{-16}{-6} \quad (\text{Divide both sides of the equation by } -6.)$$

$$= 2\frac{2}{3} \quad (\text{Simplify the fraction.})$$

Example 11

In each case, write an equation and solve it.

a Four is added to a number, m , and the result is multiplied by 3. The final result is -6 .

b Eight is subtracted from a number, x , and the result is multiplied by 2. The final result is 10.

(continued on next page)

Solution

a $(m + 4) \times 3 = -6$

$$3(m + 4) = -6$$

$$3m + 12 = -6 \quad (\text{Expand the brackets.})$$

$$3m = -18 \quad (\text{Subtract 12 from both sides.})$$

$$m = -6 \quad (\text{Divide both sides by 3.})$$

b $(x - 8) \times 2 = 10$

$$2(x - 8) = 10$$

$$2x - 16 = 10 \quad (\text{Expand the brackets.})$$

$$2x = 26 \quad (\text{Add 16 to both sides.})$$

$$x = 13 \quad (\text{Divide both sides by 2.})$$

Exercise 6D

Example 10

1 Solve each equation for x .

a $2(x + 1) = 9$

b $5(x - 3) = 12$

c $3(2x + 4) = 14$

d $3(2x - 1) = 8$

e $5(3 + 2x) = 19$

f $5(5 - 3x) = 4$

g $6(x - 2) = 14$

h $4(2 - 3x) = 15$

i $5(3x - 2) = 6$

j $-5(2x + 2) = 6$

k $-3(4x - 5) = 10$

l $-3(2 - 3x) = 4$

Example 11

2 In each case, write an equation and solve it.

a Six is added to a number, x , and the result is multiplied by 3. The final result is -10 .

b Six is subtracted from a number, m , and the result is multiplied by 3. The final result is 5.

c Ten is added to a number, p , and the result is multiplied by 2. The final result is 4.

d Five is subtracted from a number, n , and the result is multiplied by 3. The final result is 14.

e Three is subtracted from a number, x , and the result is multiplied by 2. The final result is -10 .

f Three is added to a number, x , and the result is multiplied by -2 . The final result is -2 .

g Six is subtracted from a number, x , and the result is multiplied by -4 . The final result is -10 .



Like terms



If a boy has 3 pencil cases with the same number, x , of pencils in each, he has $3x$ pencils in total.



If he is given 2 more pencil cases with x pencils in each of the 5 cases, then he has $3x + 2x = 5x$ pencils in total. This is correct as the number of pencils in each of the five cases is x . $3x$ and $2x$ are said to be **like terms**.

If Janna has x packets of chocolates, each containing y chocolates, then she has $x \times y = xy$ chocolates. If David has twice as many chocolates as Janna, he has $2 \times xy = 2xy$ chocolates. Together they have $2xy + xy = 3xy$ chocolates.

The terms $2xy$ and xy are **like terms**. The pronumerals are the same and have the same exponents in each term.

The distributive law can be used to explain the addition and subtraction of like terms.

$$\begin{aligned} 2xy + xy &= 2 \times xy + 1 \times xy \\ &= (2 + 1)xy \\ &= 3xy \end{aligned}$$

Example 12

Which of these pairs consist of like terms?

a $3x, 5x$

b $4x^2, 8x$

c $4x^2y, 12x^2y$

d $ab, 2ba$

e $3mn^2, 5nm^2$

Solution

- a** $5x$ and $3x$ are like terms.
- b** These are not like terms because the powers of x differ.
- c** These are like terms because each is a number times x^2 times y .
- d** These are like terms because $ab = ba$.
- e** These are unlike terms because the powers of n and m are different.

Adding and subtracting like terms

Like terms can be added and subtracted, as shown in the example below.

Example 13

Simplify each expression by collecting like terms.

a $4x + 3x - 2x$

b $2x + 6x - 3x + 8x$

c $11m - 2m + 8n - 6n$

d $4m + 2n + 5m + 6n$

e $6m + 2n - 3m - n$

f $12m - 2n + 6m - 4n$

Solution

a $4x + 3x - 2x = 5x$

b $2x + 6x - 3x + 8x = 13x$

c $11m - 2m + 8n - 6n = 9m + 2n$

d $4m + 2n + 5m + 6n = 4m + 5m + 2n + 6n$
 $= 9m + 8n$

e $6m + 2n - 3m - n = 6m - 3m + 2n - n$
 $= 3m + n$

f $12m - 2n + 6m - 4n = 12m + 6m - 2n - 4n$
 $= 18m - 6n$

Example 14

Simplify each expression by collecting like terms together.

a $2x^2 + 3x^2 + 5x^2$

b $3xy + 2xy$

c $4x^2 - 3x^2$

d $2x^2 + 3x + 4x$

e $4x^2y - 3x^2y + 3xy^2$

Solution

a $2x^2 + 3x^2 + 5x^2 = 10x^2$

b $3xy + 2xy = 5xy$

c $4x^2 - 3x^2 = 1x^2 = x^2$

d $2x^2 + 3x + 4x = 2x^2 + 7x$

e $4x^2y - 3x^2y + 3xy^2 = x^2y + 3xy^2$

Example 15

Expand the brackets and collect like terms.

a $2(x - 6) + 5x$

b $3 + 3(x - 1)$

c $5 + x - 2(3x - 4)$



Solution

$$\begin{aligned}\mathbf{a} \quad 2(x - 6) + 5x &= 2x - 12 + 5x \\ &= 7x - 12\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 3 + 3(x - 1) &= 3 + 3x - 3 \\ &= 3x\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad 5 + x - 2(3x - 4) &= 5 + x - 6x + 8 \\ &= 13 - 5x\end{aligned}$$

Example 16

Solve these equations by first collecting like terms.

$$\mathbf{a} \quad 2x + 3 = 5x + 1$$

$$\mathbf{b} \quad 4(5 - 2x) + 12x = 22$$

$$\mathbf{c} \quad 2(3x - 2) - 4(x + 1) = 2$$

$$\mathbf{d} \quad 5(3x - 2) = 6x + 3$$

Solution

$$\mathbf{a} \quad 2x + 3 = 5x + 1$$

$$2x + 3 - 2x = 5x + 1 - 2x \quad (\text{Subtract } 2x \text{ from both sides.})$$

$$3 = 3x + 1 \quad (\text{Collect like terms.})$$

$$2 = 3x \quad (\text{Subtract 1 from both sides.})$$

$$x = \frac{2}{3} \quad (\text{Divide both sides by 3.})$$

$$\mathbf{b} \quad 4(5 - 2x) + 12x = 22$$

$$20 - 8x + 12x = 22 \quad (\text{Expand the brackets.})$$

$$20 + 4x = 22 \quad (\text{Collect like terms.})$$

$$4x = 2 \quad (\text{Subtract 20 from both sides.})$$

$$x = \frac{1}{2} \quad (\text{Divide both sides by 4.})$$

$$\mathbf{c} \quad 2(3x - 2) - 4(x + 1) = 2$$

$$6x - 4 - 4x - 4 = 2 \quad (\text{Expand the brackets.})$$

$$2x - 8 = 2 \quad (\text{Collect like terms.})$$

$$2x = 10 \quad (\text{Add 8 to both sides.})$$

$$x = 5 \quad (\text{Divide both sides by 2.})$$

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d $5(3x - 2) = 6x + 3$
 $15x - 10 = 6x + 3$ (Expand the brackets.)
 $9x = 13$ (Collect like terms.)
 $x = 1\frac{4}{9}$ (Divide both sides by 9.)



Exercise 6E

Example 12

1 Which of these pairs contain like terms and which do not?

- | | | |
|------------------------------------|-------------------------------------|--------------------------------|
| a $11a$ and $4a$ | b $6b$ and $-2b$ | c $12m$ and $5m$ |
| d $14p$ and $5p$ | e $7p$ and $-3q$ | f $-6a$ and $7b$ |
| g $4mn$ and $7mn$ | h $3pq$ and $-2pq$ | i $6ab$ and $-7b$ |
| j $4ab$ and $5a$ | k $11a^2$ and $5a^2$ | l $6x^2$ and $-7x^2$ |
| m $4b^2$ and $-3b$ | n $2y^2$ and $2y$ | o $-5a^2b$ and $9a^2b$ |
| p $6mn^2$ and $11mn^2$ | q $6l^2s$ and $17ls^2$ | r $12d^2e$ and $14de^2$ |
| s $-4x^2y^2$ and $12x^2y^2$ | t $-5a^2bc^2$ and $8a^2bc^2$ | |

Example 13

2 Simplify these expressions; that is, collect like terms.

- | | | |
|-------------------------------|-------------------------------|----------------------------|
| a $3x + 4x$ | b $5x - 6x$ | c $4x + 7x - 8x$ |
| d $4x - 7x + 8x$ | e $-11x + 10x$ | f $3x + 13x - 20x$ |
| g $6x - 11x + 17x$ | h $-12m + 13m + 50m$ | i $5n + 6n - 20n$ |
| j $67x - 70x + 100x$ | k $4n - 5n + 10n$ | l $-10p - 30p - 5p$ |
| m $5x - 10x - 15x$ | n $80p - 100p + 20p$ | |
| o $10m + 3m + 4n + 7n$ | p $10m - 3m + 7n - 3n$ | |
| q $3m + 2n + 4m + 6n$ | r $3x + 4y - 2x + y$ | |
| s $5x + 2y - 3x - y$ | t $5m - 2n + 6m - 7n$ | |
| u $5q - 2p + 2q - 5r$ | v $5a - 7b + 2a - 11b$ | |

Example 14

3 Collect like terms.

- | | |
|---------------------------------|------------------------------|
| a $45xy + 32xy + 16xy$ | b $19xy + 6xy - 4xy$ |
| c $8xy^2 + 9xy^2 - y^2x$ | d $4x + 3x + 3y + 7y$ |

e $6v - 11v + 7z - 14z$

f $6y - 11x + 10y + 15x$

g $7x^3 + 6x^2 + 4y^3 - x^2$

h $8x^2 - 12x^2 + y^2 + 12y^2$

i $2x^2 + x^2 - 5xy + 7yx$

j $11x^2 - 20x^2 - 5x^2 + y^2 - 10y^2$

Example 15**4** Expand the brackets in each expression, and collect like terms.

a $12 + 7(x + 4)$

b $3(3 + x) + 2x$

c $5(x + 2) + 2(x + 3)$

d $2(7 + 5x) + 4(x + 6)$

e $9(3 - x) + 7(x + 4)$

f $3(2x + 7) + 2(x - 5)$

g $10(x - 4) - 2(x + 1)$

h $4(3 + x) - 3(2x + 3)$

i $11(x - 1) - 5(x - 3)$

j $7(2x + 4) - 3(2 - 4x)$

k $5x + 3(2x + 7)$

l $2x(3a - 4) + 6(2ax + 4x)$

Example 16**5** Solve each equation for x .

a $3x + 2 = x - 4$

b $5x - 4 = 2x + 8$

c $6x + 2 = 2x + 8$

d $3x - 4 = x + 6$

e $11x + 2 = x + 10$

f $3x + 1 = 2x - 1$

6 Solve each equation for x .

a $3(x - 2) + x + 4 = 10$

b $2(x - 5) + 3x = 10$

c $x + 5 + 2(x - 4) = 9$

d $2x + 6 + x - 1 = 25$

7 Solve these equations by first expanding the brackets.

a $4(x - 2) = 10$

b $2(x + 3) = x - 9$

c $5(x - 3) - 4(x + 8) = 2x$

d $3(x + 8) + 5 - 3x = 2x$

e $13 = 6(x - 4) + (5 + 3)x$

f $2.5(x + 2^2) = 6(3x - 2) - 4(x + 1.5) - 3$

6F Problem-solving using algebra



The algebra that has been introduced in the previous sections of this chapter can be used to solve problems that are difficult to solve otherwise. In many cases, algebra simplifies the problem into a form to which the methods for solving equations can be applied.

Example 17

In each case, write an equation and solve it.

a A number has 3 added to it, and the result is multiplied by 4. The final result is -10 .

b A number is divided by 7, and 6 is subtracted from it. The result is -15 .

Solution

a Let m be the number.

$$(m + 3) \times 4 = -10$$

$$4m + 12 = -10 \quad (\text{Expand the brackets.})$$

$$4m = -22 \quad (\text{Subtract 12 from both sides.})$$

$$m = -\frac{22}{4} \quad (\text{Divide both sides by 4.})$$

$$= -\frac{11}{2}$$

$$= -5\frac{1}{2}$$

The number is $-5\frac{1}{2}$.

b Let n be the number.

$$\frac{n}{7} - 6 = -15$$

$$\frac{n}{7} = -9 \quad (\text{Add 6 to both sides.})$$

$$n = -63 \quad (\text{Multiply both sides by 7.})$$

The number is -63 .

Example 18

Multiplying a number by 4 and adding 6 gives the same result as multiplying the number by 3 and subtracting 4. Find the number.

Solution

Let x be the number. The resulting equation is

$$4x + 6 = 3x - 4.$$

$$4x + 6 - 3x = 3x - 4 - 3x \quad (\text{Subtract } 3x \text{ from both sides.})$$

$$x + 6 = -4$$

$$x = -10 \quad (\text{Subtract 6 from both sides.})$$

Hence the number is -10 .

Check: $\text{LHS} = 4 \times (-10) + 6$
 $= -40 + 6$
 $= -34$

$$\begin{aligned} \text{RHS} &= 3 \times (-10) - 4 \\ &= -34 \end{aligned}$$

Example 19

Five less than a number, multiplied by 3, is equal to the sum of 2 times the number and 12. Find the number.

Solution

Let x be the number. Then from the information given:

$$3(x - 5) = 2x + 12$$

$$3x - 15 = 2x + 12 \quad (\text{Expand the brackets.})$$

$$3x = 2x + 27 \quad (\text{Add 15 to both sides.})$$

$$x = 27 \quad (\text{Subtract } 2x \text{ from both sides.})$$

Hence the number is 27.

Example 20

John is thinking of getting a mobile phone, but is having trouble deciding between two plans. One costs \$30 a month, includes 10 free minutes, and charges 50 cents a minute after that. The other costs \$10 a month and charges \$1 a minute. After how many minutes do the two plans cost the same?

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Solution

Let x be the number of minutes per month at which the cost of the two plans is the same. Then from the information given:

Cost of the first plan per month is $30 + 0.5(x - 10)$

Cost of the second plan per month is $10 + x$

$$30 + 0.5(x - 10) = 10 + x$$

$$30 + 0.5x - 5 = 10 + x \quad (\text{Expand the brackets.})$$

$$25 + 0.5x = 10 + x \quad (\text{Collect like terms.})$$

$$15 + 0.5x = x \quad (\text{Subtract 10 from both sides.})$$

$$15 = 0.5x \quad (\text{Subtract } 0.5x \text{ from both sides.})$$

$$30 = x \quad (\text{Multiply both sides by 2.})$$

The two plans cost the same amount when John has used exactly 30 minutes.

Example 21

The average of eight numbers is 56. A ninth number is added and the average is then 60. What is the ninth number?

Solution

If the average of eight numbers is 56, then the sum of these numbers is 448.

Let n be the ninth number. Then $\frac{n + 448}{9} = 60$.

Solving for n :

$$\frac{n + 448}{9} = 60$$

$$n + 448 = 540 \quad (\text{Multiply both sides of the equation by 9.})$$

$$n = 92 \quad (\text{Subtract 448 from both sides of the equation.})$$

The ninth number is 92.

Exercise 6F

In each of the following problems introduce a pronumeral, write an equation and solve it.

Example 17

- 1 In each case, write an equation in x and solve it.
 - a A number has 6 added to it, and the result is 24.
 - b A number is multiplied by 5, and the result is 35.

- c A number is multiplied by 3, and the result is 37.
- d A number is multiplied by 6, and then 4 is added. The result is 48.
- e A number is divided by 6, and the result is 20.
- f A number is divided by 7, and 5 is subtracted from it. The result is 33.

Example 18

- 2** Multiplying a number by 3 and adding 8 gives the same result as multiplying the number by 2 and subtracting 8. Find the number.

Example 19

- 3** Multiplying a number by 2 and subtracting 6 gives the same result as multiplying the number by 10 and adding 8. Find the number.

Example 20

- 4** Anthony goes to the fair with his family. They ride together on the Ferris Wheel. They buy one adult and three child tickets for \$12. If the child tickets are half the cost of the adult tickets, how much does one child ticket cost?
- 5** I start with a number, multiply it by 5 and add 7, and I end up with 28. What was the number I started with?
- 6** The sum of two numbers is 42, and one of the numbers is five times the other. Find the two numbers.
- 7** The sum of two numbers is twice the difference, and one number is five more than the other. Find the two numbers.
- 8** Five times a number minus two times the number is equal to six times the same number minus 12. What is the number?
- 9** In class tests this year, Juan has so far scored 75, 69, 81 and 87. If he wants to get an average of 80 after his next test, what score does Juan need to get?
- 10** If Amy has an average score of 67 after four tests, and after doing a fifth test her average increases to 72, what score did she get on the fifth test?
- 11** The local movie theatre sells popcorn in small, medium and large containers. The small size costs \$1 less than the medium size, and the large size costs twice as much as the medium size. A group of friends go to the movies and decide to buy one container of each size, and they pay \$13. How much does the medium-size popcorn cost?
- 12** Licia bought her lunch from the school canteen for \$3.00. She had a sausage roll, a caramel milkshake and an apple. She paid 60 cents more for the milkshake than for the fruit, and 30 cents more for the sausage roll than for the milkshake. Write an expression for how much she spent, and use it to find the cost of each item.

Example 21



The **any-order property of multiplication** was discussed in Chapter 1. In Chapter 3, index notation was used to simplify expressions. For example, $x \times x \times x = x^3$ and $y \times y = y^2$.

Here is another example using index notation.

$$\begin{aligned} 3x \times 2y \times 2xy &= 3 \times x \times 2 \times y \times 2 \times x \times y \\ &= 3 \times 2 \times 2 \times x \times x \times y \times y \\ &= 12x^2y^2 \end{aligned}$$

Example 22

Simplify each expression.

a $5 \times 2a$

b $3a \times 2a$

c $5xy \times 2xy$

d $6x^2y \times 3x$

Solution

a $5 \times 2a = 10a$

b $3a \times 2a = 3 \times 2 \times a \times a$
 $= 6a^2$

c $5xy \times 2xy = 5 \times 2 \times x \times x \times y \times y$
 $= 10x^2y^2$

d $6x^2y \times 3x = 6 \times 3 \times x \times x \times x \times y$
 $= 18x^3y$

In Section 2C we learnt to simplify fractions by cancelling common factors. We can do the same dealing with algebraic fractions, provided that the pronumeral we cancel is not equal to zero.

Example 23

Simplify:

a $\frac{8x}{2}$

b $\frac{7xy}{x}$

c $\frac{42xyz}{xz}$

d $\frac{56abc}{35c}$

e $\frac{xy}{3xyz}$

Solution

a $\frac{8x}{2} = \frac{\cancel{8}^4 x}{\cancel{2}_1}$
 $= 4x$

b $\frac{7xy}{x} = \frac{7\cancel{x}^1 y}{\cancel{x}_1}$
 $= 7y$

c $\frac{42xyz}{xz} = \frac{42\cancel{x}^1 y \cancel{z}^1}{\cancel{x}_1 \cancel{z}_1}$
 $= 42y$

$$\begin{aligned} \text{d } \frac{56abc}{35c} &= \frac{\cancel{5}^8 \cancel{6}^1 ab \cancel{c}^1}{\cancel{3}^5 \cancel{5}^1 \cancel{c}^1} \\ &= \frac{8ab}{5} \end{aligned}$$

$$\text{e } \frac{xy}{3xyz} = \frac{1}{3z}$$

Example 24

Simplify:

$$\text{a } \frac{x}{x}$$

$$\text{b } \frac{x^2}{x}$$

$$\text{c } \frac{x^3}{x^2}$$

$$\text{d } \frac{x^2}{x^3}$$

Solution

$$\begin{aligned} \text{a } \frac{x}{x} &= \frac{\cancel{x}^1}{\cancel{x}^1} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{b } \frac{x^2}{x} &= \frac{x \times x}{x} \\ &= \frac{x \times \cancel{x}^1}{\cancel{x}^1} \\ &= x \end{aligned}$$

$$\begin{aligned} \text{c } \frac{x^3}{x^2} &= \frac{x \times x \times x}{x \times x} \\ &= \frac{x \times \cancel{x}^1 \times \cancel{x}^1}{\cancel{x}^1 \times \cancel{x}^1} \\ &= x \end{aligned}$$

$$\begin{aligned} \text{d } \frac{x^2}{x^3} &= \frac{x \times x}{x \times x \times x} \\ &= \frac{1}{x} \end{aligned}$$

Example 25

Simplify:

$$\text{a } \frac{60p^2q}{12p}$$

$$\text{b } \frac{50x^2y^2}{20xa}$$

Solution

$$\text{a } \frac{60\cancel{p}^2 q}{12\cancel{p}^1} = 5pq$$

$$\text{b } \frac{50\cancel{x}^2 y^2}{20\cancel{x}^1 a} = \frac{5xy^2}{2a}$$



Exercise 6G

Example 22

1 Simplify, using index notation:

a $2a \times 6b$

b $3x \times 4x$

c $5xy \times 10$

d $7ab \times 11$

e $5a \times 2a$

f $6c \times 11c$

g $4m \times 6n$

h $7mn \times 11mn$

i $4n^2 \times 3$

j $7m^2 \times 6m$

k $11m^2 \times 2m^2$

l $7a^2b \times 5$

2 Rewrite each expression without brackets.

a $(5n)^2$

b $(4z)^2$

c $(16z)^2$

d $(3c^2)^2$

3 Simplify:

a $2x \times 4x$

b $7x \times 3x$

c $3x \times 2x$

d $2xy \times 3x$

e $4xy \times 2xy$

f $6xy \times 2x^2y$

Example 23

4 Simplify:

a $\frac{6x}{3}$

b $\frac{8xy}{y}$

c $\frac{24xyz}{xz}$

d $\frac{mnp}{5m}$

e $\frac{9zx}{3z}$

f $\frac{18xy}{y}$

g $\frac{18xyz}{yz}$

h $\frac{72abc}{16c}$

Example 24

5 Simplify:

a $\frac{4x}{xy}$

b $\frac{a^2}{a}$

c $\frac{a^3}{a^2}$

d $\frac{4ab}{ab}$

e $\frac{3a^2b}{a}$

f $\frac{a^3}{a}$

g $\frac{a}{a^2}$

h $\frac{a^2}{a^3}$

i $\frac{a}{a}$

j $\frac{a^2b}{ab}$

k $\frac{a^2b^2}{ab}$

l $\frac{ab^2}{ab}$

Example 25

6 Simplify:

a $\frac{48p^2q}{36p}$

b $\frac{100a^2b^2}{5ba}$

c $\frac{200a^3b^2}{10a}$

7 A rectangle has length x cm and width $\frac{x}{3}$ cm.

a What is the area of the rectangle?

b Find the area if $x = 9$.

- 8 A triangle has base $\frac{2x}{3}$ cm and height $\frac{x}{4}$ cm.
- What is the area of the triangle in terms of x ?
 - Find the area if $x = 24$.
- 9 A rectangle has length x cm and area $\frac{6x^2}{5}$ cm². Find its width.
- 10 A rectangle has length $2x$ cm and width $\frac{5x}{3}$ cm.
- What is the area of the rectangle?
 - Find the area if $x = 27$.
- 11 A square that has side length $8x$ cm is divided into y rectangles of equal area. What is the area of each of these rectangles?
- 12 A floor of a room is rectangular in shape, has length $10x$ metres and width $7x$ metres.
- The floor is to be tiled with rectangular tiles each of length $\frac{x}{3}$ metres and width $\frac{x}{4}$ metres.
How many tiles are needed to tile this floor area?



Review exercise

1 Evaluate each expression for $a = 4$, $b = -2$, $c = -3$ and $d = 5$.

a $d - 3$	b $c + 5$	c $3d - 8$	d $5b + 6$
e $10 - 2b$	f $7 - 3a$	g $3(a + 2)$	h $7(1 - c)$
i $\frac{a}{b}$	j $ad - bc$	k b^2	l $\frac{ac + 7}{5 - 2b}$

2 Substitute $x = -3$ in each expression and evaluate it.

a $2x + 3$	b $1 - 2x$	c $4x + 3$	d $1 + x$
e x^2	f x^3	g $2 - x^2$	h $4 + x^2$
i $-1 + x^3$	j $x^3 - 8$	k $(2 + x)^3$	l $(2 - x)^3$

3 Substitute $m = 2$, $n = -3$ and $p = 11$ in each expression and evaluate it.

a $m^2 + p^2$	b $m - n$	c $p - n$	d $n - p$
e mnp	f $m^2n^2p^2$	g $m - 2p$	h $p^2 + m^2 - n$

4 Expand the brackets in each expression.

a $3(a + 4)$	b $6(3 - x)$	c $x(2 + y)$	d $-2(11 - x)$
e $14(x - 3) - 10(x - 3)$	f $3(x + 10) - 4(5 - x)$		
g $2a(3 - x) + 3x(a + 7)$	h $-5(x + 6) + 3(2x - 4)$		
i $-2(x - 3) + 4(3 - x)$	j $-7(2x - 4) + 5(4 - x)$		

5 Solve each equation. Check your solutions.

a $2x - 3 = -11$	b $x + 6 = -12$	c $4x - 6 = -10$
d $3 - a = 6$	e $2m - 6 = -8$	f $10 - 2m = -4$

6 Solve each equation. Check each answer.

a $x + 3x - 5 = 11$	b $3(a + 4) = 9$	c $2x + 1 = 3(x - 4)$
d $x + 3 = 4x - 1$	e $2x + 6 = 3 - x$	f $x + 2 = 4(x - 5)$

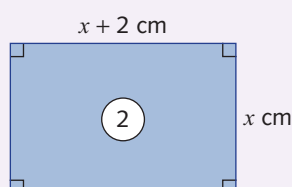
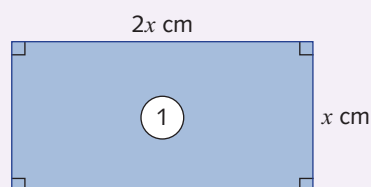
7 Expand the brackets where necessary and simplify each of these expressions.

a $(x^2y)^2 \times (2xy)^3$	b $(-2xy)^2 \times x^3y^2$	c $\frac{x^2y^2}{xy}$
d $\frac{m^3n^2}{m^2n}$	e $(-4x^2y)^2 \times xy$	f $\frac{20a^2b^2}{5a}$

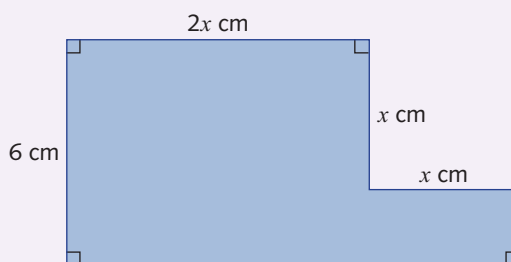
8 In a theme park, the cost of an adult ticket is 1.5 times that of a child ticket. If a family of 2 adults and 3 children paid \$48, how much does an adult ticket cost?



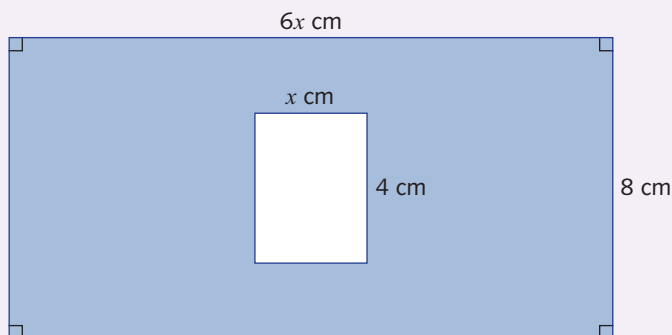
- 9 In a strict monastery there are 100 monks, either senior or junior. If a senior monk is allowed to have 3 slices of bread for each meal, and a junior monk is only allowed $\frac{1}{3}$ of a slice, and altogether they consume 60 slices of bread during lunch, how many senior monks are there?
- 10 The perimeter of rectangle ① is numerically equal to the perimeter of rectangle ②. Find the value of x .



- 11 The perimeter of the shaded region is 36 cm.
Find the value of x .



- 12 The area of the shaded region is 121 cm^2 .
Find the value of x .





Challenge exercise

- 1 a Carry out the following operations on the number 5.
 - Add 3.
 - Multiply the result by 9.
 - Subtract 2.
 - Add the number you started with (in this case 5).
 - Subtract 25.
 - Divide by 10.
- b Repeat the procedure with another number.
- c Let the number be x and apply same the operations. State your result. What is the final number?
- 2 If a painted wooden cube is cut in half along each edge (making 8 smaller cubes), then each of these cubes is painted on three faces. If a painted wooden cube is cut into thirds along each edge (making 27 smaller cubes), 8 of these cubes are painted on three faces, 12 of these cubes are painted on two faces, 6 are painted on one face only, and one is not painted at all. If a painted wooden cube is cut into four parts along each edge, how many smaller cubes are there, and how many are painted on three, two, one and no faces respectively? Find an expression for how many cubes are painted on three, two, one and no faces if the large cube is cut into n parts along each edge.
- 3 Paul, Kate and Sarah's mother left a bowl of jellybeans on their kitchen counter. Paul took one-third of the jellybeans, but then threw back 4 because he didn't like the black ones. Sarah ate 6 jellybeans and then took one-quarter of what remained. Kate took one-third of what remained, and then picked out and ate the last 4 black ones because she really liked them. If after all this there were 6 jellybeans left, how many were there to begin with? If there were 12 jellybeans left, how many were there to begin with? If there were x jellybeans left, how many were there to begin with?
- 4 A primary school is planning to hold a fete, but needs help in figuring out how much to charge for entry. They intend to have an 'all you can eat' sausage sizzle, and they think each adult will eat about \$3 worth of sausages, and each child \$2 worth. The cost of cleaning after the event will be \$300. They want an adult ticket to cost twice as much as a child ticket, and they expect that, on average, 1.7 parents and 2.3 children will come from each family. How much will they need to charge for adult and child tickets if they are expecting 120 families to attend and they don't want to make a profit or a loss? (Round to the nearest cent.)
- 5 Andrew and a group of his friends have decided they want to race go-karts on the weekend, and Andrew wants to work out which of the two local go-kart places will be cheaper. Kart-Kingdom charges \$15 per person to do safety training, and then 50 cents per lap that you do. Go-Mobile charges 80 cents per lap, and the safety training is free. After how many laps does Go-Mobile become more expensive than Kart-Kingdom?
- 6 Find all the two-digit integers such that when you reverse the digits and add that number to the original number, you get 55.
- 7 A two-digit number has the property that the sum of the number and its digits is 63. What is the original number?
- 8 Is there a two-digit integer with the property that 11 times the sum of its digits is equal to itself?



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CHAPTER

7

Number and Algebra

Percentages

We encounter percentages often, and they can be very useful in many aspects of our lives. For example, we see discounts offered in shops everywhere – as smart shoppers, we definitely need to know how to do percentage calculations if we want the best deal.

The word 'percentage' comes from the Latin *per centum*, meaning 'per hundred'. A percentage is another way of writing a fraction with a denominator of 100. The symbol for percentage is %. For example:

$$8\% = \frac{8}{100}, \quad 25\% = \frac{25}{100}, \quad 99\% = \frac{99}{100}, \quad 150\% = \frac{150}{100}$$

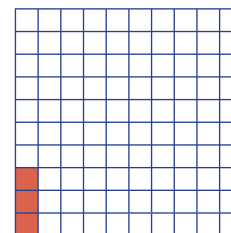
In this chapter, you will learn how to perform several different types of useful calculations involving percentages.

Also, we review the previous study of percentages and consider more applications of percentages, such as discounts, profits and loss, interest and GST.

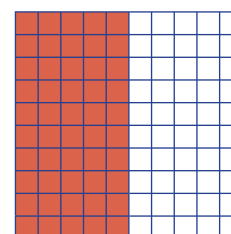


To understand the basic idea of percentages, we need only look at the meaning of ‘out of a hundred’.

A square that has been cut into 100 smaller squares can be used to model percentages. If we colour in three of them, we say that ‘three out of a hundred’ or ‘three per cent’ are coloured in.



If we colour 50 of them, we say that ‘50 out of a hundred’ or ‘50 per cent’ are coloured in. Half of the squares are coloured in.



Converting percentages to fractions

A percentage is a fraction with a denominator of 100. To convert a percentage to its fraction equivalent, write it as a fraction with a denominator of 100 and then simplify. For example:

$$\begin{aligned} 65\% &= \frac{65}{100} \\ &= \frac{13}{20} \end{aligned}$$

$$\begin{aligned} 150\% &= \frac{150}{100} \\ &= 1\frac{1}{2} \end{aligned}$$

For percentages such as $12\frac{1}{2}\%$ or $33\frac{1}{3}\%$, an extra step is required to convert the numerators to whole numbers:

$$\begin{aligned} 12\frac{1}{2}\% &= \frac{12\frac{1}{2}}{100} \\ &= \frac{25}{200} \\ &= \frac{1}{8} \end{aligned}$$

(Multiply the numerator and denominator of the fraction by 2.)

$$\begin{aligned} 33\frac{1}{3}\% &= \frac{33\frac{1}{3}}{100} \\ &= \frac{100}{300} \\ &= \frac{1}{3} \end{aligned}$$

(Multiply the numerator and denominator of the fraction by 3.)

Converting percentages to decimals

A percentage can be converted to a decimal by writing it as a fraction with a denominator of 100 and then converting to a decimal.

$$\begin{aligned} 65\% &= \frac{65}{100} \\ &= 0.65 \end{aligned}$$

$$\begin{aligned} 150\% &= \frac{150}{100} \\ &= 1.5 \end{aligned}$$

For a percentage such as 37.5%, divide 37.5 by 100 in one step:

$$\begin{aligned} 37.5\% &= \frac{37.5}{100} \\ &= 0.375 \end{aligned}$$

Converting fractions and decimals to percentages

We know from the meaning of the word ‘percentage’ that fractions with a denominator of 100 convert directly to percentages. For example:

$$\frac{2}{100} = 2\%,$$

$$\frac{37}{100} = 37\%,$$

$$\frac{175}{100} = 175\%,$$

$$\frac{100}{100} = 1 = 100\%$$

Equivalent fractions can be used for some fractions whose denominators are not 100 to find the corresponding percentage:

$$\begin{aligned} \frac{2}{10} &= \frac{20}{100} \\ &= 20\% \end{aligned}$$

$$\begin{aligned} \frac{3}{20} &= \frac{15}{100} \\ &= 15\% \end{aligned}$$

$$\begin{aligned} \frac{3}{5} &= \frac{60}{100} \\ &= 60\% \end{aligned}$$

$$\begin{aligned} \frac{3}{2} &= \frac{150}{100} \\ &= 150\% \end{aligned}$$

More generally, to convert a fraction to a percentage, multiply by 100%, which is the same as multiplying by 1.

$$\begin{aligned} \frac{2}{5} &= \frac{2}{5} \times \frac{100}{1}\% \\ &= 40\% \end{aligned}$$

$$\begin{aligned} \frac{2}{3} &= \frac{2}{3} \times \frac{100}{1}\% \\ &= 66\frac{2}{3}\% \end{aligned}$$

To convert a decimal to a percentage, the procedure is the same – multiply by 100%, which is the same as multiplying by 1.

$$\begin{aligned} 0.6 &= 0.6 \times 100\% \\ &= 60\% \end{aligned}$$

$$\begin{aligned} 3.2 &= 3.2 \times 100\% \\ &= 320\% \end{aligned}$$

Here are some commonly used percentages and their fraction equivalents. It can be very useful to know these. Try to learn them in both directions, but you can quickly work them out if you forget them.

For example, $\frac{1}{5} = 20\%$ and 37.5% is $\frac{3}{8}$.

Fraction	Percentage
$\frac{1}{2}$	50%
$\frac{1}{4}$	25%
$\frac{1}{3}$	$33\frac{1}{3}\%$
$\frac{3}{4}$	75%
$\frac{2}{3}$	$66\frac{2}{3}\%$
$\frac{1}{5}$	20%
$\frac{2}{5}$	40%

Fraction	Percentage
$\frac{3}{5}$	60%
$\frac{4}{5}$	80%
$\frac{1}{8}$	$12\frac{1}{2}\%$
$\frac{3}{8}$	$37\frac{1}{2}\%$
$\frac{5}{8}$	$62\frac{1}{2}\%$
$\frac{7}{8}$	$87\frac{1}{2}\%$
$\frac{1}{1}$	100%

Percentage of a quantity

We often talk about a percentage of a particular quantity. For example:

- 20% of all suitcases are brown.
- 52% of voters in a recent election voted for a particular political party.
- 80% of the students at a particular school access the internet at least once a day.

In such cases, we may want to know the actual number of objects or people involved. The following example shows how to do calculations of this type.

Example 1

32% of people interviewed watched the Australian Open tennis final. Calculate how many of the people interviewed watched the final if:

- 300 people were interviewed
- 250 people were interviewed.

Solution

- a** If 300 people were interviewed, then 32 of every 100 people interviewed watched the final. That is, $32 \times 3 = 96$ people watched it. This can be set out as follows:

32% of 300 people interviewed watched the final.

Number of people who watched = 32% of 300

$$\begin{aligned} &= \frac{32}{100} \times \frac{300}{1} \\ &= 96 \end{aligned}$$

- b** 32% of 250 people interviewed watched the final.

Number of people who watched = 32% of 250

$$\begin{aligned} &= \frac{32}{100} \times \frac{250}{1} \\ &= \frac{8}{25} \times \frac{250}{1} \\ &= 80 \end{aligned}$$

Expressing one quantity as a percentage of another

Sometimes we want to describe one quantity as a percentage of another.

Example 2

There are 50 people in a swimming club and 35 of them go to squad training. Calculate the percentage of the club members who go to squad training.

Solution

35 out of the 50 or $\frac{35}{50}$ go to squad training.

$$\begin{aligned} \text{Percentage going to squad training} &= \frac{35}{50} \times 100\% \\ &= 70\%. \end{aligned}$$

Thus 70% of the swimming club members go to squad training.

Comparing fractions using percentages

One useful way to compare your results in a French test, in which you scored $\frac{16}{20}$, and a German test, in which you scored $\frac{42}{50}$, is to change both scores to percentages. In this case, $\frac{16}{20} = 80\%$ and $\frac{42}{50} = 84\%$, so you did better in the German test.



Percentages

- A **percentage** is a fraction with denominator of 100.

For example, $5\% = \frac{5}{100}$.

- To convert a percentage to a fraction, write the percentage as a fraction with a denominator of 100 and then simplify.
- To convert a percentage to a decimal, write the percentage as a fraction with a denominator of 100 and then convert to a decimal.
- To convert a fraction or a decimal to a percentage, multiply by 100%.
- To express one quantity as a percentage of another, write the first quantity as a fraction of the second, and then convert to a percentage by multiplying by 100%.
- A useful way to compare two fractions is to convert them both to percentages.

Example 3

a Convert 45% to a fraction.

b Convert 135% to a fraction.

c Convert 45% to a decimal.

d Convert 135% to a decimal.

Solution

$$\begin{aligned}\mathbf{a} \quad 45\% &= \frac{45}{100} \\ &= \frac{9}{20}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 135\% &= \frac{135}{100} \\ &= 1\frac{7}{20}\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad 45\% &= \frac{45}{100} \\ &= 0.45\end{aligned}$$

$$\begin{aligned}\mathbf{d} \quad 135\% &= \frac{135}{100} \\ &= 1.35\end{aligned}$$

Example 4

Express each fraction as a percentage.

a $\frac{27}{100}$

b $\frac{127}{1000}$

Solution

a $\frac{27}{100} = 27\%$

b $\frac{127}{1000} = \frac{127}{1000} \times \frac{100}{1}\%$
 $= \frac{127}{10}\%$
 $= 12.7\%$

Example 5

Express each fraction as a percentage.

a $\frac{4}{5}$

b $\frac{37}{50}$

c $2\frac{4}{5}$

Solution

a $\frac{4}{5} = \frac{4}{5} \times \frac{100}{1}\%$
 $= \frac{4}{\cancel{5}^1} \times \frac{\cancel{100}^{20}}{1}\%$
 $= 80\%$

b $\frac{37}{50} = \frac{37}{50} \times \frac{100}{1}\%$
 $= \frac{37}{\cancel{50}^1} \times \frac{\cancel{100}^2}{1}\%$
 $= 74\%$

c $2\frac{4}{5} = \frac{14}{5} \times \frac{100}{1}\%$
 $= \frac{14}{\cancel{5}^1} \times \frac{\cancel{100}^{20}}{1}\%$
 $= 280\%$

Example 6

Write $7\frac{1}{4}\%$ as a fraction.

Solution

$$7\frac{1}{4}\% = \frac{7\frac{1}{4} \times 4}{100 \times 4}$$
$$= \frac{29}{400}$$

Example 7

What is 20% of 415?

Solution

$$\begin{aligned} 20\% \text{ of } 415 &= \frac{20}{100} \times \frac{415}{1} \\ &= \frac{1}{5} \times \frac{415}{1} \\ &= 83 \end{aligned}$$

Example 8

What is 7% of 197?

Solution

$$\begin{aligned} 7\% \text{ of } 197 &= \frac{7}{100} \times \frac{197}{1} \\ &= \frac{1379}{100} \\ &= 13.79 \end{aligned}$$

Example 9

- a** There are 200 Year 8 students in a school. Of these, 112 are girls. What percentage of the students are girls?
- b** A rugby league team played 20 matches in a season. They lost 8 of the matches. What percentage of the matches did they lose?

Solution

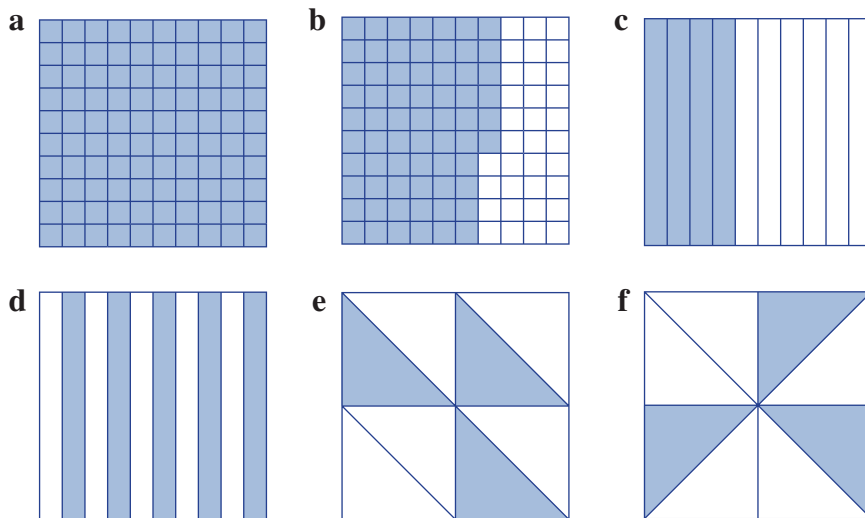
$$\begin{aligned} \text{a Percentage of girls in the school} &= \frac{112}{200} \times 100\% \\ &= \frac{112}{2} \% \\ &= 56\% \end{aligned}$$

$$\begin{aligned} \text{b Percentage of matches lost} &= \frac{8}{20} \times 100\% \\ &= 40\% \end{aligned}$$



Exercise 7A

1 What percentage of each square is shaded?



Example 3a, b

2 Convert each percentage to a fraction.

- a** 10% **b** 25% **c** 85% **d** 50%
- e** 40% **f** 20% **g** 125% **h** 630%
- i** 205% **j** 138% **k** 496% **l** 43 125%

Example 3c, d

3 Convert each percentage to a decimal.

- a** 55% **b** 37% **c** 19% **d** 80%
- e** 30% **f** 10% **g** 250% **h** 148%
- i** 3912% **j** 3% **k** 7% **l** 1%

Example 4

4 Express each fraction as a percentage.

- a** $\frac{23}{100}$ **b** $\frac{99}{100}$ **c** $\frac{11}{100}$ **d** $\frac{150}{100}$ **e** $\frac{230}{100}$
- f** $\frac{999}{100}$ **g** $\frac{42}{10}$ **h** $\frac{67}{10}$ **i** $\frac{14}{10}$ **j** $\frac{9}{10}$
- k** $\frac{5}{10}$ **l** $\frac{1}{10}$ **m** $\frac{670}{1000}$ **n** $\frac{810}{1000}$ **o** $\frac{330}{1000}$
- p** $\frac{999}{1000}$ **q** $\frac{123}{1000}$ **r** $\frac{702}{1000}$ **s** $\frac{9999}{1000}$ **t** $\frac{1234}{1000}$

5 Express each fraction as a percentage.

a $\frac{2}{4}$

b $\frac{49}{50}$

c $\frac{4}{25}$

d $\frac{1}{20}$

e $\frac{3}{5}$

f $\frac{19}{25}$

g $1\frac{1}{4}$

h $\frac{1}{40}$

i $3\frac{17}{25}$

j $\frac{6}{5}$

k $\frac{10}{4}$

l $\frac{25}{20}$

m $\frac{3}{40}$

n $2\frac{2}{5}$

o $\frac{17}{40}$

6 Write each decimal as a percentage.

a 0.65

b 0.23

c 0.75

d 0.5

e 0.3

f 0.6

g 1.5

h 1.1

i 1.8

j 1

k 0.07

l 0.02

m 1.05

n 4.09

o 7.03

7 Write each percentage as a fraction.

a $2\frac{1}{2}\%$

b $7\frac{1}{2}\%$

c $8\frac{1}{4}\%$

d $37\frac{1}{2}\%$

e $12\frac{1}{2}\%$

f $67\frac{1}{2}\%$

g $87\frac{1}{2}\%$

h $66\frac{2}{3}\%$

8 Complete the table below, showing the decimal, fraction and percentage equivalents in each row.

Decimal	Fraction	Percentage
0.5		50%
	$\frac{1}{4}$	
		75%
0.4		
		100%
0.367		
	$\frac{29}{100}$	
0.403		
	$\frac{3}{8}$	
	$\frac{5}{4}$	
2.75		
		200%

9 Express each quantity as a percentage of the total.

- a** 30 cents out of a dollar
- b** 250 m out of 1 km
- c** 160 women in a train carrying 200 people
- d** 16 teeth filled out of 24
- e** \$10 discount on a \$200 jumper
- f** 18 out of 45 people in a restaurant ordering pizza

10 Calculate these amounts.

- a** 50% of 468 **b** 25% of 144 **c** $12\frac{1}{2}\%$ of 1360
- d** $33\frac{1}{3}\%$ of 8730 **e** 20% of 500 **f** 40% of 455
- g** $37\frac{1}{2}\%$ of 192 **h** $87\frac{1}{2}\%$ of 960 **i** $66\frac{2}{3}\%$ of 525

Example 7, 8

11 Calculate:

- a** 20% of 100 **b** 20% of 200 **c** 20% of 1000 **d** 50% of 300
- e** 5% of 30 **f** 15% of 300 **g** 63% of 100 **h** 78% of 500
- i** 57% of 30 **j** 200% of 100 **k** 200% of 118 **l** 200% of 3.5

12 **a** Find 10% of 340. **b** Find 30% of 340. **c** Find 70% of 340.

d Find 5% of 340. **e** Find $2\frac{1}{2}\%$ of 340. **f** Find $7\frac{1}{2}\%$ of 340.

13 **a** Find 25% of 320. **b** Find 75% of 320. **c** Find $12\frac{1}{2}\%$ of 320.

14 **a** Find 10% of 456. **b** Find 30% of 456.

c Find 10% of 912. **d** Find 5% of 912.

15 Use fractions to calculate:

- a** 20% of 48 m **b** 75% of 80 lollies **c** 30% of 270 children

16 Use decimals to find:

- a** 25% of 28 songs **b** 40% of 280 stories **c** 15% of 160 students

17 Express \$1.80 as a percentage of \$5.00.

18 Express \$24.40 as a percentage of \$30.50.

19 Mike bought 72 stamps. He bought 27 more stamps than Carla. Express the number of stamps Carla bought as a percentage of Mike's.



In this section, we look at some harder examples of changing percentages to fractions and fractions to percentages. The emphasis here is on converting a fraction whose denominator does not divide into a power of 10 to a percentage.

Example 10

Write $\frac{2}{7}$ as a percentage.

Solution

$$\begin{aligned}\frac{2}{7} &= \frac{2}{7} \times 100\% \\ &= \frac{200}{7}\% \\ &= 28\frac{4}{7}\%\end{aligned}$$

Changing percentages to fractions

We now look at how to change a percentage to a fraction. For example,

$45\frac{1}{2}\% = \frac{45\frac{1}{2}}{100}$. We want to write this as a simple fraction.

$$\begin{aligned}45\frac{1}{2}\% &= \frac{45\frac{1}{2}}{100} \\ &= \frac{45\frac{1}{2} \times 2}{100 \times 2} \quad (\text{Multiply numerator and denominator by 2.}) \\ &= \frac{91}{200}\end{aligned}$$

Example 11

Write each percentage as a fraction.

a $23\frac{1}{3}\%$

b $32\frac{4}{5}\%$

c $5\frac{1}{7}\%$

Solution

$$\begin{array}{lll} \mathbf{a} \quad 23\frac{1}{3}\% = \frac{23\frac{1}{3}}{100} & \mathbf{b} \quad 32\frac{4}{5}\% = \frac{32\frac{4}{5}}{100} & \mathbf{c} \quad 5\frac{1}{7}\% = \frac{5\frac{1}{7}}{100} \\ & & \\ & = \frac{23\frac{1}{3} \times 3}{100 \times 3} & = \frac{32\frac{4}{5} \times 5}{100 \times 5} & = \frac{36}{700} \\ & = \frac{70}{300} & = \frac{164}{500} & = \frac{9}{175} \\ & = \frac{7}{30} & = \frac{41}{125} & \end{array}$$

Expressing one quantity as a percentage of another

We have already looked at how to express one quantity as a percentage of another in the previous section. Here are some examples in which the resulting percentages involve fractions.

Example 12

Express 55 as a percentage of 120.

Solution

$$\begin{aligned} \frac{55}{120} &= \frac{55}{120} \times \frac{100}{1} \% \\ &= \frac{55}{6} \times \frac{5}{1} \% \\ &= \frac{275}{6} \% \\ &= 45\frac{5}{6} \% \end{aligned}$$

Example 13

Two bakeries, Brownie and Best Bake, both bake bread. On a particular day, 27 out of 40 loaves at the Brownie Bakery were baked the previous day. At the Best Bake Bakery, 57 out of 90 loaves were baked the previous day. Which bakery was selling the greater percentage of loaves baked the previous day?

(continued on next page)

Solution

The percentage of loaves baked the previous day at the Brownie Bakery is:

$$\begin{aligned}\frac{27}{40} \times 100\% &= \frac{2700}{40}\% \\ &= \frac{135}{2}\% \\ &= 67\frac{1}{2}\%\end{aligned}$$

The percentage of loaves baked the previous day at the Best Bake Bakery is:

$$\begin{aligned}\frac{57}{90} \times 100\% &= \frac{570}{9}\% \\ &= 63\frac{1}{3}\%\end{aligned}$$

Hence the Brownie Bakery had the greater percentage of loaves baked a day earlier.

Example 14

Write 0.32 out of 4.8 as a percentage.

Solution

$$\begin{aligned}\text{First } \frac{0.32}{4.8} &= \frac{32}{480} \\ &= \frac{32}{480} \times 100\% \\ &= \frac{2}{30} \times 100\% \\ &= \frac{2}{3} \times 10\% \\ &= 6\frac{2}{3}\%\end{aligned}$$



Exercise 7B

Example 10

- 1 Express each fraction as a percentage. Your answers will contain fractions.

a $\frac{2}{3}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$ **d** $\frac{7}{9}$ **e** $\frac{11}{12}$ **f** $\frac{5}{7}$

Example 11

- 2 Express each percentage as a fraction.

a $12\frac{1}{5}\%$ **b** $6\frac{3}{4}\%$ **c** $5\frac{1}{4}\%$ **d** $15\frac{1}{6}\%$ **e** $6\frac{2}{3}\%$ **f** $5\frac{1}{12}\%$
g $12\frac{1}{6}\%$ **h** $21\frac{3}{4}\%$ **i** $42\frac{2}{7}\%$ **j** $6\frac{2}{7}\%$ **k** $44\frac{2}{3}\%$ **l** $7\frac{5}{12}\%$

- 3 Express each percentage as a fraction in simplest form.

a 200% **b** 1.5% **c** 124% **d** 210% **e** 0.5% **f** $125\frac{1}{2}\%$

- 4 Express each percentage first as a decimal and then as a fraction in simplest form.

a 12.25% **b** 6.75% **c** 5.75% **d** 8.06%
e 12.05% **f** 227% **g** 11 790% **h** 5711%

Example 12

- 5 Express the first number in each pair as a percentage of the second.

a 23, 69 **b** 35, 125 **c** 45, 135
d 456, 2000 **e** 34, 134 **f** 56, 400

- 6 In his vocabulary tests last term, Mick scored 21 out of 25, 13 out of 15, and 17 out of 20. By expressing his scores as percentages, rank Mick's efforts on the tests from best to worst.

Example 13

- 7 The total population of a state is 2 500 000. The population of the capital city is 1 500 000.

- a** What percentage of the population of the state lives in the capital city?
b What percentage of the population of the state lives outside the capital city?

- 8 A student was awarded a mark of 48 in a test out of 60 marks. What was the student's percentage score?

- 9 The number of people who enter Luna Park on a particular Sunday is 3200. Of these, 1250 purchase a ride on the rollercoaster. What percentage of the 3200 people purchase a ride on the rollercoaster?

- 10 In medieval Europe, the Black Death plague had an extremely high fatality rate. Calculate the percentage of those who died from the disease if the number of survivors in a village of 300 people infected with the disease was 15.

- 11 A drug for treating cholera is 98.5% effective.

- a** If 1200 infected people are treated with the drug, how many would you expect to recover?
b A different drug was administered, and 78 out of 80 people recovered. Using percentages, work out which drug is more effective.

- 12 a** How much larger is $\frac{1}{3}$ than 33%, as a percentage?
- b** How much larger is $\frac{1}{3}$ than 0.3, as a percentage?
- 13** A survey was conducted at three different schools to find how many students travel to school by bus.
- School A reported that $\frac{2}{7}$ of its students are bus travellers.
 - School B reported that 110 of its 400 students are bus travellers.
 - School C reported that 28.9% of its students travel by bus.
- Rank the three schools in terms of the percentage of students who travel by bus.
- 14** The bread sales on Saturday in a supermarket were:
- white bread, 135 loaves
 - wholemeal, 84 loaves
 - multigrain, 117 loaves
 - raisin bread, 24 loaves
- Express the number of loaves of each type sold as a percentage of total sales.
- 15** Express each of these situations as a percentage.
- a** In a population of 24 million people, 0.06 million work in the automobile industry.
- b** In a chess game, 5 out of 32 pieces were captured and removed from the board.
- 16** The average percentage, by weight, of body fat is 17% for men and 23% for women. Calculate the amount of fat, in kilograms, for:
- a** a woman weighing 76 kg **b** a man weighing 100 kg
- c** a woman weighing 60 kg **d** a man weighing 72 kg

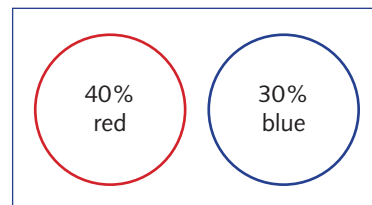


Sums and differences of percentages

In some situations, but not all, percentages can be added and subtracted. There are two conditions that must be satisfied before addition or subtraction of percentages can be undertaken.

- They are percentages of the same whole.
- There is no overlap among the groups being considered.

For example, 200 people are asked their favourite colour (only one colour is allowed). 40% reply blue and 30% reply red, so $40\% + 30\% = 70\%$ have red or blue as their favourite colour.



Also, $40\% - 30\% = 10\%$ more of the people surveyed have blue rather than red as their favourite colour. In this situation both percentages are percentages of the same 200 people and there is no overlap between the groups.

Example 15

30% of the students in a school of 1200 have blond hair and 25% have black hair. What percentage of the students have either blond or black hair?

Solution

The percentage is $30\% + 25\% = 55\%$.

We could also solve the problem by working out the actual numbers involved.

$$30\% \text{ of } 1200 = 360$$

$$25\% \text{ of } 1200 = 300$$

$$\text{Total} = 660$$

$$\begin{aligned} \text{Percentage} &= \frac{660}{1200} \times 100\% \\ &= \frac{660}{12} \% \\ &= 55\% \end{aligned}$$

However, there is no need to do it in this way because the two conditions above are satisfied. Both the percentages given in the question are percentages of the number of students in the school, and we can safely assume that no one has both black and blond hair.

Example 16

60% of people on a large railway platform are reading a newspaper and 24% are reading a novel. The other people on the platform are not reading. What is the percentage of people on the platform who are:

- a** reading? **b** not reading?

Solution

- a** $60\% + 24\% = 84\%$
84% of people on the platform are reading.
- b** Since all of the people (either reading or not reading) on the platform = 100%,
 $100\% - 84\% = 16\%$.
Hence 16% of people on the platform are not reading.

Note that in the last example there is once again no overlap between the people on the platform who are reading and those who are not reading.

In the next example, we cannot add the percentages because they are percentages of different wholes. The first, wrong, solution shows what happens if you try to do this.

Example 17

Two football teams, the Sharks and the Raiders, play a match. The Sharks club has 10000 members and the Raiders club has 12000 members. 15% of the Sharks club members and 20% of the Raiders club members attend the match. What percentage of the combined membership of the clubs attend the match?

Solution (Wrong!)

$$15\% + 20\% = 35\%$$

35% of the combined membership of the clubs attend the match.

Solution (Correct)

Number of Sharks attending = 15% of 10 000 = 1500

Number of Raiders attending = 20% of 12 000 = 2400

$$\begin{aligned}\text{Number of supporters attending the match} &= 1500 + 2400 \\ &= 3900\end{aligned}$$

$$\begin{aligned}\text{Combined membership of the clubs} &= 10\,000 + 12\,000 \\ &= 22\,000\end{aligned}$$

$$\begin{aligned}\text{Percentage of combined membership at match} &= \frac{3900}{22\,000} \times 100\% \\ &= \frac{195}{11}\% \\ &= 17\frac{8}{11}\%\end{aligned}$$

Example 18

Add the percentages 5%, 10% and 40%, and express the result as a fraction.

Solution

$$\begin{aligned}5\% + 10\% + 40\% &= 55\% \\ &= \frac{55}{100} \\ &= \frac{11}{20}\end{aligned}$$

Multiplication of percentages

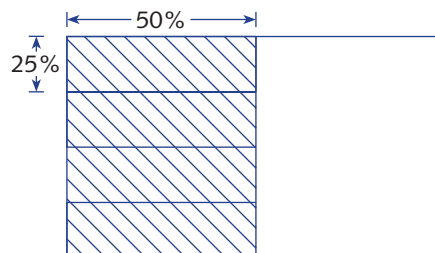
Multiplication of percentages arises in situations such as the following.

50% of a class are boys, and 25% of them have blue eyes. Hence the percentage of students in the class who are boys with blue eyes is 25% of 50%.

We can draw a diagram to illustrate this. The large cross-hatched rectangle represents the boys in the class, and the smaller blue rectangle represents the 25% of the boys who have blue eyes.

The percentage of students who are boys and have blue eyes is

$$\begin{aligned} 25\% \text{ of } 50\% &= \frac{1}{4} \text{ of } \frac{1}{2} \\ &= \frac{1}{8} \\ &= 12\frac{1}{2}\% \end{aligned}$$



Example 19

Find 20% of 70%.

Solution

$$\begin{aligned} 20\% \text{ of } 70\% &= \frac{20}{100} \times \frac{70}{100} \\ &= \frac{1}{5} \times \frac{7}{10} \\ &= \frac{7}{50} \\ &= 14\% \end{aligned}$$

Example 20

There are 65 000 spectators in a football stadium of whom 55% barrack for the Reds. Of those who barrack for the Reds, 80% are male. Find the percentage of spectators who are male and barrack for the Reds.

Solution

Percentage of spectators who are male and barrack for the Reds

$$\begin{aligned} &= 80\% \text{ of } 55\% \\ &= \frac{80}{100} \times 55\% \\ &= \frac{4}{5} \times \frac{11}{20} \\ &= \frac{44}{100} \\ &= 44\% \end{aligned}$$



Exercise 7C

Example 18

- 1 In each part, add the percentages and express the result as a fraction.

a $5\% + 20\% + 30\%$

b $0.5\% + 24\% + 65.5\%$

c $2\% + 10\% + 50\%$

d $12.5\% + 22.5\% + 2.25\%$

Example 19

- 2 Calculate:

a 20% of 50%

b 15% of 30%

c 60% of 80%

d 34% of 90%

e $12\frac{1}{2}\%$ of 80%

f $33\frac{1}{3}\%$ of 25%

g 40% of 35%

h 80% of 15%

i $66\frac{2}{3}\%$ of 50%

- 3 **a** Find 30% of 20%. **b** Find 20% of 30%.

Example 20

- 4 65% of a group of 4300 teenagers said they like healthy food.

a How many of the teenagers said they like healthy food?

b What is the percentage of the teenagers who said they do not like healthy food?

- 5 In a particular group of 600 people:

- 15% watch only the Channel Nine news every night
- 40% watch only the Channel Seven news every night
- 25% watch only the Channel Two news every night.

a How many people watch:

i the Channel Nine news? **ii** the Channel Seven news?

iii the Channel Two news?

b How many people watch none of these news broadcasts?

- 6 When a group of 500 people were asked to name their favourite sport. 34% responded 'only football' and 26% responded 'only cricket'.

a How many people responded 'only football'?

b How many people responded 'only cricket'?

c How many people did not respond 'only cricket' or 'only football'?

Example 20

- 7 15% of a herd of goats are male and 6% of the male goats are older than 4 years of age. What percentage of the total number of goats are male and older than 4 years of age?

- 8 A jar contains dark chocolate balls and milk chocolate balls. 35% of the chocolate balls are dark chocolate and 15% of the dark chocolate balls are wrapped in gold foil. What percentage of the chocolate balls are:
- milk chocolate?
 - dark chocolate in gold foil?
 - dark chocolate without gold foil?
- 9 60% of a group of students are boys, and 30% of the boys have blue eyes. What percentage of the class are boys with blue eyes?
- 10 80% of a herd of cattle are Herefords, and 15% of the Herefords are under one year of age. What percentage of the cattle are Herefords under one year of age?
- 11 40% of a group of students are going to a concert, and 23% of the students going to the concert are also going to a party afterwards. What percentage of students go to both the concert and the party afterwards?
- 12 If you travel from Goatshead to Oxtail, you have covered 45% of the total distance between the two towns when you reach the turnoff. The bridge is 60% of the way from the turnoff to Oxtail. What percentage of the total trip still remains when you get to the bridge?



- 13 Explain why the following reasoning is incorrect. 20% of the students going to one school have blond hair. 30% of the students going to a nearby school have blond hair. Therefore $20\% + 30\% = 50\%$ of the students in the two schools have blond hair.
- 14 Explain why the following reasoning is incorrect. 60% of the Queensland Reds' supporters wear red shirts. 40% of the Queensland Reds' supporters are female. Therefore $40\% \text{ of } 60\% = 24\%$ of Queensland Reds' supporters are female and wear red shirts.

7D Simple interest



Perhaps the most common everyday example of percentages is bank interest. If you invest \$4000 with a bank for 1 year at 5% per annum interest, you receive

$$\begin{aligned} 5\% \text{ of } 4000 &= \frac{1}{20} \times 4000 \\ &= \$200. \end{aligned}$$

The \$200 is the **interest** payable at the end of the year on the amount invested. The words *per annum* (p.a.) are Latin for ‘by the year’.

Interest rates are usually expressed as an annual rate of interest. For example, 5% p.a. means 5% per annum (5% per year).

The amount that you invest (\$4000 in the example above) is called the **principal** and the percentage interest per annum (5%) is called the **interest rate**. The period of the investment is called the **term** (one year).

If \$4000 was invested at 5% per annum for 2 years, you would receive \$200 in interest for each year, giving a total of \$400 for the 2 years. The interest is paid only on the principal. This is called **simple interest**. If the investment was for 3 years, you would receive $3 \times \$200 = \600 .

The **amount** at the end of the term is the principal plus the interest.



Simple interest

With simple interest, there are four important quantities to consider:

- The **principal** is the amount invested.
- The **term** is the period of time for which the money is invested.
- The **interest rate** is usually expressed as a percentage (increase) per annum.
- The **interest** is the money earned.
- The **amount** at the end of the term is the principal plus the interest.

In simple interest, the interest for each year of the term is the same – it is the principal multiplied by the interest rate.

Example 21

Cynthia deposits \$3500 in a bank for 3 years and the bank pays her simple interest at a rate of $5\frac{1}{4}\%$ per annum. How much interest does she receive?

Solution

The principal is \$3500, the interest rate is $5\frac{1}{4}\%$ p.a. The term is 3 years.

(continued on next page)

Interest per year = $5\frac{1}{4}\%$ of 3500

$$= \frac{21}{400} \times 3500$$

$$= \frac{735}{4}$$

$$= \$183.75$$

(In this question, the principal is \$3500, the term is 3 years and the rate is $5\frac{1}{4}\%$.)

The term is 3 years, so total interest received = 3×183.75
= \$551.25.



Exercise 7D

- Find the amount of interest earned by an investment at 5% p.a. for one year on a principal of:
a \$1000 **b** \$2000 **c** \$2500 **d** \$10000 **e** \$100000 **f** \$150000
- Find the amount of interest earned by an investment at 6% p.a. for one year on a principal of:
a \$1000 **b** \$2000 **c** \$2500 **d** \$10000 **e** \$100000 **f** \$150000
- \$250 000 is invested at 6% p.a. simple interest. Find the amount of the investment at the end of:
a 1 year **b** 2 years **c** 3 years
- \$125 000 is invested at 5% p.a. simple interest. Find the amount of the investment at the end of:
a 1 year **b** 2 years **c** 3 years
- Calculate the amount of an investment of \$10 000 at the end of 1 year for each of these simple interest rates.
a 5% **b** 17% **c** 23% **d** 5.5% **e** 5.25%
- Calculate the amount of an investment of \$12 000 at the end of 2 years for each of these simple interest rates.
a 6% **b** 5% **c** 5.25% **d** 12.5% **e** 8%
- Calculate the amount of an investment of \$25 000 at the end of 3 years for each of these simple interest rates.
a 12.5% **b** 6% **c** 5.25% **d** 8% **e** 7%

Example 21



Discount is an important example of the use of percentages. Shops offer discounts as incentives to buy. You will see signs such as these written on shop windows:

- ‘20% off all goods’
- ‘15% off all track pants’
- ‘All items reduced by 17%’

When a discount is expressed as a percentage, then the bigger the price the bigger the discount. For example, an offer of \$1 off the price of a box of chocolates that normally cost \$4 is a 25% discount, whereas \$1 off a \$50 pair of jeans is only a 2% discount.

The method for calculating a discounted price is shown in the following example.

Example 22

Gloria goes to the Mighty Good Emporium to buy a bicycle. The shop is offering a 20% discount on all bicycles. The bicycle Gloria wants is marked at \$460. How much does she pay for it?

Solution

$$\begin{aligned} 20\% \text{ of } \$460 &= \frac{20}{100} \times 460 \\ &= \frac{1}{5} \times 460 \\ &= 92 \end{aligned}$$

Gloria obtains a discount of \$92. She pays $(460 - 92) = \$368$.



Exercise 7E

Example 22

- 1 Calculate the discounted price in each case.
 - a A discount of 10% on a purchase of \$3210
 - b A discount of 12% on a purchase of \$5600
 - c A discount of 5% on a purchase of \$68
 - d A discount of 15% on a purchase of \$625
 - e A discount of 8% on a purchase of \$56
 - f A discount of 23% on a purchase of \$78.50

- 2 A shop has a discount sale that marks down all prices by 20%. If a customer buys a jacket originally marked at \$90, how much does he pay?
- 3 A discount of 45% is offered on all CDs. What is the discounted price on a CD marked at \$32?
- 4 Gaye buys three novels and two textbooks at a shop that is advertising 25% discount on all books. The novels are marked at \$36 each and the textbooks are marked at \$72 and \$96. What is the total price after the discount has been given?
- 5 A shopkeeper paid \$200 each for two handbags. He sold the first handbag for a profit of 25% of the cost price. He sold the second at a loss of 15% of the cost price. How much profit did the shopkeeper make on the two bags?



Percentages provide a convenient way to express an increase or decrease in a quantity.

In the newspapers or on television, you will often hear such news as:

- A company's profits for the year have increased by 20%.
- The population of a city has increased by 7% over the past two years.
- The grain production in a certain area is down by 25%.

Percentage increase

Example 23

a Increase 600 by 5%.

b Increase 850 by $6\frac{1}{2}\%$.

Solution

$$\begin{aligned}\text{a } 5\% \text{ of } 600 &= \frac{5}{100} \times 600 \\ &= 30\end{aligned}$$

$$\begin{aligned}\text{Therefore } 600 \text{ increased by } 5\% &= 600 + 30 \\ &= 630\end{aligned}$$

$$\begin{aligned}\text{b } 6\frac{1}{2}\% \text{ of } 850 &= \frac{6\frac{1}{2}}{100} \times \frac{850}{1} \\ &= \frac{13}{200} \times \frac{850}{1} \\ &= \frac{13 \times 17}{4} \\ &= \frac{221}{4} \\ &= 55\frac{1}{4}\end{aligned}$$

$$\begin{aligned}\text{Therefore } 850 \text{ increased by } 6\frac{1}{2}\% &= 850 + 55\frac{1}{4} \\ &= 905\frac{1}{4}\end{aligned}$$

Example 24

House A was valued at \$200 000 in January 2004 and at \$324 000 in January 2011.
House B was valued at \$196 000 in January 2004 and at \$280 000 in January 2011.
Find the percentage increase in the value of each house.

Solution

House A

$$\begin{aligned}\text{Increase in value of house A} &= 324\,000 - 200\,000 \\ &= 124\,000\end{aligned}$$

$$\begin{aligned}\text{Percentage increase in value} &= \frac{124\,000}{200\,000} \times 100\% \\ &= 62\%\end{aligned}$$

House B

$$\begin{aligned}\text{Increase in value of house B} &= 280\,000 - 196\,000 \\ &= \$84\,000\end{aligned}$$

$$\begin{aligned}\text{Percentage increase in value} &= \frac{84\,000}{196\,000} \times 100\% \\ &= \frac{300}{7}\% \\ &= 42\frac{6}{7}\%\end{aligned}$$

We often use percentage increase to describe such things as population increases. For example, we may read in the newspaper that the population of Queensland increased by 8% over the past five years.

Example 25

The population of a particular mob of kangaroos in a given area is increasing by 5% every year. If the population on 1 January 2005 was 660, what was the population of kangaroos on 1 January 2006?

Solution

If the population at 1 January 2005 was 660, then the population on 1 January 2006 would be $660 + (5\% \text{ of } 660)$.

$$\begin{aligned} 5\% \text{ of } 660 &= \frac{1}{20} \times 660 \\ &= \frac{66}{2} \\ &= 33 \end{aligned}$$

The total population of kangaroos on 1 January 2006 was $660 + 33 = 693$.

Example 26

A shop finds its sales for February have increased by 24% from its January figures. In January, the sales figure was \$32 000. Find the sales for February.

Solution

$$\begin{aligned} \text{Increase in sales} &= 24\% \text{ of } 32\,000 \\ &= \frac{24}{100} \times 32\,000 \\ &= \$7680 \end{aligned}$$

$$\begin{aligned} \text{February sales} &= 7680 + 32\,000 \\ &= \$39\,680 \end{aligned}$$

Percentage decrease

Everyday examples of percentage decrease are also common. For example:

- the number of pelicans in a particular breeding colony has decreased by 20% from the previous year
- the numbers of supporters at the matches of the Gunarrong Football Club are down by 30% from the previous season
- the number of tourists visiting the Big Turnip tourist site has decreased by 15% from the previous year.

Example 27

a Decrease 600 by 8%.

b Decrease 800 by $12\frac{1}{4}\%$.

Solution

$$\begin{aligned}\mathbf{a} \quad 8\% \text{ of } 600 &= \frac{8}{100} \times \frac{600}{1} \\ &= 48\end{aligned}$$

$$\begin{aligned}\text{Decreased amount} &= 600 - 48 \\ &= 552\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad 12\frac{1}{4}\% \text{ of } 800 &= \frac{12\frac{1}{4}}{100} \times \frac{800}{1} \\ &= \frac{49}{400} \times \frac{800}{1} \\ &= 98\end{aligned}$$

$$\begin{aligned}\text{Decreased amount} &= 800 - 98 \\ &= 702\end{aligned}$$

Example 28

The circulation of a newspaper on 1 January 2010 was 640 000. On 1 January 2011, the circulation was 540 000. Find the percentage decrease in circulation.

Solution

The drop in circulation was $640\,000 - 540\,000 = 100\,000$.

$$\begin{aligned}\text{Percentage decrease in circulation} &= \frac{100\,000}{640\,000} \times 100\% \\ &= \frac{5}{32} \times 100\% \\ &= \frac{5}{8} \times 25\% \\ &= 15\frac{5}{8}\%\end{aligned}$$

Example 29

The population of Gopedope was 80 000 on 1 February 1845. By 1 February 1901, the population had fallen by 45%. Find the population on 1 February 1901.

Solution

Drop in population of Gopedope = 45% of 80 000

$$\begin{aligned} &= \frac{45}{100} \times 80\,000 \\ &= 45 \times 800 \\ &= 36\,000 \end{aligned}$$

$$\begin{aligned} \text{Population of Gopedope on 1 February 1901} &= 80\,000 - 36\,000 \\ &= 44\,000 \end{aligned}$$

Example 30

Rabbit Island is home to two types of rabbits.

On 1 January 2005, there were 600 grey rabbits and 400 pink-nosed rabbits.

On 1 January 2006, there were 1260 grey and 550 pink-nosed rabbits.

- a For each type of rabbit, find the percentage increase in population.
- b Find the percentage increase in the total rabbit population.

Solution

- a Grey rabbits increased by $1260 - 600 = 660$.

$$\begin{aligned} \text{Percentage increase} &= \frac{660}{600} \times 100\% \\ &= \frac{660}{6} \% \\ &= 110\% \end{aligned}$$

Pink-nosed rabbits increased by $550 - 400 = 150$.

$$\begin{aligned} \text{Percentage increase} &= \frac{150}{400} \times 100\% \\ &= 37.5\% \end{aligned}$$

- b The total number of rabbits, increased from $600 + 400 = 1000$ by $660 + 150 = 810$.

$$\begin{aligned} \text{Percentage increase} &= \frac{810}{1000} \times 100\% \\ &= 81\% \end{aligned}$$



Exercise 7F

Example 23

1 Find the value when:

- | | |
|---|---|
| a 800 is increased by 7% | b 1000 is increased by 6% |
| c 750 is increased by 8% | d 270 is increased by 10% |
| e 370 is increased by 8% | f 4500 is increased by 9% |
| g 6000 is increased by $6\frac{1}{2}\%$ | h 20 000 is increased by $12\frac{1}{4}\%$ |
| i 5600 is increased by $11\frac{1}{4}\%$ | |

2 Calculate the percentage change (increase or decrease) that occurs when:

- | | | |
|-------------------------|-------------------------|--------------------------|
| a 20 becomes 40 | b 25 becomes 100 | c 2 becomes 10 |
| d 10 becomes 11 | e 8 becomes 10 | f 110 becomes 100 |
| g 90 becomes 120 | h 100 becomes 80 | i 48 becomes 36 |

3 Find the new amount after each percentage change.

- | | |
|---|--|
| a \$45 000 is increased by 20% | b \$60 000 is decreased by 15% |
| c \$32 000 is increased by $12\frac{1}{2}\%$ | d 42 000 hectares is increased by 24% |

Example 24

4 The circulation of a newspaper on 1 January 2009 was 720 000. On 1 January 2010, the circulation was 800 000. Find the percentage increase in circulation.

Example 26

5 The farmers in a particular region of China planted 45 000 hectares of farmland with sorghum in 2005. By 2010, the area had increased by 24%. Find the area planted with sorghum in 2010.

Example 27

6 Find the value when:

- | | |
|-------------------------------------|---|
| a 500 is decreased by 10%. | b 6250 is decreased by 8%. |
| c 5500 is decreased by 6%. | d 7000 is decreased by $6\frac{1}{2}\%$. |
| e 10 000 is decreased by 9%. | f 15 500 is decreased by $12\frac{1}{2}\%$. |

Example 29

7 The population of a city was 92 000 on 1 February 1920. By 1 February 1950, the population had fallen by 45%. Find the population at this second date.

Example 28

8 The circulation of a newspaper on 1 January 2008 was 840 000. On 1 January 2009, the circulation was 640 000. Find the percentage decrease in circulation.

9 If the annual increase in the population of a country is 2.5% and the present number of inhabitants is 2 624 000, what will the population be in a year's time?

- 10** The population of a city was 1 000 000 and it increased by $2\frac{1}{2}\%$ of this value in each of 3 successive years.

- a** What was the population at the end of the first year?
- b** What was the population at the end of 3 years?

Example 24

- 11** A house was bought for \$300 000 in 2000. It was sold again in 2005 for \$475 000. What was the percentage increase in the value of the house?

- 12** A man buys a car for \$7500 and sells it two years later for \$5000. What is his percentage loss?

- 13** Two different machines are used to produce chocolates in a chocolate shop.

Machine A produces 520 g of chocolate from 650 g of raw materials. Machine B produces 450 g of chocolates from 600 g of raw materials. The rest of the raw materials stick to the machines.

- a** What percentage, by weight, of the raw materials is turned into chocolates by machine A?
- b** What percentage, by weight, of the raw materials is turned into chocolates by machine B?
- c** Which of the machines is the more efficient?

- 14** A floor covering shop made a profit of 6% on its total costs last year. If the total costs were \$1 240 000, what was the profit?

- 15** A shoe shop had total costs of \$5 200 000 on salaries, shoes etc., and their total sales was \$4 628 000.

- a** What was the loss?
- b** What was the loss expressed as a percentage of the total costs?

- 16** Find the profit or loss for these situations.

- a** 10% loss on \$76 000
- b** 8% profit on \$80 000
- c** 50% profit on \$1 000 000
- d** 25% loss on \$420 000

- 17** Find the percentage profit or loss on costs in these situations.

- a** Costs \$12 000 and sales \$13 920
- b** Costs \$52 000 and sales \$56 680
- c** Costs \$112 000 and sales \$100 800
- d** Costs \$220 000 and sales \$200 900

- 18** A contractor made a profit of 15% for a job. The costs were \$45 000. How much did the contractor receive?



The unitary method was introduced in Section 2D. Here is a reminder of how it works.
The unitary method is very useful for doing the problems of the previous section in reverse.

Example 31

If 8 oranges cost \$5.60, how much do 3 oranges cost?

Solution

$$\begin{array}{ll} \boxed{\div 8} & 8 \text{ oranges cost } \$5.60 \\ & 1 \text{ orange costs } \frac{5.60}{8} = 0.7 \\ & \quad = 70\text{c}. \\ \boxed{\times 3} & 3 \text{ oranges cost } 3 \times 70\text{c} = 210\text{c} \\ & \quad = \$2.10. \end{array}$$

Notice that we first worked out the cost of one orange. The word ‘unit’ is another word for ‘one’ (of some object). This is why the method is called the *unitary* method.

The same idea can be used to find 100% of an amount if you are given a percentage of that amount.

Example 32

7% of an amount of money is \$84. What is the sum of money?

Solution

7% of the amount is 84.

$$\begin{array}{ll} \boxed{\div 7} & 1\% \text{ of the amount is } 12. \\ \boxed{\times 100} & 100\% \text{ of the amount is } \$1200. \end{array}$$

Example 33

8% of an amount of money is \$6000. How much is 3%?

Solution

8% of the amount is 6000.

$$\begin{array}{ll} \boxed{\div 8} & 1\% \text{ of the amount is } 750. \\ \boxed{\times 3} & 3\% \text{ of the amount is } \$2250. \end{array}$$

Example 34

A 15% discount is offered on all goods at a clothing shop. A suit has a discounted price of \$425. What was the original price of the suit?

Solution

Discounted price = $(100 - 15)\%$ of the original price
= 85% of the original price

85% of the original price = 425

$$\boxed{\div 85} \quad 1\% \text{ of the original price} = \frac{425}{85} \\ = 5$$

$$\boxed{\times 100} \quad 100\% \text{ of the original price} = 500$$

The original price of the suit was \$500.

Note: We could have used 5% as the unit here.

Alternative method

Let x be the original price.

$$85\% \text{ of } x = 425$$

$$\frac{85}{100} \times x = 425$$

$$x = 425 \times \frac{100}{85} \\ = 500$$

The original price is \$500.

Example 35

A population of ants increases by 20% in a week and the new population is 125 400. What was the population at the beginning of the week?

(continued on next page)

Solution

We can use 1% as the unit in this problem.

120% of the original population is 125 400.

$$\boxed{\div 120} \quad 1\% \text{ of the original population is } 1045.$$

$$\boxed{\times 100} \quad 100\% \text{ of the original population is } 104\,500.$$

The population at the beginning of the week was 104 500.

Sometimes the method can be modified to make our calculations simpler.

Alternative method using 20% as the unit

120% of the original population is 125 400.

$$\boxed{\div 6} \quad 20\% \text{ of the original population is } 20\,900.$$

$$\boxed{\times 5} \quad 100\% \text{ of the original population is } 104\,500.$$

The population at the beginning of the week was 104 500.

Example 36

John spends 24% of his weekly wage on rent and he spends 25% of the remainder in repayments on a motorbike. He still has \$228 left. How much does he earn in one week?

Solution

$$\begin{aligned} \text{Percentage of wage left after payment of rent} &= 100\% - 24\% \\ &= 76\% \end{aligned}$$

$$\begin{aligned} \text{Percentage of wage spent in repayments on motorbike} &= 25\% \text{ of } 76\% \\ &= \frac{1}{4} \text{ of } 76\% \\ &= 19\% \end{aligned}$$

$$\begin{aligned} \text{Percentage of wage remaining} &= 76\% - 19\% \\ &= 57\% \end{aligned}$$

$$57\% \text{ of John's wage} = 228$$

$$\begin{aligned} 1\% \text{ of John's wage} &= \frac{228}{57} \\ &= 4 \end{aligned}$$

Hence 100% of John's wage = \$400.

GST (goods and services tax)

The GST on an item is calculated as 10% of the pre-GST price of that item and is included in the price. Therefore the price including GST is 110% of the pre-GST price.

Example 37

The price of a suit, including GST, is \$440. What was the pre-GST price of the suit?

Solution

110% of the pre-GST price is 440.

$$\boxed{\div 11} \quad 10\% \text{ of the original price is } \$40.$$

$$\boxed{\times 10} \quad 100\% \text{ of the original cost is } \$400.$$

Example 38

The price of a shirt, including GST, is \$61.60. What was the original price of the shirt before GST was added?

Solution

110% of the original price is 61.60.

$$\boxed{\div 11} \quad 10\% \text{ of the original price is } 5.60.$$

$$\boxed{\times 10} \quad 100\% \text{ of the original price is } \$56.$$

Exercise 7G

Example 32

- 1 20% of the children in a swimming pool are boys. There are 8 boys. How many children are there in the swimming pool?

Example 33

- 2 a 20% of an amount of money is \$46 890. What is 80% of the amount of money?
b 5% of an amount of money is \$42 000. What is 20% of the amount of money?
c $12\frac{1}{2}\%$ of an amount of money is \$64 800. What is the total amount of money?
d $33\frac{1}{3}\%$ of an amount of money is \$45 000. What is 60% of the amount of money?

Example 34

- 3 The price of a car has been discounted by 10%. The discounted price is \$22 500. What was the price before the discount?

- 4 A park contains paved areas, lawns and garden beds. 8% of the area of the park is garden bed. There are 22 000 m² of garden bed.
- a What is the total area of the park?
 - b The lawns are 80% of the park. How many square metres of lawn are there?
- 5 Mr Ho invests an amount of money for one year at 5% per annum interest. The bank tells him that at the end of the year he will have \$126 000. How much does Mr Ho invest?
- 6 The number of students at a school increased by 12% from 2010 to 2011. In 2011, there are 1680 students. How many students were there in 2010?
- 7
- a 105% of x is 568 911. Find x .
 - b 10% of x is 45 670. Find x .
 - c 7% of w is 8470. Find w .
 - d 120% of a is 14 460. Find a .
- 8 Raylee gives 60% of her weekly wage to her mother and 25% of the remainder to her brother. She still has \$240 left. How much does she earn in one week?
- 9 Felicity spent 20% of her savings on a bicycle and 15% of the remainder on a book. What percentage of her savings did she have left?
- 10 A discount of 15% is offered on the price of a book. The discounted price is \$136. What was the original price?
- 11 For each item, the cost including GST is given. What was the cost of each item before GST was added?
- a A shirt priced at \$88
 - b A book priced at \$68.20
 - c A model aeroplane priced at \$253
 - d A chess set priced at \$93.50

Example 36

Example 37, 38

7H Simple interest II



In the previous section on simple interest, we calculated the interest and the amount given the term and the rate. In this section, we look at problems of the following types.

- Find the term given the amount, the principal and the rate.
- Find the rate given the amount, the principal and the term.

Example 39

- a** Philip invests \$20 000 for 2 years and receives \$3000 in interest. What was the interest rate per annum?
- b** Giselda invests \$20 000 for a number of years at 6.5% and receives \$5200. What is the term of her investment?

Solution

a Interest in one year $= \frac{3000}{2}$
 $= 1500$

$$\begin{aligned}\text{Interest rate} &= \frac{1500}{20\,000} \times 100\% \\ &= \frac{1500}{200} \% \\ &= 7\frac{1}{2} \%\end{aligned}$$

- b** Amount of interest each year = 6.5% of 20 000

$$\begin{aligned}&= \frac{6.5}{100} \times 20\,000 \\ &= \$1300\end{aligned}$$

$$\begin{aligned}\text{Term of the investment} &= \frac{5200}{1300} \left(\frac{\text{total interest received}}{\text{interest received per year}} \right) \\ &= 4 \text{ years}\end{aligned}$$



Exercise 7H

- 1 \$42 800 was invested at 6% simple interest for a number of years and the interest received at the end of the term was \$10 272. What was the term of the investment?
- 2 \$108 600 was invested at 8% simple interest for a number of years and at the end of the term the investment was worth \$134 664. How many years was the money invested for?

- 3 Dana invested \$96 000 for a year and received \$8400 in simple interest. What was the interest rate per annum?
- 4 Andrew invested \$40 000 for 2 years and received \$7000 in simple interest. What was the interest rate per annum?
- 5 \$450 000 was invested at 6% simple interest for a number of years and at the end of the term the investment was worth \$639 000. How many years was the money invested for?
- 6 \$10 800 was invested at 5% simple interest for a number of years and at the end of the term the investment was worth \$12 420. How many years was the money invested for?
- 7 David invested \$200 000 for a year and received \$13 000 in simple interest. What was the interest rate per annum?
- 8 Tran invested \$45 000 for two years and received \$5400 in simple interest. What was the interest rate per annum?
- 9 Elizabeth invested \$640 000 for three years and received \$96 000 in simple interest. What was the interest rate per annum?

Review exercise



- 1 Write these fractions as percentages. Use fractions in your answers if necessary.

a $\frac{23}{25}$	b $\frac{3}{4}$	c $\frac{3}{8}$	d $\frac{7}{8}$	e $\frac{3}{5}$	f $\frac{2}{3}$
g $\frac{45}{60}$	h $\frac{110}{100}$	i $\frac{48}{72}$	j $\frac{12}{96}$	k $\frac{31}{25}$	l $\frac{300}{150}$
- 2 Write each percentage as a mixed numeral with the fraction in simplest form.

a 50%	b 85%	c 43%	d 200%
e 340%	f 185%	g 12.5%	h 624%
- 3 Calculate:

a 40% of 20	b 25% of 18	c 23% of 200
d 18% of 150	e 97% of 200	f 3% of 600
g 85% of 4080	h 110% of 340 000	i 125% of 36
- 4 In a city, 33% of vehicles are utes, 18% are four-wheel drives, 27% are sedans and 22% are station wagons. If there are 29 700 vehicles in the city, how many of each type of vehicle are there?
- 5 When the kindergarten children arrived yesterday, $\frac{1}{4}$ of them were wearing skirts, $\frac{2}{5}$ were wearing shorts, $\frac{3}{10}$ were wearing dresses and the rest of the children were wearing jeans. What percentage of the children were wearing jeans?
- 6 What is the weight of fat, in grams, in one 200 g container of yoghurt, if 3% of the yoghurt, by weight, is fat?
- 7 Find:

a 12.5% of 3000	b 3.5% of 6000	c $33\frac{1}{3}\%$ of 4200
d 5.6% of 9000	e 62.5% of 4200	f 12.5% of 320 000
g 5.5% of 200	h 2.4% of 1000	i 0.1% of 78
j 15% of 140	k 99.95% of 30 000	l 1.375% of 240
- 8 The label on a 750 mL bottle of soft drink states that it contains 5% pure lemon juice.
 - a** How many millilitres of pure lemon juice does it contain?
 - b** If pure lemon juice costs 50 cents per 100 mL, what is the value of the lemon juice in the bottle?
- 9 In a group of 500 people, 22% are left-handed. How many of the group are left-handed?
- 10 In a flock of 2650 sheep, 32% have black wool. How many black sheep are there?
- 11 A breakfast cereal contains $23\frac{1}{2}\%$ oats, by weight. What is the weight of oats in a package of cereal containing 1500 g of the cereal?

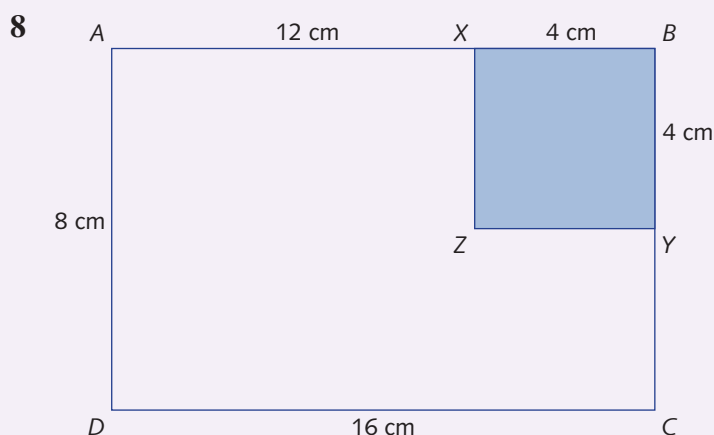


- 12 There are 16 540 high-school students in Sunview, and 60% of them attend state high schools. What is the number of state high-school students in Sunview?
- 13 It is known that watermelons are 92% water. A particular watermelon weighs 3600 g. How much does the water in this watermelon weigh?
- 14 A meat pie weighs 320 g. It is found that 73% of the pie is meat. What is the weight of the meat in the pie?
- 15 A packet of shortbread biscuits weighs 280 g. It is known that the biscuits are 12% butter, by weight. Find the weight of butter in one packet of biscuits.
- 16 In total, 14 carbon, hydrogen and nitrogen atoms make up one toxic molecule. If the number of carbon atoms is 5 more than the number of nitrogen atoms, and $42\frac{6}{7}\%$ of the atoms are carbon, find the number of each type of atom.
- 17 The price of a shirt including GST is \$110. What is the pre-GST price of the shirt?
- 18 A hotel charges a fee of 2% for a guest using a credit card. If the hotel bill is \$1250 before the fee is applied, how much is the fee?
- 19 A shop is offering a discount of 10% on all goods. If a garden table is priced at \$550, what is the discount?
- 20 The label on a 750 mL bottle of soft drink states that it contains 15% pure mango juice.
 - a How many millilitres of mango juice does it contain?
 - b If pure mango juice costs 25 cents per 100 mL, what is the value of the mango juice in the bottle?
- 21 In a clothing factory, 28% of the employees are female. There are 672 female employees. How many employees are there in total?
- 22 A sheep station has 1872 merino sheep. This is 36% of all the sheep on the station. How many sheep are there on the station?
- 23 A farm grows both barley and wheat. 30% of the barley is grown organically and 60% of the wheat is grown organically. 70% of the grain produced by the farm is wheat and the remainder is barley. What percentage of the farm's grain is organically grown?
- 24 On a test containing 30 true/false questions, David gave 50% more right answers than wrong ones. How many questions did he answer correctly?
- 25 \$52 000 was invested at 5% (simple interest) for a number of years and the interest received at the end of the term was \$15 600. What was the term of the investment?
- 26 A car salesyard is offering 15% off all new cars. If a car is priced at \$32 500, what is the discount being offered?

Challenge exercise



- 1 Fred Callum earns a salary of \$450 per week for a 40-hour week. His weekly salary is increased by 10% and his number of hours worked is decreased by 10%. Calculate his new hourly salary.
- 2 The length of a rectangle is increased by 15% and the width decreased by 10%. If the area of the original rectangle is 100 cm^2 , find the area of the new rectangle.
- 3 In a lottery, only 0.002% of tickets won prizes. If there were two prizes, how many tickets were sold?
- 4 An illness has struck the town of Abellane. On 1 January, 20% of the residents have the illness. On 1 February, 10% of the people who had the disease have recovered but 40% of those who did not have the disease now have it. What percentage of the population has the disease on 1 February?
- 5 In an election in a small island nation of 10000 people, 80% of the population voted. Of those who voted, 63% voted for the New Liberation Party. How many people did not vote for the New Liberation Party?
- 6 A dairy farmer found that the milk supplied by his cows was 5% cream and 95% skimmed milk. He wanted to know how much skimmed milk he should add to a litre of milk to reduce the percentage of cream to 4%. (*Recall:* 1 L = 1000 mL). What is the answer?
- 7 A girl makes up a cordial drink so that 10% of the drink is cordial concentrate. The rest is water. The girl has 350 mL of the drink in a jug. She now decides that she wants the cordial concentrate content to be only 8%. How much extra water does she need to put in the jug?



- a i Find the area of the shaded region XYZ .
ii Find the area of the unshaded region $AXZYCD$.
- b Find 20% of the area of rectangle $ABCD$.
- c Find 80% of the shaded area + 20% of the area of $AXZYCD$.
- d Express the answer to part c as a percentage of the area of rectangle $ABCD$.



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CHAPTER

8

Measurement and Geometry

Pythagoras' theorem

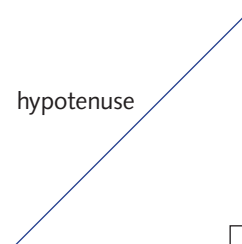
Triangles have been a source of fascination for people over many centuries. You will learn some of the intriguing facts about triangles in this and subsequent chapters.

Pythagoras' theorem is an amazing result that relates the lengths of the sides of a right-angled triangle. It is one of the most commonly used results in all of mathematics. In this chapter, we will learn about why it is so useful and important, and how to prove it.

Introduction

Among the set of all triangles, there is a special class, known as **right-angled triangles** or **right triangles**. A right-angled triangle has one and only one right angle, and the other two angles are complementary acute angles.

The longest side in a right triangle is called the **hypotenuse**. This name is derived from the Greek word 'teino', which is related to the word 'tension' in English, and has the basic meaning of *to stretch*.



It was well known early in the ancient world that if the two sides adjacent to the right angle have lengths 3 units and 4 units, then the hypotenuse has a length of 5 units.

A number of questions arise:

- Is there anything special about the numbers 3, 4 and 5?
- Are there other sets of three numbers that form the side lengths of a right-angled triangle?
- Is there some relationship between the lengths of the sides of a right-angled triangle?

These were questions that the Greeks examined in some detail. They found and proved a remarkable statement that relates the lengths of the three sides of a right-angled triangle. This work was done by the Pythagorean school of Greek thinkers, and so is known today as **Pythagoras' theorem**.

A **theorem** is a statement of an important mathematical truth.

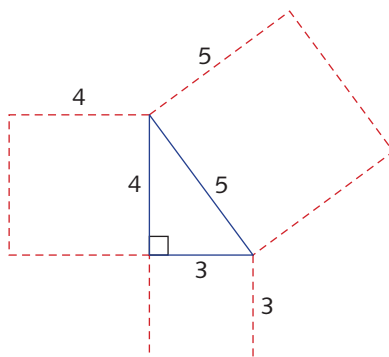
Exercise

Draw a line AB of length 12 cm, leaving yourself some space above the line you have drawn. Using a set square, draw a line AC at right angles to AB , of length 5 cm. Now join C and B to form a right-angled triangle and measure the length of the hypotenuse. You should find that the answer is 13 cm.

Exercise

Repeat the above construction, this time with $AB = 8$ cm and $AC = 6$ cm. Measuring the third side, you should find it is 10 cm. This is because this triangle is an enlargement of the 3, 4, 5 triangle.

To understand what Pythagoras and the Greeks proved, we need to look at the *squares* of the lengths of the sides of a right-angled triangle.



Lengths	3	4	5	Lengths	5	12	13	Lengths	6	8	10
Squares	9	16	25	Squares	25	144	169	Squares	36	64	100

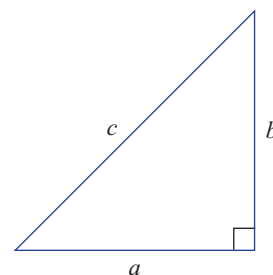
Can you see the pattern?

- In the 3, 4, 5 triangle, $9 + 16 = 25$, that is, $3^2 + 4^2 = 5^2$.
- In the 5, 12, 13 triangle, $25 + 144 = 169$, that is, $5^2 + 12^2 = 13^2$.
- And in the 6, 8, 10 triangle, $36 + 64 = 100$, that is, $6^2 + 8^2 = 10^2$.



Pythagoras' theorem

- The square of the length of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the lengths of the other two sides.
- In symbols, if we call the length of the hypotenuse c , and call the lengths of the other two sides a and b , then $a^2 + b^2 = c^2$.

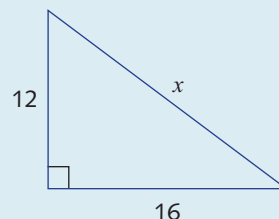




We can use Pythagoras' theorem to find the length of the hypotenuse of a right-angled triangle, without measuring, if we know the two shorter sides.

Example 1

Find the length of the hypotenuse in the right-angled triangle opposite.



Solution

Let x be the length of the hypotenuse.

Pythagoras' theorem says that

$$\begin{aligned} x^2 &= 12^2 + 16^2 \\ &= 144 + 256 \\ &= 400, \end{aligned}$$

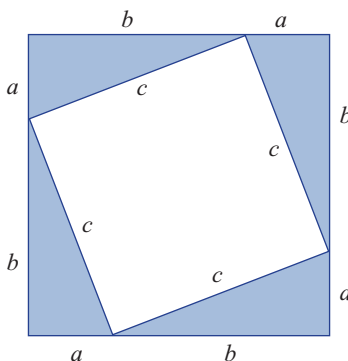
so $x = 20$.

Remember that 400 is the square of the length of the hypotenuse, so we have to take the square root in the final step to find the correct value of x .

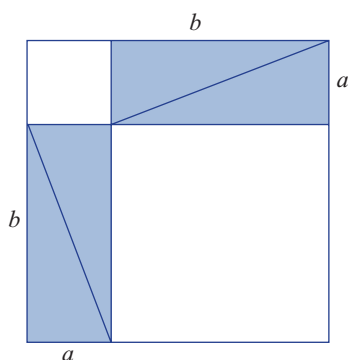
Why the theorem is true

To establish a theorem, we must give a valid proof. There are many ways to prove Pythagoras' theorem, but some are easier than others. Here is a simple geometric demonstration of why the result is true.

The diagram below shows four identical right-angled triangles, each with hypotenuse length c and the other sides of length a and b . They have been arranged within a large square of side length $a + b$, leaving uncovered the white square of side length c .



The triangles and the square, with side length c , can be rearranged within the same large square. The square at the top left has area a^2 and the square at the bottom right has area b^2 .

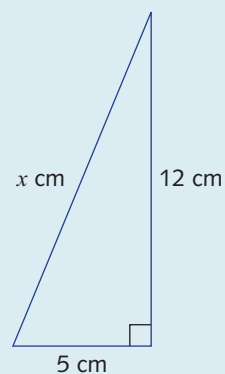


The area of the white square in the first diagram must be equal to the sum of the areas of the two white squares in the second diagram.

From this we can see that $a^2 + b^2 = c^2$.

Example 2

Use Pythagoras' theorem to find the length of the hypotenuse in the triangle opposite.



Solution

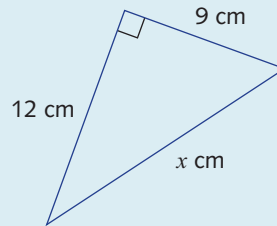
Pythagoras' theorem says that

$$\begin{aligned} x^2 &= 5^2 + 12^2 \\ &= 25 + 144 \\ &= 169, \end{aligned}$$

so $x = 13$.

Example 3

Use Pythagoras' theorem to find the value of x in the triangle opposite.



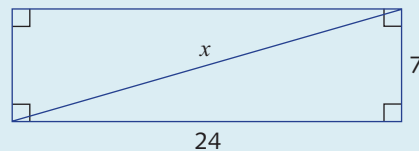
Solution

Pythagoras' theorem says that

$$\begin{aligned}x^2 &= 9^2 + 12^2 \\&= 81 + 144 \\&= 225, \\ \text{so } x &= 15.\end{aligned}$$

Example 4

Find the length of the diagonal in the rectangle opposite.



Solution

The diagonal divides the rectangle into two right-angled triangles. Pythagoras' theorem tells us that

$$\begin{aligned}x^2 &= 7^2 + 24^2 \\&= 49 + 576 \\&= 625, \\ \text{so } x &= 25.\end{aligned}$$



Exercise 8A

1 Write down these squares.

a 9^2

b 11^2

c 7^2

d 15^2

2 Find the square root of each of these numbers.

a 64

b 144

c 196

d 900

3 Calculate:

a $7^2 + 24^2$

b $9^2 + 12^2$

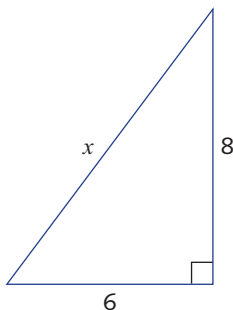
c $20^2 - 16^2$

d $17^2 - 15^2$

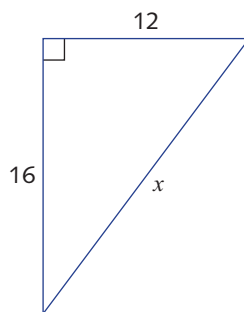
Example 1

4 Use Pythagoras' theorem to find the length of the missing side in each of these triangles.

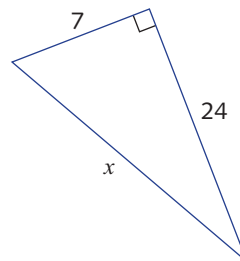
a



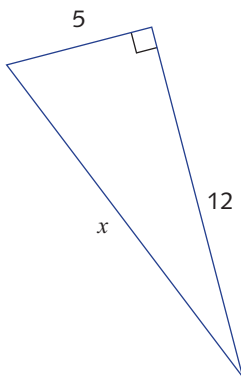
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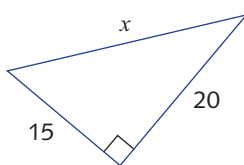
c



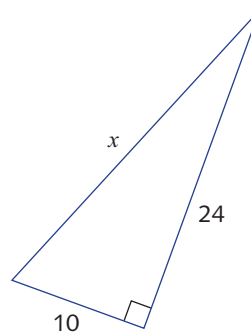
d



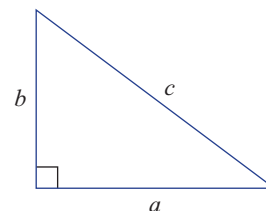
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f

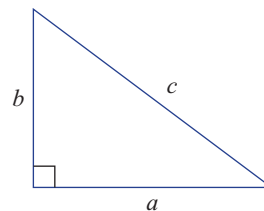


5 The triangle opposite is a right-angled triangle with hypotenuse of length c . The other two sides have lengths a and b . Apply Pythagoras' theorem to complete the table below. The first row is done for you. What pattern do you notice?



Row	a	b	a^2	b^2	$c^2 = a^2 + b^2$	c
1	3	4	9	16	25	5
2	6	8				
3	9	12				
4	12	16				
5	15	20				

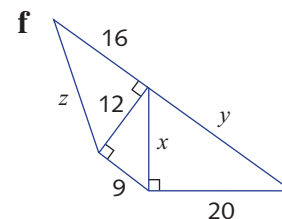
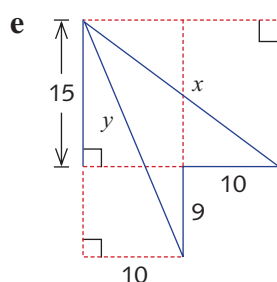
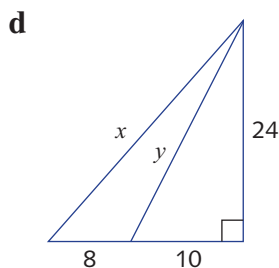
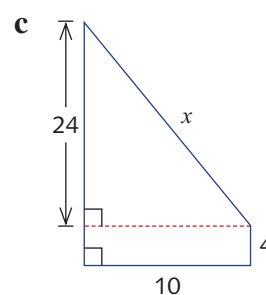
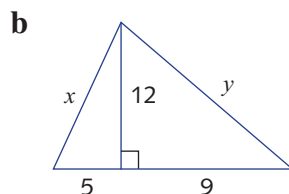
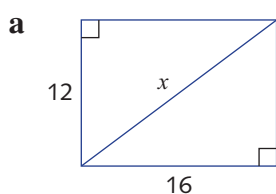
- 6 The triangle opposite is a right-angled triangle with hypotenuse of length c . The other two sides have lengths a and b . Apply Pythagoras' theorem and complete the table below. What pattern do you notice?



Row	a	b	a^2	b^2	$c^2 = a^2 + b^2$	c
1	3	4				
2	5	12				
3	7	24				
4	9	40				
5	11	60				

Example 4

- 7 Find the values of x , y and z in these diagrams.

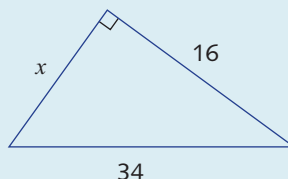




We can use Pythagoras' theorem to find the length of one of the shorter sides in a right-angled triangle, given the other two sides.

Example 5

Find the length of the missing side, marked in the diagram as x .



Solution

Applying Pythagoras' theorem,

$$x^2 + 16^2 = 34^2$$

$$x^2 + 256 = 1156$$

$$x^2 = 1156 - 256$$

$$= 900,$$

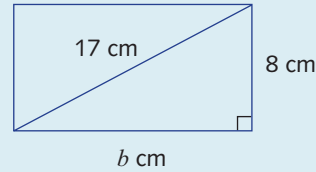
so $x = 30$ (taking the square root).

To make such calculations easier, here is a table of squares from 13 to 40. You should know the squares from 1 to 12 already.

Number	Square	Number	Square
13	169	27	729
14	196	28	784
15	225	29	841
16	256	30	900
17	289	31	961
18	324	32	1024
19	361	33	1089
20	400	34	1156
21	441	35	1225
22	484	36	1296
23	529	37	1369
24	576	38	1444
25	625	39	1521
26	676	40	1600

Example 6

A rectangle has length 8 cm and diagonal 17 cm.
What is its width?



Solution

Let b be the width, measured in cm. Then

$$17^2 = 8^2 + b^2 \quad (\text{Pythagoras' theorem})$$

$$289 = 64 + b^2$$

$$b^2 = 289 - 64$$

$$= 225,$$

$$\text{so } b = 15.$$

The width of the rectangle is 15 cm.

Converse of Pythagoras' theorem

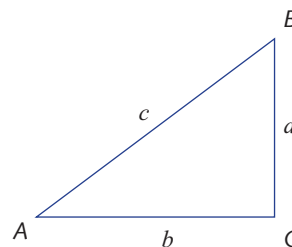
We have seen that in a right-angled triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.

Now suppose that we have a triangle in which the square of the length of the longest side equals the sum of the squares of the lengths of the other two sides. Does it follow that the triangle is right-angled? The answer is yes, and this result is called the **converse** of Pythagoras' theorem.

We will be able to prove this after we have studied congruence in Chapter 14 of *ICE-EM Mathematics Year 8 Book 2*.

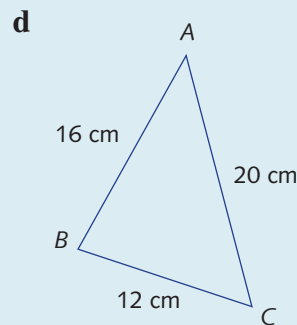
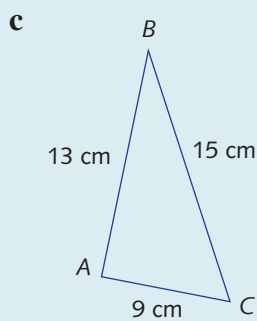
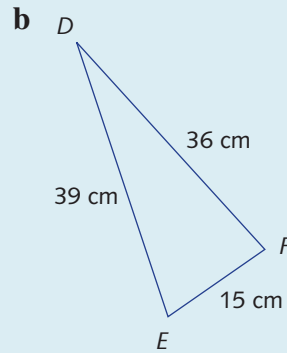
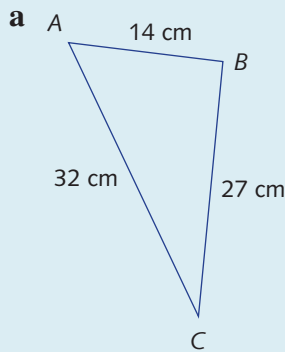
Theorem

If $c^2 = a^2 + b^2$, then $\angle ACB$ is a right angle.



Example 7

Which of the triangles below are right-angled triangles? Name the right angle in each case.



Solution

In each case, let a and b be the lengths of the two shorter sides of the triangle and let c be the length of the longest side.

If $a^2 + b^2 = c^2$, then the triangle is a right-angled triangle, with the right angle opposite the hypotenuse.

a $14^2 + 27^2 = 925$

$$32^2 = 1024$$

$$14^2 + 27^2 \neq 32^2$$

The triangle is not right-angled.

c $9^2 + 13^2 = 250$

$$15^2 = 225$$

$$9^2 + 13^2 \neq 15^2$$

Not a right-angled triangle

b $15^2 + 36^2 = 1521$

$$39^2 = 1521$$

$$15^2 + 36^2 = 39^2$$

Triangle DEF is right-angled, with $\angle DFE = 90^\circ$

d $12^2 + 16^2 = 400$

$$20^2 = 400$$

$$12^2 + 16^2 = 20^2$$

A right-angled triangle with $\angle ABC = 90^\circ$

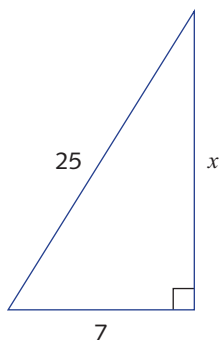


Exercise 8B

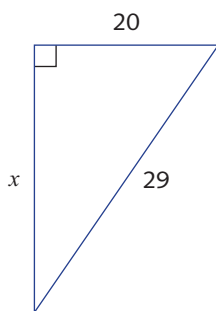
Example 5

- 1 Use Pythagoras' theorem to find the value of x in each triangle.

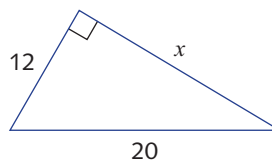
a



b



c



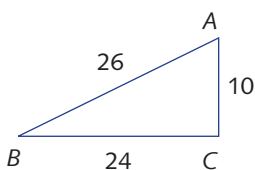
Example 6

- 2 A rectangle has one side of length 20 cm and its diagonal has length 25 cm. Find the length of the other side.

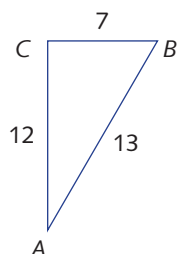
Example 7

- 3 Decide whether each of the triangles below is right-angled. If it is, name the right angle.

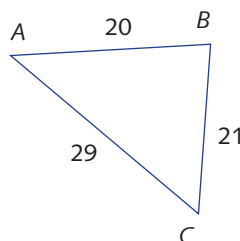
a



b

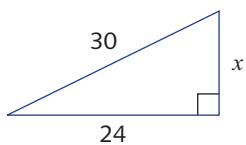


c

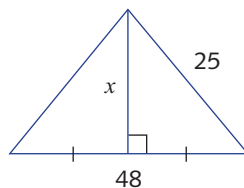


- 4 Find the values of the pronumerals in these diagrams.

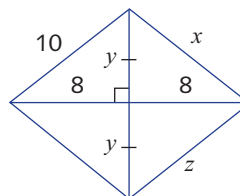
a



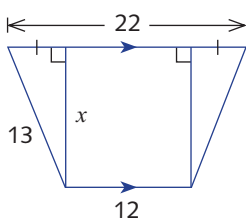
b



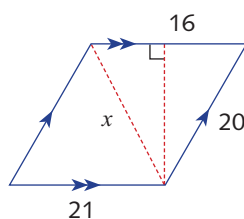
c



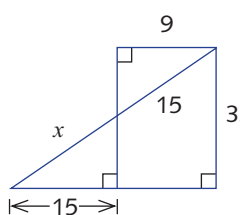
d



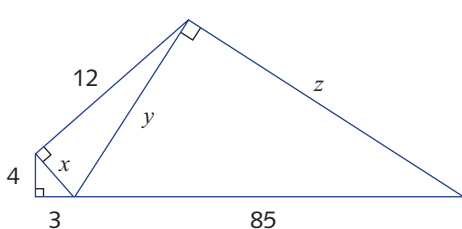
e



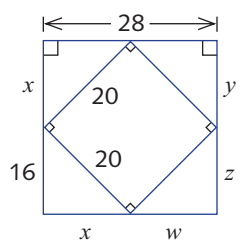
f



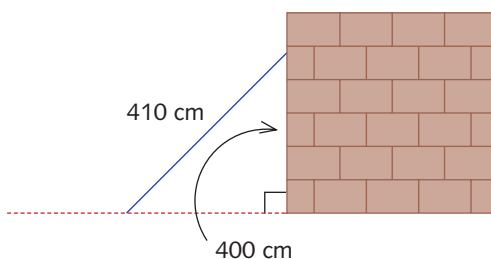
g



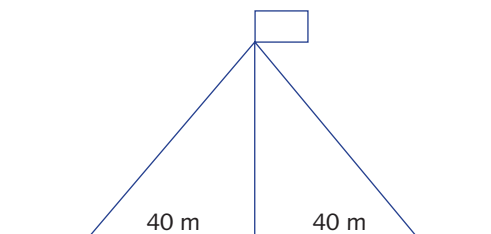
h



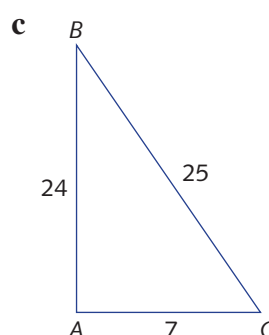
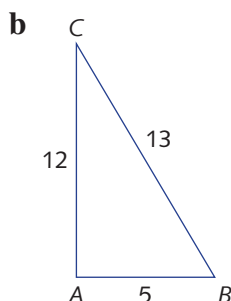
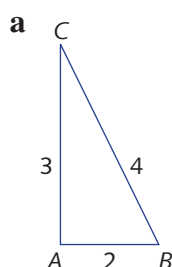
- 5 The sides of a rectangular table are 150 cm and 360 cm. What is the length of the diagonal?
- 6 A ladder of length 410 cm is leaning against a wall. It touches the wall 400 cm above the ground. What is the distance between the foot of the ladder and the wall?



- 7 A flagpole is supported by two cables of equal length as shown below. The total length of the cables is 100 m. Each cable is attached to a point on level ground 40 m away from the foot of the flagpole. The dimensions of the flag are $1.2 \text{ m} \times 2.4 \text{ m}$. Calculate the height to the top of the flag.



- 8 Which of the triangles below are right-angled triangles?



- 9 A ship sails 32 kilometres north and then 24 kilometres east. How far is it from its starting point?
- 10 **a** Find the square of the length of the diagonal of a square of side 1 cm.
- b** Find the square of the length of the diagonal of a square of side length:
- i** 2 cm **ii** 3 cm **iii** 5 cm **iv** n cm



You may have noticed that in the final step in each example in Section 8B we arrived at a perfect square, such as 25, 169 or 625. We were then able to take the square root and arrive at a whole-number answer. In many cases this will not happen.

The square root of a number that is not a perfect square cannot be written as a fraction, and is an example of what is called an **irrational number**.

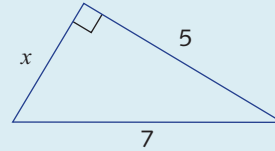
The square root of 2 is an example of an irrational number. Since $1^2 = 1$ and $2^2 = 4$, the square root of 2, which is written as $\sqrt{2}$, must be somewhere between 1 and 2. Although we cannot express the square root of 2 in exact decimal form, we can find an **approximation** to it. In fact, $\sqrt{2} \approx 1.414$, correct to 3 decimal places, and $\sqrt{2} \approx 1.41$ correct to 2 decimal places.

When we use Pythagoras' theorem to solve a problem, we often need to find the square root of a number that is not a perfect square. We can do this approximately by using a calculator, or by looking up a table. Here is a list of square roots of the numbers from 2 to 29. When the square root is not a whole number, we have given an approximation correct to 2 decimal places.

Number	Square root (2 dec. pl.)	Number	Square root (2 dec. pl.)
2	1.41	16	4
3	1.73	17	4.12
4	2	18	4.24
5	2.24	19	4.36
6	2.45	20	4.47
7	2.65	21	4.58
8	2.83	22	4.69
9	3	23	4.80
10	3.16	24	4.90
11	3.32	25	5
12	3.46	26	5.10
13	3.61	27	5.20
14	3.74	28	5.29
15	3.87	29	5.39

Example 8

Find the length, correct to 2 decimal places, of the missing side in the right-angled triangle opposite.



Solution

By Pythagoras' theorem,

$$x^2 + 5^2 = 7^2$$

$$x^2 + 25 = 49$$

$$x^2 = 49 - 25$$

$$= 24$$

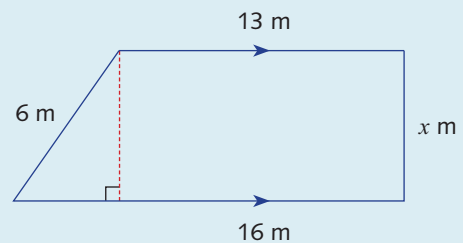
$$x = \sqrt{24}$$

$$\approx 4.90$$

(using the table)

Example 9

Find the height of the trapezium opposite.



Solution

By Pythagoras' theorem,

$$x^2 + (16 - 13)^2 = 6^2$$

$$x^2 + 9 = 36$$

$$x^2 = 36 - 9$$

$$= 27$$

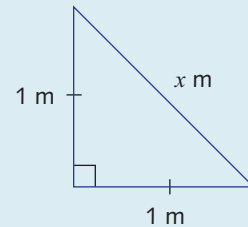
$$x = \sqrt{27}$$

$$\approx 5.20$$

(using the table)

Example 10

Find the length of the hypotenuse in the triangle opposite, correct to 2 decimal places. What type of triangle is it?



Solution

By Pythagoras' theorem,

$$1^2 + 1^2 = x^2$$

$$x^2 = 2,$$

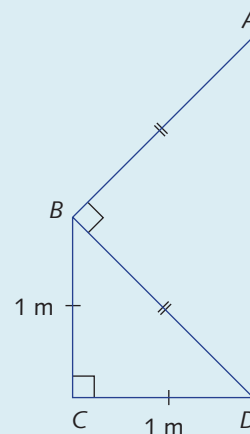
$$\text{so } x = \sqrt{2}$$

$$\approx 1.41 \text{ (to 2 decimal places).}$$

This triangle is an isosceles right-angled triangle. The length of the hypotenuse is 1.41 m, correct to 2 decimal places.

Example 11

Find the length of AD in the diagram opposite.



Solution

By Pythagoras' theorem,

$$1^2 + 1^2 = BD^2,$$

$$\text{so } BD^2 = 2.$$

(continued on next page)

Also, $BD^2 + AB^2 = AD^2$,
 so $2 \times BD^2 = AD^2$ (isosceles $\triangle ABD$).
 Hence $2 \times 2 = AD^2$
 $AD^2 = 4$,
 so $AD = 2$.
 The length of AD is 2 m.

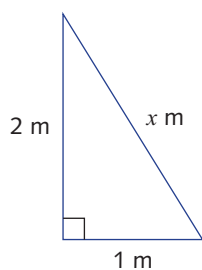
Exercise 8C

Note: In this exercise, you should use the table on page 247 to work out any square roots you need.

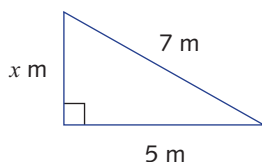
Example 8,10

- 1 Use Pythagoras' theorem to find the length of the unknown side in each diagram below. Round your answers correct to 2 decimal places.

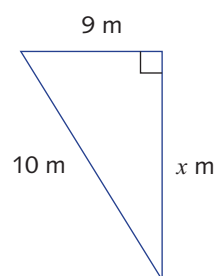
a



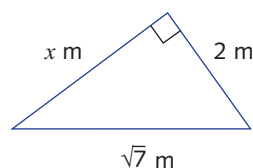
b



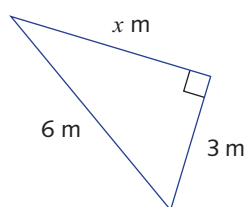
c



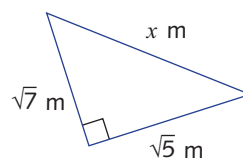
d



e

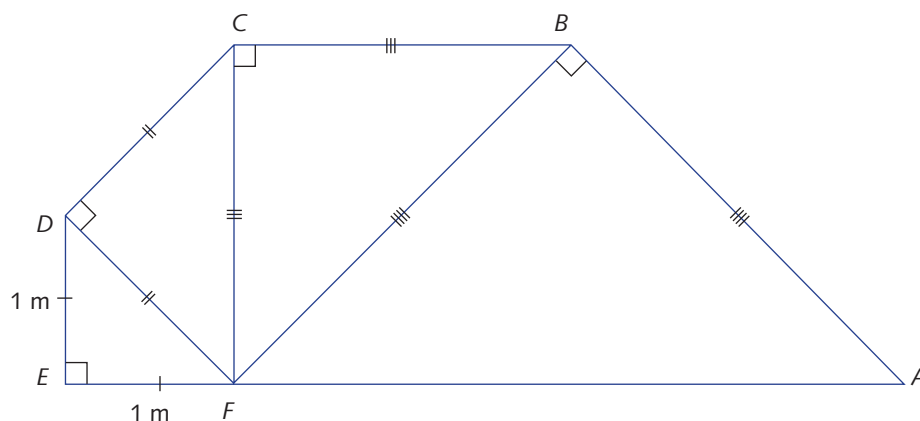


f



Example 11

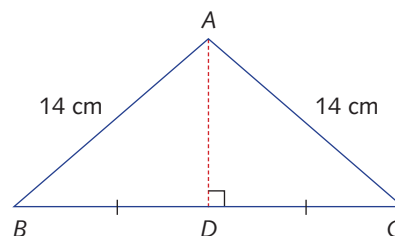
- 2 Find the length of AF in the diagram below. Only take the square root in the final step.



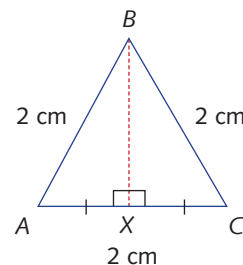
- 3 Find the length of the diagonal of a rectangle with side lengths 2 cm and 5 cm. Round off your answer correct to 2 decimal places.

- 4 Find the perimeter of a square with diagonal length 6 cm. Round your answer to 2 decimal places.

- 5 An isosceles triangle ABC has equal sides $AB = AC = 14$ cm. The side BC is 26 cm. A line AD is drawn at right angles from A to BC . This divides BC into two intervals of equal length. Find the length of AD , correct to 2 decimal places.



- 6 Triangle ABC is equilateral, with each side of length 2 cm. Find the length of BX .

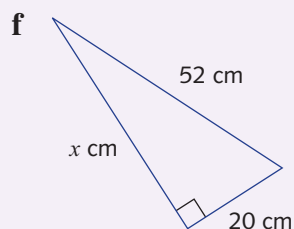
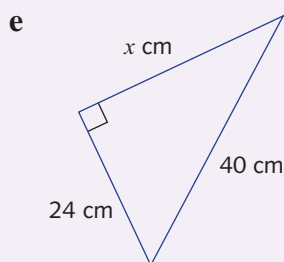
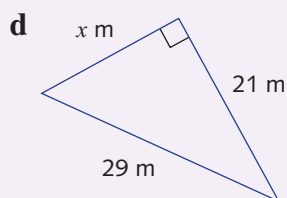
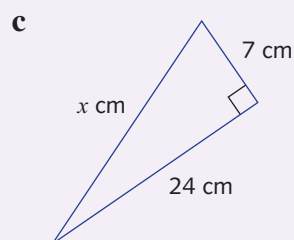
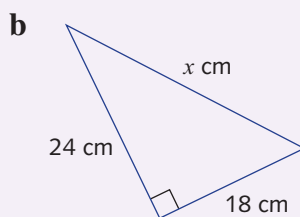
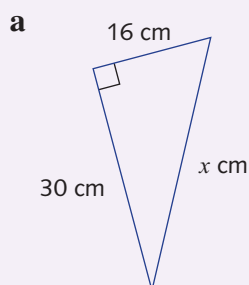


- 7 The equal sides of a right-angled isosceles triangle are 3 cm in length. What is the length of the hypotenuse?
- 8 A cross-country runner runs 3 km west, then 2 km south and then 8 km east. How far is she from her starting point? Calculate your answer in kilometres, correct to 2 decimal places.
- 9 A rod 2.5 m long is leaning against a wall. The bottom of the rod is 1.5 m from the wall. How far up the wall does the rod reach?
- 10 A man starts from a point A and walks 2 km due north to a point B and then 5 km due west to a point C. How far is C from A? Calculate your answer in kilometres, correct to 2 decimal places.
- 11 Amelia starts from a point A and walks 6 km due south to B. Amelia then walks due east to a point C which is 7 km from A. How far is C from B? Calculate your answer in kilometres, correct to 2 decimal places.
- 12 A right-angled isosceles triangle has a hypotenuse of length 6 cm. Find the lengths of the other two sides in centimetres, correct to 2 decimal places.

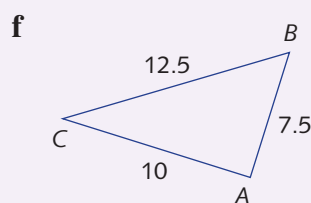
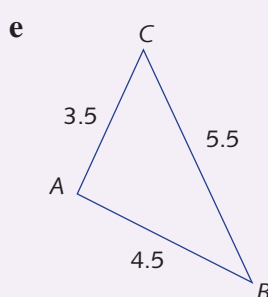
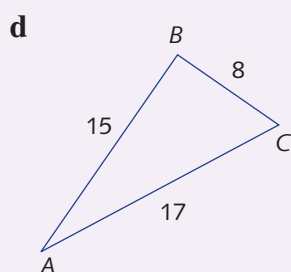
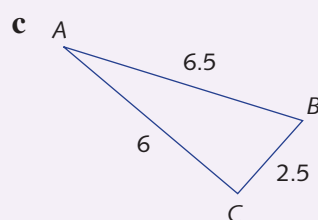
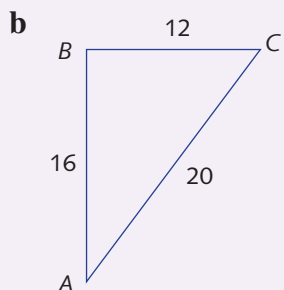
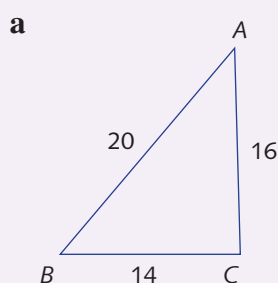


Review exercise

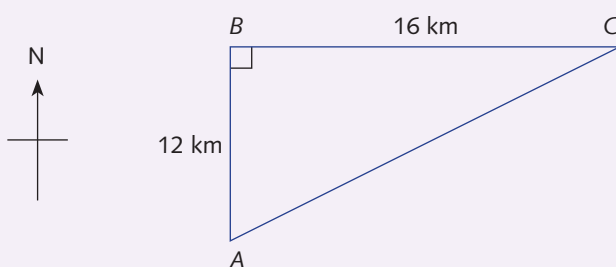
1 Find the value of x in each of these diagrams.



2 Decide whether each of the triangles below is right-angled. If it is, name the right angle.

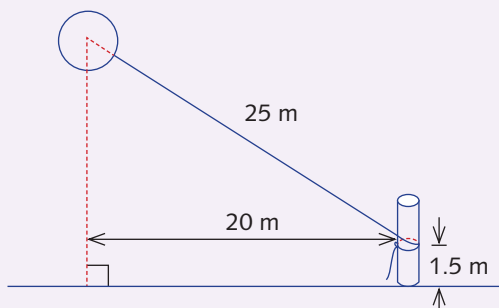


3 John starts at a point A and cycles 12 km due north. He turns east at point B , cycles 16 km and reaches point C . How far is he from his starting point?

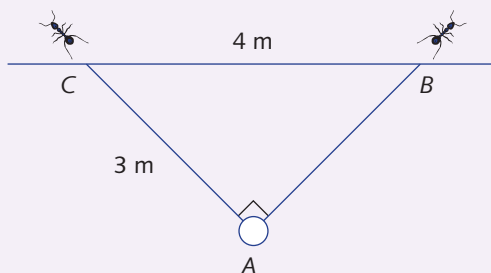




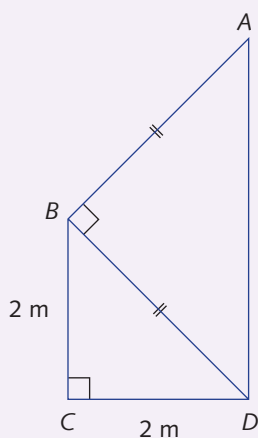
- 4 How high is the balloon off the ground if it is tied to a wooden pole at a height of 1.5 m by a cord of length 25 m?



- 5 Two ants take different paths, CA and BA respectively, while walking back to their nest at point A . How much further does one ant walk than the other? Round your answer to 2 decimal places.



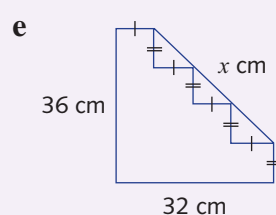
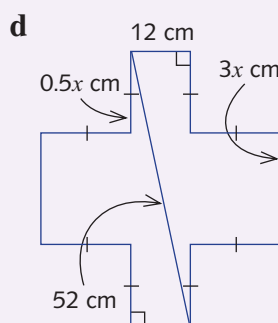
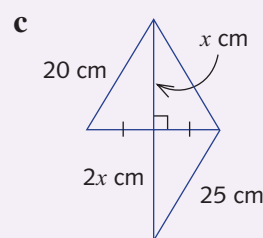
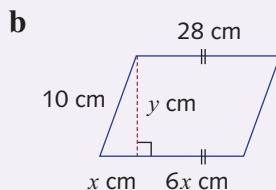
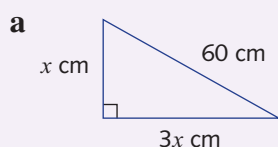
- 6 a A square has side length 3 cm. Find the length of the diagonal correct to 2 decimal places.
b The diagonal length of a square is 4 cm. Find the side length of the square.
- 7 Find the length of AD in the diagram below.



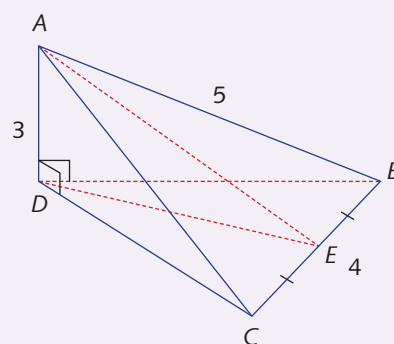


Challenge exercise

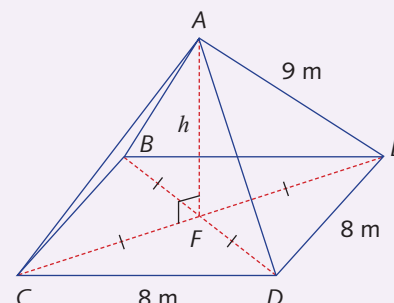
- 1 Show that the diagonal of a square of side length a is $a\sqrt{2}$.
- 2 What is the length of the longest diagonal in a cube with edges of length x m?
- 3 What is the area of an equilateral triangle with sides of length x m?
Hint: The height also divides the base into two equal parts.
- 4 Find the exact values of x and y in the following diagrams.



- 5 A right-angled triangle has hypotenuse of length 41 cm and one of the sides is 9 cm. Find the area of the triangle.
- 6 Draw an equilateral triangle of side length 4 cm. Find, correct to 2 decimal places, the height of this triangle and its area.
- 7 In the diagram to the right, $\triangle ABC$ and $\triangle DBC$ are isosceles triangles, with $AB = AC$ and $DB = DC$.
 AE and DE are the perpendicular bisectors of $\triangle ABC$ and $\triangle DBC$ respectively, so $\angle AEB = \angle DEB = 90^\circ$. Find AE , DE and DB , correct to 2 decimal places.



- 8 A pyramid has a square base $BCDE$ of side length 8 m. The sloping edges of the pyramid are of length 9 m. Find the height h .





CHAPTER

9

Review and problem-solving

Chapter 1

- 1 The population of an ant colony was 14 000 at the start of January. By the end of the month, 256 ants had died, 89 ants had moved into the colony and 123 ants had moved out. The population at the end of January was 14 298. How many ants were born between the start and the end of the month?
- 2 If each of 37 students in 24 classes ate 5 pies in one week, how many pies were eaten in total?
- 3 The cost of 20 packets of chips is \$9.50, while each packet costs 65 cents when sold individually. If you want to buy 40 packets of chips, which option is cheaper? How much do you save by taking the cheaper option?
- 4 What is the smallest number that must be subtracted from 34 657 so that the result may be exactly divided by 129?
- 5 The quotient arising from the division of a number by 53 is 29 and the remainder is 23. What is this number?
- 6 The product of two numbers is 67 267, and the smaller of them is 137. Find the sum of the two numbers.
- 7 Andrew and Bashir play marbles. At one stage of the game, Andrew has 347 marbles and Bashir has 135. Bashir wins 106 marbles from Andrew. How many marbles does each boy have now?
- 8 What number multiplied by 23 will give the same product as 391 multiplied by 37?
- 9 How many times must 43 be added to 1649 to give 4186?
- 10 At an election, there are three candidates, A, B and C. A obtains 19 878 more votes than C, and B obtains 12 435 more votes than C. A and B together obtain 137 187 votes. How many voted for C?

Chapter 2

- 1 Simplify:

a $\frac{125}{35}$

b $\frac{29}{145}$

c $\frac{289}{136}$

d $\frac{8}{36}$

e $1\frac{15}{9}$

f $\frac{6}{160}$

g $\frac{88}{99}$

h $\frac{96}{108}$

- 2 Evaluate the following additions and subtractions. Write each answer as a mixed numeral, with the fractional part in simplest form.

a $4\frac{1}{2} + 3\frac{2}{5}$

b $7\frac{2}{3} - 5\frac{1}{4}$

c $6\frac{5}{8} + 2\frac{5}{6}$

d $9\frac{3}{10} - \frac{3}{4}$

e $5\frac{7}{12} + 3\frac{4}{9}$

f $8\frac{2}{9} - 6\frac{2}{3}$

3 Evaluate:

a $120 \times \frac{75}{100}$ **b** $120 \div \frac{75}{100}$ **c** $5\frac{1}{4} \times \frac{2}{3}$ **d** $5\frac{1}{4} \div \frac{2}{3}$

e $2\frac{7}{9} \times 7\frac{1}{2}$ **f** $2\frac{7}{9} \div 7\frac{1}{2}$ **g** $8 \div 1\frac{5}{7}$ **h** $8 \times 1\frac{5}{7}$

- 4** A rectangular garden uses a straight river bank as its longer side. The other sides are fenced. If the longer side is 25 m, and the shorter side is $\frac{1}{3}$ of the longer side, what is the total length of fencing material?
- 5** If a year (365 days) is divided into lunar months of 28 days, how many months will there be in a year? Express your answer as a mixed numeral.
- 6** There were 60 people at a party. If $\frac{1}{5}$ of the people wore red, $\frac{1}{4}$ wore blue and the rest wore white, how many people wore white?
- 7** Horatia has saved \$1500. If she spends $\frac{2}{5}$ of the money on rent and shopping, and $\frac{2}{9}$ of the remaining money on petrol, how much does she have left?
- 8** Order each set of fractions from smallest to largest.
- a** $\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{2}, \frac{4}{5}$ **b** $\frac{11}{25}, \frac{4}{5}, \frac{13}{20}, \frac{1}{4}, \frac{6}{10}$
- 9** A balloon shop had 120 balloons in the morning, 24 of which were filled with helium.
- a** What fraction of the balloons were filled with helium?
- b** During the day, $\frac{1}{8}$ of the helium balloons were sold and 12 of the ordinary balloons were accidentally popped. What fraction of the balloons left at the end of the day were filled with helium?
- 10** If Alice can swim 100 m backstroke in 1 minute 20 seconds and she can swim butterfly at $\frac{9}{8}$ times her backstroke speed, and freestyle at $\frac{10}{9}$ times her butterfly speed, how fast can she swim 100 m freestyle?
- 11** Calculate:
- a** 1.2×3.8 **b** $1.3 + 2.5 \times 0.6$ **c** $7.1 + 4.5 \div 1.5$
- d** $13.6 - 7.85$ **e** $12.5 \times 3.1 - 0.8742$ **f** $92.8 \times 87.1 + 0.51$

Chapter 3

1 Find all the factors of:

a 12 **b** 17 **c** 270 **d** 135 **e** 58 **f** 144

2 List all the prime numbers between 100 and 160.

- 3** Find the lowest common multiple (LCM) of:
- a** 5 and 35 **b** 18 and 48 **c** 170 and 66 **d** 12, 25 and 75
- 4** Find the highest common factor (HCF) of:
- a** 17 and 25 **b** 96 and 39 **c** 150 and 24 **d** 380, 190 and 650
- 5** Find the result of multiplying the HCF and the LCM of each pair of numbers below. Compare the result in each case to the product of the original pair. What do you notice?
- a** 24 and 76 **b** 102 and 54
- 6** Write in the box the number needed to make each statement true.
- a** $2^8 \div 2^5 = 2^{\square}$ **b** $5^3 \times 5^2 = 5^{\square}$ **c** $4^3 \times 4^2 = 2^{\square}$
- d** $6^7 \div 3^7 = \square^7$ **e** $(3^2)^2 = \square$ **f** $\square^5 \div 6^5 = 1$
- g** $3^3 \times 3^{\square} = 81$ **h** $(5^{\square})^2 = 5^{16}$ **i** $(7^{13} \div 7^8)^{\square} = 7^{15}$
- 7** Evaluate:
- a** 8^2 **b** $\sqrt{121}$ **c** $\sqrt{56^2}$ **d** $\sqrt{5^4}$
- e** $(8^2)^2$ **f** $\sqrt{2209}$ **g** 25^3
- 8** Copy and fill in the missing digits so that the numbers in each set are:
- a** divisible by 3: 2567__, 199__4, __56
- b** divisible by 6: 7365__, 1__08, 222__
- c** divisible by 3, 4 and 5: 10__0, 12198__, 59__20
- 9** Fill in the spaces so that the number __4__20 is divisible by 11.
- 10** Find the prime factorisations of:
- a** 210 **b** 5040 **c** 436 **d** 1386
- 11** Evaluate:
- a** $2^2 \times 7^3$ **b** 11×13^2 **c** $2^3 \times 7 \times 9 \times 15$
- d** Write the product in part **c** as a product of powers of primes.
- 12** Using prime factorisation, find the HCF and LCM of:
- a** 12960 and 3600 **b** 1750 and 1176
- 13** Calculate mentally:
- a** 25×16 **b** 125×32 **c** 75×40 **d** 5×874

Chapter 4

1 Calculate:

a $-15 + 23$

b $25 - (-61)$

c $37 - 90 + 23$

d $23 - 28 - 6$

e $-213 - (-1129) + 81$

f $9 - (16 + (-28))$

2 Evaluate:

a $3 \times (-5 + 9) - 6$

b $-3 \times -9 - 12$

c $-21 \div 3 + 9 \times -2$

d $-12 \times (-4) - (-8 - 2) \times 5$

e $\frac{-95 - 6 \times 8}{8 - (-2)}$

f $\frac{28 \times 8}{5} - \frac{8 \times (-12)}{6}$

g $-\frac{6}{13} + \frac{5}{8}$

h $\frac{7}{12} \times \left(-\frac{9}{7}\right) + \frac{5}{3}$

3 The maximum temperature for Thredbo today was -8°C , while Mount Kosciusko had a top temperature of -24.3°C . How much warmer than Mount Kosciusko was Thredbo today?

4 Plot these points on the Cartesian plane.

$A(-4, 2)$

$B(3, -10)$

$C(9, 8)$

$D(-7, 0)$

5 Points A , B and C have coordinates $(1, 0)$, $(3, 2)$ and $(5, 0)$ respectively. Plot the points and join them with straight lines. What is the area of the triangle ABC ?

6 Arrange these numbers in order from smallest to largest.

$-\frac{9}{8}, \frac{12}{8}, 0.2, -0.02, -\frac{7}{9}, \frac{1}{0.4}$

7 Calculate:

a $2\frac{3}{4} - 6\frac{2}{5}$

b $12.8 - (-10.6) \times \frac{2}{3}$

c $12 \times \frac{8}{9} \times \left(-\frac{7}{6}\right)$

8 Sam started the week with a debt of \$360.50. He worked hard and earned \$3100, but had \$250 stolen. He won \$80.20 in the lottery and had to pay an electricity bill of \$210. After paying his debt and his electricity bill, how much did Sam have left?

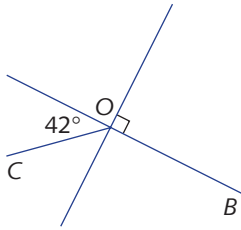
9 Mauna Kea is Hawaii's tallest volcano and the world's highest mountain when measured from its base on the sea floor. The height above sea level is 2405 metres. The base of Mauna Kea on the sea floor is 7286 metres below sea level. What is the height of Mauna Kea from sea floor to summit?

10 In a maths competition, Harry scored 20. Each correct answer was worth 10 marks. For each incorrect answer, 2.5 marks were deducted. If Harry answered 5 questions correctly, how many incorrect answers did he have?

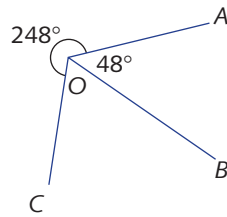
Chapter 5

1 Find the value of $\angle BOC$ in each diagram below.

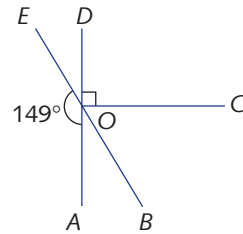
a



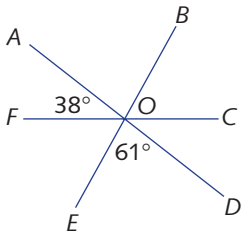
b



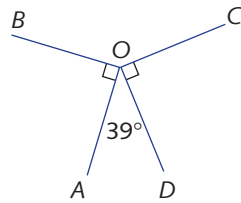
c



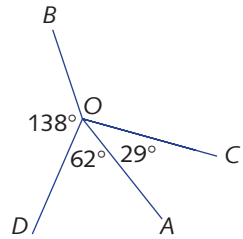
d



e

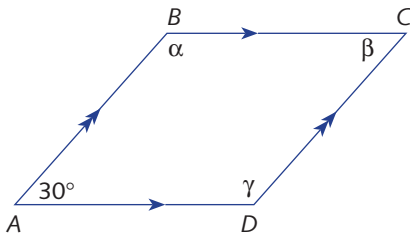


f

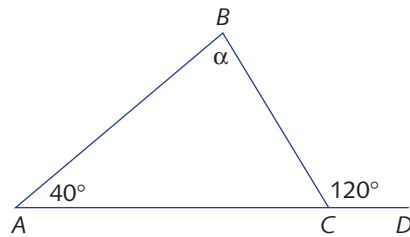


2 Find the values of α , β and γ in these diagrams.

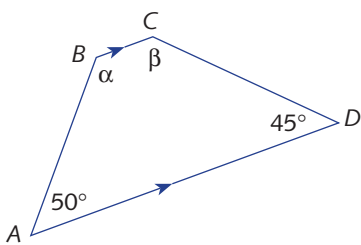
a



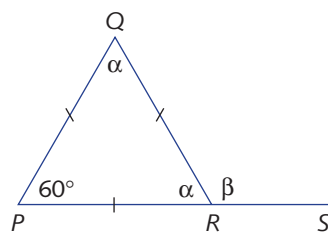
b



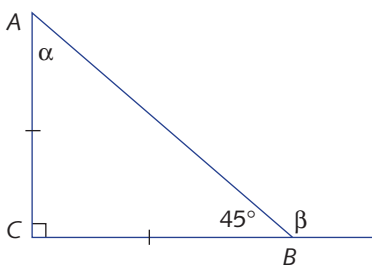
c



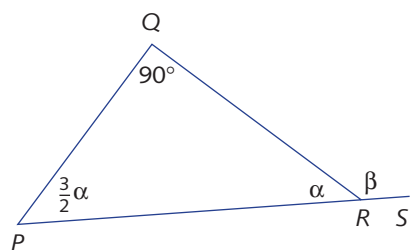
d



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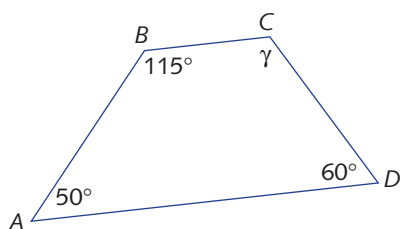


f

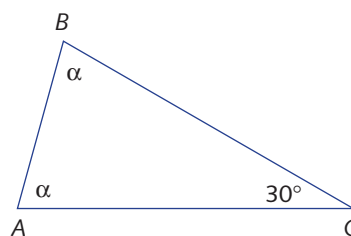


- 3 For each diagram below, write down an equation involving the pronumerals and then solve the equation.

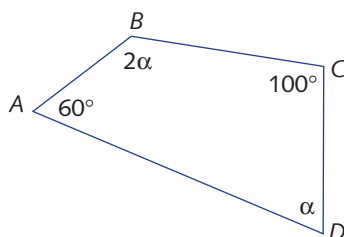
a



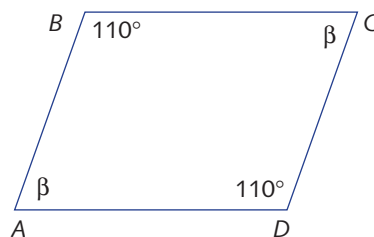
b



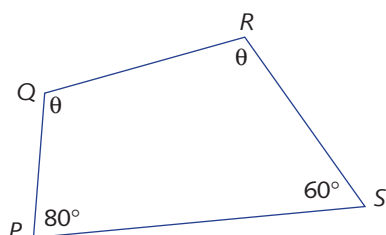
c



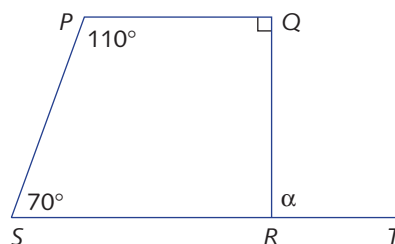
d



e



f



Chapter 6

- 1 Evaluate each expression for $a = 2$, $b = 1$ and $c = 3$.

a $a + b + c$

b abc

c $a + b - c$

d $\frac{a + b}{c}$

e $a^2 + b^2$

f $c^3 - a^3$

g $c - a - b$

h $\frac{a^2 + b^2}{a^2 - b^2}$

- 2 Substitute $x = -1$ for each expression and evaluate it.

a x^3

b x^5

c $(2x)^2$

d $(2x)^3$

e $-x$

f $x^3 + 5$

g $x^2 + 5$

h $\frac{2 - x}{x^2 + 2}$

- 3 Solve:

a $m + 3 = 8$

b $m - 3 = 11$

c $2m = 8$

d $\frac{m}{4} = 6$

e $2x - 4 = 11$

f $\frac{x}{4} - 8 = 7$

g $5 - x = 11$

h $2x - 4 = -6$

i $\frac{x}{5} + 8 = -7$

4 Expand brackets.

a $2(x - 3)$

b $-3(x + 4)$

c $5(x - 2)$

d $a(x + 3)$

e $-3(x - 4)$

f $-4(3 - x)$

5 Collect like terms to simplify each of these expressions.

a $2x - 3y + 5x + 2y$

b $x - y + 2x + 4y$

c $5x - 7x + 6x$

d $3x - 11x + 8x$

e $2xy - 8xy + 6xy$

f $6x^2y - 2yx^2 + x^2y$

6 Evaluate each expression for $a = -3$, $b = \frac{2}{5}$ and $c = 10$.

a $a + b - c$

b abc

c $b^2(c + a)$

d $(c^2 + a^2) - 2b$

e $\frac{ca^2}{30}$

f $\frac{a^2b}{c - b}$

g $a^2 - b^2 - c$

h $\frac{a}{b}(a - c)$

7 Solve each equation.

a $2x + 32 = 6$

b $5b - 12 = 3$

c $53 + 4a = 97$

d $\frac{14}{b} + 23 = 10$

e $\frac{6x}{5} = 120$

f $\frac{7x + 2}{5} = 6$

8 Expand brackets in each of these expressions and collect like terms.

a $23x + 7(x + y)$

b $\frac{1}{4}(52y + x) + 2x$

c $3y(4x + 2) + 8x(y + 3)$

d $27 - 5x + x(4x + 6)$

e $\frac{x}{3}(9 - x) + 6x(x + 2)$

f $32x + 7x(y - 3) + 2x - 5y(x - 1)$

g $a(x - 10 + 3y) - 2a(x + y)$

h $4xy(3 + x) - 3yx(6 + 5y)$

i $18a(x - 4y + a) - 5a(x - 3y)$

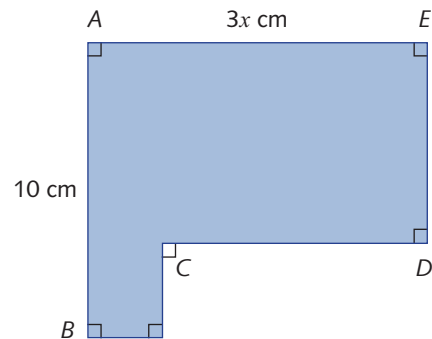
9 Prove that these statements are true.

a Select any whole number and add 14. Double the result and then subtract 8 from the number. Divide by 2 and then subtract the original number. The result is always 10.

b Start with any number. Add 5 and then triple that number. Subtract 9. Divide by 3. Subtract the original number. The result is always 2.

10 The entry fee for the Melbourne Show is \$20 per adult and \$8 per child. The cost of each ride is \$5 per person. Stephen's family (2 parents and 3 children) went to the Show and spent \$164 on rides and entrance tickets. If the whole family always went on rides together, how many rides did they go on?

- 11** Adrian has a habit of reading a book before he sleeps. He reads the same number of pages every night, Monday to Friday. He reads three times more pages on Saturday night than on a weeknight. He does not read any pages on Sunday night. If Adrian took 7 weeks to finish a book that had 280 pages, how many pages does he read on a weeknight, and how many on a Saturday night?
- 12** The perimeter of the figure shown opposite is 43 cm. Find the value of x .



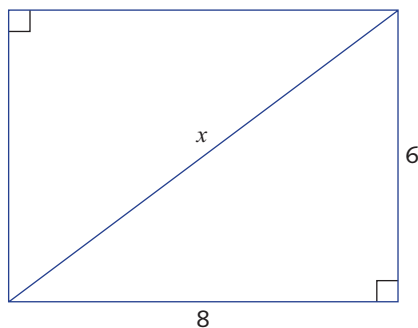
Chapter 7

- 1** Express each fraction as a percentage.
- a** $\frac{2}{5}$ **b** $\frac{5}{6}$ **c** $\frac{7}{8}$ **d** $\frac{7}{9}$
- e** $\frac{5}{7}$ **f** $\frac{3}{40}$ **g** $\frac{8}{11}$ **h** $\frac{7}{12}$
- 2** Express each percentage as a fraction.
- a** $8\frac{1}{2}\%$ **b** $16\frac{2}{3}\%$ **c** $15\frac{1}{3}\%$ **d** $8\frac{6}{9}\%$ **e** $4\frac{2}{7}\%$ **f** $5\frac{1}{2}\%$
- 3** Express each percentage as a fraction in simplest form.
- a** 150% **b** 2.5% **c** 136% **d** 204% **e** 0.25% **f** 5.25%
- 4** Complete each percentage addition and express the result as a fraction.
- a** $15\% + 25\% + 14\%$ **b** $2.5\% + 24\% + 65.5\%$
- c** $2\% + 70\% + 28\%$ **d** $45\% - 10\% + 7\%$
- 5** 20% of a class of 30 are aged 13 years or under. How many of the class are over 13 years of age?
- 6** Find:
- a** 25% of 5000 **b** 12.5% of 16 000 **c** $66\frac{2}{3}\%$ of 4200
- d** 6.6% of 9000 **e** 87.5% of 4200 **f** 7% of 320 000
- 7** Find the new amount after each of the percentage changes.
- a** \$155 000 increased by 20% **b** \$60 000 decreased by 5%
- c** \$32 000 increased by 15% **d** 108 000 hectares increased by 24%

- 8 Calculate the total value of an investment of \$16 000 at the end of 2 years for each simple interest rate.
- a 5% b 6.5% c 5.75%
- 9 Calculate the total value of an investment of \$45 000 at the end of 3 years for each simple interest rate.
- a 8% b 6% c 5.25%
- 10 Calculate the discounted price in each case.
- a A discount of 10% on a purchase of \$4520
- b A discount of 15% on a purchase of \$27
- c A discount of 5% on a purchase of \$86
- d A discount of $12\frac{1}{2}\%$ on \$240
- e A discount of $2\frac{1}{2}\%$ on \$480
- 11 Calculate:
- a 30% of 50% b 20% of 30% c 50% of 80% d 20% of 90%
- 12 a 20% of a number is 1020. Find the number.
- b 25% of a number is 600. Find the number.
- c 15% of a number is 330. Find the number.
- 13 The price of a book pre-GST is \$30. Assume a GST rate of 10%. What is the price of the book including GST?

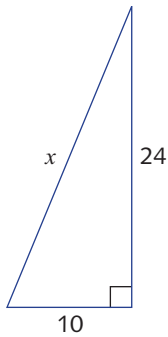
Chapter 8

- 1 Find the length of the diagonal in the rectangle shown below.

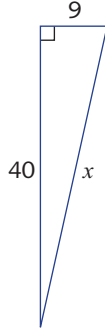


2 Use Pythagoras' theorem to find the missing side lengths in the triangles below.

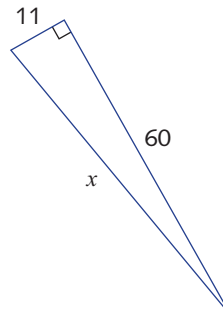
a



b

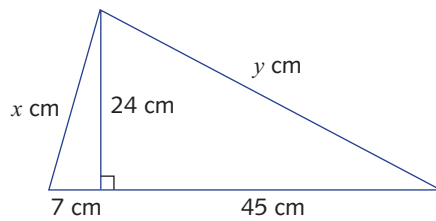


c

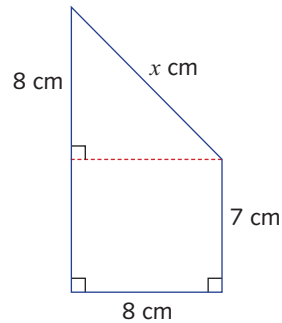


3 Find the values of x and y in the diagrams below.

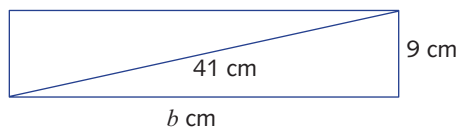
a



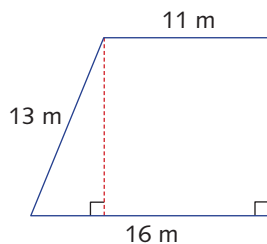
b



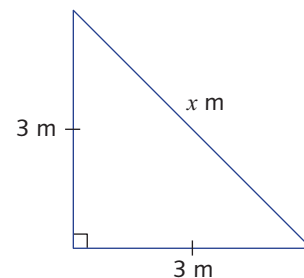
4 A rectangle has width 9 cm and diagonal 41 cm as shown below. What is its length?



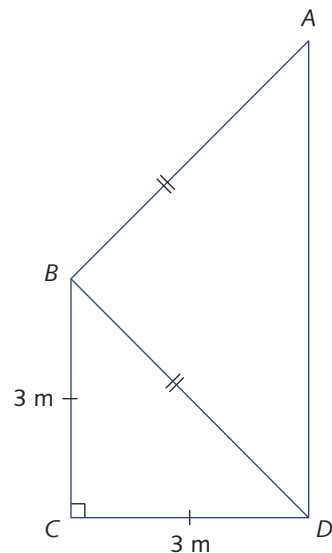
5 Find the height of the trapezium shown below.



6 Find the length of the hypotenuse, correct to 2 decimal places, in the triangle to the right.

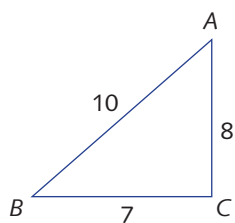


7 Find the length of the side AD in the diagram below.

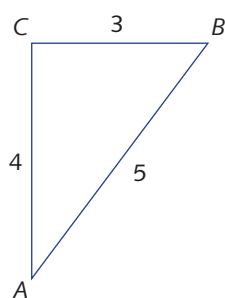


8 For each triangle shown below, decide whether it is a right-angled triangle. If it is, name the right angle.

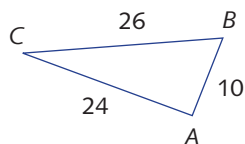
a



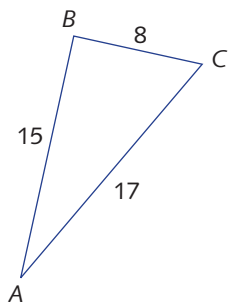
b



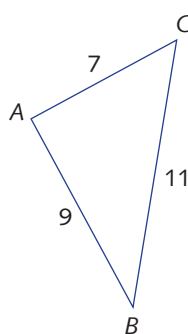
c



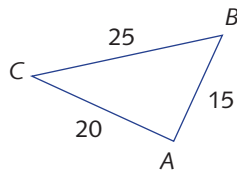
d



e



f



9B Problem-solving

Kaprekar's routine

Choose a set of three digits. Repeats are allowed, but the digits are not *all* allowed to be the same (so 176 and 838 are okay but 444 is not). Now carry out the following steps.

Step 1: Make the largest three-digit number you can out of them.

Step 2: Then reverse this number to get the smallest number possible. This may involve putting a zero at the front of a number.

Step 3: Then subtract the smaller number from the larger one. If the result of the subtraction is two digits, then a 0 is placed to the left of the number.

Now go back to Step 1 with this number and repeat the process until you notice something special happening. For example, if you choose 680, the first step gives 860, step 2 gives 068, step 3 gives $860 - 68 = 792$ and then you start again with 792 . . . $972 - 279 = \dots$

1 Describe what eventually happens.

2 What do you notice about the numbers produced on the way?

We call the number at which you finally arrive a **self-producing integer**. With some sets of three digits, you get it straight away. Other sets take a few iterations but never more than six.

If the set you start with contains *four* digits instead of three, you get a different self-producing integer.

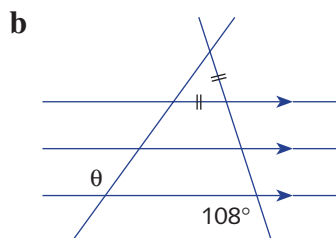
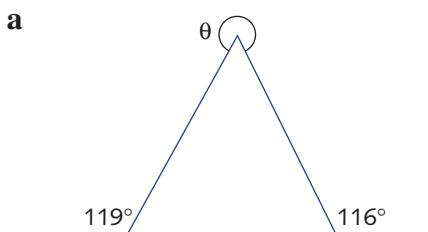
3 Find what it is.

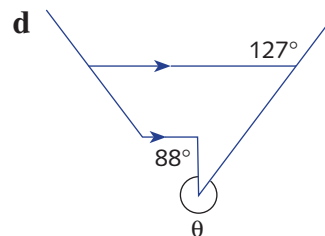
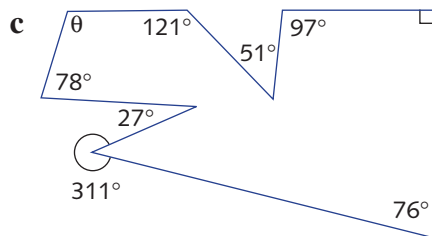
4 Find out what happens if you start with a set of *five* digits. (Don't be in too much of a hurry to report on your findings here, because there is a bit more to it than meets the eye. It might be useful to have a partner to help you. Also, if you have been doing all your subtractions without the aid of a calculator up until now, well, now might be a good time to get one out!)

5 *Challenge:* There are three self-producing integers with 10 digits. This is probably a job for a computer program if you feel like having a bash at it.

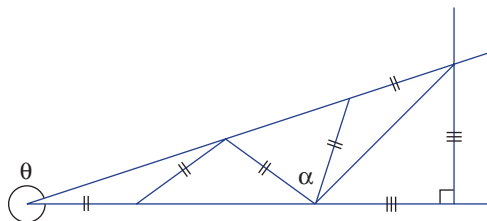
Angle chasing

1 Find θ in each diagram below.





- 2 What pairs of whole number values can θ and α take in the diagram below? Explain the procedure you followed to obtain your answer.



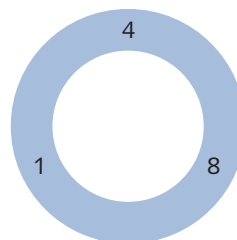
Multiples

- 1 Take a three-digit multiple of 37:

$$13 \times 37 = 481$$

Arrange the digits of 481 clockwise in a circle.

Reading around the circle clockwise, we obtain the numbers 481, 814 and 148.



- a** Show that each of these is a multiple of 37.

- b** Find out what happens for the three-digit multiples of 37 below:

185, 259, 296, 740, 814 and 925

- 2 Now look at a multiple of 41 that has five digits:

$$1679 \times 41 = 68\,839$$

- a** Show that each of the numbers 68839, 88396, 83968, 39688 and 96883 is a multiple of 41.

- b** Try some more five-digit multiples of 41 to see what happens.

- 3 **a** Work out each of these multiplications.

i $37 \times 5 \times 3 = \square$ **ii** $37 \times 2 \times 3 = \square$ **iii** $37 \times 7 \times 3 = \square$

What do you notice?

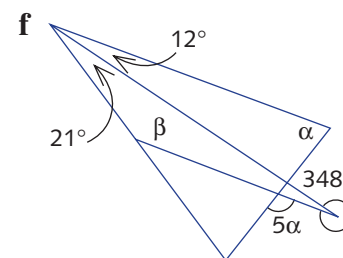
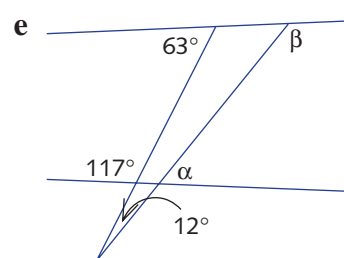
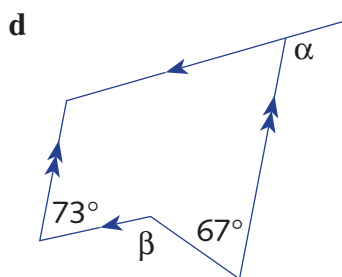
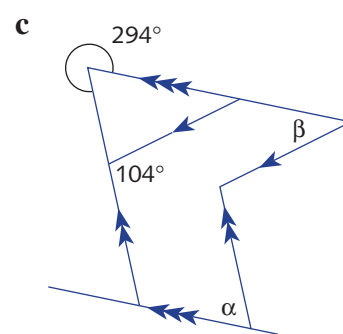
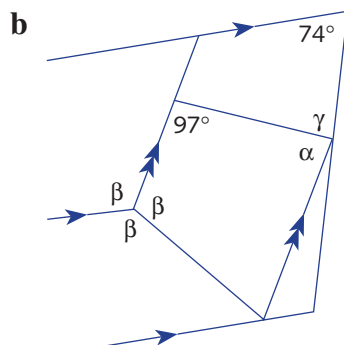
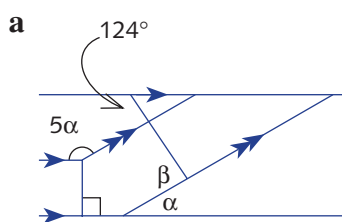
- b** Can you predict the value of $37 \times 9 \times 3$? Can you explain what happens?

- c** Now try $37 \times 13 \times 3$. Can you predict the value of $37 \times 17 \times 3$? Can you explain what happens?

- d** Show that 41 is a factor of 11 111. Can you make up a similar problem to part **c** based on this? (*Hint*: Use the form $41 \times a \times b = \underline{\hspace{2cm}}$. Replace b with a suitable number and then choose other numbers in turn to replace a .)

Geometry challenge

1 Find the values of the pronumerals in each of these diagrams.



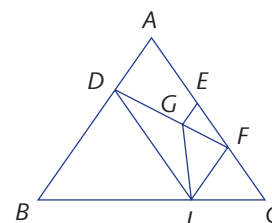
2 The triangle ABC opposite is an isosceles triangle, with $AB = AC$ and $\angle ABC = 55^\circ$.
 $AD \parallel EG \parallel FI$ and $DI \parallel AF$.

a Find $\angle BAC$ and $\angle ACB$.

b Find $\angle FIC$ and prove that $\triangle FIC$ is isosceles.

c Prove that $DB = DI$.

d If $GD = GI$ and $\angle FGI = 34^\circ$, find $\angle GIF$ and $\angle EFG$.



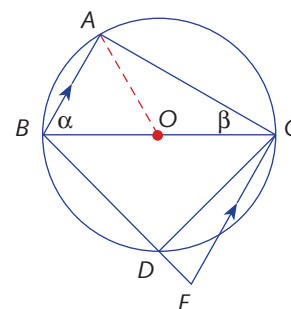
3 O is the centre of the circle shown opposite.

a Let $\angle ABC$ be α . Express $\angle AOC$ in terms of α .

b Let $\angle ACB$ be β . Express $\angle AOB$ in terms of β .

c Prove that $\triangle ABC$ is a right-angled triangle.
 Hence name two other right-angled triangles in the diagram.

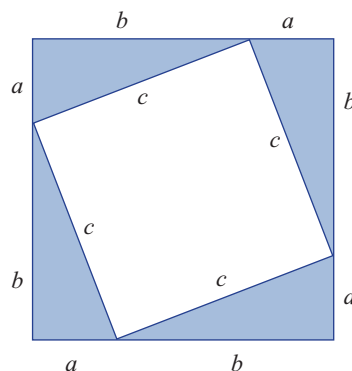
d If $\angle ECD = 25^\circ$, find $\angle ABD$.



Pythagoras' theorem

In Chapter 8 we gave a proof of Pythagoras' theorem that relied on rearranging triangles and comparing areas in two diagrams. We now give a proof using just one diagram.

- a Show that $(a + b)^2 = a^2 + 2ab + b^2$.
- b Show that the total area of the four triangles is $2ab$.
- c Hence, show $c^2 = a^2 + b^2$.



Pythagorean triples

In Chapter 8 we encountered a number of right-angled triangles whose side lengths are positive integers. Two such triangles are the ones with side lengths 3, 4 and 5, and 5, 12 and 13.

The numbers 3, 4, 5 are an example of a **Pythagorean triple**. This means that

$$5^2 = 3^2 + 4^2.$$

In a similar way, 5, 12, 13 is a Pythagorean triple because

$$13^2 = 5^2 + 12^2.$$

In general, the set of numbers a , b , c is a Pythagorean triple if a , b and c are positive integers and $a^2 + b^2 = c^2$. The name comes from the fact that a right-angled triangle can be formed with side lengths a , b and c , where c is the length of the hypotenuse.

It is said that Pythagoras invented the following method for finding some of these triples.

Step 1: Take an odd number as the length of one side.

Step 2: Square it and subtract 1.

Step 3: Halve the result (this gives the second side).

Step 4: Add 1 to this result to get the length of the hypotenuse.

For example:

Step 1: Take 11 as the length of the first side.

Step 2: Square it to obtain 121 and subtract 1 to obtain 120.

Step 3: Halve the result to obtain 60 (the length of the second side).

Step 4: Add 1 to get 61 (the length of the hypotenuse).

Check that $11^2 + 60^2 = 61^2$.

Produce some more Pythagorean triples using this method, starting with some other odd numbers.

Not all Pythagorean triples can be produced in this way – an example is the triple 15, 8, 17. Can you see why this triple does not fit the pattern?

Plato, the Greek philosopher and mathematician, recorded the following method for producing Pythagorean triples.

Step 1: Start with a number divisible by 4 as the first side length.

Step 2: Square it, divide by 4 and subtract 1 (this is the second side).

Step 3: Square the original number, divide by 4 and add 1 (this is the hypotenuse).

For example:

Step 1: Start with 8.

Step 2: Square it to obtain 64, divide by 4 to obtain 16, and subtract 1 to obtain 15.

Step 3: Square 8 to obtain 64, divide by 4 to obtain 16, and add 1 to obtain 17.

Generate some other triples using Plato's method, starting with 12, 16, 20, . . .

Again, not all Pythagorean triples can be produced in this way. For example, 77, 36, 85 is a Pythagorean triple. Can you see why it does not fit Plato's method or Pythagoras' method?

The complete story

Here is a remarkable fact: there is a procedure for producing all possible Pythagorean triples!

To start the explanation, notice that if we have a Pythagorean triple, such as 3, 4, 5, we can get infinitely many others, all related to it, by multiplying each of the numbers by the same factor. For example, 9, 12, 15 (multiplying by 3) is also a Pythagorean triple.

A Pythagorean triple is said to be **primitive** if the three numbers have no common factor other than 1. For example, 3, 4, 5 is primitive but 6, 8, 10 is not. Do the above methods produce primitive Pythagorean triples?

Generating all possible Pythagorean triples

The procedure that produces all of the primitive Pythagorean triples a, b, c uses algebra. Here it is:

Start with any two positive integers u and v , where $v > u$, u and v have no common factor other than 1, and one of them is odd and the other is even. Now calculate:

$$a = v^2 - u^2$$

$$b = 2uv$$

$$c = u^2 + v^2$$

Then a, b, c is a primitive Pythagorean triple, and every primitive Pythagorean triple can be produced in this way.

Use this method to reproduce all of the primitive Pythagorean triples you created using Pythagoras' method and Plato's method.

Sports percentages

It is suitable to use a calculator or spreadsheet for this exercise.

A common use of percentages is to decide how sporting teams are placed on a ladder or league table. When you look at a ladder, you expect to see the teams with the most wins at the top and those with fewest wins near the bottom. If matches are drawn, half the points for a win are given to both teams involved. The points for wins and draws are called **premiership points**. A win is often worth four points and a draw is worth two.

A problem arises when two or more teams have the same number of premiership points. They cannot both occupy the same rung on the ladder, so a way of deciding which team is to be placed higher is needed. This is where percentages come in.

The total number of points (or goals) scored by a team in the season to date is expressed as a percentage of the total number of points its opponents have scored against it. The formula is:

$$\text{percentage} = \frac{\text{points for}}{\text{points against}} \times 100\%$$

When a team has scored more points than its opponents have scored against it to date, its percentage at that stage will be more than 100%. If it has been outscored by its opponents overall, its percentage will be less than 100%.

This method does not favour teams that are high-scoring, but that allow their opponents to score highly against them, over low-scoring teams with good defensive set-ups. It is commonly used in some football codes, netball, basketball, ten-pin bowling and volleyball. For other sports, this method is less appropriate due to the scoring systems involved. For this reason, it is not usually used for hockey, soccer, cricket or softball.

The table below is a netball ladder after five games of a season have been played. Only two of the percentages have been calculated.

	TEAM	Wins	Losses	Draws	Goals For	Goals Against	Percentage	Prem. Points
1	Pelicans	5	–	–	115	80	143.8%	20
2	Cassowaries	4	1	–	126	90		16
3	Jabirus	3	2	–	104	96		12
4	Hawks	2	3	–	130	104		8
5	Brolgas	2	3	–	105	112		8
6	Kookaburras	2	3	–	90	120		8
7	Emus	1	4	–	77	105	73.3%	4
8	Eagles	1	4	–	85	125		4

Activity 1

Your first task is to calculate the percentages of the other six teams, and confirm that the teams are in the right order. Remember, if a team has more premiership points, it will be placed above a team with fewer premiership points, even though its percentage may not be as high.

The teams then play the sixth round of games. The results are:

- Brolgas (27) beat Jabirus (16)
- Emus (20) beat Kookaburras (15)
- Hawks (30) beat Eagles (28)
- Cassowaries (31) beat Pelicans (27).

These results change the teams' numbers of wins, losses, goals for and goals against for the season, and their premiership points. It may also change their places on the ladder.

Activity 2

Update the ladder. This means you will need to work out all the new figures and percentages for each team. For example, the Brolgas' Goals For will increase by 27 and their Goals Against will increase by 16. The Jabirus' Goals For will go up by 16 and their Goals Against will go up by 27. A calculator may help you work out the percentages.

Activity 3

- 1 Which team's percentage did not change? Can you explain why?
- 2 How many of the teams that won had their percentages go down? Which ones were they?
- 3 Is it possible to lose and have your percentage increase? Give an example of a team that had this happen.
- 4 Add up the Goals For column. Add up the Goals Against column. Explain your results.

Arithmetic modulo 7 means that you divide a number by 7 and just look at the remainder. The quotient gets discarded. For example, $15 = 7 \times 2 + 1$, so we write

$$15 \equiv 1 \pmod{7} \quad \text{Read this as '15 is congruent to 1, modulo 7'.$$

Such an equation is called a **congruence**.

Thinking in modulo 7 can relate to the days of the week.

Today is Monday. What day of the week will it be in 15 days time?

In 14 days time it will be Monday again, so in 15 days time it will be Tuesday.

Today is Monday 4 May. What day of the week will it be on 4 June?

There are 31 days in May, and $31 = 7 \times 4 + 3$. In 28 days time it will be Monday again, so in 31 days time it will be Thursday.

There are seven possible remainders 0, 1, 2, 3, 4, 5 and 6 after division by 7, and there are seven days in the week.

Arithmetic modulo 5 means that you divide the number by 5 and just look at the remainder. Someone counted on the fingers of one hand the people coming into the Wombat Heights Football Ground, but forgot to count how many times he'd gone around his hand:

1, 2, 3, 4, 0, 1, 2, 3, 4, 0, 1, 2, 3, . . . 2, 3.

'I don't know how many people went into the Football Ground today, but I do know that the number ended in a 3 or an 8.'

Arithmetic modulo 12 and arithmetic modulo 24 are related to counting the hours around a 12-hour clock and around a 24-hour clock.

Dividing 90 by 12 and by 24 gives

$$90 \equiv 6 \pmod{12} \quad \text{and} \quad 90 \equiv 18 \pmod{24}$$

'It is now 8:30 p.m. What will my 12-hour watch and my 24-hour clock say in 90 hours time?'

'My 12-hour watch will say 2:30, and my 24-hour clock will say 14:30.'

Arithmetic modulo n where n is any whole number greater than one is possible.

There are three possible remainders 0, 1, 2 after division by 3. Similarly, there are ten possible remainders 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 after division by 10.

In each case every whole number is congruent to one of the remainders. The plural of modulus is moduli.

Here are some more congruences, using various moduli:

$$20 \equiv 2 \pmod{3} \quad 67 \equiv 7 \pmod{10}$$

$$42 \equiv 0 \pmod{7} \quad 35 \equiv 1 \pmod{2}$$

$$100 \equiv 1 \pmod{11} \quad 80 \equiv 8 \pmod{9}$$

We can use any whole number greater than 1 as the **modulus**.

Congruence with negative integers

Here is how to divide -10 by 7 and get a whole number remainder less than 7 :

$$-10 = 7 \times -2 + 4, \text{ so } -10 \equiv 4 \pmod{7}$$

Here are two everyday interpretations of this idea:

Arithmetic modulo 7 can also be used with negative integers.

Today is Monday. What day of the week was it 10 days ago?

Fourteen days ago it was also Monday, so 10 days ago it was Friday.

Today is Monday 4 May. What day of the week was it on 4 April?

There are 30 days in April, and $-30 = 7 \times (-5) + 5$. Thirty-five days ago it was also Monday, so 30 days ago it was Saturday.

Arithmetic modulo 12 and **arithmetic modulo 24** can be used with negative integers.

Here is an example with 12-hour and 24-hour clocks.

‘It is now 8:30 p.m. What did my 12-hour watch and my 24-hour clock say 31 hours ago?’

Dividing -31 by 12 and by 24 gives

$$-31 = 12 \times -3 + 5 \quad \text{and} \quad -31 = 24 \times -2 + 17$$

Writing this in congruences gives

$$-31 \equiv 5 \pmod{12} \quad \text{and} \quad -31 \equiv 17 \pmod{24}$$

‘My 12-hour watch said 1:30, and my 24-hour clock said 13:30.’

Here are some more congruences involving negative integers and various moduli:

$$-20 \equiv 1 \pmod{3} \quad -67 \equiv 3 \pmod{10}$$

$$-42 \equiv 0 \pmod{7} \quad -35 \equiv 1 \pmod{2}$$

$$-100 \equiv 10 \pmod{11} \quad -80 \equiv 1 \pmod{9}$$

Congruence modulo a whole number

The integers 9 and 23 are both congruent to 2 modulo 7 , so we say that they are congruent to each other:

$$9 \equiv 23 \pmod{7}$$

We could have checked this directly simply by subtracting 9 from 23 :

$$23 - 9 = 14, \text{ which is a multiple of } 7$$

Working with days of the week this becomes:

‘It will be the same day of the week in 9 days time and in 23 days time.’

This provides us with the standard definition of congruence modulo 7 . Two integers are called **congruent modulo 7** if their difference is a multiple of 7 . By the division algorithm, every integer is congruent modulo 7 to one, and only one, of the seven remainders $0, 1, 2, 3, 4, 5$ and 6 .

The integers 9 and -33 are also congruent modulo 7, because

$$9 - (-33) = 9 + 33 = 42, \text{ which is a multiple of 7.}$$

‘It will be the same day of the week in 9 days time as it was 33 days ago.’

On the number line below, we have marked both the multiples of 7 and all the integers congruent to 2 modulo 7.



Each integer congruent to 2 modulo 7 is 2 more than a multiple of 7, no matter whether the integer is positive or negative.

Here are some more congruences. Check them by subtracting the integers:

$$\begin{aligned} 10 &\equiv 40 \pmod{3} & -27 &\equiv -67 \pmod{10} \\ -42 &\equiv 21 \pmod{7} & 30 &\equiv 514 \pmod{2} \\ -100 &\equiv -540 \pmod{11} & 30 &\equiv -51 \pmod{9} \end{aligned}$$



Modular Arithmetic

- Two integers are called **congruent modulo 7** if their difference is a multiple of 7.
- Every integer is congruent modulo 7 to one, and only one, whole number less than 7, that is, to 0, 1, 2, 3, 4, 5 or 6
- The **modulus** can be any whole number greater than 1.



Exercise 9C

1 Copy and complete:

a $27 = 7 \times \underline{\quad} + \underline{\quad}$, hence $27 \equiv \underline{\quad} \pmod{7}$

b $359\,412\,294 = 10 \times \underline{\quad} + \underline{\quad}$, hence $359\,412\,294 \equiv \underline{\quad} \pmod{10}$

c $53 = 12 \times \underline{\quad} + \underline{\quad}$, hence $53 \equiv \underline{\quad} \pmod{12}$

d $3 = 8 \times \underline{\quad} + \underline{\quad}$, hence $3 \equiv \underline{\quad} \pmod{8}$

2 Copy and complete:

a $36 \equiv \underline{\quad} \pmod{2}$

b $100 \equiv \underline{\quad} \pmod{12}$

c $156 \equiv \underline{\quad} \pmod{25}$

d $94 \equiv \underline{\quad} \pmod{9}$

3 Find the first number over 50 that is congruent to 1:

a modulo 2

b modulo 8

c modulo 10

d modulo 3

e modulo 19

f modulo 15

g modulo 30

h modulo 66

- 4 Roger says ‘She loves me, she loves me not, she loves me, she loves me not. . .’ as he counts the petals of a flower. The flower has 47 petals. Is happiness assured for Roger or not?
- 5 Six children – Adam, Bessie, Charles, Doreen, Edwin and Florence – choose chocolates in turn from a box of 80 chocolates. Adam starts, and they keep going in alphabetical order. Who gets the last chocolate?
- 6 It is 4 p.m. now. What will the time be in 110 hours?
- 7 If the first of November is a Sunday, what dates of November are Wednesdays? What day of the week is the 27 November?
- 8 In what years between 1890 and 1920 could one have truthfully said, ‘Next year is a leap year’?
- 9
 - a What is 52 348 181 modulo 4? Is there an easier way than dividing?
 - b What is 52 348 181 modulo 100? Is there an easier way than dividing?
 - c What is 52 348 181 modulo 50? Is there an easier way than dividing?
 - d What is 52 348 181 modulo 25? Is there an easier way than dividing?
- 10
 - a What is the time 27 hours after 11 a.m.?
 - b What day of the week is 176 days after a Wednesday?
 - c A man is facing north. He turns through 90° in a clockwise direction 57 times. In which direction is he facing now?
- 11
 - a Write -13 as a multiple of 5 plus remainder, where the remainder is 0, 1, 2, 3 or 4.
 - b Hence copy and complete, ‘ $-13 \equiv \dots \pmod{5}$.’
 - c Mark the multiples of 5 between -26 and 14 on a number line. Mark also -13 and all integers between -26 and 14 congruent to -13 modulo 5.
- 12
 - a Mark on the number line all integers between -10 and 5 congruent to 2 modulo 5.
 - b Mark on the number line all integers between -10 and 5 congruent to 3 modulo 5.
- 13 Copy and complete:
 - a $-20 = 7 \times \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$, hence $-20 \equiv \underline{\hspace{1cm}} \pmod{7}$
 - b $-30 = 8 \times \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$, hence $-30 \equiv \underline{\hspace{1cm}} \pmod{8}$
 - c $-65 \equiv \underline{\hspace{1cm}} \pmod{11}$ d $-70 \equiv \underline{\hspace{1cm}} \pmod{9}$
 - e $-100 \equiv \underline{\hspace{1cm}} \pmod{11}$ f $-72 \equiv \underline{\hspace{1cm}} \pmod{6}$
- 14
 - a Divide the numbers $-16, -11, -1, 0, 4, 15$ and 18 by 4. Hence decide which of the numbers are congruent mod 4.
 - b Divide the numbers $-14, -30, 5, 18, 45, 53$ and 73 by 12. Hence decide which of the numbers are congruent mod 12.

Addition

The wonderful thing about adding in modular arithmetic is that you can simply add the remainders. For example,

$$31 = 7 \times 4 + 3 \text{ and } 30 = 7 \times 4 + 2, \text{ so } 31 + 30 = 7 \times 8 + 5;$$

and writing this in modulo 7 arithmetic,

$$30 + 31 \equiv 3 + 2 \equiv 5 \pmod{7}$$

Check this by adding the integers first: $30 + 31 = 61 \equiv 5 \pmod{7}$.

Today is Monday 4 May. What day of the week will it be on 4 July?

May has 31 days and June has 30 days, and $31 + 30 \equiv 5 \pmod{7}$, so it will be Saturday.

When the remainders add to more than 7, we reduce the remainder modulo 7:

$$31 + 30 + 31 \equiv 3 + 2 + 3 \equiv 8 \equiv 1 \pmod{7}$$

This is because $31 + 30 + 31 = 7 \times 12 + 8 = 7 \times 13 + 1$.

Today is Monday 4 May. What day of the week will it be on 4 August?

Since $31 + 30 + 31 \equiv 1 \pmod{7}$, it will be Tuesday.

Here are some more additions with various moduli.

$$10 + 25 \equiv 1 + 1 \equiv 2 \pmod{3} \qquad 26 + 39 \equiv 6 + 9 \equiv 5 \pmod{10}$$

$$9 + 75 \equiv 2 + 5 \equiv 0 \pmod{7} \qquad -11 + 21 \equiv 1 + 1 \equiv 0 \pmod{2}$$

$$-100 + 12 \equiv 10 + 1 \equiv 0 \pmod{11} \qquad 30 + 50 \equiv 3 + 5 \equiv 8 \pmod{9}$$

Check them by adding the integers first. For example, $10 + 25 = 35 \equiv 2 \pmod{3}$.

Multiplication

Multiplication of integers is repeated addition, so we can use the same method for multiplying in modular arithmetic. For example,

$$10 \times 12 \equiv 3 \times 5 \equiv 1 \pmod{7} \text{ and } 65 \times 12 \equiv 2 \times 5 \equiv 3 \pmod{7}$$

Check the first calculation by multiplying the integers first: $10 \times 12 = 120 \equiv 1 \pmod{7}$.

You can check the second if you feel the need to do so.

A mine organises its shifts in 12-day cycles, with the first cycle beginning on a Monday. On what days of the week will the 11th cycle and the 66th cycle begin?

Since $10 \times 12 \equiv 1 \pmod{7}$, the 11th cycle will begin on a Tuesday.

Since $65 \times 12 \equiv 3 \pmod{7}$, the 66th cycle will begin on a Thursday.

Here are some more multiplications with various moduli.

$$10 \times 25 \equiv 1 \times 1 \equiv 1 \pmod{3}$$

$$25 \times 38 \equiv 5 \times 8 \equiv 0 \pmod{10}$$

$$9 \times 75 \equiv 2 \times 5 \equiv 3 \pmod{7}$$

$$-10 \times 25 \equiv 0 \times 1 \equiv 0 \pmod{2}$$

$$-100 \times 12 \equiv 10 \times 1 \equiv 10 \pmod{11}$$

$$30 \times 50 \equiv 3 \times 5 \equiv 6 \pmod{9}$$

Check at least some of them by multiplying the integers first.

Powers

Taking powers is repeated multiplication, so once again we can use the same method for taking powers in modular arithmetic. For example,

$$10^3 \equiv 3^3 \equiv 27 \equiv 6 \pmod{7} \text{ and } 30^6 \equiv 2^6 \equiv 64 \equiv 1 \pmod{7}$$

Here are some similar calculations:

$$14^4 \equiv 2^4 \equiv 16 \equiv 1 \pmod{3} \qquad 25^3 \equiv 5^3 \equiv 125 \equiv 5 \pmod{10}$$

$$4^{10} \equiv (4^2)^5 \equiv 16^5 \equiv 0^5 \equiv 0 \pmod{8} \qquad (-11)^6 \equiv 1^6 \equiv 1 \pmod{2}$$

$$(-3)^8 \equiv 9^4 \equiv 4^2 \equiv 5 \pmod{11} \qquad (-2)^9 \equiv (-8)^3 \equiv 1^3 \equiv 1 \pmod{9}$$

Fibonacci sequences in modular arithmetic

Fibonacci was a mediaeval Italian mathematician who studied, amongst other things, a curious sequence of numbers which is now named after him. The Fibonacci sequence starts with 1 and 1, then every term is the sum of the two previous terms:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \dots$$

The numbers soon get very big and take a bit of calculating. But now try writing out the sequence in arithmetic modulo 8. The rules are the same – the first two terms are 1 and 1, then every term is the sum modulo 8 of the two previous terms:

$$1, 1, 2, 3, 5, 0, 5, 5, 2, 7, 1, 0, 1, 1, \dots$$

We get a zero every 6 terms, and the whole thing repeats itself after 12 terms. The final questions in the following exercises continue this experimentation.



Exercise 9D

- 1 Perform these additions in modulo arithmetic. It is best to reduce the numbers *before* you add them, but do the ones marked ✓ both ways for comparison.

✓a $16 + 2 \pmod{5}$	✓b $902 + 30 \pmod{9}$
✓c $7 + 7 \pmod{2}$	✓d $6 + 8 + 3 + 15 \pmod{3}$
e $4 + 4 + 8 + 7 + 6 \pmod{5}$	f $1 + 2 + 3 + 4 + 5 \pmod{4}$
✓g $1 + 3 + 5 + 7 + 9 \pmod{5}$	h $79 + 79 + 79 + 79 \pmod{80}$

- i $243 + 527 + 384 + 22 + 3 \pmod{5}$
- j $3\,628\,598 + 21\,242\,522 + 376\,554\,004 \pmod{100}$
- 2 Reduce the numbers first before you multiply, but do those marked ✓ both ways.
✓a $7 \times 8 \pmod{5}$ ✓b $11 \times 12 \pmod{8}$
✓c $4 \times 4 \times 4 \pmod{3}$ d $9 \times 8 \pmod{12}$
e $247 \times 3482 \pmod{10}$ f $552 \times 661 \times 776 \pmod{5}$
- 3 If you have a string of numbers to multiply, reduce them all first, but also reduce the product at each step as you go along.
a $26 \times 13 \times 17 \times 35 \times 62 \pmod{3}$ b $10 \times 23 \times 59 \times 44 \times 33 \pmod{6}$
c $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \pmod{8}$ d $1 \times 3 \times 5 \times 7 \times 9 \pmod{8}$
- 4 Reducing as you go along is really important when taking powers. You can also group the factors in pairs, or in threes, if that will make the calculations easier.
a $5 \times 5 \times 5 \times 5 \times 5 \times 5 \pmod{7}$ b $4 \times 4 \times 4 \times 4 \pmod{7}$
c $16 \times 16 \times 16 \times 16 \times 16 \pmod{7}$ d $23^8 \pmod{5}$
e $5^6 \pmod{11}$ f $4^7 \pmod{12}$
g $2^{12} \pmod{12}$ h $9^{26} \pmod{10}$
- 5 These questions involve both sums and products. Reduce the numbers to start with, then keep reducing the numbers after each step so they do not get too big.
a $14 \times 5 + 21 \times 4 \pmod{3}$ b $15 \times 4 + 25 \times 23 \pmod{6}$
c $10^5 + 2^5 \pmod{11}$ d $3 + 3^2 + 3^3 + 3^4 \pmod{5}$
- 6 Perform these additions, then explain what general result they illustrate.
a $2 + 2 + 2 \pmod{3}$ b $1 + 1 + 1 + 1 \pmod{4}$
c $3 + 3 + 3 + 3 + 3 \pmod{5}$ d $5 + 5 + 5 + 5 + 5 + 5 \pmod{6}$
- 7 My watch has 6 functions, which the silver button goes through in order when I push it. If I push the button 74 times, how many more times must I push the button to get it back to the start? Use modulo arithmetic.
- 8 A satellite goes around the Earth once every 7 hours. If it is 9 a.m. now, what time will it be in 247 revolutions? (Don't calculate 7×247 , use modulo arithmetic.)
- 9 It is Wednesday today. Christmas is 280 days away. Then 25 days holiday at the beach. Then 211 days at work. Then overseas for 143 days. What day of the week is the last day overseas? Use modulo arithmetic.

- 10** Eric has a packet of 37 Smarties. He counts them out on his five fingers, over and over again, 19 times in fact. What finger does he end on? (Don't calculate 37×19 , use modulo arithmetic.)
- 11** A dancer turns 60° 9 times, then 40° 12 times, then 90° 13 times. What angle does he make with his original position? Use modulo arithmetic.
- 12** **a** Prove that every power of 10 is equal to 1 modulo 9.
b Hence prove that every whole number is congruent modulo 9 to the sum of its digits.
c Hence prove that every whole number is congruent modulo 3 to the sum of its digits.
- 13** **a** Prove that every odd power of 10 is congruent to -1 modulo 11, and that every even power of 10 is congruent to 1 modulo 11.
b Hence prove that every whole number such as 123456 is congruent modulo 11 to the alternating sum of its digits, where alternating sum means $6 - 5 + 4 - 3 + 2 - 1$ with the units taken as positive.
- 14** Write out the Fibonacci sequences using the following moduli. Continue each sequence until it repeats itself. In each case, say how often the zeroes occur, and how long it takes for the whole thing to repeat itself.
- a** modulo 3 **b** modulo 2 **c** modulo 4 **d** modulo 5
e modulo 6 **f** modulo 7 **g** modulo 9 **h** modulo 11
i modulo 10 (this one takes quite a while – don't give up)

Answers to exercises

Chapter 1 answers

Exercise 1A

- 1 a 78 b 103 c 157 d 138 e 100 f 140
2 a 61 b 126 c 81 d 223 e 65 f 96
3 a 18 b 26 c 48 d 68 e 58 f 75
4 a 1810 b 9330 c 10 125 d 1580 e 1699 f 789 g 876 h 2479 i 1087
5 a 375 b 1493 c 1019 d 1075 e 339 f 898 g 952 h 10780
6 2675 people 7 14 000 people
8 9904 cats 9 51 805 people
10 821 chairs, 1055 chairs and tables altogether
11 258 900 people are 15 and over.
12 775 409 900 L, 218 008 100 L difference
13 a 69 mm b 197 mm c 128 mm
14 10071 15 16 229 290 Roman Catholics
16 2002 263 17 590 black balls

Exercise 1B

- 1 a 630 b 13 200 c 540 d 770
2 a 693 b 504 c 672 d 1666
3 a 107 b 88 c 106 d 494
4 a 23 remainder 4 b 49 c 66 remainder 8 d 113 remainder 1
5 a 4942 b 16 588 c 14 694 d 29 988 e 38 016
f 31 325 g 4860 h 22 464 i 28 336 j 33 396
6 a 278 b 135 c 1078 d 83 e 675
f 136 g 8677 h 4360 i 857 remainder 6 j 813
7 a 63 remainder 9 b 65 remainder 10 c 146 remainder 8 d 503 remainder 3
e 122 f 212 remainder 14 g 83 remainder 4 h 247 remainder 15
8 11 136 students 9 840 seats 10 792 apartments
11 1428 cars 12 103 200 tomatoes 13 3315 lights
14 3145 people 15 Plan A costs \$33.00, Plan B costs \$35.76.
16 561 858 990 km 17 \$520 18 17 bags
19 a 1553 bags with 5 left over b 345 mega-bags with 9 left over
20 a 384 weeks b 288 weeks c 192 weeks d 24 weeks, 18 weeks, 12 weeks
21 53 times

Review exercise

- 1 a 60 b 57 c 62 d 61 e 30 f 97
g 90 h 75 i 90 j 41 k 53 l 55
m 94 n 82 o 36 p 45 q 140 r 140
s 80 t 130 u 150
2 a 22 b 4 c 28 d 8 e 93 f 19
g 75 h 109 i 194 j 5 k 260 l 75
3 a 900 b 7800 c 700 d 420 e 760 f 1600
g 600 h 4700 i 0

- 4 12 979 km²
 5 a $231 + 264 + 136 = 631$ b $843 + 637 + 567 = 2047$ c $376 \times 9 = 3384$
 6 5930 dogs 7 551 girls 8 24 chocolates 9 45 boys
 10 54 boxes 11 74 people 12 16 380 kg 13 35 trees

Challenge exercise

- 1 2.5 km 2 There are at least 384 solutions. One is $98\,765 + 1234 = 99\,999$.
 3 22 birds, 14 beasts 4 $n \rightarrow 2n + 4 \rightarrow 10n + 32 \rightarrow 100n + 320 - 320 \rightarrow n$.
 Hence you get the number you started with.
 5 a 5 b 21 779 c 402, 529
 6 a 108 b 21 c 35 d 42
 7 a 7307 b 243 8 $69\,237 \div 231 = 299$ remainder 168

Chapter 2 answers

Exercise 2A

- 1 a $\frac{2}{3}$ b $\frac{1}{2}$ c $\frac{6}{7}$ d $\frac{5}{6}$ e $1\frac{2}{7}$ f 4
 g $\frac{5}{7}$ h $\frac{3}{8}$ i $\frac{4}{5}$ j $\frac{2}{3}$ k $\frac{2}{3}$ l $\frac{3}{4}$
 m $1\frac{1}{4}$ n $1\frac{1}{3}$ o $1\frac{1}{4}$ p $2\frac{1}{3}$
 2 a yes b no c yes d yes e no f yes
 3 a $\frac{2}{5} < \frac{3}{4} < \frac{4}{5} < \frac{9}{10} < \frac{11}{10} < \frac{5}{4}$ b $\frac{7}{10} < \frac{11}{15} < \frac{3}{4} < \frac{23}{30} < \frac{4}{5} < \frac{5}{6}$
 4 $\frac{21}{63}, \frac{42}{63}, \frac{9}{63}, \frac{18}{63}, \frac{27}{63}, \frac{7}{63}, \frac{14}{63}, \frac{35}{63}, \frac{24}{63}$
 5 a $\frac{2}{3} < \frac{5}{6} < \frac{7}{8}$ b $\frac{2}{3} < \frac{11}{16} < \frac{3}{4}$ c $\frac{2}{3} < \frac{31}{45} < \frac{11}{12}$ d $\frac{7}{12} < \frac{5}{8} < \frac{2}{3}$ e $\frac{25}{54} < \frac{1}{2} < \frac{5}{9}$ f $\frac{31}{48} < \frac{2}{3} < \frac{11}{16}$

Exercise 2B

- 1 a 2 b $1\frac{3}{5}$ c $\frac{1}{2}$ d $2\frac{2}{5}$ e $2\frac{1}{3}$ f $1\frac{3}{7}$
 2 a $\frac{1}{2}$ b $\frac{1}{6}$ c $1\frac{1}{5}$ d $\frac{17}{19}$ e $\frac{3}{5}$ f $\frac{5}{6}$
 3 a $1\frac{5}{12}$ b $1\frac{89}{110}$ c $1\frac{2}{15}$ d $\frac{19}{90}$ e $1\frac{55}{72}$ f $1\frac{11}{35}$
 g $\frac{23}{24}$ h $\frac{73}{56}$ i $\frac{11}{10}$ j $\frac{5}{3}$ k $\frac{35}{24} = 1\frac{11}{24}$
 4 a $\frac{1}{6}$ b $\frac{1}{12}$ c $\frac{5}{33}$ d $\frac{4}{35}$ e $\frac{1}{24}$ f $\frac{1}{72}$
 g $\frac{1}{90}$ h $\frac{3}{35}$ i $\frac{1}{12}$ j $\frac{13}{60}$ k $\frac{4}{55}$
 5 a $5\frac{13}{15}$ b $2\frac{11}{12}$ c $4\frac{1}{20}$ d $6\frac{1}{6}$ e $3\frac{13}{15}$ f $8\frac{39}{40}$
 g $4\frac{49}{55}$ h $9\frac{23}{40}$ i $14\frac{17}{40}$
 6 a $2\frac{1}{4}$ b $10\frac{7}{8}$ c $13\frac{7}{24}$ d $25\frac{7}{8}$ e $10\frac{11}{12}$ f $49\frac{27}{110}$
 7 $4\frac{5}{8}$ km 8 $2\frac{11}{20}$ km
 9 $\frac{11}{12}$ of a litre of water 10 $\frac{7}{8}$ 11 $\frac{4}{7}$ 12 $\frac{7}{30}$

13 $\frac{7}{12}$

14 $\frac{7}{36}$

15 $\frac{1}{2}$

16 a $\frac{11}{24}$

b $\frac{7}{12}$

c $\frac{17}{24}$

d $\frac{7}{24}$

Exercise 2C

1 a $\frac{5}{24}$

b $\frac{33}{64}$

c 10

d $\frac{33}{100}$

e $13\frac{1}{3}$

f $13\frac{7}{11}$

2 a $\frac{1}{6}$

b $\frac{1}{77}$

c $\frac{1}{4}$

d $\frac{7}{10}$

e $\frac{1}{14}$

f $\frac{1}{6}$

g $\frac{2}{3}$

h $\frac{49}{60}$

i 2

j $\frac{2}{5}$

k $\frac{7}{13}$

l $\frac{2}{5}$

m $\frac{35}{96}$

n $\frac{1}{4}$

o $\frac{3}{5}$

3 a $2\frac{4}{5}$

b $8\frac{8}{15}$

c $22\frac{7}{50}$

d $3\frac{5}{33}$

e $2\frac{2}{3}$

f $2\frac{7}{10}$

g $\frac{3}{5}$

h $2\frac{17}{32}$

i $5\frac{5}{9}$

4 847 students

5 a 12 L

b $6\frac{1}{4}$ km

c 6250 m

d $37\frac{1}{2}$ m

e 48 kg

f 480 mm

g 60 m

h $87\frac{1}{2}$ km

6 a $1\frac{23}{32}$

b $2\frac{17}{30}$

c $1\frac{1}{6}$

d $4\frac{1}{6}$

e $\frac{3}{40}$

f $\frac{5}{77}$

g $\frac{15}{88}$

h $\frac{1}{36}$

i $1\frac{1}{15}$

j 24

k 25

l $2\frac{1}{2}$

7 a $5\frac{5}{8}$

b $1\frac{11}{15}$

c $3\frac{1}{2}$

d $8\frac{1}{3}$

e $\frac{8}{15}$

f $1\frac{13}{20}$

g $\frac{4}{5}$

h $\frac{44}{75}$

i $\frac{64}{75}$

8 28 table tops

9 $10\frac{10}{19}$ seconds

10 a $\frac{1}{2}$

b 3

11 a $\frac{3}{7}$

b 270

12 360

13 a 4500

b $\frac{1}{4}$

14 \$63

15 780 ha

16 \$2400

17 2025 L

18 42 km

19 $\frac{87}{40}$ m

Exercise 2D

1 a 360

b 605

c 600

2 a \$128

b 90 kg

c \$2205

d 2875 m^3

3 a \$1448

b 968 students

4 28 students

5 a 250 litres

b \$540

c \$175

d $2\frac{1}{2}$ hours = 150 minutes

6 96 goals

7 $28\frac{4}{5}$ hectare

8 $\frac{1}{6}$ litre of milk

Exercise 2E

1 a $2\frac{1}{10}$

b $5\frac{23}{1000}$

c $6\frac{71}{100}$

d $2\frac{3}{500}$

e $\frac{19}{250}$

f $5\frac{17}{25}$

g $\frac{17}{2000}$

h $2\frac{1}{125}$

i $16\frac{7}{8}$

j $23\frac{5}{8}$

- 2 a 51.75 b 0.875 c 36.12 d 112.34 e 87.195
- 3 a $2.35 < 2.435 < 2.5 < 2.5834 < 2.83$ b $18.0099573 < 18.02 < 18.1 < 18.1002 < 18.21$
c $6.6 < 6.66 < 60.006 < 60.66 < 66.06$ d $47.682 < 55.16 < 55.2 < 55.24 < 56.001$
- 4 $4.31\left(4\frac{31}{100}\right) < 4.6\left(4\frac{3}{5}\right) < 4.625\left(4\frac{5}{8}\right) < 4.75\left(4\frac{3}{4}\right)$

Exercise 2F

- 1 a 47.57 b 129.454 c 519.42 d 217.28
- 2 a 23 b 0.3 c 0.26 d 0.26 e 0.026 f 0.75
g 0.561 h 26300 i 250 j 0.5267 k 323000 l 4600
- 3 a 39.41 b 114.335 c 3841.2 d 3193.61 e 1349.52 f 2225.92
- 4 a 0.21 b 0.048 c 0.063 d 0.0042 e 0.00024 f 0.004
g 37.8048 h 0.000122
- 5 a 1.188 b 1.84 c 1.37 d 8.54 e 4.85 f 8.64
- 6 a 40 b 4000000 c 12.6 d 902.1 e 35 f 70000
g 80000 h 12 i 5632
- 7 0.3 million = 300000, 60000
- 8 4.97 m 9 1.5666 million = 1566600
- 10 a 1.756 kg b 3.073 kg c 0.7024 kg d 1.4926 kg e 0.371394 kg f 0.3512 kg
- 11 64190000 kg 12 219576335 kg
- 13 perimeter 87.42 m, area 440.7354 m²
- 14 a 0.04 b 76.93 c 4 d 510.92 e 28.40526 f 56.5
- 15 a 68955.351 g b 71039.252 g c 93309 g

Exercise 2G

- 1 a 0.4 b 0.8 c 0.12 d 0.875 e 4.15 f 7.375
g 45.75 h 7.85 i 3.4 j 1.12
- 2 a $0.\dot{2}8571\dot{4}$ b $0.\dot{4}2857\dot{1}$ c $0.\dot{6}$ d $0.\dot{2}$ e $0.\dot{2}\dot{7}$ f $0.\dot{4}$
g $0.\dot{6}\dot{3}$ h $0.\dot{8}$ i $0.\dot{1}5384\dot{6}$
- 3 a $0.1\dot{6}$ b $0.8\dot{3}$ c $0.41\dot{6}$ d $0.58\dot{3}$ e $0.91\dot{6}$ f $0.\dot{4}$
- 4 a $64.\dot{5}$ b $78.8\dot{3}$ c $45.\dot{1}8$ d $0.91\dot{6}$ e $3.\dot{2}8571\dot{4}$
- 5 a 463.15 b 7.3 c 79.50 d 0.065 e 8.0 f 86
- 6 a 0.29 b 0.83 c 0.57 d 0.58 e 0.44
- 7 a 0.75 b 0.7467 c 0.74674675

Review exercise

- 1 a $\frac{22}{33}, \frac{10}{15}$ b $\frac{35}{42}, \frac{20}{24}$ c $\frac{18}{21}, \frac{6}{7}$ d $\frac{17}{34}, \frac{112}{224}$
- 2 a $\frac{3}{2} > \frac{8}{6} > \frac{5}{4} > \frac{10}{12} > \frac{2}{3} > \frac{2}{4}$ b $3\frac{1}{8} > 3\frac{1}{9} > 2\frac{6}{7} > 2\frac{5}{6} > 2\frac{3}{7} > 2\frac{2}{9}$
- 3 a $2\frac{1}{2}$ b $\frac{5}{6}$ c $1\frac{1}{4}$ d $\frac{3}{40}$
- 4 a $11\frac{1}{4}$ b $3\frac{1}{13}$ c 11 d $7\frac{1}{32}$ e 3
f $\frac{5}{56}$ g $\frac{5}{9}$ h $15\frac{1}{2}$ i $2\frac{21}{32}$

- 5 a $\frac{3}{4}$ b $1\frac{1}{12}$ c $\frac{1}{12}$ d $3\frac{8}{15}$ e $5\frac{17}{20}$ f $5\frac{19}{20}$
g $9\frac{9}{14}$ h $6\frac{1}{12}$ i $8\frac{3}{20}$ j $2\frac{1}{8}$ k $1\frac{3}{10}$ l $1\frac{7}{10}$
6 a $3\frac{5}{12}$ b $4\frac{1}{8}$ c 3 d $7\frac{3}{10}$ e 14 f $3\frac{1}{4}$
7 $\frac{2}{15}$
8 \$4.50 for transport, \$1.80 for telephone calls and \$10.80 on clothes; $\frac{1}{20}$ of his pocket money is saved.
9 $\frac{4}{15}$ 10 2060 11 8520 12 21
13 a 0.0897 b 654.079 c 3.141 59 d 3.142 e 2.718 281 828 5 f 2.7183
14 a 45.6 b 0.28 c 10 305.2 d 0.000 43 e 2.487 f 9.712
g 0.000 003 h 0.034 i 1008.36 j 0.028 94
15 a 38.025 b 34.8495 c 460.912 d 5.94 e 48.781 f 94.43
g 203.7 h 0.246 i 0.000 690 j 200 k 1675 l $0.1\dot{3}$
16 a \$5.63 b \$9.80 c \$32.63
17 2 18 6, \$86.69
19 a 9 cents b 60 cents c 2 cents
20 a 15 b $\frac{1}{5}, 12$ c $\frac{1}{20}$
21 \$8.50 22 $\frac{1}{4}$ 23 1 hour 24 150 cm
25 a $26\frac{2}{3}$ kg b $1386\frac{2}{3}$ kg
26 a $\frac{1}{8}$ b $\frac{3}{11}$ c $\frac{5}{18}$ d $\frac{6}{11}$ e $\frac{7}{23}$ f $\frac{49}{69}$ g 1 h 1
27 a $2\frac{1}{7}$ b $\frac{8}{11}$ c $\frac{5}{12}$ d $4\frac{1}{2}$
28 6 mL, 66.64 kg

Challenge exercise

- 1 15 2 3.36 metres 3 $\frac{1}{8}$
4 a i $\frac{1}{6}$ ii $\frac{1}{20}$ iii $\frac{1}{56}$ iv $\frac{1}{90}$
b When subtracting consecutive unit fractions, the answer is a unit fraction whose denominator is the product of the denominators of the two fractions.
c $\frac{1}{3} - \frac{1}{4}$
5 a $\frac{5}{8}$ b $\frac{1}{4}$ c $\frac{1}{2}$ d $\frac{1}{2}$ e $\frac{1}{2}$ f $\frac{1}{4}$ g $1\frac{4}{5}$ h $\frac{5}{8}$
6 e $0.\dot{0}37, 0.\dot{3}7\dot{0}, 0.\dot{7}0\dot{3}$. Then $0.\dot{0}7\dot{4}, 0.1, 0.1\dot{4}\dot{8}$.
7 b $0.\dot{0}09\dot{9}$ c $0.\dot{0}99\dot{0}, 0.\dot{9}90\dot{0}, 9.\dot{9}00\dot{9}$ Hence $\frac{99}{100} = 0.\dot{9}00\dot{9}$ d $0.\dot{0}19\dot{8}, 0.\dot{0}29\dot{7}, 0.\dot{0}39\dot{6}$
8 210 seconds

Chapter 3 answers

Exercise 3A

- 1 a factors of 8: 1, 8, 2, 4; factors of 12: 1, 12, 2, 6, 3, 4; HCF: 4
b factors of 6: 1, 6, 2, 3; factors of 20: 1, 20, 2, 10, 4, 5; HCF: 2
c factors of 14: 1, 14, 2, 7; factors of 9: 1, 9, 3; HCF: 1

- d** factors of 20: 1, 20, 2, 10, 4, 5; factors of 22: 1, 22, 2, 11; HCF: 2
e factors of 1: 1; factors of 8: 1, 8, 2, 4; HCF: 1
f factors of 5: 1, 5; factors of 15: 1, 15, 3, 5; HCF: 5
- 2 a** 6 **b** 3 **c** 12 **d** 1 **e** 4 **f** 13 **g** 6 **h** 1 **i** 10
- 3 a** multiples of 4: 4, 8, 12, 16, 20; multiples of 6: 6, 12, 18, 24, 30; LCM: 12
b multiples of 10: 10, 20, 30, 40, 50, 60; multiples of 12: 12, 24, 36, 48, 60; LCM: 60
c multiples of 3: 3, 6, 9, 12, 15, 18, 21; multiples of 7: 7, 14, 21; LCM: 21
d multiples of 5: 5, 10, 15, 20; multiples of 10: 10, 20, 30, 40; LCM: 10
e multiples of 1: 1, 2, 3, 4, 5; multiples of 5: 5, 10, 15, 20, 25; LCM: 5
f multiples of 6: 6, 12, 18, 24, 30; multiples of 15: 15, 30, 45, 60; LCM: 30
- 4 a** 24 **b** 72 **c** 17 **d** 60 **e** 49 **f** 180 **g** 12 **h** 72 **i** 120
- 5 a i** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30
ii 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 **iii** 5, 10, 15, 20, 25, 30
b 7, 11, 13, 17, 19, 23, 29 **c** 30, 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46, 48, 49, 50
- 6 a** 2 **b** 2, 3 **c** 3, 5 **d** 2, 3, 5 **e** 2, 11 **f** 2, 19 **g** 5
- 7 a** $10 = 2 \times 5$, $14 = 2 \times 7$ **b** $30 = 2 \times 3 \times 5$, $42 = 2 \times 3 \times 7$ **c** $8 = 2^3$, $9 = 3^2$
- 8 a** $1^2 + 2^2$ **b** $2^2 + 3^2$ **c** $1^2 + 4^2$ **d** $2^2 + 5^2$ **e** $1^2 + 6^2$ **f** $4^2 + 5^2$
- 9** 12 m **10** 20 days **11** 12:24 p.m. **12** 7, 11, 13
- 13** $140 = 3 + 137$, $142 = 3 + 139$, $144 = 5 + 139$, $146 = 7 + 139$, $148 = 11 + 137$,
 $150 = 11 + 139$, $152 = 13 + 139$, $154 = 17 + 137$, $156 = 17 + 139$, $158 = 19 + 139$, $160 = 11 + 149$
(Note: There may be other possibilities.)
- 14** 6, 10, 14, 15, 21, 35 *(Note: There may be other possibilities.)*
- 15** The factor that is squared to produce a square appears only once in the list of factors; all other factors appear in pairs.

Exercise 3B

- 1** power; base; index *or* exponent
- 2 a** 16 **b** 81 **c** 144 **d** 10000 **e** 4000000 **f** 625
- 3 a** 7^2 , 7^3 , 7^4 **b** 2^7 , 3^7 , 4^7 **c** $49 = 7^2$, $144 = 12^2$, $8 = 2^3$
(Note: There may be other possibilities.)
d i 64^1 **ii** 8^2 **iii** 4^3 **iv** 2^6
- 4 a** 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 **b** 5, 25, 125, 625
- 5 a** 2^5 **b** $3^4 \times 4^2$ **c** 3^1 or 3
- 6 a** 7^5 **b** 11^6 **c** 11^6 **d** 3^9 **e** 13^7 **f** 10^5
- 7 a** $7^1 = 7$ **b** 11^3 **c** 5^3 **d** 3^2 **e** $6^1 = 6$ **f** 6
g 5^5 **h** 7^9 **i** 5^5 **j** 2^9 **k** 2^{12} **l** 5^4
- 8 a** 2^6 **b** 3^{10} **c** 2^{10} **d** 11^6 **e** 5^6 **f** 7^8
- 9 a** 3^{13} **b** 8^7 **c** 5^8 **d** 10^{45} **e** 6^4 **f** 2^8 **g** 7^9 **h** 4^2 **i** 12^{12}
j 11^{63} **k** 4^{16} **l** 10^4
- 10 a** $10^4 = 10000$ **b** $20^3 = 8000$ **c** $10^3 \times 2^2 = 4000$
- 11 a** 1 **b** 1 **c** 2 **d** 0 **e** 7 **f** 36 **g** 7 **h** 1 **i** 1

Exercise 3C

- 1 a** 48 **b** 26 **c** 24 **d** 4 **e** 5832 **f** 72 **g** 100 **h** 5 **i** 10000 **j** 10
k 10000 **l** 190
- 2 a** 35 **b** 11 **c** 24 **d** 48 **e** 20 **f** 4 **g** 50 **h** 80 **i** 2 **j** 8
k 45 **l** 5
- 3 a** 56 **b** 137 **c** 40 **d** 13 **e** 57 **f** 32 **g** 574 **h** 1384 **i** 24
- 4 a** $4 \times (3 + 7) = 40$ **b** $(70 - 20) \div 5 = 10$ **c** $(2^2 + 6 \times 4) \div 2^2 = 7$
d $4 + 3^2 - 2 = 11$ **e** $(3 + 2^2) \times 2^3 - 7^2 = 7$ **f** $(4 + 4^4) \div (2^3 + 2) = 26$
- 5 a** $4 \times (2 + 7) = 36$ **b** $4 \times 2 + 7 = 15$ **c** $20 - (2 + 2) - (2 + 2) = 12$

$$\mathbf{d} \quad 20 - (2 + 2 - 2 + 2) = 16$$

$$\mathbf{g} \quad (2^2 + 2) \times 8 + 2^2 = 12$$

$$\mathbf{j} \quad (11 - 2 + 3)^2 = 144$$

$$\mathbf{e} \quad (70 - 20) \div 5 = 10$$

$$\mathbf{h} \quad 2^2 + 2 \times (8 \div 2^2) = 8$$

$$\mathbf{k} \quad (11 - 2) + 3^2 = 18$$

$$\mathbf{f} \quad 70 - (20 \div 5) = 66$$

$$\mathbf{i} \quad (2^2 + 2 \times 8 \div 2)^2 = 144$$

$$\mathbf{l} \quad 11 - (2 + 3^2) = 0$$

Exercise 3D

- 1 **a** 2896 is divisible by 2, 4 and 8. **b** 56374 is divisible by 2.
c 1858732 is divisible by 2 and 4. **d** 280082 is divisible by 2.
- 2 **a** 3765 is divisible by 5. **b** 68275 is divisible by 5 and 25.
c 444300 is divisible by 5 and 25. **d** 556875 is divisible by 5, 25 and 125.
- 3 **a** 5679 is divisible by 3 and 9. **b** 7425 is divisible by 3, 9 and 11.
c 71643 is divisible by 3 and 11. **d** 1727 is divisible by 11.
- 4 **a** 3798 is divisible by 6. **b** 5772 is divisible by 6 and 12.
c 9909 is divisible by neither 6 nor 12. **d** 48882 is divisible by 6.
- 5 **a** 672 is divisible by 2, 3, 4, 6, 8 and 12.
b 49395 is divisible by 3 and 5.
c 136290 is divisible by 2, 3, 5, 6, 10 and 11.
d 242010000437000361 is divisible by 3.
- 6 **a** 301032 **b** 301030 **c** 321030 **d** 301032 **e** 321330
f 391930 **g** 391732 **h** 321432 **i** 321231
(Note: There are many other possibilities.)
- 7 **a** 3 **b** 6 **c** 10 **d** 2 and 4
- 8 **a** 36450 is divisible by 15 and 18.
b 21942 is divisible by 18.
c 2041200 is divisible by 12, 15 and 18.
d 2007000000 is divisible by 12, 15 and 18.

Exercise 3E

- 1 **a** 2^3 **b** $2^3 \times 3$ **c** $3^2 \times 5$ **d** $3^4 \times 11$ **e** $3 \times 5 \times 7^2$ **f** $2^3 \times 3 \times 11^2$
- 2 **a** 36 **b** 432 **c** 27 **d** 1620 **e** 9 **f** 6480
- 3 **a** 9 **b** 180 **c** 7 **d** 336 **e** 120 **f** 216
g 18 **h** 4 **i** 35
- 4 **a** 15 (cube root) **b** 27 (square root), 9 (cube root) **c** 66 (square root)
- 5 **a** $3^2 \times 5^2$, 15 (square root) **b** $2^2 \times 3^4$, 18 (square root) **c** 2^9 , 8 (cube root)
d $2^6 \times 3^3$, 12 (cube root) **e** $2^3 \times 3^2 \times 5$ **f** $2^4 \times 11^2$, 44 (square root)
g $2^3 \times 3^3$, 6 (cube root) **h** $2^3 \times 11^3$, 22 (cube root)
- 6 30
- 7 $360 = 5 \times 3^2 \times 2^3$, 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360

Review exercise

- 1 C 2 D
- 3 **a** $8 + 88$ **b** $32 + 8$ **c** $176 + 8$ **d** $200 + 280$ **e** $8 + 288$
(Note: There are other possibilities.)
- 4 **a** 24 **b** 90 **c** 48 **d** 360 **e** 30 **f** 180 **g** 540 **h** 1020 **i** 600
- 5 **a** 3 **b** 15 **c** 1 **d** 10 **e** 7 **f** 4 **g** 6 **h** 6 **i** 21
- 6 **a** 36 **b** 121 **c** 2500 **d** 961 **e** 7569 **f** 20449

- 7 a 8^6 b 2^4 c 3^{15} d 4^{28} e 10^{12} f 7^{17} g 5^6 h 8^4 i 12^{14}
- 8 a B b C
- 9 a 36 seconds b 9 times
- 10 $28 = 2 \times 2 \times 7$; 29 is prime; $30 = 2 \times 3 \times 5$; 31 is prime; $32 = 2 \times 2 \times 2 \times 2 \times 2$; $33 = 3 \times 11$; $34 = 2 \times 17$; $35 = 5 \times 7$; $36 = 2 \times 2 \times 3 \times 3$; 37 is prime
- 11 a 2^8 , 16 b $3^2 \times 7^2$, 21 c $2^2 \times 17^2$, 34 d $3^2 \times 29^2$, 87 e $5^2 \times 3^6$, 135 f $2^8 \times 3^4$, 144
- 12 a $3^2 + 8^2$ b $4^2 + 11^2$ c $2^2 + 15^2$
- 13 a 64 is divisible by 4. b 1336 is divisible by 4.
c 3972 is divisible by both 3 and 4, and so also by 12.
- 14 a 21212 b 22210 c 10800 d 11132 e 24000 f 10010
(Note: There are many other possibilities!)
- 15 a 103 b 6553 c 39529 d 53353 e 616137 f 1
- 16 10:24 a.m.
- 17 a 4 b 4 c 4 d 4
- 18 a 6 b 6 c 6 d 6
- 19 a 1, 3, 37, 111 b 1, 7, 11, 13, 77, 91, 143, 1001 c 1, 11, 101, 1111
- 20 a 324 b 120

Challenge exercise

- 1 2340 2 328 020; there are 16 such numbers.
- 3 a i 187979 ii 24453 iii 18204978 iv 183704778
b The long multiplication algorithm makes the process clear.

$$\begin{array}{r}
 5 \ 3 \ 8 \ 9 \\
 \times \quad 1 \ 1 \\
 \hline
 5 \ 3 \ 8 \ 9 \\
 5 \ 3 \ 8 \ 1 \ 9 \ 0 \\
 \hline
 5 \ 9 \ 2 \ 7 \ 9
 \end{array}$$

- 4 301 or 721 5 19 employees, each works 7 hours 6 34 and 35
- 7 24000000000 hours later it will be 7 p.m., so 4 hours before that it will be 3 p.m.
- 8 25201 9 31 10 128 and 78125
- 11 a $2^9 \times 3^5 \times 5^1 \times 7^3 \times 13^1 \times 17^2 \times 19^1$

- b 11 students. For ages that are not prime numbers, we use the factors and multiply. Ensure you have used each prime factor only as often as it occurs in the prime factorisation.

Age	Prime factors	Number of students
13	13	1
14	$(2 \times 7)(2 \times 7)(2 \times 7)$	3
15	(3×5)	1
16	$(2 \times 2 \times 2 \times 2)$	1
17	(17)	2
18	$(2 \times 3 \times 3)(2 \times 3 \times 3)$	2
19	19	1

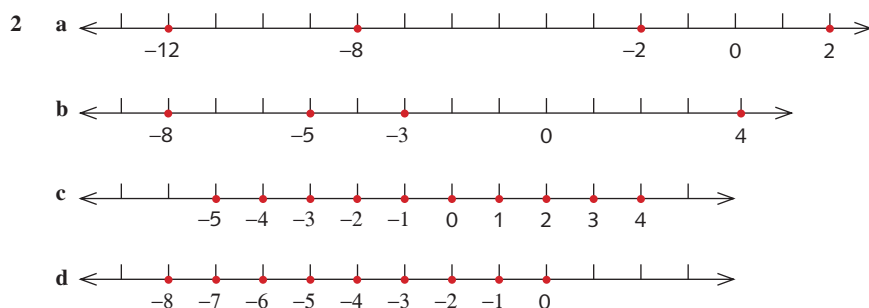
- 12 360 13 a 501 b 5541

- 14 a $m = 6, n = 4$ b $a = 1, b = 2, c = 4, d = 5, e = 3$ c $a = 3, b = 2, c = 4, d = 5, e = 6, f = 1$
 15 66 or 318 or 402 16 $729 = 3^6$

Chapter 4 answers

Exercise 4A

- 1 a $-21, -20, -19, -18, -17$
 b $-113, -112, -111, -110, -109, -108, -107, -106$
 c $-7, -6, -5, -4, -3, -2, -1, 0, 1, 2$
 d $-123, -122, -121, -120, -119, -118$



- 3 $-7, -4, -1$ 4 $3, -1, -5$ 5 $-29, -22, -15, -8$
 6 a 0°C b -25°C c -5°C d -15°C
 7 a -6 b 3 c -34 d 5 e 72 f -67
 g 456 h -10000 i 3500 j -7800 k 45000 l -76000

Exercise 4B

- 1 a -1 b 4 c -4 d 4 e 5 f 4 g 9 h 7 i -8 j 0 k -7
 l -9 m 0 n -37 o 39 p -6 q -63 r -90 s 255 t 66
 2 a -11 b -2 c -8 d -8 e -5 f -10 g -6 h -3 i -7 j -2 k -16
 l -38 m -27 n -19 o -6 p -93 q -5 r -96 s -36 t -9 u -128 v -280
 3 a -400 b -420 c -170 d 115 e -465 f 200 g -1200 h -1000 i -4600 j -2000
 k -500 l -7500
 4 a 6 b 14 c 0 d 10 e 0 f 7 g 2 h -28

Exercise 4C

- 1 a 5 b -5 c 7 d -18 e 4 f -10 g -9 h -17 i 7 j -8
 k -17 l -16
 2 a -3 b -6 c -11 d 16 e -20 f -2 g -2 h 17 i -16 j -9
 k -8 l 41
 3 a -1 b -2 c -3 d -3 e -5 f 9 g -3 h -4 i -9
 4 a -40 b -18 c 13 d -2 e 4 f 13 g -29 h 78 i 20 j 28
 k -20 l 60
 5 a 356 b -604 c 132 d -6105
 6 16°C 7 An increase of 11°C

8

Minimum	Maximum	Increase
-11°C	6°C	17°C
-16°C	7°C	23°C
-35°C	-4°C	31°C
-25°C	-15°C	10°C
-7°C	-2°C	5°C
-13°C	2°C	15°C
-11°C	5°C	16°C

9 -10°C

10 a 35 points

b 0 points

c -5 points

Exercise 4D

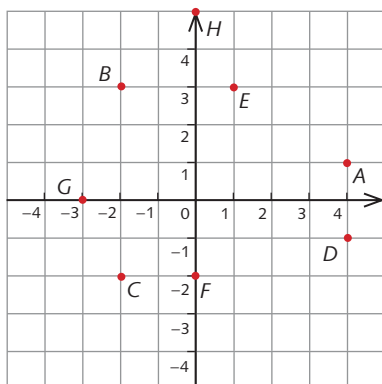
- 1 a -12 b -14 c -70 d -48 e -216 f -15 g -24
h -60 i -77 j -60 k 140 l -64 m 140 n 196
o -171 p -100 q 180 r 108 s -300 t 300
- 2 a -6 b -14 c -7 d -9 e -30 f -6 g -18
h -5 i -7 j -40 k 13 l 9 m 5 n 11
o 8 p 18 q 4 r 60 s -28 t -35
- 3 a -4 b 6 c -4 d -3
- 4 a 5 b -5 c -6 d 15 e -34 f -122 g -16
h -99 i -7 j -30 k 4 l -1441
- 5 a 84 b 280 c 2160 d 3600 e -160 f -16 g 4950 h -600
- 6 a $2 \times (-25) = -50$ b $5 \times (-15) = -75$ c $-7 \times (-9) = 63$ d $-9 \times (-8) = 72$
e $80 \div (-8) = -10$ f $-45 \div (-9) = 5$ g $-321 \div (-107) = 3$ h $5664 \div (-8) = -708$

Exercise 4E

- 1 a 81 b -49 c 75 d -144 e -6400 f 169
g -216 h 81 i -3125 j 117649 k -1
- 2 a 9 b 35 c -14 d 21 e -18 f 175 g -62 h -150
- 3 a 24 b -22 c -20 d -54 e -45 f 30 g -87 h -560
- 4 a 24 b 27 c 7 d -59 e -20 f 33 g 120 h -162 i 136 j -652 k -100
- 5 a 90 b 14 c -35 d -48 e -56 f -349 g -51 h -188
- 6 a -1 b -36 c 18 d -16
- 7 a 370 b -400000 c -2900 d 4000
- 8 -\$1200 9 a \$24000 b -\$16000

Exercise 4F

1



2 $A(1, 1)$, $B(2, -3)$, $C(0, 6)$, $D(-4, 0)$, $E(-4, -2)$, $F(-4, 5)$, $G(4, -4)$

3 a square, 9 cm^2

b triangle, 18 cm^2

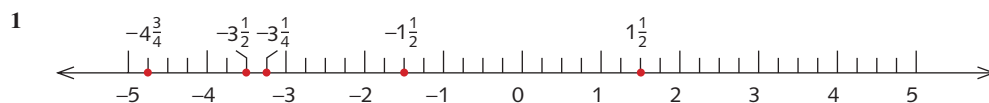
c trapezium, 71.5 cm^2

d right-angled triangle, 7.5 cm^2

e AO is perpendicular to BC .

f parallelogram, 8 cm^2

Exercise 4G



2 a $-2 < -\frac{7}{5} < -1 < \frac{1}{4} < \frac{1}{2} < 1 < -\frac{5}{3}$

b $-\frac{11}{5} < -\frac{7}{4} < \frac{14}{5} < \frac{7}{2} < -\frac{11}{3}$

c $-\frac{11}{12} < -\frac{4}{5} < -\frac{2}{3} < \frac{12}{13}$

d $-2 < -1\frac{11}{13} < -\frac{15}{13} < -1$

3 a $2\frac{1}{2}$ b $2\frac{1}{2}$ c -1 d $-\frac{1}{3}$ e $-1\frac{1}{2}$ f $-\frac{1}{15}$ g $-\frac{1}{2}$ h $-\frac{3}{5}$ i $2\frac{1}{4}$ j $-3\frac{3}{5}$

k $-2\frac{5}{18}$ l $-2\frac{1}{14}$ m $-\frac{1}{2}$ n $1\frac{1}{2}$ o $-\frac{11}{15}$ p $-\frac{5}{12}$ q $-1\frac{7}{20}$ r $-\frac{5}{12}$ s $-\frac{11}{20}$ t $-\frac{17}{35}$

u $2\frac{17}{36}$ v $-3\frac{31}{35}$ w $-4\frac{9}{91}$ x $\frac{37}{56}$

4 a $-1367\frac{1}{2}$ b $-2107\frac{1}{3}$ c $789\frac{1}{7}$

5 a $-\frac{3}{4}$ b $\frac{5}{24}$ c $\frac{55}{108}$ d $1\frac{7}{20}$ e $-\frac{11}{12}$ f $-\frac{5}{6}$ g $-1\frac{1}{2}$ h $-1\frac{2}{9}$ i $-\frac{5}{9}$

j $-\frac{7}{12}$ k $-\frac{7}{16}$ l $-\frac{5}{6}$

6 a -6 b $-1\frac{1}{8}$ c $-\frac{3}{5}$ d $-\frac{7}{12}$ e $-\frac{9}{11}$ f $1\frac{1}{2}$ g $-\frac{9}{10}$ h $4\frac{2}{7}$ i 4 j $\frac{1}{14}$

k $\frac{27}{28}$ l $-7\frac{1}{7}$

7 a $-\frac{19}{30}$ b $-3\frac{23}{72}$ c $\frac{7}{8}$ d $-1\frac{7}{12}$ e $-\frac{1}{4}$ f $-2\frac{1}{4}$ g $5\frac{1}{2}$ h 0 i -5

8 a $2\frac{1}{2}$ b 0 c 2 d $-6\frac{1}{2}$ e 0 f $-1\frac{9}{20}$

9 a $\frac{1}{4}$ b $-\frac{1}{8}$ c $\frac{1}{16}$ d $-\frac{1}{32}$ e $\frac{1}{2}$ f -2 g -3 h 3

Review exercise

1 a 23 b -14 c 15 d -95 e -16 f -68 g -180 h -60 i 70 j -80
k -160 l 85

2 -10

3 a 25°C b -25°C

4 a 27°C b -27°C

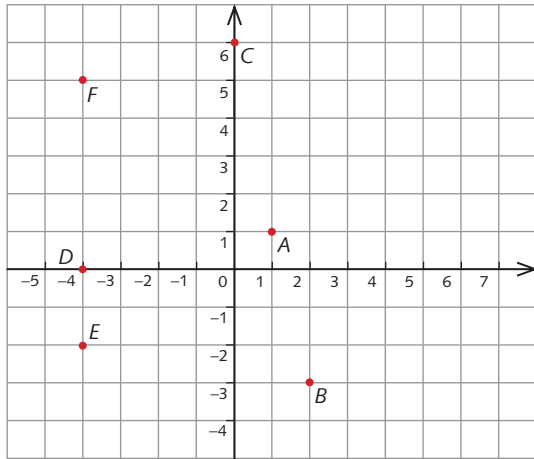
5 a -250 b -396 c -1750 d 2040 e -48 f 1150 g 3200 h 500 i -440 j -2400
k 128 l -200

6 a -25 b -4 c -7 d 17 e 4 f 5 g 8 h 150 i -3 j -6
k 8 l -104

7 a 4 b -63 c -60 d 60 e -168 f -16 g 16 h 49 i -90

8 -56 9 -6

10



11 $A(4, 4)$, $B(-4, 3)$, $C(-4, -3)$, $D(3, -3)$, $E(1, 1)$, $F(0, -4)$, $G(-2, 0)$

12 a $2\frac{1}{2}$ b $4\frac{1}{2}$ c $-\frac{10}{13}$ d $-\frac{1}{4}$ e $-1\frac{1}{4}$ f $5\frac{6}{11}$

13 a $-\frac{1}{6}$ b -30 c $-1\frac{1}{7}$ d 1 e $-\frac{3}{10}$ f $\frac{1}{3}$ g $-\frac{1}{10}$ h $-\frac{5}{24}$ i $\frac{1}{36}$

14 644 V

15 a 1 b -1 c 1 d -1 e 1

Clearly, -1 to an even power is 1, whereas -1 to an odd power is -1 .

16 220 m

17 a $12\frac{1}{2}$ b -3 c -500 d 0

18 a $\frac{5}{8}$ b $\frac{7}{8}$ c $-\frac{1}{8}$

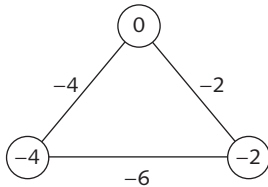
19 a $2\frac{1}{4}$ b $-4\frac{3}{4}$ c $4\frac{3}{4}$ d $-4\frac{3}{8}$ e $-\frac{5}{14}$ f $-2\frac{4}{5}$ g $\frac{3}{14}$ h $1\frac{5}{14}$

20 a 11 b -11 c 10 d $\frac{1}{8}$

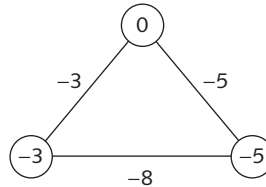
21 a 2 b -2 c $-\frac{2}{3}$ d $\frac{4}{9}$

Challenge exercises

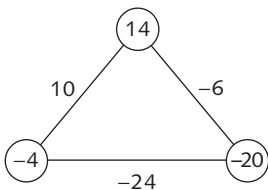
1 a



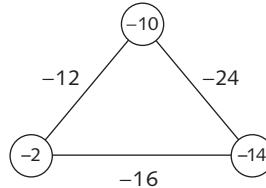
b



c



d



2 a $4 + (-5) - (-2) = 1$

b $4 + (-2) - (-5) = 7$

c $-2 + (-5) - (4) = -11$

3 a $-3 + 3 - (-4) = 4$

b $-3 + (-4) - 3 = -10$

c $-4 + 3 - (-3) = 2$

4 -2

5 a $(2 + -3) \times (3 + 4) = -7$ b $2 + (-3 \times 3) + (4 \times 2) = 1$ c $((2 - 5) \times 6 + 7) \times 6 - 5 = -71$

6

8	-6	4
-2	2	6
0	10	-4

7 a i -3 ii -3 b -50
 8 100 9 -6 10 a -10 b -50 11 -21
 12 a $\frac{1}{3}$ b $\frac{1}{10}$ c $-\frac{1}{10}$ d $-\frac{4}{45}$

Chapter 5 answers

Note: In the answers for this chapter, reasons have only been given for selected questions.
 Note that in geometry there are often two or more different valid arguments leading to the same conclusion.

Exercise 5A

- 1 a 152° (straight angle $\angle COB$) b 65° (adjacent angles at O) c 136° (adjacent angles at O)
 d 28° (straight angle $\angle XO B$) e 180° (straight angle $\angle AOB$) f 125° (adjacent angles at O)
 g 246° (revolution at O) h 130° (revolution at O) i 55° (straight angle $\angle BOJ$)
 j 188° (revolution at O) k 30° (revolution at O) l 60° (revolution at O)
 2 a $\angle SQT$ b $\angle VQU$ c $\angle AQB$ d $\angle NQO$
 3 a $\alpha = 56^\circ$ (vertically opposite angles at B)
 b $\beta = 90^\circ$ (vertically opposite angles at K),
 $\theta = 90^\circ$ (straight angle)
 c $\alpha = 48^\circ$, $\beta = 132^\circ$ d $\theta = 72^\circ$ e $\alpha = 55^\circ$, $\beta = 90^\circ$, $\gamma = 35^\circ$, $\theta = 55^\circ$ f $\alpha = \beta = \gamma = \theta = 60^\circ$
 g $\alpha = 65^\circ$, $\beta = 115^\circ$, $\gamma = 20^\circ$, $\theta = 125^\circ$ h $\alpha = 50^\circ$, $\beta = 55^\circ$
 4 a $\theta = 17^\circ$ (vertically opposite angles at V) b $\beta = 27\frac{1}{2}^\circ$ (straight angle $\angle JKL$) c $\alpha = 60^\circ$
 d $\alpha = 72^\circ$ e $\beta = 135^\circ$ f $\gamma = 33\frac{3}{4}^\circ$, $\theta = 135^\circ$ g $\alpha = 92\frac{1}{2}^\circ$ h $\alpha = 18^\circ$

Exercise 5B

- 1 a corresponding b alternate c co-interior d alternate e co-interior f corresponding
 g co-interior h alternate i corresponding j co-interior k corresponding l alternate
 2 a $\theta = 126^\circ$ (co-interior angles, $AB \parallel CD$)
 b $\theta = 136^\circ$ (alternate angles, $PQ \parallel RS$)
 c $\theta = 66^\circ$ (corresponding angles, $AB \parallel CD$)
 d $\gamma = 69^\circ$ (corresponding angles, $BK \parallel CL$)
 e $\alpha = 34^\circ$ (alternate angles, $RS \parallel TU$)
 f $\beta = 66^\circ$ (co-interior angles, $LM \parallel ON$)
 g $\theta = 57^\circ$ (co-interior angles, $DG \parallel EF$)
 h $\alpha = 28^\circ$ (alternate angles, $FG \parallel LM$)
 i $\alpha = 43^\circ$ (corresponding angles, $BX \parallel CY$)
 j $\gamma = 133^\circ$ (co-interior angles, $AX \parallel BY$)
 k $\beta = 77^\circ$ (alternate angles, $CA \parallel BD$)
 l $\beta = 90^\circ$ (corresponding angles, $RU \parallel ST$)

- 3 a $\beta = 107^\circ$ (co-interior angles, $FI \parallel GH$),
 $\gamma = 107^\circ$ (corresponding angles, $FG \parallel IH$)
b $\alpha = \beta = 64^\circ, \gamma = 116^\circ$ (various arguments)
c $\alpha = \beta = \gamma = \theta = 112^\circ$ (various arguments)
d $\alpha = 119^\circ$ (co-interior angles, $PQ \parallel SR$),
 $\beta = 61^\circ$ (co-interior angles, $PS \parallel QR$),
 $\gamma = 119^\circ$
e $\theta = 90^\circ$ (co-interior angles, $MN \parallel PO$),
 $\gamma = 108^\circ$ (co-interior angles, $MN \parallel PO$)
f $\gamma = 37^\circ$ (alternate angles, $BC \parallel ML$),
 $\theta = 78^\circ$ (alternate angles, $BC \parallel ML$)
g $\alpha = \beta = \gamma = 133^\circ$ (various arguments)
h $\alpha = 74^\circ$ (corresponding angles, $AC \parallel BX$),
 $\beta = 35^\circ$ (alternate angles, $AC \parallel BX$)
- 4 a $\angle PBV = 81^\circ$ (alternate angles, $PQ \parallel VB$),
 $\angle AVB = 81^\circ$ (alternate angles, $AV \parallel BP$)
b $\angle WAV = 123^\circ$ (co-interior angles, $WB \parallel AV$),
 $\angle AVB = 57^\circ$ (co-interior angles, $AW \parallel BV$)
c $\angle RAV = 45^\circ$ (alternate angles, $RS \parallel VA$),
 $\angle AVB = 135^\circ$ (co-interior angles, $RA \parallel BV$)
d $\angle QAV = 28^\circ$ (alternate angles, $PQ \parallel AV$),
 $\angle AVB = 28^\circ$ (alternate angles, $AQ \parallel BC$)

Exercise 5C

- 1 a $\alpha = 73^\circ$ (straight angle $\angle ABC$),
 $\theta = 107^\circ$ (alternate angles, $AC \parallel DE$)
b $\theta = 100^\circ$ (corresponding angles, $AX \parallel BY$),
 $\beta = 80^\circ$ (straight angle $\angle MAB$)
c $\alpha = 35^\circ$ (vertically opposite angles at Q),
 $\beta = 35^\circ$ (corresponding angles, $AB \parallel CD$),
 $\gamma = 145^\circ$ (straight angle $\angle CRD$)
d $\alpha = \beta = \gamma = 55^\circ, \theta = 125^\circ$
e $\alpha = 146^\circ$ (co-interior angles, $AB \parallel CD$),
 $\theta = 124^\circ$ (revolution at D)
f $\alpha = 66^\circ, \beta = 26^\circ, \gamma = 88^\circ$
g $\alpha = 73^\circ, \beta = 34^\circ, \gamma = 73^\circ$
h $\alpha = 90^\circ, \beta = 55^\circ, \gamma = 35^\circ$
- 2 a $\angle PON = 125^\circ$ (co-interior angles, $PA \parallel ON$),
 $\angle QON = 135^\circ$ (co-interior angles, $QB \parallel ON$),
 $\angle POQ = 100^\circ$ (revolution at O)
b 116° c 133° d 63°
- 3 a 100° b 54° c 130° d 88° e 76° f 42° g 88° h 40°
- 4 a $\gamma = 46^\circ$ b $\theta = 64^\circ$ c $\alpha = 90^\circ$ d $\theta = 18^\circ$
e $\beta = 30^\circ$ f $\beta = 66^\circ$ g $\gamma = 90^\circ, \theta = 62^\circ$ h $\alpha = 18^\circ$

Exercise 5D

- 1 a $HJ \parallel KM$ (co-interior angles are supplementary)
b $AB \parallel CZ$ (alternate angles are equal)
c $FP \parallel GQ$ (corresponding angles are equal)

- d** $AD \parallel EH$ (corresponding angles are equal)
e $RS \parallel UT$ (co-interior angles are supplementary),
 $RU \parallel ST$ (co-interior angles are supplementary)
f $DE \parallel GF$ (alternate angles are equal),
 $DG \parallel EF$ (alternate angles are equal)
g $DF \parallel AC$ (corresponding angles are equal),
 $AV \parallel BF$ (alternate angles are equal)
h $KL \parallel NM$ (co-interior angles are supplementary)
2 a $AB \parallel CD$ (corresponding angles are equal),
 $\beta = 54^\circ$ (corresponding angles, $AB \parallel CD$)
b $AB \parallel CD$ (alternate angles are equal),
 $\alpha = 100^\circ$ (alternate angles, $AB \parallel CD$)
c $AB \parallel CD$ (alternate angles are equal),
 $\gamma = 28^\circ$ (alternate angles, $AB \parallel CD$)
d $AB \parallel CD$ (co-interior angles are supplementary),
 $\alpha = 109^\circ$ (co-interior angles, $AB \parallel CD$)
3 a $\alpha = 66^\circ$ **b** $\theta = 53^\circ$ **c** $\gamma = 13^\circ$ **d** $\beta = 37^\circ$

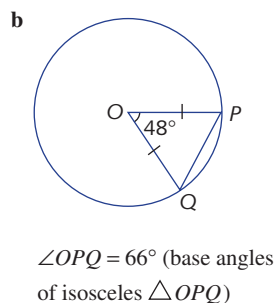
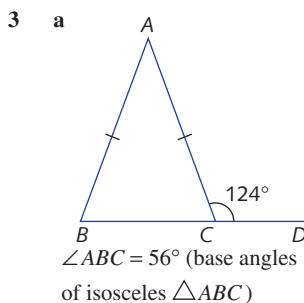
Exercise 5E

- 1 a** 90° **b** 30° **c** 58° **d** 28° **e** 94°
2 a $\theta = 38^\circ$ (angle sum of $\triangle ABJ$)
b $\beta = 57^\circ$ (angle sum of $\triangle CTU$)
c $\alpha = 65^\circ$ (angle sum of $\triangle ABC$)
d $\beta = 136^\circ$ (angle sum of $\triangle IBT$)
e $\angle BIR = 110^\circ$ (vertically opposite angles at I),
 $\beta = 27^\circ$ (angle sum of $\triangle BIR$)
f $\angle POQ = 69^\circ$ (angle sum of $\triangle OPQ$),
 $\alpha = 69^\circ$ (vertically opposite angles at O)
g $\theta = 90^\circ$
h $\gamma = 60^\circ$
3 a $\alpha = 120^\circ$ (exterior angle of $\triangle ABC$)
b $\gamma = 25^\circ$ (exterior angle of $\triangle EGL$)
c $\beta = 45^\circ$ (exterior angle of $\triangle ETX$)
d $\angle AOT = 58^\circ$ (exterior angle of $\triangle AOT$),
 $\alpha = 58^\circ$ (vertically opposite angles at O)
e $\theta = 23^\circ$ (exterior angle of $\triangle ADE$)
f $\angle LKC = 94^\circ$ (straight angle $\angle JKL$),
 $\theta = 118^\circ$ (exterior angle of $\triangle CKL$)
g $\beta = 159^\circ$
h $\gamma = 115^\circ$
4 a $\alpha = 45^\circ$ (straight angle $\angle LMN$),
 $\beta = 135^\circ$ (exterior angle of $\triangle MNT$)
b $\theta = 108^\circ, \gamma = 107^\circ$ **c** $\alpha = 75^\circ, \beta = 51^\circ$ **d** $\theta = 95^\circ$ **e** $\gamma = 100^\circ$
f $\alpha = 120^\circ, \theta = 84^\circ$ **g** $\alpha = 98^\circ, \beta = 115^\circ$ **h** $\alpha = 81^\circ, \beta = 116^\circ$
5 a $\alpha = 15^\circ$ **b** $\alpha = 30^\circ$ **c** $\beta = 26\frac{1}{2}^\circ$ **d** $\alpha = 40^\circ$
e $\theta = 62^\circ$ **f** $\theta = 18^\circ$ **g** $\beta = 67^\circ$ **h** $\gamma = 14^\circ$
6 a $\alpha = 58^\circ$ (alternate angles, $AR \parallel CD$),
 $\beta = 28^\circ$ (alternate angles, $CA \parallel DR$),
 $\gamma = 94^\circ$ (angle sum of $\triangle ADR$)

- b** $\alpha = 50^\circ$ (co-interior angles, $FI \parallel GH$),
 $\beta = 50^\circ$ (alternate angles, $FI \parallel GH$),
 $\gamma = 10^\circ$ (angle sum of $\triangle FIH$)
c $\alpha = 75^\circ, \theta = 30^\circ$ **d** $\alpha = 55^\circ, \beta = \gamma = 60^\circ$ **e** $\beta = \theta = 55^\circ$
f $\alpha = \beta = 58^\circ, \gamma = 42^\circ$ **g** $\alpha = 36^\circ, \beta = 54^\circ, \gamma = 90^\circ$ **h** $\alpha = 152^\circ, \beta = 68^\circ, \gamma = 84^\circ$
7 **a** $\angle APQ = 97^\circ$ (angle sum of $\triangle APQ$),
 $\angle AVB = 97^\circ$ (corresponding angles, $PQ \parallel VB$)
b 25° **c** 107° **d** 70° **e** 36° **f** 119° **g** 8° **h** 72°

Exercise 5F

- 1** **a** $\alpha = 65^\circ$ (base angles of isosceles $\triangle ABC$)
 $\beta = 50^\circ$ (angle sum of $\triangle ABC$)
b $\angle A = \gamma$ (base angles of isosceles $\triangle ABC$),
 $\gamma = 45^\circ$ (angle sum of $\triangle ABC$)
c $\alpha = \beta$ (base angles of isosceles $\triangle FGH$),
 $\alpha = \beta = 66^\circ$ (exterior angle of $\triangle FGH$)
d $\alpha = 36^\circ$ (angle sum of isosceles $\triangle PQR$)
e $\alpha = \beta = 60^\circ$ (equilateral $\triangle FGH$)
f $\alpha = 42^\circ$ (angle sum of $\triangle ABM$),
 $\angle C = 48^\circ$ (base angles of isosceles $\triangle ABC$),
 $\beta = 42^\circ$ (angle sum of $\triangle ACM$)
2 **a** isosceles: $AB = AC$ (opposite angles of $\triangle ABC$ are equal)
b not isosceles: $\angle A = 52^\circ$ (angles are all different)
c isosceles: $\angle C = 64^\circ$ (angle sum of $\triangle ABC$),
 $AB = AC$ (opposite angles of $\triangle ABC$ are equal)
d isosceles: $\angle A = 36^\circ$ (exterior angle of $\triangle ABC$),
 $AC = BC$ (opposite angles of $\triangle ABC$ are equal)
e not isosceles: $\angle C = 76^\circ$ and $\angle BAC = 38^\circ$
f not isosceles



- 4** **a** $\alpha = 39^\circ$ **b** $\alpha = 72^\circ, \beta = 90^\circ$ **c** $\alpha = 25^\circ, x = 7$ **d** $\alpha = 45^\circ, x = 17$
e $\beta = 45^\circ, y = 11$ **f** $\alpha = \gamma = 60^\circ, \beta = 120^\circ, x = y = 15$
5 **a** $\alpha = 110^\circ$ **b** $\alpha = 66^\circ$ **c** $\beta = 36^\circ$ **d** $\alpha = 56^\circ, \beta = 62^\circ$
e $\beta = 31^\circ$ **f** $\alpha = 90^\circ$

Exercise 5G

- 1** **a** $\alpha = 68^\circ$ (angle sum of quadrilateral $PQRS$)
b $\alpha = 90^\circ$ (angle sum of quadrilateral $ABCD$)
c $\alpha = 110^\circ$ (angle sum of quadrilateral $ABCD$)
d $\beta = 65^\circ$ (angle sum of quadrilateral $PQRS$)
e $\alpha = 62^\circ$ (angle sum of quadrilateral $ABCD$)

- 2 a 30° b 90° c 110° d 240°
 3 a $\alpha = \gamma = 104^\circ, \beta = 76^\circ$ b $\alpha = 90^\circ, \beta = 126^\circ$ c $\alpha = 124^\circ$ d $\alpha = 90^\circ$
 4 a $\alpha = 136^\circ, \beta = \gamma = 44^\circ$ b $\alpha = 163^\circ$ c $\alpha = 50^\circ, \beta = 130^\circ, \gamma = 101^\circ, \theta = 40^\circ$
 d $\alpha = 99^\circ$ e $\alpha = 50^\circ, \beta = 57^\circ$ f $\beta = 49^\circ$
 5 a $\beta = 67^\circ$ b $\alpha = 72^\circ$ c $\alpha = 135^\circ$ d $\alpha = 42^\circ$ e $\alpha = 17^\circ$ f $\alpha = 79^\circ, \beta = 138^\circ$
 6 a $90^\circ, 45^\circ, 45^\circ$ b $135^\circ, 22.5^\circ, 22.5^\circ$

Review exercise

- 1 a $\theta = 75^\circ$ (corresponding angles, $AB \parallel SD$)
 b $\beta = 90^\circ$ (co-interior angles, $QP \parallel SR$)
 c $\alpha = 55^\circ$ (corresponding angles, $AB \parallel CD$)
 d $\alpha = 20^\circ$ (vertically opposite angles),
 $\beta = 20^\circ$ (corresponding angles, $AB \parallel CD$),
 $\gamma = 160^\circ$ (straight angle $\angle CRD$)
 e $\beta = 90^\circ$ (co-interior angles, $LM \parallel ON$)
 f $\alpha = 71^\circ$ (co-interior angles, $AB \parallel CD$),
 $\beta = 109^\circ$ (co-interior angles, $AD \parallel BC$)
 2 a 60° b 27° c 16° d 6°
 3 a 53° b 104° c 46° d 165°
 4 a $\theta = 110^\circ$ b $\alpha = 33^\circ$
 5 a $\alpha = 100^\circ$ b $\alpha = 26^\circ$ c $\alpha = 126^\circ, \beta = 37^\circ$ d $\alpha = 106^\circ, \beta = 20^\circ$
 e $\alpha = 80^\circ, \beta = 40^\circ, \gamma = 60^\circ$ f $\alpha = 111^\circ, \beta = 69^\circ$
 6 a $\alpha = 112^\circ$ b $\alpha = 78^\circ, \beta = 58^\circ$ c $\alpha = 20^\circ$ d $\alpha = 76^\circ, \beta = 104^\circ, \gamma = 52^\circ$
 e $\alpha = 36^\circ$ f $\alpha = 130^\circ, \beta = 50^\circ, \gamma = 130^\circ$
 7 a 82° b 73° c 76° d 32°
 8 If any angle is reflex, then the angle sum is greater than 180° .
 9 a $\alpha = 60^\circ$ (angle sum of $\triangle ABC$)
 b $\beta = 80^\circ$ (exterior angle of $\triangle ABC$),
 $\gamma = 170^\circ$ (exterior angle of $\triangle ABX$)
 c $\beta = 85^\circ$ (alternate angles, $AE \parallel CB$),
 $\gamma = 70^\circ$ (exterior angle of $\triangle ABC$)
 10 360°

Challenge exercise

- 1 $\alpha = 95^\circ$ 2 a $\alpha = 40^\circ$ b $\alpha = \theta + (\gamma - \beta)$ c $\alpha = (180 - \theta) + (\beta - \gamma)$
 3 $\alpha = 2\beta$
 4 a 360° b 540° c 720° d 900° e 1080° f 1800°
 sum of internal angles = (number of sides $- 2$) $\times 180^\circ$
 5 $\triangle AFG, \triangle ABC, \triangle BDK, \triangle CEL, \triangle JBC, \triangle JBH, \triangle JCI$
 6 $\angle ABC = 71^\circ$ 7 30° or 150°

Chapter 6 answers

Exercise 6A

- 1 a 7 b 8 c 1 d 7 e 18 f 2 g 5 h $1\frac{3}{4}$
 2 a -3 b 12 c -1 d -3 e -2 f -22 g 1 h -50
 3 a 9 b 9 c -9 d -27 e 27 f 81 g 27

- 4 a 3 b 3 c -1 d -1 e 29 f -7 g 3 h 1 i -5 j -5
k 4 l -8
- 5 a $\frac{1}{9}$ b $3\frac{8}{9}$ c -7 d 7
- 6 a 4 b -2 c -7 d 2 e 12 f 5 g -3 h 4 i 3 j -4 k -9
- 7 a 7 b 9 c 35 d -17 e 133 f -9 g 3 h 88 i -722

Exercise 6B

- 1 a $m = 7$ b $a = 3$ c $x = 50$ d $m = 2\frac{3}{4}$ e $x = 3$
f $m = 2$ g $x = 18$ h $x = 20$ i $n = 4$
- 2 a $x = -3$ b $x = -10\frac{1}{2}$ c $x = -2\frac{1}{2}$ d $x = -5$ e $x = -3\frac{1}{2}$
f $x = -\frac{4}{5}$ g $x = 54$ h $x = 60$ i $x = 40$
- 3 a 3 b 18 c -18 d -6 e -4 f 14
g -3 h 14 i 6
- 4 a -3 b -3 c -5 d $-5\frac{1}{3}$ e $8\frac{1}{3}$ f $2\frac{5}{9}$
g $3\frac{2}{5}$ h $8\frac{1}{3}$ i $5\frac{2}{3}$
- 5 a 36 b 30 c -80 d -20 e -18 f -32
- 6 a 3 b $2\frac{2}{5}$ c 2 d 4 e -4 f $-4\frac{3}{4}$
g $-4\frac{1}{5}$ h $8\frac{1}{5}$ i $7\frac{7}{10}$
- 7 a 4 b $5\frac{2}{5}$ c 5 d $1\frac{3}{5}$ e 12 f 45
- 8 a $x = 9$ b $a = 12$ c $z = 8$ d $b = 10\frac{7}{11}$ e $x = 24$ f $x = 70$
- 9 a $a + 5 = 21$, $a = 16$ b $7x = 35$, $x = 5$ c $5z = 37$, $z = 7\frac{2}{5}$ d $5m + 3 = 50$, $m = 9\frac{2}{5}$
e $\frac{n}{6} = 10$, $n = 60$ f $\frac{p}{3} - 5 = 23$, $p = 84$
- 10 a 1 b 1 c $-1\frac{1}{2}$ d $19\frac{1}{4}$ e $15\frac{2}{5}$
- 11 a $20 - x = 10$, $x = 10$ b $6 - 2m = 20$, $m = -7$ c $\frac{n}{8} + 6 = 20$, $n = 112$
d $7p + 10 = 60$, $p = 7\frac{1}{7}$ e $6 - x = -10$, $x = 16$ f $15 - 7y = -6$, $y = 3$
g $\frac{k}{10} - 7 = -1$, $k = 60$
- 12 a $x + 6 = 11$, $x + 10 = 15$ b $2x - 3 = 5$, $2x + 2 = 10$ c $7x + 6 = 10$, $7x = 4$ d $\frac{x}{3} = 4$, $x + 2 = 14$
- 13 a $\frac{x}{3} - 5 = 11$, $\frac{x}{3} = 16$ b $2x - 11 = 3$, $2x + 9 = 23$ c $5 - 2x = 7$, $-2x = 2$ d $x + 6 = 11$, $2x = 10$

Exercise 6C

- 1 a $6a - 2$ b $30p + 35$ c $12 - 4x$ d $3a - 6$ e $42 - 14x$ f $21 - 3x$
g $-20 + 4x$ h $-42 + 12p$ i $-15x + 6$ j $-24x + 12$ k $-6 + 3x$ l $-6x - 2$
m $-20p + 35$ n $-12 + 4x$ o $-2a + 4$ p $42 + 14x$
- 2 a $2x + 6$, 16 b $10x + 20$, 70 c $2x - 4$, 6 d $6x - 18$, 12
- 3 a 3 b 0 c 19 d 9
- 4 a 2 b -28 c -54 d 0
- 5 a -32 b -16 c 5 d -30 e -10 f -30

Exercise 6D

- 1 a $3\frac{1}{2}$ b $5\frac{2}{5}$ c $\frac{1}{3}$ d $1\frac{5}{6}$ e $\frac{2}{5}$ f $1\frac{2}{5}$ g $4\frac{1}{3}$ h $-\frac{7}{12}$ i $1\frac{1}{15}$ j $-1\frac{3}{5}$
- k $\frac{5}{12}$ l $1\frac{1}{9}$
- 2 a $3(6+x) = -10$, $x = -9\frac{1}{3}$ b $3(m-6) = 5$, $m = 7\frac{2}{3}$ c $2(10+p) = 4$, $p = -8$
- d $3(n-5) = 14$, $n = 9\frac{2}{3}$ e $2(x-3) = -10$, $x = -2$ f $-2(3+x) = -2$, $x = -2$
- g $-4(x-6) = -10$, $x = 8\frac{1}{2}$

Exercise 6E

- 1 like: a, b, c, d, g, h, k, l, o, p, s, t
unlike: e, f, i, j, m, n, q, r
- 2 a $7x$ b $-x$ c $3x$ d $5x$ e $-x$ f $-4x$
g $12x$ h $51m$ i $-9n$ j $97x$ k $9n$ l $-45p$
m $-20x$ n 0 o $13m+11n$ p $7m+4n$ q $7m+8n$ r $x+5y$
s $2x+y$ t $11m-9n$ u $7q-2p-5r$ v $7a-18b$
- 3 a $93xy$ b $21xy$ c $16xy^2$ d $7x+10y$ e $-5v-7z$ f $16y+4x$
g $7x^3+4y^3+5x^2$ h $-4x^2+13y^2$ i $3x^2+2xy$ j $-14x^2-9y^2$
- 4 a $7x+40$ b $5x+9$ c $7x+16$ d $14x+38$ e $-2x+55$ f $8x+11$
g $8x-42$ h $-2x+3$ i $6x+4$ j $26x+22$ k $11x+21$ l $18ax+16x$
- 5 a -3 b 4 c $1\frac{1}{2}$ d 5 e $\frac{4}{5}$ f -2
- 6 a 3 b 4 c 4 d $6\frac{2}{3}$
- 7 a $4\frac{1}{2}$ b -15 c -47 d $14\frac{1}{2}$ e $2\frac{9}{14}$ f $2\frac{16}{23}$

Exercise 6F

- 1 a $x+6=24$, $x=18$ b $5x=35$, $x=7$ c $3x=37$, $x=12\frac{1}{3}$
d $6x+4=48$, $x=7\frac{1}{3}$ e $\frac{x}{6}=20$, $x=120$ f $\frac{x}{7}-5=33$, $x=266$
- 2 -16 3 $-1\frac{3}{4}$ 4 $\$2.40$ 5 $4\frac{1}{5}$
- 6 7 and 35 7 $2\frac{1}{2}$ and $7\frac{1}{2}$ 8 4 9 88
- 10 92 11 $\$3.50$
- 12 Let $\$x$ be the cost of the apple. Then $\$(x+0.60)$ is the cost of the milkshake, and $\$(x+0.60+0.30)$ is the cost of the sausage roll, so $x+(x+0.60)+(x+0.60+0.30)=3.00$, and $x=0.50$. Hence the apple costs $\$0.50$, the milkshake costs $\$1.10$ and the sausage roll costs $\$1.40$.

Exercise 6G

- 1 a $12ab$ b $12x^2$ c $50xy$ d $77ab$ e $10a^2$ f $66c^2$ g $24mn$ h $77m^2n^2$ i $12n^2$ j $42m^3$
k $22m^4$ l $35a^2b$
- 2 a $25n^2$ b $16z^2$ c $256z^2$ d $9c^4$
- 3 a $8x^2$ b $21x^2$ c $6x^2$ d $6x^2y$ e $8x^2y^2$ f $12x^3y^2$
- 4 a $2x$ b $8x$ c $24y$ d $\frac{np}{5}$ e $3x$ f $18x$ g $18x$ h $\frac{9}{2}ab$

- 5 a $\frac{4}{y}$ b a c a d 4 e $3ab$ f a^2 g $\frac{1}{a}$ h $\frac{1}{a}$ i 1 j a
 k ab l b
- 6 a $\frac{4}{3}pq$ b $20ab$ c $20a^2b^2$
- 7 a $\frac{x^2}{3} \text{ cm}^2$ b 27 cm^2 8 a $\frac{x^2}{12} \text{ cm}^2$ b 48 cm^2
- 9 $\frac{6x}{5} \text{ cm}$ 10 a $\frac{10x^2}{3} \text{ cm}^2$ b 2430 cm^2
- 11 $\frac{64x^2}{y} \text{ cm}^2$ 12 840 tiles

Review exercise

- 1 a 2 b 2 c 7 d -4 e 14 f -5 g 18 h 28 i -2 j 14
 k 4 l $-\frac{5}{9}$
- 2 a -3 b 7 c -9 d -2 e 9 f -27 g -7 h 13 i -28 j -35
 k -1 l 125
- 3 a 125 b 5 c 14 d -14 e -66 f 4356 g -20 h 128
- 4 a $3a + 12$ b $18 - 6x$ c $2x + xy$ d $2x - 22$ e $4x - 12$ f $7x + 10$
 g $6a + ax + 21x$ h $x - 42$ i $18 - 6x$ j $-19x + 48$
- 5 a $x = -4$ b $x = -18$ c $x = -1$ d $a = -3$ e $m = -1$ f $m = 7$
- 6 a $x = 4$ b $a = -1$ c $x = 13$ d $x = \frac{4}{3}$ e $x = -1$ f $x = 7\frac{1}{3}$
- 7 a $8x^7y^5$ b $4x^5y^4$ c xy d mn e $16x^5y^3$ f $4ab^2$
- 8 $\$12$ 9 10 10 $x = 2$
- 11 $x = 4$ 12 $x = 2\frac{3}{4}$

Challenge exercise

- 1 a $8, 72, 70, 75, 50, 5$ c x
- 2 64 smaller cubes; 0 faces: 8 ; 1 face: 24 ; 2 faces: 24 ; 3 faces: 8
 In general, 0 faces: $(n-2)^3$; 1 face: $6(n-2)^2$; 2 faces: $12(n-2)$; 3 faces: 8
- 3 6 left: 33 ; 12 left: 51 . In general, $3x + 15$
- 4 child ticket: $\$2.14$; adult ticket: $\$4.28$
- 5 50 laps 6 $14, 41, 23, 32, 50$ 7 54
- 8 No. Let the two-digit integer be $10a + b$,
 where a and b are the digits of the tens and units places.
 Then
 $11(a + b) = 10a + b$
 $11a + 11b = 10a + b$,
 so $a + 10b = 0$, which is impossible because $0 < a < 10$ and $0 < b < 10$.

Chapter 7 answers

Exercise 7A

- 1 a 100% b 66% c 40% d 50% e 37.5% f 37.5%
- 2 a $\frac{1}{10}$ b $\frac{1}{4}$ c $\frac{17}{20}$ d $\frac{1}{2}$ e $\frac{2}{5}$ f $\frac{1}{5}$ g $1\frac{1}{4}$ h $6\frac{3}{10}$ i $2\frac{1}{20}$ j $1\frac{19}{50}$
 k $4\frac{24}{25}$ l $431\frac{1}{4}$

- 3 a 0.55 b 0.37 c 0.19 d 0.8 e 0.3 f 0.1 g 2.5 h 1.48 i 39.12 j 0.03
k 0.07 l 0.01
- 4 a 23% b 99% c 11% d 150% e 230% f 999% g 420% h 670% i 140% j 90%
k 50% l 10% m 67% n 81% o 33% p 99.9% q 12.3% r 70.2% s 999.9% t 123.4%
- 5 a 50% b 98% c 16% d 5% e 60% f 76% g 125% h 2.5% i 368% j 120%
k 250% l 125% m 7.5% n 240% o 42.5%
- 6 a 65% b 23% c 75% d 50% e 30% f 60% g 150% h 110% i 180% j 100%
k 7% l 2% m 105% n 409% o 703%
- 7 a $\frac{1}{40}$ b $\frac{3}{40}$ c $\frac{33}{400}$ d $\frac{3}{8}$ e $\frac{1}{8}$ f $\frac{5}{8}$ g $\frac{7}{8}$ h $\frac{2}{3}$

8

Decimal	Fraction	Percentage
0.5	$\frac{1}{2}$	50%
0.25	$\frac{1}{4}$	25%
0.75	$\frac{3}{4}$	75%
0.4	$\frac{2}{5}$	40%
1	$\frac{1}{1} = \frac{2}{2} = \dots$	100%
0.367	$\frac{367}{1000}$	36.7%
0.29	$\frac{29}{100}$	29%
0.403	$\frac{403}{1000}$	40.3%
0.375	$\frac{3}{8}$	37.5%
1.25	$\frac{5}{4}$	125%
2.75	$\frac{11}{4}$	275%
2.0	2	200%

- 9 a 30% b 25% c 80% d $66\frac{2}{3}\%$ e 5% f 40%
- 10 a 234 b 36 c 170 d 2910 e 100 f 182 g 72 h 840 i 350
- 11 a 20 b 40 c 200 d 150 e 1.5 f 45 g 63 h 390 i 17.1 j 200
k 236 l 7
- 12 a 34 b 102 c 238 d 17 e 8.5 f 25.5
- 13 a 80 b 240 c 40
- 14 a 45.6 b 136.8 c 91.2 d 45.6
- 15 a 9.6 m b 60 lollies c 81 children
- 16 a 7 songs b 112 stories c 24 students
- 17 36% 18 80% 19 62.5%

Exercise 7B

- 1 a $66\frac{2}{3}\%$ b $16\frac{2}{3}\%$ c $83\frac{1}{3}\%$ d $77\frac{7}{9}\%$ e $91\frac{2}{3}\%$ f $71\frac{3}{7}\%$

- 2 a $\frac{61}{500}$ b $\frac{27}{400}$ c $\frac{21}{400}$ d $\frac{91}{600}$ e $\frac{1}{15}$ f $\frac{61}{1200}$ g $\frac{73}{600}$ h $\frac{87}{400}$ i $\frac{74}{175}$ j $\frac{11}{175}$
 k $\frac{67}{150}$ l $\frac{89}{1200}$
- 3 a 2 b $\frac{3}{200}$ c $1\frac{6}{25}$ d $2\frac{1}{10}$ e $\frac{1}{200}$ f $1\frac{51}{200}$
- 4 a $0.1225 = \frac{49}{400}$ b $0.0675 = \frac{27}{400}$ c $0.0575 = \frac{23}{400}$ d $0.0806 = \frac{403}{5000}$ e $0.1205 = \frac{241}{2000}$
 f $2.27 = 2\frac{27}{100}$ g $117.9 = 117\frac{9}{10}$ h $57.11 = 57\frac{11}{100}$
- 5 a $33\frac{1}{3}\%$ b 28% c $33\frac{1}{3}\%$ d $22\frac{4}{5}\%$ e $25\frac{25}{67}\%$ f 14%
- 6 2nd test $86\frac{2}{3}\%$, 3rd test 85%, 1st test 84%
- 7 a 60% b 40%
- 8 80% 9 $39\frac{1}{16}\%$ 10 95%
- 11 a 1182 b The first drug is more effective ($98.5 > 97.5$).
- 12 a $\frac{1}{3}\%$ b $3\frac{1}{3}\%$
- 13 school C (highest), school A, school B
- 14 white: 37.5%; wholemeal: $23\frac{1}{3}\%$; multigrain: 32.5%; raisin: $6\frac{2}{3}\%$
- 15 a 0.25% b $15\frac{5}{8}\%$
- 16 a 17.48 kg b 17 kg c 13.8 kg d 12.24 kg

Exercise 7C

- 1 a $\frac{11}{20}$ b $\frac{9}{10}$ c $\frac{31}{50}$ d $\frac{149}{400}$
- 2 a 10% b $4\frac{1}{2}\%$ c 48% d $30\frac{3}{5}\%$ e 10% f $8\frac{1}{3}\%$ g 14% h 12% i $33\frac{1}{3}\%$
- 3 a 6% b 6%
- 4 a 2795 b 35%
- 5 a i 90 ii 240 iii 150 b 120
- 6 a 170 b 130 c 200
- 7 0.9%
- 8 a 65% b 5.25% c 29.75%
- 9 18% 10 12% 11 9.2% 12 22%
- 13 They are not percentages of the same whole.
- 14 The 40% of females come from all of the supporters, not just from those who wear red shirts.

Exercise 7D

- 1 a \$50 b \$100 c \$125 d \$500 e \$5000 f \$7500
- 2 a \$60 b \$120 c \$150 d \$600 e \$6000 f \$9000
- 3 a \$15 000 b \$30 000 c \$45 000
- 4 a \$6250 b \$12 500 c \$18 750
- 5 a \$10 500 b \$11 700 c \$12 300 d \$10 550 e \$10 525
- 6 a \$13 440 b \$13 200 c \$13 260 d \$15 000 e \$13 920
- 7 a \$34 375 b \$29 500 c \$28 937.50 d \$31 000 e \$30 250

Exercise 7E

- 1 a \$2889 b \$4928 c \$64.60 d \$531.25 e \$51.52 f \$60.45
2 \$72 3 \$17.60 4 \$207 5 \$20

Exercise 7F

- 1 a 856 b 1060 c 810 d 297 e $399\frac{3}{5}$ f 4905
g 6390 h 22 450 i 6230
2 a 100% increase b 300% increase c 400% increase d 10% increase e 25% increase
f $9\frac{1}{11}\%$ decrease g $33\frac{1}{3}\%$ increase h 20% decrease i 25% decrease
3 a \$54 000 b \$51 000 c \$36 000 d 52 080 hectares
4 $16\frac{2}{3}\%$ 5 55 800 hectares
6 a 450 b 5750 c 5170 d 6545 e 9100 f $13\,562\frac{1}{2}$
7 50 600 8 $23\frac{17}{21}\%$ 9 2 689 600
10 a 1 025 000 b 1 075 000
11 $46\frac{2}{13}\%$ 12 $33\frac{1}{3}\%$
13 a 80% b 75% c machine A
14 \$74 400 15 a \$572 000 b 11%
16 a 16% profit b 9% profit c 10% loss d 5% loss
17 a \$7600 loss b \$6400 profit c \$500 000 profit d \$105 000 loss
18 \$51 750

Exercise 7G

- 1 40
2 a \$187 560 b \$168 000 c \$518 400 d \$81 000
3 \$25 000 4 a 275 000 m² b 220 000 m²
5 \$120 000 6 1500
7 a 541 820 b 456 700 c 121 000 d 12 050
8 \$800 9 68% 10 \$160
11 a \$80 b \$62 c \$230 d \$85

Exercise 7H

- 1 4 years 2 5 years 3 8.75% 4 8.75% 5 7 years
6 3 years 7 6.5% 8 6% 9 5%

Review exercise

- 1 a 92% b 75% c $37\frac{1}{2}\%$ d $87\frac{1}{2}\%$ e 60% f $66\frac{2}{3}\%$
g 75% h 110% i $66\frac{2}{3}\%$ j $12\frac{1}{2}\%$ k 124% l 200%
2 a $\frac{1}{2}$ b $\frac{17}{20}$ c $\frac{43}{100}$ d 2 e $3\frac{2}{5}$ f $1\frac{17}{20}$
g $\frac{1}{8}$ h $6\frac{6}{25}$
3 a 8 b 4.5 c 46 d 27 e 194 f 18 g 3468 h 374 000 i 45
4 utes: 9801; four-wheel drives: 5346; sedans: 8019; station wagons: 6534
5 5% 6 6 g

- 7 a 375 b 210 c 1400 d 504 e 2625 f 40000 g 11 h 24 i 0.078
j 21 k 29985 l 3.3
- 8 a 37.5 mL b 19c
- 9 110 10 848 11 352.5 g 12 9924
- 13 3312 g 14 233.6 g 15 33.6 g 16 6 carbon, 1 nitrogen and 7 hydrogen atoms
- 17 \$100 18 \$25 19 \$55
- 20 a 112.5 mL b 28c
- 21 2400 22 5200 23 51% 24 18 25 6 years
- 26 \$4875

Challenge exercise

- 1 \$13.75 2 103.5 cm² 3 100000 4 50% 5 4960
- 6 250 mL 7 87.5 mL
- 8 a i 16 cm² ii 112 cm² b 25.6 cm² c 35.2 cm² d 27.5%

Chapter 8 answers

Exercise 8A

- 1 a 81 b 121 c 49 d 225
- 2 a 8 b 12 c 14 d 30
- 3 a 625 b 225 c 144 d 64
- 4 a 10 b 20 c 25 d 13 e 25 f 26

5

Row	a	b	a^2	b^2	$c^2 = a^2 + b^2$	c
1	3	4	9	16	25	5
2	6	8	36	64	100	10
3	9	12	81	144	225	15
4	12	16	144	256	400	20
5	15	20	225	400	625	25

Pattern: a increases by 3, b by 4 and c by 5

6

Row	a	b	a^2	b^2	$c^2 = a^2 + b^2$	c
1	3	4	9	16	25	5
2	5	12	25	144	169	13
3	7	24	49	576	625	25
4	9	40	81	1600	1681	41
5	11	60	121	3600	3721	61

Pattern: a increases by 2; the difference between b and c is 1.

- 7 a $x = 20$ b $x = 13, y = 15$ c $x = 26$
d $x = 30, y = 26$ e $x = 25, y = 26$ f $x = 15, y = 25, z = 20$

Exercise 8B

- 1 a 24 b 21 c 16 2 15 cm
- 3 a yes, $\angle ACB$ b no c yes, $\angle ABC$
- 4 a $x = 18$ b $x = 7$ c $x = z = 10, y = 6$ d $x = 12$ e $x = 13$
f $x = 25$ g $x = 5, y = 13, z = 84$ h $x = z = 12, w = y = 16$
- 5 390 cm 6 90 cm 7 31.2 m

- 8 a no b yes b yes
 9 40 km
 10 a 2 cm b i 8 cm ii 18 cm iii 50 cm iv $2n^2$

Exercise 8C

- 1 a 2.24 m b 4.90 m c 4.36 m d 1.73 m e 5.20 m f 3.46 m
 2 4 m 3 5.39 cm 4 16.97 cm 5 5.20 cm 6 $\sqrt{3}$ cm ≈ 1.73 cm
 7 $3\sqrt{2}$ cm ≈ 4.24 cm 8 5.39 km 9 2 m 10 5.39 km 11 3.61 km
 12 4.24 cm

Review exercise

- 1 a 34 cm b 30 cm c 25 cm d 20 m e 32 cm f 48 cm
 2 a no b yes, $\angle ABC$ c yes, $\angle ACB$ d yes, $\angle ABC$ e no f yes, $\angle CAB$
 3 20 km 4 15 m 5 2.65 m
 6 a 4.24 cm b 2.82 cm 7 4 m

Challenge exercise

- 1 $\sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$ 2 $\sqrt{3}x$ m 3 $\frac{\sqrt{3}}{4}x^2$ m²
 4 a $x = \sqrt{360} \approx 18.96$ cm b $x = 4$ cm, $y = \sqrt{84} \approx 9.16$ cm
 c $x = \sqrt{75} \approx 8.65$ cm d $x = \sqrt{160} \approx 12.64$ cm e $x = \sqrt{1305} \approx 36.12$ cm
 5 180 cm² 6 height = 3.46 cm, area = 6.93 cm²
 7 $AE = \sqrt{21} \approx 4.58$, $DE = \sqrt{12} \approx 3.64$, $DB = 4$ 8 7 m

Chapter 9 answers

9A Review

Chapter 1

- 1 588 2 4440 3 Buy 2 lots of 20 packets for \$19.00, \$7 saved
 4 85 5 1560 6 628
 7 Barry has 241 marbles, Andrew has 241 marbles.
 8 629 9 59 10 52437

Chapter 2

- 1 a $3\frac{4}{7}$ b $\frac{1}{5}$ c $2\frac{1}{8}$ d $\frac{2}{9}$ e $2\frac{2}{3}$ f $\frac{3}{80}$ g $\frac{8}{9}$ h $\frac{8}{9}$
 2 a $7\frac{9}{10}$ b $2\frac{5}{12}$ c $9\frac{11}{24}$ d $8\frac{11}{20}$ e $9\frac{1}{36}$ f $1\frac{5}{9}$
 3 a 90 b 160 c $3\frac{1}{2}$ d $7\frac{7}{8}$ e $20\frac{5}{6}$ f $\frac{10}{27}$ g $4\frac{2}{3}$ h $13\frac{5}{7}$
 4 $41\frac{2}{3}$ metres 5 $13\frac{1}{28}$ 6 33 7 \$700
 8 a $\frac{1}{2} < \frac{2}{3} < \frac{3}{4} < \frac{4}{5} < \frac{5}{6}$ b $\frac{1}{4} < \frac{11}{25} < \frac{6}{10} < \frac{13}{20} < \frac{4}{5}$
 9 a $\frac{1}{5}$ b $\frac{1}{5}$ 10 1 minute 4 seconds
 11 a 4.56 b 2.8 c 10.1 d 5.75 e 37.8758 f 8083.39

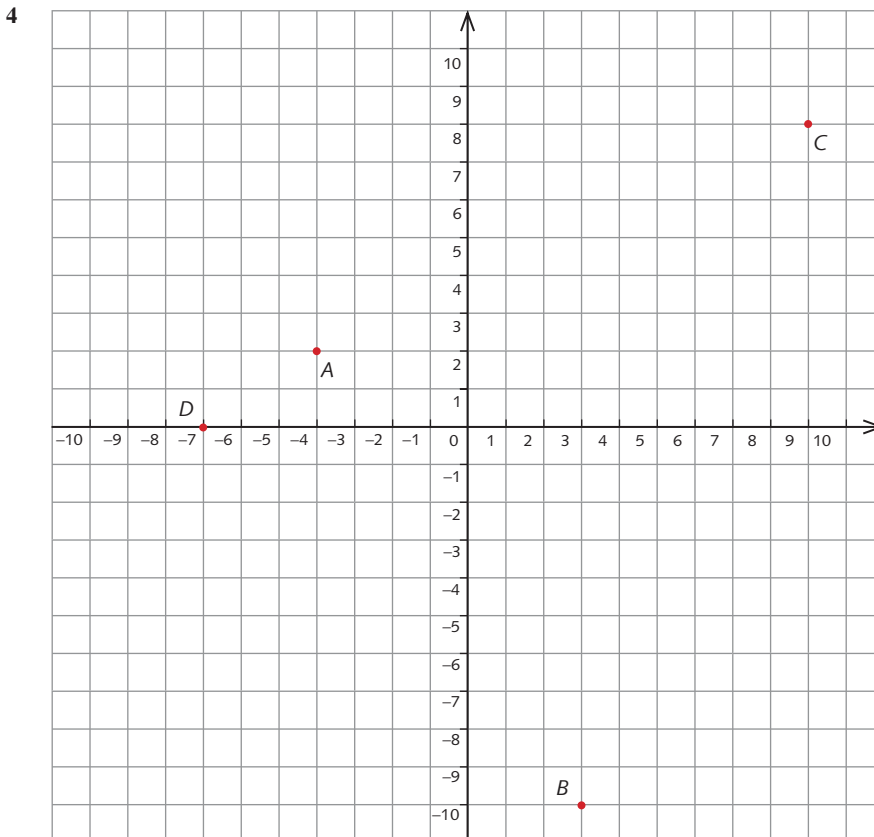
Chapter 3

- 1 a 1, 2, 3, 4, 6, 12 b 1, 17
 c 1, 2, 3, 5, 6, 9, 10, 15, 18, 27, 30, 45, 54, 90, 135, 270
 d 1, 3, 5, 9, 15, 27, 45, 135 e 1, 2, 29, 58 f 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144
- 2 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157
- 3 a 35 b 144 c 5610 d 300
- 4 a 1 b 3 c 6 d 10
- 5 a product of 24 and 76 = 1824, $\text{HCF} \times \text{LCM} = 4 \times 456 = 1824$
 b product of 102 and 54 = 5508, $\text{HCF} \times \text{LCM} = 6 \times 918 = 5508$
- 6 a 3 b 5 c 10 d 2 e 81 f 6 g 1 h 8 i 3
- 7 a 64 b 11 c 56 d 25 e 4096 f 47 g 15625
- 8 For example:
 a 25 671, 19914, 456 b 73 656, 1008, 2226 c 1020, 121 980, 59 220
 (Note: There are more possibilities.)
- 9 There are eight possibilities; two of them are 84 920 and 94 820.
- 10 a $2 \times 3 \times 5 \times 7$ b $2^4 \times 3^2 \times 5 \times 7$ c $2^2 \times 109$ d $2 \times 3^2 \times 7 \times 11$
- 11 a 1372 b 1859 c 7560 d $2^3 \times 3^3 \times 5 \times 7$
- 12 a $\text{HCF} = 720$, $\text{LCM} = 64\,800$ b $\text{HCF} = 14$, $\text{LCM} = 147\,000$
- 13 a 400 b 4000 c 3000 d 4370

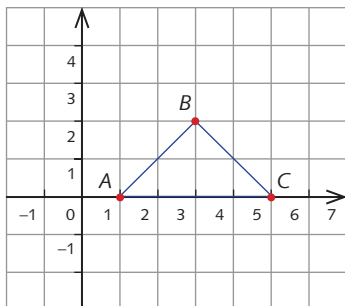
Chapter 4

- 1 a 8 b 86 c -30 d -11 e 997 f 21
- 2 a 6 b 15 c -25 d 98 e $-14\frac{3}{10}$ f $60\frac{4}{5}$
 g $\frac{17}{104}$ h $\frac{11}{12}$

3 16.3°C



5



$$\begin{aligned}\text{Area of } \triangle ABC &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 4 \times 2 \\ &= 4 \text{ unit squares}\end{aligned}$$

6 $-\frac{9}{8} < -\frac{7}{9} < -0.02 < 0.2 < \frac{12}{8} < \frac{1}{0.4}$

7 a $-3\frac{13}{20}$ b $19\frac{13}{15}$ c $-12\frac{4}{9}$

8 \$2359.70 9 9691 metres 10 12

Chapter 5

- 1 a 138° b 64° c 59° d 81° e 141° f 131°
 2 a $\alpha = 150^\circ, \beta = 30^\circ, \gamma = 150^\circ$ b $\alpha = 80^\circ$ c $\alpha = 130^\circ, \beta = 135^\circ$ d $\alpha = 60^\circ, \beta = 120^\circ$
 e $\alpha = 45^\circ, \beta = 135^\circ$ f $\alpha = 36^\circ, \beta = 144^\circ$
 3 a $\gamma = 135^\circ$ b $\alpha = 75^\circ$ c $\alpha = 66\frac{2}{3}^\circ$ d $\beta = 70^\circ$ e $\theta = 110^\circ$ f $\alpha = 90^\circ$

Chapter 6

- 1 a 6 b 6 c 0 d 1 e 5 f 19 g 0 h $1\frac{2}{3}$
 2 a -1 b -1 c 4 d -8 e 1 f 4 g 6 h 1
 3 a $m = 5$ b $m = 14$ c $m = 4$ d $m = 24$ e $x = 7\frac{1}{2}$ f $x = 60$
 g $x = -6$ h $x = -1$ i $x = -75$
 4 a $2x - 6$ b $-3x - 12$ c $5x - 10$ d $ax + 3a$ e $-3x + 12$ f $-12 + 4x$
 5 a $7x - y$ b $3x + 3y$ c $4x$ d 0 e 0 f $5x^2y$
 6 a $-12\frac{3}{5}$ b -12 c $1\frac{3}{25}$ d $108\frac{1}{5}$ e 3 f $\frac{3}{8}$
 g $-1\frac{4}{25}$ h $97\frac{1}{2}$
 7 a $x = -13$ b $b = 3$ c $a = 11$ d $b = -1\frac{1}{13}$ e $x = 100$ f $x = 4$
 8 a $30x + 7y$ b $13y + \frac{9}{4}x$ c $20xy + 6y + 24x$ d $4x^2 + x + 27$ e $15x + \frac{17x^2}{3}$ f $2xy + 13x + 5y$
 g $ay - ax - 10a$ h $4x^2y - 15xy^2 - 6xy$ i $13ax - 57ay + 18a^2$
 9 a $\frac{2(x+14)-8}{2} - x = x + 14 - 4 - x = 10$ b $\frac{3(x+5)-9}{3} - x = x + 5 - 3 - x = 2$
 10 4 rides
 11 5 pages on a week night, 15 pages on Saturday night
 12 $3\frac{5}{6}$ cm

Chapter 7

- 1 a 40% b $83\frac{1}{3}\%$ c 87.5% d $77\frac{7}{9}\%$ e $71\frac{3}{7}\%$
 f 7.5% g $72\frac{8}{11}\%$ h $58\frac{1}{3}\%$
 2 a $\frac{17}{200}$ b $\frac{1}{6}$ c $\frac{23}{150}$ d $\frac{13}{150}$ e $\frac{3}{70}$ f $\frac{11}{200}$

- 3 a $1\frac{1}{2}$ b $\frac{1}{40}$ c $1\frac{9}{25}$ d $2\frac{1}{25}$ e $\frac{1}{400}$ f $\frac{21}{400}$
- 4 a $\frac{27}{50}$ b $\frac{23}{25}$ c 1 d $\frac{18}{25}$ 5 24
- 6 a 1250 b 2000 c 2800 d 594 e 3675 f 22400
- 7 a \$186 000 b \$57 000 c \$36 800 d 133 920 hectares
- 8 a \$17 600 b \$18 080 c \$17 840
- 9 a \$55 800 b \$53 100 c \$52 087.50
- 10 a \$4068 b \$22.95 c \$81.70 d \$210 e \$468
- 11 a 15% b 6% c 40% d 18%
- 12 a 5100 b 2400 c 1100 13 \$33

Chapter 8

- 1 10 2 a 26 b 41 c 61
- 3 a $x = 25$ cm, $y = 51$ cm b $\sqrt{128}$ cm
- 4 40 cm 5 12 m 6 4.23 m 7 6 m
- 8 a no b yes, $\angle BCA$ c yes, $\angle BAC$ d yes, $\angle ABC$ e no f yes, $\angle BAC$

9B Problem solving

Kaprekar's routine

- Eventually a subtraction produces either 495 or 594. Since $954 - 459 = 495$, repeating the process will keep on producing 495.
- The numbers that are produced by this process all have '9' in the middle and two other digits whose sum is nine (for example, 297, 693, 198 and so on). Their digit sum is obviously always 18. As the process is repeated, the outside numbers approach 4 and 5; for example, $891 \rightarrow 792 \rightarrow 693$ and so on.
- 6174 ($= 7641 - 1467$) This number is known as **Kaprekar's constant**.
- There is no five-digit self-producing integer. However, there are three different cyclic patterns that numbers with five digits can fall into. Two have four members ($63954 \rightarrow 61974 \rightarrow 82962 \rightarrow 75933$ and so on, and $62964 \rightarrow 71973 \rightarrow 83952 \rightarrow 74943$ and so on), the other has two members ($59994 \rightarrow 53955$ and so on).
- Challenge:* 6333176664, 9753086421, 9975084201

Angle chasing

- 1 a 305° b 126° c 107° d 325°
- 2 $\theta = 342^\circ$, $\alpha = 72^\circ$

Multiples

- 1 a $481 = 13 \times 37$, $148 = 4 \times 37$, $814 = 22 \times 37$
b $185 = 5 \times 37$, $851 = 23 \times 37$, $518 = 14 \times 37$
 $259 = 7 \times 37$, $592 = 16 \times 37$, $925 = 25 \times 37$
 $296 = 8 \times 37$, $962 = 26 \times 37$, $629 = 17 \times 37$
 $740 = 20 \times 37$, $74 = 2 \times 37$, $407 = 11 \times 37$
 $814 = 22 \times 37$, $481 = 13 \times 37$, $148 = 4 \times 37$
 $925 = 25 \times 37$, $259 = 7 \times 37$, $592 = 16 \times 37$
- 2 a $68839 = 1679 \times 41$, $88396 = 2156 \times 41$, $83968 = 2048 \times 41$, $39688 = 968 \times 41$, $96883 = 2363 \times 41$
b They are all multiples of 41.
- 3 a i 555 ii 222 iii 777
The product turns out to be a three-digit number with all digits the same as the middle multiplicand.
b 999 , $37 \times 3 = 111$
c 1443, 1887. The product turns out to be a four-digit number, with the first and last digits the same as the middle multiplicand and each of the two digits in the middle is the sum of the outer digits.
d $41 \times 5 \times 271 = 55555$, $41 \times 2 \times 271 = 22222$, $41 \times 7 \times 271 = 77777$,
 $41 \times 9 \times 271 = 99999$, $41 \times 13 \times 271 = 144443$, $41 \times 17 \times 271 = 188887$

Geometry challenge

- 1 a $\alpha = 30^\circ, \beta = 86^\circ$ b $\alpha = 83^\circ, \beta = 120^\circ, \gamma = 83^\circ$ c $\alpha = 66^\circ, \beta = 38^\circ$
d $\alpha = 107^\circ, \beta = 140^\circ$ e $\alpha = 51^\circ, \beta = 129^\circ$ f $\alpha = 30^\circ, \beta = 147^\circ$
- 2 a $\angle BAC = 70^\circ, \angle ACB = 55^\circ$
b $\angle FIC = 55^\circ$, $\triangle FIC$ is isosceles triangle because base angles are equal.
c $\angle DIB = 55^\circ$, $\triangle DIB$ is isosceles triangle because base angles are equal thus $DB = DI$.
d $\angle GIF = 53^\circ, \angle EFG = 17^\circ$
- 3 a $\angle AOC = 2\alpha$ b $\angle AOB = 2\beta$
c $2\alpha + 2\beta = 180^\circ$ (straight angle BOC) so $\alpha + \beta = 90^\circ$. Hence $\angle BAC = 90^\circ$.
 $\triangle BDC$ and $\triangle CDE$ are right-angled triangles.
d 115°

Pythagoras' theorem

- a $(a+b)^2 = (a+b)(a+b) = a(a+b) + b(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$
- b Each of the 4 triangles has area $\frac{1}{2}ab$
- c $c^2 + 2ab = a^2 + 2ab + b^2$. Hence $c^2 = a^2 + b^2$

Pythagorean triples

The two methods always produce primitive triples.

Sports percentages

Activity 1

140%, 108.3%, 125%, 93.8%, 75%, 68%

Activity 2

	Team	Wins	Losses	Goals For	Goals Against	Percentage (%)	Points
1	Cassowaries	5	1	157	117	134.2	20
2	Pelicans	5	1	142	111	127.9	20
3	Hawks	3	3	160	132	121.2	12
4	Brolgas	3	3	132	128	103.1	12
5	Jabirus	3	3	120	123	97.6	12
6	Emus	2	4	97	120	80.8	8
7	Kookaburras	2	4	105	140	75.0	8
8	Eagles	1	5	113	155	72.9	4

Activity 3

- 1 The Kookaburras' percentage did not change, because their old percentage happened to be equal to the ratio of goals in the latest game.
- 2 The Hawks' and Cassowaries' percentages went down even though they won.
- 3 Yes; the Eagles are an example.
- 4 They both sum to 1026, because each goal scored contributes 1 to both the Goals For and Goals Against columns.

Exercise 9C

- 1 a $27 = 7 \times 3 + 6$, hence $27 \equiv 6 \pmod{7}$.
b $359412294 = 10 \times 35941229 + 4$, hence $359412294 \equiv 4 \pmod{10}$.
c $53 = 12 \times 4 + 5$, hence $53 \equiv 5 \pmod{12}$.
d $3 = 8 \times 0 + 3$, hence $3 \equiv 3 \pmod{8}$.

- 2 a 0 b 4 c 6 d 4
 3 a 51 b 57 c 51 d 52 e 58 f 61 g 61 h 67
 4 Yes, because $47 \equiv 1 \pmod{2}$.
 5 Bessie, because $80 \equiv 2 \pmod{6}$.
 6 6 a.m.
 7 4, 11, 18, 25; Friday
 8 1891, 1895, 1903, 1907, 1911, 1915, 1919 (1900 is not a leap year)
 9 a 1 (look at the last two digits) b 81 (look at the last two digits)
 c 31 (look at the last two digits) d 6 (look at the last two digits)
 10 a 2 p.m. b Thursday c East
 11 a $-13 = 5 \times (-3) + 2$ b $-13 \equiv 2 \pmod{5}$
 13 a $-20 = 7 \times (-3) + 1$, hence $-20 \equiv 1 \pmod{7}$.
 b $-30 = 8 \times (-4) + 2$, hence $-30 \equiv 2 \pmod{8}$.
 c 1 d 2 e 10 f 0
 14 a $-16 \equiv 0 \equiv 4$; $-1 \equiv 15$ b $-30 \equiv 18$; $5 \equiv 53$

Exercise 9D

- 1 a 3 b 5 c 0 d 2 e 4 f 3 g 0 h 76 i 4 j 24
 2 a 1 b 4 c 1 d 0 e 4 f 2
 3 a 1 b 0 c 0 d 1
 4 a 1 b 4 c 4 d 1 e 5 f 4 g 4 h 1
 5 a 1 b 5 c 9 d 0
 6 They are all zero. When you add an integer to itself a number of times equal to the modulus, then the sum is zero.
 7 4 times, because $74 \equiv 2 \pmod{6}$.
 8 10 a.m., because $7 \times 247 \equiv 1 \pmod{24}$.
 9 Wednesday, because $280 + 25 + 211 + 143 \equiv 0 + 4 + 1 + 3 \equiv 1 \pmod{7}$.
 10 3rd finger, because $37 \times 19 \equiv 3 \pmod{5}$.
 11 30° , because $60 \times 9 + 40 \times 12 + 90 \times 13 \equiv 30 \pmod{360}$.
 12 a $10^n \equiv 1^n \equiv 1 \pmod{9}$.
 b An example will demonstrate the result better than a general argument:
 $2467 = 2 \times 10^3 + 4 \times 10^2 + 6 \times 10 + 7 \equiv 2 \times 1 + 4 \times 1 + 6 \times 1 + 7 \equiv 2 + 4 + 6 + 7 \pmod{9}$.
 c If two numbers are congruent modulo 9, then they are congruent modulo 3.
 13 a $10 \equiv -1 \pmod{11}$, so $100 \equiv 1 \pmod{11}$, so $1000 \equiv -1 \pmod{11}$...
 b $2467 \equiv 2 \times 10^3 + 4 \times 10^2 + 6 \times 10 + 7 \equiv 2 \times (-1) + 4 \times 1 + 6 \times (-1) + 7 \equiv -2 + 4 - 6 + 7 \pmod{11}$.
 14 a 1, 1, 2, 0, 2, 2, 1, 0, 1, 1... length = 8 b 1, 1, 0, 1, 1... length = 3 c 1, 1, 2, 3, 1, 0, 1, 1... length = 6
 d 1, 1, 2, 3, 0, 3, 3, 1, 4, 0, 4, 4, 3, 2, 0, 2, 2, 4, 1, 0, 1, 1... length = 20
 e 1, 1, 2, 3, 5, 2, 1, 3, 4, 1, 5, 0, 5, 5, 4, 3, 1, 4, 5, 3, 2, 5, 1, 0, 1, 1... length = 24
 f 1, 1, 2, 3, 5, 1, 6, 0, 6, 6, 5, 4, 2, 6, 1, 0, 1, 1... length = 16
 g 1, 1, 2, 3, 5, 8, 4, 3, 7, 1, 8, 0, 8, 8, 7, 6, 4, 1, 5, 6, 2, 8, 1, 0, 1, 1... length = 24
 h 1, 1, 2, 3, 5, 8, 2, 10, 1, 0, 1, 1... length = 10
 i 1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0, 7, 7, 4, 1, 5, 6, 1, 7, 8, 5, 3, 8, 1, 9, 0, 9, 9, 8, 7, 5, 2, 7, 9, 6, 5, 1, 6, 7, 3, 0, 3, 3, 6, 9, 5, 4, 9, 3, 2, 5, 7, 2, 9, 1, 0, 1, 1... length = 60
 In each sequence, the zeroes cycle regularly, and the cycle length of the sequence is a multiple of the cycle length of the zeroes.