

TOPIC 8

Algebra

8.1 Overview

Why learn this?

Humankind could not have travelled to the moon without algebra. Without algebra there would be no television, no iPod, no iPad — nothing electrical at all. A knowledge of algebra also makes possible complex geometry, which is reflected in structures ranging from the Colosseum to the simple suburban house. Algebra is the fundamental building block of mathematics.

You need a knowledge of algebra to succeed in mathematics at school. The further you advance in your studies, the more useful you will find algebra.

What do you know?

assess on

- 1 THINK** List what you know about algebra. Use a thinking tool such as a concept map to show your list.
- 2 PAIR** Share what you know with a partner and then with a small group.
- 3 SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of algebra.

Learning sequence

- 8.1** Overview
- 8.2** Using variables
- 8.3** Substitution
- 8.4** Working with brackets
- 8.5** Substituting positive and negative numbers
- 8.6** Number laws and variables
- 8.7** Simplifying expressions
- 8.8** Multiplying and dividing expressions with variables
- 8.9** Expanding brackets
- 8.10** Factorising
- 8.11** Review

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8.2 Using variables

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eLesson
Using variables
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- A **variable** (or **pronumeral**) is a letter or symbol that represents a value in an algebraic expression or equation.
- In algebraic expressions such as $a + b$, the variables represent any number.
- In algebraic equations such as $x + y = 9$, variables are referred to as **unknowns** because the variable represents a specific value that is not yet known.
- When we write expressions with variables, the multiplication sign is omitted.
For example, $8n$ means ' $8 \times n$ ' and $12ab$ means ' $12 \times a \times b$ '.
- The division sign is rarely used. For example, $y \div 6$ is usually written as $\frac{y}{6}$.

WORKED EXAMPLE 1

Suppose we use b to represent the number of ants in a nest.

- Write an expression for the number of ants in the nest if 25 ants died.
- Write an expression for the number of ants in the nest if the original ant population doubled.
- Write an expression for the number of ants in the nest if the original population increased by 50.
- What would it mean if we said that a nearby nest contained $b + 100$ ants?
- What would it mean if we said that another nest contained $b - 1000$ ants?
- Another nest in very poor soil contains $\frac{b}{2}$ ants. How much smaller than the original is this nest?

THINK

- The original number of ants (b) must be reduced by 25.
- The original number of ants (b) must be multiplied by 2. It is not necessary to show the $\times$ sign.
- 50 must be added to the original number of ants (b).
- This expression tells us that the nearby nest has 100 more ants.
- This expression tells us that the nest has 1000 fewer ants.
- The expression $\frac{b}{2}$ means $b \div 2$, so this nest is half the size of the original nest.

WRITE

- $b - 25$
- $2b$
- $b + 50$
- The nearby nest has 100 more ants.
- This nest has 1000 fewer ants.
- This nest is half the size of the original nest.

Exercise 8.2 Using variables

assess on

INDIVIDUAL PATHWAYS

PRACTISE

 Questions:
1–8, 12

CONSOLIDATE

 Questions:
1–9, 12, 13

MASTER

 Questions:
1–14

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REFLECTION

List some reasons for using variables instead of numbers.

FLUENCY

- 1 **WE1** Suppose we use x to represent the number of ants in a nest.
- Write an expression for the number of ants in the nest if 420 ants were born.
 - Write an expression for the number of ants in the nest if the original ant population tripled.
 - Write an expression for the number of ants in the nest if the original ant population decreased by 130.
 - What would it mean if we said that a nearby nest contained $x + 60$ ants?
 - What would it mean if we said that a nearby nest contained $x - 90$ ants?
 - Another nest in very poor soil contains $\frac{x}{4}$ ants. How much smaller than the original is this nest?

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 Alternative expressions
used to describe the
four operations

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- 2 Suppose x people are in attendance at the start of an Aussie Rules football match.
- If a further y people arrive during the first quarter, write an expression for the number of people at the ground.
 - Write an expression for the number of people at the ground if a further 260 people arrive prior to the second quarter commencing.

- c At half-time 170 people leave. Write an expression for the number of people at the ground after they have left.
 - d In the final quarter a further 350 people leave. Write an expression for the number of people at the ground after they have left.
- 3 The canteen manager at Browning Industries orders m Danish pastries each day. Write a paragraph that could explain the table below.

Time	Number of Danish pastries
9.00 am	m
9.15 am	$m - 1$
10.45 am	$m - 12$
12.30 pm	$m - 12$
1.00 pm	$m - 30$
5.30 pm	$m - 30$



- 4 Imagine that your cutlery drawer contains a knives, b forks and c spoons.
- a Write an expression for the total number of knives and forks you have.
 - b Write an expression for the total number of items in the drawer.
 - c You put 4 more forks in the drawer. Write an expression for the number of forks now.
 - d Write an expression for the number of knives in the drawer after 6 knives are removed.
- 5 If y represents a certain number, write expressions for the following numbers.
- a A number 7 more than y .
 - b A number 8 less than y .
 - c A number that is equal to five times y .
 - d The number formed when y is subtracted from 14.
 - e The number formed when y is divided by 3.
 - f The number formed when y is multiplied by 8 and 3 is added to the result.
- 6 Using a and b to represent numbers, write expressions for:
- a the sum of a and b
 - b the difference between a and b
 - c three times a subtracted from two times b
 - d the product of a and b
 - e twice the product of a and b
 - f the sum of $3a$ and $7b$
 - g a multiplied by itself
 - h a multiplied by itself and the result divided by 5.

- 7 If tickets to a basketball match cost \$27 for adults and \$14 for children, write an expression for the cost of:
- a y adult tickets b d child tickets c r adult and h child tickets.



UNDERSTANDING

- 8 Naomi is now t years old.
- a Write an expression for her age in 2 years' time.
- b Write an expression for Steve's age if he is g years older than Naomi.
- c How old was Naomi 5 years ago?
- d Naomi's father is twice her age. How old is he?
- 9 James is travelling by train into town one particular evening and observes that there are t passengers in his carriage. He continues to take note of the number of people in his carriage each time the train departs from a station, which occurs every 3 minutes. The table below shows the number of passengers.

Time	Number of passengers
7.10 pm	t
7.13 pm	$2t$
7.16 pm	$2t + 12$
7.19 pm	$4t + 12$
7.22 pm	$4t + 7$
7.25 pm	t
7.28 pm	$t + 1$
7.31 pm	$t - 8$
7.34 pm	$t - 12$



- a Write a paragraph explaining what happened.
- b When did passengers first begin to alight the train?
- c At what time did the carriage have the most number of passengers?
- d At what time did the carriage have the least number of passengers?

REASONING

- 10 A microbiologist places m bacteria onto an agar plate. She counts the number of bacteria at approximately 3-hour intervals. The results are shown in the table below.

Time	Number of bacteria
9.00 am	m
Noon	$2m$
3.18 pm	$4m$
6.20 pm	$8m$
9.05 pm	$16m$
Midnight	$32m - 1240$



- Explain what happens to the number of bacteria in the first 5 intervals.
 - What might be causing the number of bacteria to increase in this way?
 - What is different about the last bacteria count?
 - What may have happened to cause this?
- 11 n represents an even number.
- Is the number $n + 1$ odd or even?
 - Is $3n$ odd or even?
 - Write expressions for:
 - the next three even numbers that are greater than n
 - the even number that is 2 less than n .

PROBLEM SOLVING

- 12 If the side of a square tile box is x cm long and the height is h cm, write expressions for the total surface area and the volume of the tile box.
- 13 If a rectangular tile box has the same width and height as the square tile box in question 12 but is one and a half times as long, write expressions for the total surface area and the volume of the tile box.
- 14 If the square tile box in question 12 has a side length of 20 cm and both boxes in questions 12 and 13 have a height of 15 cm, calculate the surface area and volume of the square tile box and the surface area and volume of the rectangular tile box using your expressions.



CHALLENGE 8.1

Bill was describing his age to a group of people using two different algebraic expressions. Four times a certain number minus one and three times a certain number plus three, both reveal Bill's age. What is the number used and what is Bill's age?



8.3 Substitution

- If the value of a variable (or variables) is known, it is possible to **evaluate** (work out the value of) an expression by using **substitution**. The variable is replaced with the number.
- Substitution can also be used with a formula or rule.

WORKED EXAMPLE 2

Find the value of the following expressions if $a = 3$ and $b = 15$.

a $6a$

b $7a - \frac{2b}{3}$

THINK

- a**
- 1 Substitute the variable (a) with its correct value and replace the multiplication sign.
 - 2 Evaluate and write the answer.
- b**
- 1 Substitute each variable with its correct value and replace the multiplication signs.
 - 2 Perform the first multiplication.
 - 3 Perform the next multiplication.
 - 4 Perform the division.
 - 5 Perform the subtraction and write the answer.

WRITE

a $6a = 6 \times 3$

$$= 18$$

b $7a - \frac{2b}{3} = 7 \times 3 - \frac{2 \times 15}{3}$

$$= 21 - \frac{2 \times 15}{3}$$

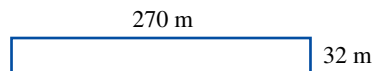
$$= 21 - \frac{30}{3}$$

$$= 21 - 10$$

$$= 11$$

WORKED EXAMPLE 3

The formula for finding the area (A) of a rectangle of length l and width w is $A = l \times w$. Use this formula to find the area of the rectangle at right.



THINK

- 1 Write the formula.
- 2 Substitute each variable with its value.
- 3 Perform the multiplication and state the correct units.

WRITE

$A = l \times w$

$$= 270 \times 32$$

$$= 8640 \text{ m}^2$$



Exercise 8.3 Substitution

INDIVIDUAL PATHWAYS

REFLECTION

Can any value be substituted for a variable in every expression?

PRACTISE

Questions:
1–4, 6, 8, 13

CONSOLIDATE

Questions:
1a–l, 2a–l, 3a–l, 4–8, 11, 13

MASTER

Questions:
1m–t, 2i–p, 3i–p, 4–13

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FLUENCY

1 **WE2** Find the value of the following expressions, if $a = 2$ and $b = 5$.

a $3a$

b $7a$

c $6b$

d $\frac{a}{2}$

e $a + 7$

f $b - 4$

g $a + b$

h $b - a$

i $5 + \frac{b}{5}$

j $3a + 9$

k $2a + 3b$

l $\frac{8}{a}$

m $\frac{25}{b}$

n ab

o $2ab$

p $7b - 30$

q $6b - 4a$

r $\frac{ab}{5}$

s $\frac{15}{a} + \frac{7}{b}$

t $\frac{9}{a} - \frac{3}{b}$

2 Substitute $x = 6$ and $y = 3$ into the following expressions and evaluate.

a $6x + 2y$

b $\frac{x}{3} + \frac{y}{3}$

c $3xy$

d $\frac{24}{x} - \frac{9}{y}$

e $\frac{12}{x} + 4 + y$

f $3x - y$

g $2.5x$

h $\frac{7x}{2}$

i $3.2x + 1.7y$

j $11y - 2x$

k $\frac{13y}{3} - 2x$

l $\frac{4xy}{15}$

m $4.8x - 3.5y$

n $8.7y - 3x$

o $12.3x - 9.6x$

p $\frac{3x}{9} - \frac{y}{12}$

3 Evaluate the following expressions, if $d = 5$ and $m = 2$.

a $d + m$

b $m + d$

c $m - d$

d $d - m$

e $2m$

f md

g $5dm$

h $\frac{md}{10}$

i $-3d$

j $-2m$

k $6m + 5d$

l $\frac{3md}{2}$

m $25m - 2d$

n $\frac{7d}{15}$

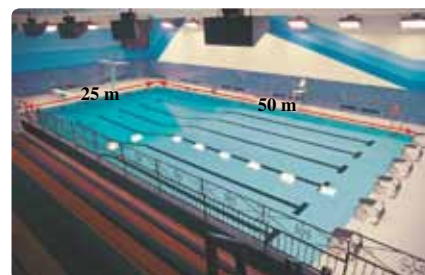
o $4dm - 21$

p $\frac{15}{d} - m$

4 **WE3** The formula for finding the perimeter (P) of a rectangle of length l and width w is $P = 2l + 2w$. Use this formula to find the perimeter of the rectangular swimming pool at right.

5 The formula for the perimeter (P) of a square of side length l is $P = 4l$. Use this formula to find the perimeter of a square of side length 2.5 cm.

6 The formula $c = 0.1a + 42$ is used to calculate the cost in dollars (c) of renting a car for one day from Poole's Car Hire Ltd, where a is the number of kilometres travelled



on that day. Find the cost of renting a car for one day if the distance travelled is 220 kilometres.

- 7 The area (A) of a rectangle of length l and width w can be found using the formula $A = lw$. Find the area of the rectangles below:

- a length 12 cm, width 4 cm
- b length 200 m, width 42 m
- c length 4.3 m, width 104 cm.

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Spreadsheet
Substitution
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UNDERSTANDING

- 8 The formula $F = \frac{9}{5}C + 32$ is used to convert temperatures measured in degrees Celsius to an approximate Fahrenheit value. F represents the temperature in degrees Fahrenheit ($^{\circ}\text{F}$) and C the temperature in degrees Celsius ($^{\circ}\text{C}$).
- a Find F when $C = 100^{\circ}\text{C}$.
 - b Convert 28°C to Fahrenheit.
 - c Water freezes at 0°C . What is the freezing temperature of water in Fahrenheit?
- 9 The formula $D = 0.6T$ can be used to convert distances in kilometres (T) to the approximate equivalent in miles (D). Use this rule to convert the following distances to miles:
- a 100 kilometres
 - b 248 kilometres
 - c 12.5 kilometres.

REASONING

- 10 Ben says that $\frac{4x^2}{2x} = 2x$. Emma says that is not correct if $x = 0$. Explain Emma's reasoning.

PROBLEM SOLVING

- 11 If $s = ut + \frac{1}{2}at^2$, evaluate s if $u = 5$, $t = 10$ and $a = 9.81$.
- 12 The width of a cuboid is x cm.
- a If the length is 5 cm more than the width and the height is 2 cm less than the width, find the volume, $V \text{ cm}^3$, of the cuboid in terms of x .
 - b Evaluate V if x equals 10.
 - c Explain why x cannot equal 1.5.
- 13 On the space battleship RAN *Fantasie*, there are p Pletons, each with 2 legs, $(p - 50)$ Argors, each with 3 legs, and $(2p + 35)$ Kleptors, each with 4 legs.
- a Find the total number of legs, L , on board the *Fantasie*, in terms of p , in simplified form.
 - b If $p = 200$, find L .

8.4 Working with brackets

- Brackets are grouping symbols. The expression $3(a + 5)$ can be thought of as 'three groups of $(a + 5)$ ', or $(a + 5) + (a + 5) + (a + 5)$.
- When substituting into an expression with brackets, remember to place a multiplication sign ($\times$) next to the brackets. For example, $3(a + 5)$ is thought of as $3 \times (a + 5)$.
- Following operation order, evaluate the brackets first and then multiply by the value outside of the brackets.

WORKED EXAMPLE 4

- a** Substitute $r = 4$ and $s = 5$ into the expression $5(s + r)$ and evaluate.
b Substitute $t = 4$, $x = 3$ and $y = 5$ into the expression $2x(3t - y)$ and evaluate.

THINK

- a**
- 1 Place the multiplication sign back into the expression.
 - 2 Substitute the variables with their correct values.
 - 3 Evaluate the expression in the pair of brackets first.
 - 4 Perform the multiplication and write the answer.
- b**
- 1 Place the multiplication signs back into the expression.
 - 2 Substitute the variables with their correct values.
 - 3 Perform the multiplication inside the pair of brackets.
 - 4 Perform the subtraction inside the pair of brackets.
 - 5 Perform the multiplication and write the answer.

WRITE

a $5(s + r) = 5 \times (s + r)$
 $= 5 \times (5 + 4)$
 $= 5 \times 9$
 $= 45$

b $2x(3t - y) = 2 \times x \times (3 \times t - y)$
 $= 2 \times 3 \times (3 \times 4 - 5)$
 $= 2 \times 3 \times (12 - 5)$
 $= 2 \times 3 \times 7$
 $= 42$



Exercise 8.4 Working with brackets

INDIVIDUAL PATHWAYS

PRACTISE

Questions:
1–4, 7

CONSOLIDATE

Questions:
1a–l, 2a–l, 3–5, 7

MASTER

Questions:
1j–p, 2j–p, 3–8

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REFLECTION

Is operation order followed when substituting values for variables?

FLUENCY

- 1 WE4** Substitute $r = 5$ and $s = 7$ into the following expressions and evaluate.

- | | | |
|-----------------------|------------------------|-------------------------|
| a $3(r + s)$ | b $2(s - r)$ | c $7(r + s)$ |
| d $9(s - r)$ | e $s(r + 3)$ | f $s(2r - 5)$ |
| g $3r(r + 1)$ | h $rs(3 + s)$ | i $11r(s - 6)$ |
| j $2r(s - r)$ | k $s(4 + 3r)$ | l $7s(r - 2)$ |
| m $s(3rs + 7)$ | n $5r(24 - 2s)$ | o $5sr(sr + 3s)$ |
| p $8r(12 - s)$ | | |

2 Evaluate each of the expressions below, if $x = 3$, $y = 5$ and $z = 9$.

- a $xy(z - 3)$ b $\frac{12}{x}(z - y)$ c $\frac{z}{3}\left(\frac{2y}{10} + x - 2\right)$
 d $(x + y)(z - y)$ e $(z - 3)4x$ f $zy(17 - xy)$
 g $\frac{y}{5}(7 - x + 3)$ h $(8 - y)(z + x)$ i $\left(7 - \frac{12}{x}\right)4y$
 j $\frac{6}{x}(xz + y - 3)$ k $(y + 2)\frac{z}{x}$ l $2x(xyz - 105)$
 m $12(y - 1)(z + 3)$ n $(3x - 7)\left(\frac{27}{x} + 7\right)$ o $-2(4x + 1)\left(\frac{36}{z} - 3\right)$
 p $-3(2y - 11)\left(\frac{z}{x} + 8\right)$

3 The formula for the perimeter (P) of a rectangle of length l and width w is $P = 2l + 2w$. This rule can also be written as $P = 2(l + w)$. Use the rule to find the perimeter of rectangular comic covers with the following measurements.

- a $l = 20$ cm, $w = 11$ cm
 b $l = 27.5$ cm, $w = 21.4$ cm

UNDERSTANDING

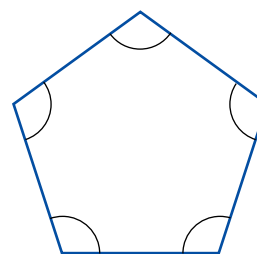
4 **MC** When $a = 8$ and $b = 12$ are substituted into the expression $\frac{a}{6}(15 - b + 9)$, the expression is equal to

- A 32 B 16 C $21\frac{1}{3}$
 D 24 E 27

5 A rule for finding the sum of the interior angles in a many-sided figure such as a pentagon is $S = 180(n - 2)^\circ$, where S represents the sum of the angles inside the figure and n represents the number of sides. The diagram at right shows the interior angles in a pentagon.

Use the rule to find the sum of the interior angles for the following figures:

- a a hexagon (6 sides) b a pentagon
 c a triangle d a quadrilateral (4 sides)
 e a 20-sided figure.

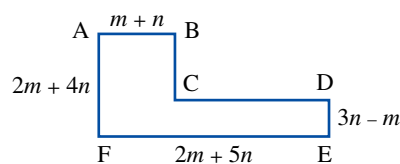


REASONING

6 The dimensions of the figure shown are given in terms of m and n . Write, in terms of m and n , an expression for:

- a the length of CD b the length of BC
 c the perimeter of the figure.

Show all of your working.



PROBLEM SOLVING

- 7 a** Find an expression for the area of a triangle whose base length is $(m + n)$ cm and whose height is $(m - n)$ cm.
- b** If $m = 15$ and $n = 6$, find the area of the triangle.
- c** Show that $m > n$.
- d** Explain what happens to the triangle as m and n move closer in value.
- 8** It can be shown that $(x - a)(x - a) = x^2 - 2ax + a^2$. By substitution, show that this is true if:
- a** $x = 4, a = 1$
- b** $x = 3p, a = 2p$
- c** $x = 0$

8.5 Substituting positive and negative numbers

- If the variable you are substituting for has a negative value, simply remember the following rules for directed numbers:
 - For addition and subtraction, signs that occur together can be combined.

Same signs positive	for example, $7 + +3 = 7 + 3$
	and $7 - -3 = 7 + 3$
Different signs negative	for example, $7 - +3 = 7 - 3$
	and $7 + -3 = 7 - 3$
 - For multiplication and division.

Same signs positive	for example, $+7 \times +3 = +21$
	and $-7 \times -3 = +21$
Different signs negative	for example, $+7 \times -3 = -21$
	and $-7 \times +3 = -21$

WORKED EXAMPLE 5

- a** Substitute $m = 5$ and $n = -3$ into the expression $m - n$ and evaluate.
- b** Substitute $m = -2$ and $n = -1$ into the expression $2n - m$ and evaluate.
- c** Substitute $a = 4$ and $b = -3$ into the expression $5ab - \frac{12}{b}$ and evaluate.

THINK

- a**
- 1 Substitute the variables with their correct value.
 - 2 Combine the two negative signs and add.
 - 3 Write the answer.
- b**
- 1 Replace the multiplication sign.
 - 2 Substitute the variables with their correct values.
 - 3 Perform the multiplication.
 - 4 Combine the two negative signs and add.
 - 5 Write the answer.

WRITE

a $m - n = 5 - -3$

$$= 5 + 3$$

$$= 8$$

b $2n - m = 2 \times n - m$

$$= 2 \times -1 - -2$$

$$= -2 - -2$$

$$= -2 + 2$$

$$= 0$$

- c 1 Replace the multiplication signs.
- 2 Substitute the variables with their correct values.
- 3 Perform the multiplications.
- 4 Perform the division.
- 5 Combine the two negative signs and add.
- 6 Write the answer.

$$\begin{aligned}
 c \quad 5ab - \frac{12}{b} &= 5 \times a \times b - \frac{12}{b} \\
 &= 5 \times 4 \times -3 - \frac{12}{-3} \\
 &= -60 - \frac{12}{-3} \\
 &= -60 - -4 \\
 &= -60 + 4 \\
 &= -56
 \end{aligned}$$

Exercise 8.5 Substituting positive and negative numbers

assess on

INDIVIDUAL PATHWAYS

PRACTISE

Questions:
1–4

CONSOLIDATE

Questions:
1a–l, 2, 3a–l, 4, 6, 7

MASTER

Questions:
1m–t, 2g–l, 3m–t, 4–8

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REFLECTION

What can you say
about the sign of x^2 ?

FLUENCY

- 1 **WE5a** Substitute $m = 6$ and $n = -3$ into the following expressions and evaluate.

- | | | |
|-------------------------|-------------------------------|------------------|
| a $m + n$ | b $m - n$ | c $n - m$ |
| d $n + m$ | e $3n$ | f $-2m$ |
| g $2n - m$ | h $n + 5$ | i $2m + n - 4$ |
| j $11n + 20$ | k $-5n - m$ | l $\frac{m}{2}$ |
| m $\frac{mn}{9}$ | n $\frac{4m}{n - 5}$ | o $\frac{4m}{n}$ |
| p $\frac{12}{2n}$ | q $\frac{9}{n} + \frac{m}{2}$ | r $6mn - 1$ |
| s $-\frac{3n}{2} + 1.5$ | t $14 - \frac{mn}{9}$ | |

- 2 **WE5b** Substitute $x = 8$ and $y = -3$ into the following expressions and evaluate.

- | | | |
|---|------------------------|--|
| a $3(x - 2)$ | b $x(7 + y)$ | c $5y(x - 7)$ |
| d $2(3 - y)$ | e $(y + 5)x$ | f $xy(7 - x)$ |
| g $(3 + x)(5 + y)$ | h $5(7 - xy)$ | i $\frac{x}{2}(5 - y)$ |
| j $\left(\frac{x}{4} - 1\right)\left(\frac{2y}{6} + 4\right)$ | k $\frac{9}{y}(6 - x)$ | l $3(x - 1)\left(\frac{y}{3} + 2\right)$ |

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Digital docs

SkillSHEET
Order of operations II
doc-6923
SkillSHEET
Order of operations with
brackets
doc-6924
SkillSHEET
Operations with directed
numbers
doc-6925

- 3 **WE5c** Substitute $a = -4$ and $b = -5$ into the following expressions and evaluate.

a $a + b$	b $a - b$	c $b - 2a$	d $2ab$
e $12 - ab$	f $-2(b - a)$	g $a - b - 4$	h $3a(b + 4)$
i $\frac{4}{b}$	j $\frac{8}{a}$	k $\frac{16}{4a}$	l $\frac{6b}{5}$
m $45 + 4ab$	n $8ab - 3b$	o $\frac{a}{2} + \frac{3b}{5}$	p $2.5b$
q $11a + 6b$	r $(a - 5)(8 - b)$	s $(9 - a)(b - 3)$	t $1.5b + 2a$

UNDERSTANDING

- 4 If $p = -2$ and $q = -3$, evaluate $\frac{3(-pq - p^2)}{q + 2p}$.

REASONING

- 5 Consider the expression $1 - 5x$. If x is a negative integer, explain why the expression will have a positive value.

PROBLEM SOLVING

- 6 Consider the equation $(a - b)(a + b) = a^2 + b^2$.
- a** By substituting $a = -3$ and $b = -2$, show that this is true.
 - b** By substituting $a = -q$ and $b = -2q$, show that this is true.
- 7 If $x = -2r$ is substituted into $\frac{(r - x)(x + r)}{(r - 2x)}$, will the answer be positive or negative if:
- a** $r > 0$?
 - b** $r < 0$?
- 8 A circle is cut out of a square.
- a** If the side length of the square is x , the radius of the circle is $0.25x$ and the area of the circle is approximately $0.196x^2$, find an expression for the remaining area.
 - b** Calculate the area when $x = 2$ by substituting into your expression. Give your answer to 3 decimal places.
 - c** What is the largest radius the circle can have?

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8.6 Number laws and variables

- When dealing with any type of number, we must obey particular rules.

Commutative Law

- The **Commutative Law** refers to the order in which two numbers may be added, subtracted, multiplied or divided.
- The Commutative Law holds true for addition and multiplication because the order in which two numbers are added or multiplied does not affect the result.

$$3 + 2 = 2 + 3$$

$$3 \times 2 = 2 \times 3$$

- Since variables take the place of numbers, the Commutative Law holds true for the addition and multiplication of variables.

$$x + y = y + x$$

$$x \times y = y \times x$$

- The Commutative Law does not hold true for subtraction or division because the results obtained are different.

$$3 - 2 \neq 2 - 3$$

$$3 \div 2 \neq 2 \div 3$$

- Since variables take the place of numbers, the Commutative Law does not hold true for the subtraction and division of variables.

$$x - y \neq y - x$$

$$x \div y \neq y \div x$$

WORKED EXAMPLE 6

Find the value of the following expressions if $x = 4$ and $y = 7$. Comment on the results obtained.

a i $x + y$ **ii** $y + x$ **b i** $x - y$ **ii** $y - x$

c i $x \times y$ **ii** $y \times x$ **d i** $x \div y$ **ii** $y \div x$

THINK

- a i** **1** Substitute each variable with its correct value.

- 2** Evaluate and write the answer.

- ii** **1** Substitute each variable with its correct value.

- 2** Evaluate and write the answer.

- 3** Compare the result with the answer obtained in part **a i**.

- b i** **1** Substitute each variable with its correct value.

- 2** Evaluate and write the answer.

- ii** **1** Substitute each variable with its correct value.

- 2** Evaluate and write the answer.

- 3** Compare the result with the answer obtained in part **b i**.

- c i** **1** Substitute each variable with its correct value.

- 2** Evaluate and write the answer.

WRITE

a i $x + y = 4 + 7$

$$= 11$$

ii $y + x = 7 + 4$

$$= 11$$

The same result is obtained; therefore, order is not important when adding two terms.

b i $x - y = 4 - 7$

$$= -3$$

ii $y - x = 7 - 4$

$$= 3$$

Two different results are obtained; therefore, order is important when subtracting two terms.

c i $x \times y = 4 \times 7$

$$= 28$$





ii 1 Substitute each variable with its correct value.

2 Evaluate and write the answer.

3 Compare the result with the answer obtained in part c i.

d i 1 Substitute each variable with its correct value.

2 Evaluate and write the answer.

ii 1 Substitute each variable with its correct value.

2 Evaluate and write the answer.

3 Compare the result with the answer obtained in part d i.

$$ii \quad y \times x = 7 \times 4$$

$$= 28$$

The same result is obtained; therefore, order is not important when multiplying two terms.

$$d \quad i \quad x \div y = 4 \div 7$$

$$= \frac{4}{7} (\approx 0.57)$$

$$ii \quad y \div x = 7 \div 4$$

$$= \frac{7}{4} (1.75)$$

Two different results are obtained; therefore, order is important when dividing two terms.

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Associative Law

- The **Associative Law** refers to the order in which three numbers may be added, subtracted, multiplied or divided, taking two at a time.
Note: The Associative Law refers to the order in which the addition (or other operation) is performed, and this order is indicated by the use of brackets. The order in which the variables are written does not change.
- Like the Commutative Law, the Associative Law holds true for addition and multiplication of numbers.

$$5 + (10 + 3) = (5 + 10) + 3$$

$$5 \times (10 \times 3) = (5 \times 10) \times 3$$

- Since variables take the place of numbers, the Associative Law holds true for the addition and multiplication of variables.

$$x + (y + z) = (x + y) + z$$

$$x \times (y \times z) = (x \times y) \times z$$

- Like the Commutative Law, the Associative Law does not hold for subtraction and division of numbers.

$$5 - (10 - 3) \neq (5 - 10) - 3$$

$$5 \div (10 \div 3) \neq (5 \div 10) \div 3$$

- Since variables take the place of numbers, the Associative law does not hold true for the subtraction and division of variables.

$$x - (y - z) \neq (x - y) - z$$

$$x \div (y \div z) \neq (x \div y) \div z$$

WORKED EXAMPLE 7

Find the value of the following expressions if $x = 12$, $y = 6$ and $z = 2$. Comment on the results obtained.

a i $x + (y + z)$

ii $(x + y) + z$

b i $x - (y - z)$

ii $(x - y) - z$

c i $x \times (y \times z)$

ii $(x \times y) \times z$

d i $x \div (y \div z)$

ii $(x \div y) \div z$

THINK

WRITE

- a i** **1** Substitute each variable with its correct value.

- 2** Evaluate the expression in the pair of brackets.

- 3** Perform the addition and write the answer.

- ii** **1** Substitute each variable with its correct value.

- 2** Evaluate the expression in the pair of brackets.

- 3** Perform the addition and write the answer.

- 4** Compare the result with the answer obtained in part **a i**.

- b i** **1** Substitute each variable with its correct value.

- 2** Evaluate the expression in the pair of brackets.

- 3** Perform the subtraction and write the answer.

- ii** **1** Substitute each variable with its correct value.

- 2** Evaluate the expression in the pair of brackets.

- 3** Perform the subtraction and write the answer.

- 4** Compare the result with the answer obtained in part **b i**.

- c i** **1** Substitute each variable with its correct value.

- 2** Evaluate the expression in the pair of brackets.

- 3** Perform the multiplication and write the answer.

- ii** **1** Substitute each variable with its correct value.

a i $x + (y + z) = 12 + (6 + 2)$

$$= 12 + 8$$

$$= 20$$

ii $(x + y) + z = (12 + 6) + 2$

$$= 18 + 2$$

$$= 20$$

The same result is obtained; therefore, order is not important when adding 3 terms.

b i $x - (y - z) = 12 - (6 - 2)$

$$= 12 - 4$$

$$= 8$$

ii $(x - y) - z = (12 - 6) - 2$

$$= 6 - 2$$

$$= 4$$

Two different results are obtained; therefore, order is important when subtracting 3 terms.

c i $x \times (y \times z) = 12 \times (6 \times 2)$

$$= 12 \times 12$$

$$= 144$$

ii $(x \times y) \times z = (12 \times 6) \times 2$



◀	2	Evaluate the expression in the pair of brackets.	$= 72 \times 2$
	3	Perform the multiplication and write the answer.	$= 144$
	4	Compare the result with the answer obtained in part c i.	The same result is obtained; therefore, order is not important when multiplying 3 terms.
d i	1	Substitute each variable with its correct value.	d i $x \div (y \div z) = 12 \div (6 \div 2)$
	2	Evaluate the expression in the pair of brackets.	$= 12 \div 3$
	3	Perform the division and write the answer.	$= 4$
ii	1	Substitute each variable with its correct value.	ii $(x \div y) \div z = (12 \div 6) \div 2$
	2	Evaluate the expression in the pair of brackets.	$= 2 \div 2$
	3	Perform the division and write the answer.	$= 1$
	4	Compare the result with the answer obtained in part d i.	Two different results are obtained; therefore, order is important when dividing 3 terms.

Identity Law

- The **Identity Law** for addition states that when zero is added to any number, the original number remains unchanged. For example, $5 + 0 = 0 + 5 = 5$.
- The Identity Law for multiplication states that when any number is multiplied by one, the original number remains unchanged. For example, $3 \times 1 = 1 \times 3 = 3$.
- Since variables take the place of numbers:

$$x + 0 = 0 + x = x$$

$$x \times 1 = 1 \times x = x$$

Inverse Law

- The **Inverse Law** for addition states that when a number is added to its additive inverse (opposite sign), the result is zero. For example, $5 + -5 = 0$.
- The Inverse Law for multiplication states that when a number is multiplied by its multiplicative inverse (reciprocal), the result is one. For example, $3 \times \frac{1}{3} = 1$.
- Since variables take the place of numbers:

$$x + -x = -x + x = 0$$

$$x \times \frac{1}{x} = \frac{1}{x} \times x = 1$$

Exercise 8.6 Number laws and variables

assessment

INDIVIDUAL PATHWAYS

PRACTISE

 Questions:
1–6, 10

CONSOLIDATE

 Questions:
1a–e, 2a–e, 3a–h, 4a–d,
5a–d, 6, 7, 9, 10

MASTER

 Questions:
1e–h, 2e–h, 3g–l, 4c–f, 5c–f,
6–11

REFLECTION

 The Commutative Law does not hold for subtraction. What can you say about the results of $x - a$ and $a - x$?

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FLUENCY

- 1 **WE6a,b** Find the value of the following expressions if $x = 3$ and $y = 8$. Comment on the results obtained.

- | | |
|---------------|--------------|
| a i $x + y$ | ii $y + x$ |
| b i $3x + 2y$ | ii $2y + 3x$ |
| c i $5x + 2y$ | ii $2y + 5x$ |
| d i $8x + y$ | ii $y + 8x$ |
| e i $x - y$ | ii $y - x$ |
| f i $2x - 3y$ | ii $3y - 2x$ |
| g i $4x - 5y$ | ii $5y - 4x$ |
| h i $3x - y$ | ii $y - 3x$ |

- 2 **WE6c,d** Find the value of the following expressions if $x = -2$ and $y = 5$. Comment on the results obtained.

- | | |
|--------------------|-------------------|
| a i $x \times y$ | ii $y \times x$ |
| b i $6x \times 3y$ | ii $3y \times 6x$ |
| c i $4x \times y$ | ii $y \times 4x$ |
| d i $7x \times 5y$ | ii $5y \times 7x$ |
| e i $x \div y$ | ii $y \div x$ |
| f i $10x \div 4y$ | ii $4y \div 10x$ |
| g i $6x \div 3y$ | ii $3y \div 6x$ |
| h i $7x \div 9y$ | ii $9y \div 7x$ |

- 3 Indicate whether each of the following is true or false for all values of the variables.

- | | | |
|-----------------------------------|---------------------------------|---|
| a $a + 5b = 5b + a$ | b $6x - 2y = 2y - 6x$ | c $7c + 3d = -3d + 7c$ |
| d $5 \times 2x \times x = 10x^2$ | e $4x \times -y = -y \times 4x$ | f $4 \times 3x \times x = 12x \times x$ |
| g $\frac{5p}{3r} = \frac{3r}{5p}$ | h $-7i - 2j = 2j + 7i$ | i $-3y \div 4x = 4x \div -3y$ |
| j $-2c + 3d = 3d - 2c$ | k $\frac{0}{3s} = \frac{3s}{0}$ | l $15 \times -\frac{2x}{3} = \frac{2x}{3} \times -15$ |

- 4 **WE7a,b** Find the value of the following expressions if $x = 3$, $y = 8$ and $z = 2$. Comment on the results obtained.

- | | |
|----------------------|---------------------|
| a i $x + (y + z)$ | ii $(x + y) + z$ |
| b i $2x + (y + 5z)$ | ii $(2x + y) + 5z$ |
| c i $6x + (2y + 3z)$ | ii $(6x + 2y) + 3z$ |
| d i $x - (y - z)$ | ii $(x - y) - z$ |

e i $x - (7y - 9z)$ **ii** $(x - 7y) - 9z$

f i $3x - (8y - 6z)$ **ii** $(3x - 8y) - 6z$

- 5 WE7c,d** Find the value of the following expressions if $x = 8$, $y = 4$ and $z = -2$.
Comment on the results obtained.

a i $x \times (y \times z)$ **ii** $(x \times y) \times z$

b i $x \times (-3y \times 4z)$ **ii** $(x \times -3y) \times 4z$

c i $2x \times (3y \times 4z)$ **ii** $(2x \times 3y) \times 4z$

d i $x \div (y \div z)$ **ii** $(x \div y) \div z$

e i $x \div (2y \div 3z)$ **ii** $(x \div 2y) \div 3z$

f i $-x \div (5y \div 2z)$ **ii** $(-x \div 5y) \div 2z$

UNDERSTANDING

- 6** Indicate whether each of the following is true or false for all values of the variables.

a $a - 0 = 0$ **b** $a \times 1\,000\,000 = 0$ **c** $15t \times -\frac{1}{15t} = 1$

d $3d \times \frac{1}{3d} = 1$ **e** $\frac{8x}{9y} \div \frac{8x}{9y} = 1$ **f** $\frac{11t}{0} = 0$

- 7 MC** The value of the expression $x \times (-3y \times 4z)$ when $x = 4$, $y = 3$ and $z = -3$ is:

A 108 **B** -432 **C** 432

D 112 **E** -108

- 8 MC** The value of the expression $(x - 8y) - 10z$ when $x = 6$, $y = 5$ and $z = -4$ is:

A -74 **B** 74 **C** -6

D 6 **E** -36

REASONING

- 9 a** If $x = -1$, $y = -2$ and $z = -3$, find the value of:

i $-x - y - z$

ii $-x - (y + z)$

- b** Comment on the answers with special reference to the Associative Law.

PROBLEM SOLVING

- 10** Evaluate each of the following expressions for $x = -3$, $y = 2$ and $z = -1$.

a $2x - (3y + 2z)$

b $x \times (y - 2z)$

- 11 a** State the additive inverse of $(3p - 4q)$.

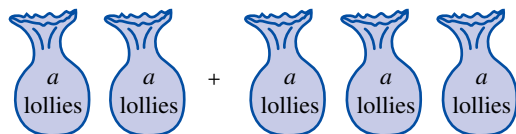
- b** State the multiplicative inverse of $(3p - 4q)$.

- c** Evaluate **a** and **b** when $p = -1$ and $q = 3$.

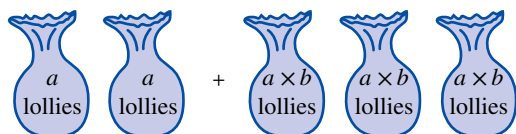
- d** What happens if you multiply a term or pronumeral by its multiplicative inverse?

8.7 Simplifying expressions

- Expressions can often be written in a more simple form by collecting (adding or subtracting) like terms.
- Like terms** are terms that contain exactly the same variables, raised to the same power. To understand why $2a + 3a$ can be added but $2a + 3ab$ can *not* be added, consider the following identical bags of lollies, each containing a lollies.



So, $2a + 3a = 5a$. Therefore we have 5 bags each containing a lollies.
Then consider the following 2 bags containing a lollies and 3 bags containing $a \times b$ lollies.



So $2a + 3ab$ cannot be added as they are not identical and we do not have any further information.

So, $2a + 3ab = 2a + 3ab$. Therefore all we can say is that we have 2 bags containing a lollies and 3 bags containing ab lollies.

For example:

$3x$ and $4x$ are like terms.

$3x$ and $3y$ are not like terms.

$3ab$ and $7ab$ are like terms.

$7ab$ and $8a$ are not like terms.

$2bc$ and $4cb$ are like terms.

$8a$ and $3a^2$ are not like terms.

$3g^2$ and $45g^2$ are like terms.

WORKED EXAMPLE 8

Simplify the following expressions.

a $3a + 5a$

b $7ab - 3a - 4ab$

c $2c - 6 + 4c + 15$

THINK

1 Write the expression and check that the two terms are like terms, that is, they contain the same variables.

2 Add the like terms and write the answer.

1 Write the expression and check for like terms.

2 Rearrange the terms so that the like terms are together. Remember to keep the correct sign in front of each term.

3 Subtract the like terms and write the answer.

1 Write the expression and check for like terms.

2 Rearrange the terms so that the like terms are together. Remember to keep the correct sign in front of each term.

3 Simplify by collecting like terms and write the answer.

WRITE

a $3a + 5a$

$= 8a$

b $7ab - 3a - 4ab$

$= 7ab - 4ab - 3a$

$= 3ab - 3a$

c $2c - 6 + 4c + 15$

$= 2c + 4c - 6 + 15$

$= 6c + 9$



Exercise 8.7 Simplifying expressions

INDIVIDUAL PATHWAYS

REFLECTION

What do you need to remember when checking for like terms?

PRACTISE

Questions:
1, 2, 3a–i, 4

CONSOLIDATE

Questions:
1, 2a–o, 3a–l, 4, 7

MASTER

Questions:
1m–x, 2m–v, 3m–u, 4–7

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Combining like terms

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SkillSHEET

Simplifying fractions

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FLUENCY

1 **WE8a** Simplify the following expressions.

a $4c + 2c$

b $2c - 5c$

c $3a + 5a - 4a$

d $6q - 5q$

e $-h - 2h$

f $7x - 5x$

g $3a - 7a - 2a$

h $-3f + 7f$

i $4p - 7p$

j $-3h + 4h$

k $11b + 2b + 5b$

l $7t - 8t + 4t$

m $9m + 5m - m$

n $x - 2x$

o $7z + 13z$

p $5p + 3p + 2p$

q $9g + 12g - 4g$

r $18b - 4b - 11b$

s $13t - 4t + 5t$

t $-11j + 4j$

u $-12l + 2l - 5l$

v $13m - 2m - 4m + m$

w $m + 3m - 4m$

x $t + 2t - t + 8t$

2 **WE8b,c** Simplify the following expressions.

a $3x + 7x - 2y$

b $3x + 4x - 12$

c $11 + 5f - 7f$

d $3u - 4u + 6$

e $2m + 3p + 5m$

f $-3h + 4r - 2h$

g $11a - 5b + 6a$

h $9t - 7 + 5$

i $12 - 3g + 5$

j $6m + 4m - 3n + n$

k $5k - 5 + 2k - 7$

l $3n - 4 + n - 5$

m $2b - 6 - 4b + 18$

n $11 - 12h + 9$

o $12y - 3y - 7g + 5g - 6$

p $8h - 6 + 3h - 2$

q $11s - 6t + 4t - 7s$

r $2m + 13l - 7m + l$

s $3h + 4k - 16h - k + 7$

t $13 + 5t - 9t - 8$

u $2g + 5 + 5g - 7$

v $17f - 3k + 2f - 7k$

UNDERSTANDING

3 Simplify the following expressions.

a $x^2 + 2x^2$

b $3y^2 + 2y^2$

c $a^3 + 3a^3$

d $d^2 + 6d^2$

e $7g^2 - 8g^2$

f $3y^3 + 7y^3$

g $2b^2 + 5b^2$

h $4a^2 - 3a^2$

- i** $g^2 - 2g^2$
k $11x^2 - 6 + 12x^2 + 6$
m $3a^2 + 2a + 5a^2 + 3a$
o $6t^2 - 6g - 5t^2 + 2g - 7$
q $12ab + 3 + 6ab$
s $4fg + 2s - fg + s$
u $18ab^2 - 4ac + 2ab^2 - 10ac$
- j** $a^2 + 4 + 3a^2 + 5$
l $12s^2 - 3 + 7 - s^2$
n $11b - 3b^2 + 4b^2 + 12b$
p $11g^3 + 17 - 3g^3 + 5 - g^2$
r $14xy + 3xy - xy - 5xy$
t $11ab + ab - 5$

REASONING

- 4** Rose owns an art gallery and sells items supplied to her by various artists. She receives a commission for all items sold. She uses the following method to keep track of the money she owes the artists when their items are sold.

- Ask the artist how much they want for the item.
- Add 50% to that price, then mark the item for sale at this new price.
- When the item sells, take one-third of the sale price as commission, then return the balance to the artist.

Use algebra to show that this method does return the correct amount to the artist.

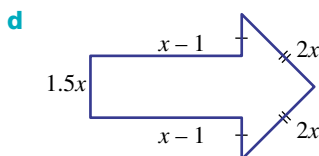
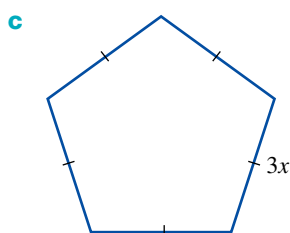
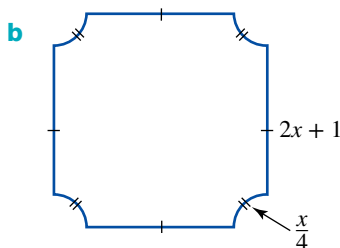
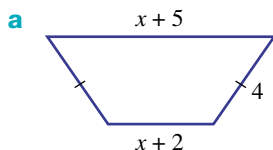
- 5** Explain, using mathematical reasoning and with diagrams if necessary, why the expression $2x + 2x^2$ cannot be simplified.

PROBLEM SOLVING

- 6** Three members of a fundraising committee are making books of tickets to sell for a raffle. Each book of tickets contains t tickets. If the first person has 14 books of tickets, the second person has 12 books of tickets and the third person has 13 books of tickets, write an expression for:

- a** the number of tickets that the first person has
b the number of tickets that the second person has
c the number of tickets that the third person has
d the total number of tickets for the raffle.

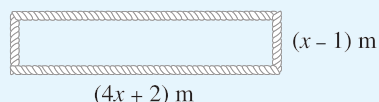
- 7** Write an expression for the total perimeter of the following shapes.





CHALLENGE 8.2

If 32 metres of rope are required to make the rectangular shape shown in the figure, what is the value of x ?



8.8 Multiplying and dividing expressions with variables

Multiplying variables

- When we multiply variables (as already stated) the Commutative Law holds, so order is not important. For example:

$$3 \times 6 = 6 \times 3$$

$$6 \times w = w \times 6$$

$$a \times b = b \times a$$

- The multiplication sign ($\times$) is usually omitted for reasons of convention.

$$3 \times g \times h = 3gh$$

$$2 \times x^2 \times y = 2x^2y$$

- Although order is not important, conventionally the variables in each term are written in alphabetical order. For example,

$$2 \times b^2 \times a \times c = 2ab^2c$$

WORKED EXAMPLE 9

Simplify the following.

a $5 \times 4g$

b $-3d \times 6ab \times 7$

THINK

- a
- 1 Write the expression and replace the hidden multiplication signs.
 - 2 Multiply the numbers.
 - 3 Remove the multiplication sign.
- b
- 1 Write the expression and replace the hidden multiplication signs.
 - 2 Place the numbers at the front.
 - 3 Multiply the numbers.
 - 4 Remove the multiplication signs and place the variables in alphabetical order.

WRITE

a $5 \times 4g$
 $= 5 \times 4 \times g$

$= 20 \times g$

$= 20g$

b $-3d \times 6ab \times 7$
 $= -3 \times d \times 6 \times a \times b \times 7$

$= -3 \times 6 \times 7 \times d \times a \times b$

$= -126 \times d \times a \times b$

$= -126abd$

Dividing expressions with variables

- When dividing expressions with variables, rewrite the expression as a fraction and simplify by cancelling.
- Remember that when the same variable appears as a factor on both the numerator and denominator, it may be cancelled.

WORKED EXAMPLE 10

a Simplify $\frac{16f}{4}$.

b Simplify $15n \div 3n$.

THINK

- a
- 1 Write the expression.
 - 2 Simplify the fraction by cancelling 16 with 4 (divide both by 4).
 - 3 No need to write the denominator since we are dividing by 1.
- b
- 1 Write the expression and then rewrite it as a fraction.

WRITE

a $\frac{16f}{4} = \frac{4\cancel{16}f}{\cancel{4}_1}$

$= \frac{4f}{1}$

$= 4f$

b $15n \div 3n = \frac{15n}{3n}$
 $= \frac{\cancel{15}^5n}{\cancel{3}_1n}$





- | | |
|---|-----------------|
| <p>2 Simplify the fraction by cancelling 15 with 3 and n with n.</p> | $= \frac{5}{1}$ |
| <p>3 No need to write the denominator since we are dividing by 1.</p> | $= 5$ |

WORKED EXAMPLE 11

Simplify $-12xy \div 27y$.

THINK

- 1** Write the expression and then rewrite it as a fraction.
- 2** Simplify the fraction by cancelling 12 with 27 (divide both by 3) and y with y .

WRITE

$$\begin{aligned}
 -12xy \div 27y &= -\frac{12xy}{27y} \\
 &= -\frac{\overset{4}{\cancel{12}}x\cancel{y}}{\underset{9}{\cancel{27}}\cancel{y}} \\
 &= -\frac{4x}{9}
 \end{aligned}$$

WORKED EXAMPLE 12

Simplify the following.

- | | |
|---|---|
| <p>a $3m^3 \times 2m$</p> | <p>b $5p^{10} \times 3p^3$</p> |
| <p>c $36x^7 \div 12x^4$</p> | <p>d $\frac{6y^3 \times 4y^8}{12y^4}$</p> |

THINK

- | | |
|--|--|
| <p>a 1 Write the problem.</p> <p>2 The order is not important when multiplying, so place the numbers first.</p> <p>3 Multiply the numbers.</p> <p>4 Check to see if the bases are the same. They are both m.</p> <p>5 Simplify by adding the indices.</p> | <p>WRITE</p> <p>a $3m^3 \times 2m$</p> <p>$= 3 \times 2 \times m^3 \times m$</p> <p>$= 6 \times m^3 \times m$</p> <p>$= 6 \times m^{3+1}$</p> <p>$= 6m^4$</p> |
| <p>b 1 Write the problem.</p> <p>2 The order is not important when multiplying, so place the numbers first.</p> <p>3 Multiply the numbers.</p> <p>4 Check to see if the bases are the same. They are both p.</p> <p>5 Simplify by adding the indices.</p> | <p>b $5p^{10} \times 3p^3$</p> <p>$= 5 \times 3 \times p^{10} \times p^3$</p> <p>$= 15 \times p^{10} \times p^3$</p> <p>$= 15 \times p^{10+3}$</p> <p>$= 15p^{13}$</p> |

c 1 Write the problem and express it as a fraction.

2 Divide the numbers.

3 Check to see if the bases are the same. They are both x .

4 Subtract the powers.

d 1 Write the problem.

2 Multiply the numbers in the numerator. Simplify the numbers in index form in the numerator.

3 Divide the numbers and subtract the powers.

$$36x^7 \div 12x^4$$

$$= \frac{36x^7}{12x^4}$$

$$= \frac{3x^7}{x^4}$$

$$= 3x^{7-4}$$

$$= 3x^3$$

$$\frac{6y^3 \times 4y^8}{12y^4}$$

$$= \frac{24y^{11}}{12y^4}$$

$$= 2y^{11-4}$$

$$= 2y^7$$

Exercise 8.8 Multiplying and dividing expressions with variables

assess on

INDIVIDUAL PATHWAYS

PRACTISE

Questions:
1–5, 9, 10

CONSOLIDATE

Questions:
1a–p, 2a–l, 3a–l, 4a–l, 5a–l,
6a–l, 7a–e, 8–10

MASTER

Questions:
1m–z, 2m–t, 3m–t, 4m–u, 5m–t,
6m–r, 7e–j, 8–11

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REFLECTION

How are multiplication and division of expressions with variables similar to multiplication and division of numbers?

FLUENCY

1 WE9 Simplify the following.

a $4 \times 3g$

b $7 \times 3h$

c $4d \times 6$

d $3z \times 5$

e $6 \times 5r$

f $5t \times 7$

g $4 \times 3u$

h $7 \times 6p$

i $7gy \times 3$

j $2 \times 11ht$

k $4x \times 6g$

l $10a \times 7h$

m $9m \times 4d$

n $3c \times 5h$

o $9g \times 2x$

p $2.5t \times 5b$

q $13m \times 12n$

r $6a \times 12d$

s $2ab \times 3c$

t $4f \times 3gh$

u $2 \times 8w \times 3x$

v $11ab \times 3d \times 7$

w $16xy \times 1.5$

x $3.5x \times 3y$

y $11q \times 4s \times 3$

z $4a \times 3b \times 2c$

2 WE10 Simplify the following.

a $\frac{8f}{2}$

b $\frac{6h}{3}$

c $\frac{15x}{3}$

d $9g \div 3$

e $10r \div 5$

f $4x \div 2x$

g $8r \div 4r$

h $\frac{16m}{8m}$

i $14q \div 21q$

j $\frac{3x}{6x}$

k $\frac{12h}{14h}$

l $50g \div 75g$

$$\text{m } \frac{8f}{24f}$$

$$\text{n } 35x \div 70x$$

$$\text{o } 24m \div 36m$$

$$\text{p } y \div 34y$$

$$\text{q } 27h \div 3h$$

$$\text{r } \frac{20d}{48d}$$

$$\text{s } \frac{64q}{44q}$$

$$\text{t } 81l \div 27l$$

3 Simplify the following.

$$\text{a } \frac{15fg}{3}$$

$$\text{b } 12cd \div 4$$

$$\text{c } \frac{8xy}{12}$$

$$\text{d } 24cg \div 24$$

$$\text{e } \frac{11xy}{11x}$$

$$\text{f } \frac{9pq}{18q}$$

$$\text{g } \frac{21ab}{28b}$$

$$\text{h } \frac{9dg}{12g}$$

$$\text{i } \frac{5jk}{kj}$$

$$\text{j } 55rt \div 77t$$

$$\text{k } \frac{10mxy}{35mx}$$

$$\text{l } 36bc \div 27c$$

$$\text{m } 13xy \div x$$

$$\text{n } \frac{16cd}{40cd}$$

$$\text{o } 14abc \div 7bc$$

$$\text{p } 3gh \div 6h$$

$$\text{q } \frac{132mnp}{60np}$$

$$\text{r } \frac{11ad}{66ad}$$

$$\text{s } 18adg \div 45ag$$

$$\text{t } \frac{bh}{7h}$$

4 Simplify the following.

$$\text{a } 3 \times -5f$$

$$\text{b } -6 \times -2d$$

$$\text{c } 11a \times -3g$$

$$\text{d } -9t \times -3g$$

$$\text{e } -5t \times -4dh$$

$$\text{f } 6 \times -3st$$

$$\text{g } -3 \times -2w \times 7d$$

$$\text{h } -4a \times -3b \times 2c \times e$$

$$\text{i } 11ab \times -3f$$

$$\text{j } 3as \times -3b \times -2x$$

$$\text{k } -5h \times -5t \times -3q$$

$$\text{l } 4 \times -3w \times -2 \times 6p$$

$$\text{m } -7a \times 3b \times g$$

$$\text{n } 17ab \times -3gh$$

$$\text{o } -3.5g \times 2h \times 7$$

$$\text{p } 5h \times 8j \times -k$$

$$\text{q } 75x \times 1.5y$$

$$\text{r } 12rt \times -3z \times 4p$$

$$\text{s } 2ab \times 3c \times 5$$

$$\text{t } -4w \times 34x \times 3$$

$$\text{u } -3ab \times -5cd \times -6ef$$

5 **WE11** Simplify the following.

$$\text{a } \frac{-4a}{8}$$

$$\text{b } \frac{-11ab}{33b}$$

$$\text{c } 60jk \div -5k$$

$$\text{d } -3h \div -6dh$$

$$\text{e } \frac{-32g}{40gl}$$

$$\text{f } -12xy \div 48y$$

$$\text{g } \frac{12ab}{-14ab}$$

$$\text{h } \frac{6fgh}{30ghj}$$

$$\text{i } -4xyz \div 6yz$$

$$\text{j } \frac{-rt}{6rt}$$

$$\text{k } -5mn \div 20n$$

$$\text{l } -14st \div -28$$

$$\text{m } 34ab \div -17ab$$

$$\text{n } \frac{-ab}{-3a}$$

$$\text{o } \frac{-7dg}{35gh}$$

$$\text{p } -60mn \div 55mnp$$

$$\text{q } \frac{28def}{18d}$$

$$\text{r } -72xyz \div 28yz$$

$$\text{s } \frac{54pq}{36pqr}$$

$$\text{t } -\frac{121oc}{132oct}$$

UNDERSTANDING

6 **WE12** Simplify the following.

$$\text{a } 2a \times a$$

$$\text{b } -5p \times -5p$$

$$\text{c } -5 \times 3x \times 2x$$

$$\text{d } ab \times 7a$$

$$\text{e } 3b^2 \times 2cd$$

$$\text{f } -5xy \times 4 \times 8x$$

$$\text{g } 7pq \times 3p \times 2q$$

$$\text{h } 5m \times n \times 6nt \times -t$$

$$\text{i } -3 \times xyz \times -3z \times -2y$$

$$\text{j } -7a \times -3b \times -2c^2$$

$$\text{k } 2mn \times -3 \times 2n \times 0$$

$$\text{l } w^2x \times -9z^2 \times 2xy^2$$

$$\text{m } 2a^4 \times 3a^7$$

$$\text{p } \frac{25p^{12} \times 4q^7}{15p^2 \times 8q^2}$$

$$\text{n } 2x^2y^3 \times x^3$$

$$\text{q } \frac{8x^3 \times 7y^2 \times 2z^2}{6x \times 14y}$$

$$\text{o } 20m^{12} \div 2m^3$$

$$\text{r } \frac{a \times ab \times 3b^2}{5a^2b^2}$$

7 Simplify the following.

$$\text{a } \frac{3}{a} \times \frac{2}{a}$$

$$\text{b } \frac{5b}{2} \times \frac{4b}{3}$$

$$\text{c } w \times \frac{5}{w^2}$$

$$\text{d } \frac{3rk}{2s} \times \frac{6st}{5rt}$$

$$\text{e } -\frac{15gt}{10ag} \times \frac{2g}{5t}$$

$$\text{f } -\frac{4ht}{3dk} \times \frac{-12hk}{9dt}$$

$$\text{g } \frac{5t}{gn} \div \frac{1}{g}$$

$$\text{h } \frac{-9th}{4g} \div \frac{tg}{6h}$$

$$\text{i } \frac{4xy}{7wz} \div \frac{x}{14z}$$

$$\text{j } \frac{-10f}{3w} \div \frac{5}{-9wz}$$

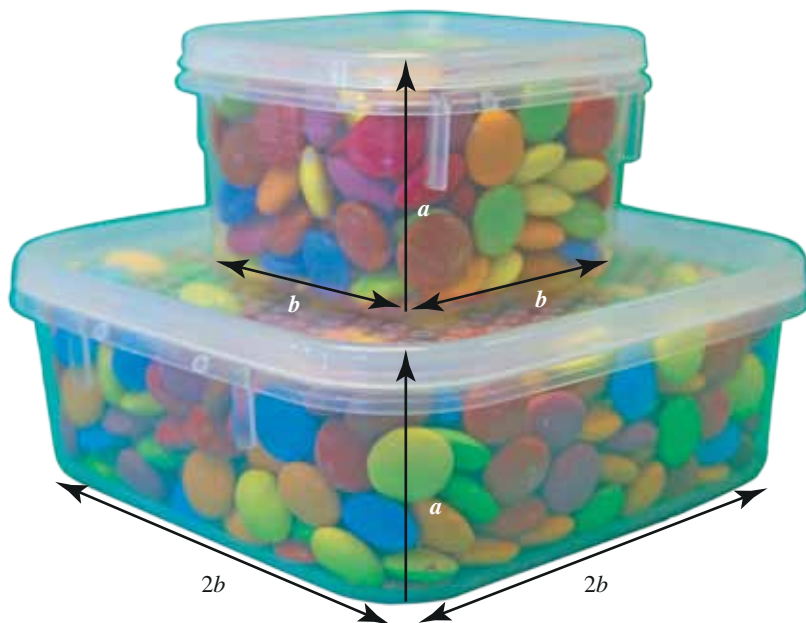
REASONING

- 8 The student's working below shows incorrect cancelling. Explain why the working is not correct.

$$\frac{3 + 5\cancel{x}^1}{2\cancel{x}^1}$$

PROBLEM SOLVING

- 9 Evaluate $\frac{2x - (3y + 2z)}{-(-x - y) + z}$ if $x = 2$, $y = -1$ and $z = -4$.
- 10 Evaluate $\frac{x \times (y - 2z)}{3x - (y - z)}$ if $x = -2$, $y = 5$ and $z = -1$.
- 11 a Write an expression for the volume of each of the containers shown.



(Hint: The volume of a container is found by multiplying the length by the width by the height.)

- b How many times will the contents of the smaller container fit inside the larger container? (Hint: Divide the volumes.)

8.9 Expanding brackets

The Distributive Law

- The **Distributive Law** is the name given to the following process.

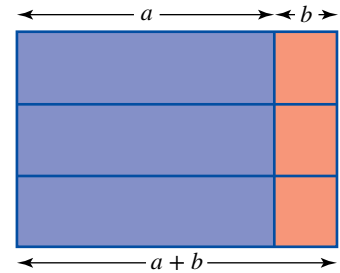
$$3(5 + 8) = 3 \times 5 + 3 \times 8$$

This is because the number out in front is *distributed* to each of the terms in the bracket.

- Since variables take the place of numbers, the Distributive Law also holds true for algebraic expressions.

$$a(b + c) = ab + ac$$

- The Distributive Law can be demonstrated using the concept of area. As can be seen in the diagram at right, $3(a + b) = 3a + 3b$
- We can think of $3(a + b) = (a + b) + (a + b) + (a + b)$
Collecting like terms, $a + a + a + b + b + b = 3a + 3b$
- An expression containing a bracket multiplied by a number can be written in expanded or factorised form.



Factorised form = Expanded form

$$\overbrace{3(a + b)} \quad = \quad \overbrace{3a + 3b}$$

Expanding and factorising are the inverse of each other.

- The Distributive Law can be used when the terms inside the brackets are either added or subtracted.

$$a(b - c) = ab - ac$$

- The Distributive Law is not used when the terms inside the brackets are multiplied or divided. You can see this with numbers $2(4 \times 5) = 2 \times 4 \times 5$; *not* $(2 \times 4) \times (2 \times 5)$.
- When simplifying expressions, we can leave the result in either factorised form or expanded form, but not a combination of both.

WORKED EXAMPLE 13

Use the Distributive Law to expand the following expressions.

a $3(a + 2)$

b $x(x - 5)$

THINK

- a**
- 1 Write the expression.
 - 2 Use the Distributive Law to expand the brackets.
 - 3 Simplify by multiplying.
- b**
- 1 Write the expression.
 - 2 Use the Distributive Law to expand the brackets.
 - 3 Simplify by multiplying.

WRITE

$$\begin{aligned} \mathbf{a} \quad 3(a + 2) &= 3(a + 2) \\ &= 3 \times a + 3 \times 2 \\ &= 3a + 6 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad x(x - 5) &= x(x - 5) \\ &= x \times x + x \times -5 \\ &= x^2 - 5x \end{aligned}$$

- Some expressions can be simplified further by collecting like terms after any brackets have been expanded.

WORKED EXAMPLE 14

Expand the expressions below and then simplify by collecting any like terms.

a $3(x - 5) + 4$

b $4(3x + 4) + 7x + 12$

c $2x(3y + 3) + 3x(y + 1)$

d $4x(2x - 1) - 3(2x - 1)$

THINK

- a**
- 1 Write the expression.
 - 2 Expand the brackets.
 - 3 Collect the like terms (-15 and 4).
- b**
- 1 Write the expression.
 - 2 Expand the brackets.
 - 3 Rearrange so that the like terms are together. (Optional)
 - 4 Collect the like terms.
- c**
- 1 Write the expression.
 - 2 Expand the brackets.
 - 3 Rearrange so that the like terms are together. (Optional)
 - 4 Simplify by collecting the like terms.
- d**
- 1 Write the expression.
 - 2 Expand the brackets. Take care with negative terms.
 - 3 Simplify by collecting the like terms.

WRITE

a $3(x - 5) + 4$

$$\begin{aligned} &= 3 \times x + 3 \times -5 + 4 \\ &= 3x - 15 + 4 \\ &= 3x - 11 \end{aligned}$$

b $4(3x + 4) + 7x + 12$

$$\begin{aligned} &= 4 \times 3x + 4 \times 4 + 7x + 12 \\ &= 12x + 16 + 7x + 12 \\ &= 12x + 7x + 16 + 12 \end{aligned}$$

$$= 19x + 28$$

c $2x(3y + 3) + 3x(y + 1)$

$$\begin{aligned} &= 2x \times 3y + 2x \times 3 + 3x \times y + 3x \times 1 \\ &= 6xy + 6x + 3xy + 3x \\ &= 6xy + 3xy + 6x + 3x \\ &= 9xy + 9x \end{aligned}$$

d $4x(2x - 1) - 3(2x - 1)$

$$\begin{aligned} &= 4x \times 2x + 4x \times -1 - 3 \times 2x - 3 \times -1 \\ &= 8x^2 - 4x - 6x + 3 \\ &= 8x^2 - 10x + 3 \end{aligned}$$

Exercise 8.9 Expanding brackets

assess on

INDIVIDUAL PATHWAYS

PRACTISE

Questions:
1–4, 8

CONSOLIDATE

Questions:
1a–l, 2a–l, 3a–l, 4a–f, 5a–f, 8

MASTER

Questions:
1m–t, 2m–r, 3j–p, 4f–j, 5f–j, 6–9

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REFLECTION

Why doesn't the Distributive Law apply when there is a multiplication sign inside the brackets, that is for $a(b \times c)$?

FLUENCY

1 WE13 Use the Distributive Law to expand the following expressions.

a $3(d + 4)$

b $2(a + 5)$

c $4(x + 2)$

d $5(r + 7)$

e $6(g + 6)$

f $2(t - 3)$

g $7(d + 8)$

h $9(2x - 6)$

- | | | | |
|----------------------|----------------------|-----------------------|-----------------------|
| i $12(4 + c)$ | j $7(6 + 3x)$ | k $45(2g + 3)$ | l $1.5(t + 6)$ |
| m $11(t - 2)$ | n $3(2t - 6)$ | o $t(t + 3)$ | p $x(x + 4)$ |
| q $g(g + 7)$ | r $2g(g + 5)$ | s $3f(g + 3)$ | t $6m(n - 2m)$ |

2 Expand the following.

- | | | | |
|-------------------------|-------------------------|------------------------|-------------------------|
| a $3(3x - 2)$ | b $3x(x - 6y)$ | c $5y(3x - 9y)$ | d $50(2y - 5)$ |
| e $-3(c + 3)$ | f $-5(3x + 4)$ | g $-5x(x + 6)$ | h $-2y(6 + y)$ |
| i $-6(t - 3)$ | j $-4f(5 - 2f)$ | k $9x(3y - 2)$ | l $-3h(2b - 6h)$ |
| m $4a(5b + 3c)$ | n $-3a(2g - 7a)$ | o $5a(3b + 6c)$ | p $-2w(9w - 5z)$ |
| q $12m(4m + 10)$ | r $-3k(-2k + 5)$ | | |

3 **WE14** Expand the expressions below and then simplify by collecting any like terms.

- | | |
|--------------------------------|--|
| a $7(5x + 4) + 21$ | b $3(c - 2) + 2$ |
| c $2c(5 - c) + 12c$ | d $6(v + 4) + 6$ |
| e $3d(d - 4) + 2d^2$ | f $3y + 4(2y + 3)$ |
| g $24r + r(2 + r)$ | h $5 - 3g + 6(2g - 7)$ |
| i $4(2f - 3g) + 3f - 7$ | j $3(3x - 4) + 12$ |
| k $-2(k + 5) - 3k$ | l $3x(3 + 4r) + 9x - 6xr$ |
| m $12 + 5(r - 5) + 3r$ | n $12gh + 3g(2h - 9) + 3g$ |
| o $3(2t + 8) + 5t - 23$ | p $24 + 3r(2 - 3r) - 2r^2 + 5r$ |

4 Expand the following and then simplify by collecting like terms.

- | | |
|-----------------------------------|-----------------------------------|
| a $3(x + 2) + 2(x + 1)$ | b $5(x + 3) + 4(x + 2)$ |
| c $2(y + 1) + 4(y + 6)$ | d $4(d + 7) - 3(d + 2)$ |
| e $6(2h + 1) + 2(h - 3)$ | f $3(3m + 2) + 2(6m - 5)$ |
| g $9(4f + 3) - 4(2f + 7)$ | h $2a(a + 2) - 5(a^2 + 7)$ |
| i $3(2 - t^2) + 2t(t + 1)$ | j $m(n + 4) - mn + 3m$ |

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UNDERSTANDING

5 Simplify the following expressions by removing the brackets and then collecting like terms.

- | | |
|------------------------------------|-------------------------------------|
| a $3h(2k + 7) + 4k(h + 5)$ | b $6n(3y + 7) - 3n(8y + 9)$ |
| c $4g(5m + 6) - 6(2gm + 3)$ | d $11b(3a + 5) + 3b(4 - 5a)$ |
| e $5a(2a - 7) - 5(a^2 + 7)$ | f $7c(2f - 3) + 3c(8 - f)$ |
| g $7x(4 - y) + 2xy - 29$ | h $11v(2w + 5) - 3(8 - 5vw)$ |
| i $3x(3 - 2y) + 6x(2y - 9)$ | j $8m(7n - 2) + 3n(4 + 7m)$ |

REASONING

- 6 a** Using the concept of area as shown above, explain with diagrams and mathematical reasoning why $5(6 - 2) = 5 \times 6 - 5 \times 2$.
- b** Using the concept of area as shown above, explain with diagrams and mathematical reasoning why $4(x - y) = 4 \times x - 4 \times y$.
- 7** Expressions of the form $(a + b)(c + d)$ can be expanded by using the Distributive Law twice. Distribute one of the factors over the other; for example,



$(a + b)(c + d)$. The expression can then be fully expanded following Worked example 13.

- a** $(x + 1)(x + 2)$ **b** $(a + 3)(a + 4)$
c $(c + 2)(c - 3)$ **d** $(y + 4)(y - 4)$
e $(u - 2)(u - 3)$ **f** $(k - 5)(k - 2)$

PROBLEM SOLVING

- 8** The price of a pair of jeans is \$50. During a sale, the price of the jeans is discounted by \$ d .
a Write an expression to represent the sale price of the jeans.
b If you buy three pairs of jeans during the sale, write an expression to represent the total purchase price:
 i containing brackets **ii** in expanded form (without brackets).
c Write an expression to represent the total change you would receive from \$200 for the three pairs of jeans purchased during the sale.
9 A triptych is a piece of art that is divided into three sections or panels. The middle panel is usually the largest panel and is flanked by two related panels.



- a** Write a simplified expression for the area of the three paintings (excluding the frame).
b Write a simplified expression for the combined area of the triptych.
c The value of f is $m + 102.5$. Substitute $(m + 102.5)$ into your combined area formula and simplify the expression.
d The actual value of m is 122.5 cm. Sketch the shape of the three paintings in your workbook and show the actual measurements of each, including length, width and area.

8.10 Factorising

- **Factorising** is the opposite process to expanding.
- Factorising involves identifying the highest common factors of the algebraic terms.
- To find the highest common factor of the algebraic terms:
 1. Find the highest common factor of the number parts.
 2. Find the highest common factor of the variable parts.
 3. Multiply these together.

WORKED EXAMPLE 15

Find the highest common factor of $6x$ and 10 .

THINK

- 1** Find the highest common factor of the number parts.
Break 6 down into factors.
Break 10 down into factors.
The highest common factor is 2.
- 2** Find the highest common factor of the variable parts.
There isn't one, because only the first term has a variable part.

WRITE

$$\begin{aligned} 6 &= 3 \times 2 \\ 10 &= 5 \times 2 \\ \text{HCF} &= 2 \end{aligned}$$

The HCF of $6x$ and 10 is 2.

WORKED EXAMPLE 16

Find the highest common factor of $14fg$ and $21gh$.

THINK

- 1** Find the highest common factor of the number parts.
Break 14 down into factors.
Break 21 down into factors.
The highest common factor is 7.
- 2** Find the highest common factor of the variable parts.
Break fg down into factors.
Break gh down into factors.
Both contain a factor of g .
- 3** Multiply these together.

WRITE

$$\begin{aligned} 14 &= 7 \times 2 \\ 21 &= 7 \times 3 \\ \text{HCF} &= 7 \end{aligned}$$

$$\begin{aligned} fg &= f \times g \\ gh &= g \times h \\ \text{HCF} &= g \end{aligned}$$

The HCF of $14fg$ and $21gh$ is $7g$.

- To factorise an expression, place the highest common factor of the terms outside the brackets and the remaining factors for each term inside the brackets.

WORKED EXAMPLE 17

Factorise the expression $2x + 6$.

THINK

- 1** Break down each term into its factors.
- 2** Write the highest common factor outside the brackets.
Write the other factors inside the brackets.
- 3** Remove the multiplication sign.

WRITE

$$\begin{aligned} 2x + 6 \\ &= 2 \times x + 2 \times 3 \\ &= 2 \times (x + 3) \\ &= 2(x + 3) \end{aligned}$$

WORKED EXAMPLE 18
Factorise $12gh - 8g$.
THINK

- 1 Break down each term into its factors.
- 2 Write the highest common factor outside the brackets.
Write the other factors inside the brackets.
- 3 Remove the multiplication signs.

WRITE

$$\begin{aligned}
 12gh - 8g &= 4 \times 3 \times g \times h - 4 \times 2 \times g \\
 &= 4 \times g \times (3 \times h - 2) \\
 &= 4g(3h - 2)
 \end{aligned}$$

Exercise 8.10 Factorising

assessment
INDIVIDUAL PATHWAYS
PRACTISE

 Questions:
1–4, 12

CONSOLIDATE

 Questions:
1, 2, 3a–l, 4a–l, 5, 6, 12, 13

MASTER

 Questions:
1, 2, 3j–r, 4m–v, 5–13

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REFLECTION

What strategies will you use to find the highest common factor?

FLUENCY

- 1 **WE15** Find the highest common factor of the following.

- | | | | |
|-------------|--------------|--------------|----------------|
| a 4 and 6 | b 6 and 9 | c 12 and 18 | d 13 and 26 |
| e 14 and 21 | f $2x$ and 4 | g $3x$ and 9 | h $12a$ and 16 |

- 2 **WE16** Find the highest common factor of the following.

- | | | |
|----------------------|---------------------|----------------------|
| a $2gh$ and $6g$ | b $3mn$ and $6mp$ | c $11a$ and $22b$ |
| d $4ma$ and $6m$ | e $12ab$ and $14ac$ | f $24fg$ and $36gh$ |
| g $20dg$ and $18ghq$ | h $11gl$ and $33lp$ | i $16mnp$ and $20mn$ |
| j $28bc$ and $12c$ | k $4c$ and $12cd$ | l x and $3xz$ |

- 3 **WE17** Factorise the following expressions.

- | | | |
|---------------|-------------|--------------|
| a $3x + 6$ | b $2y + 4$ | c $5g + 10$ |
| d $8x + 12$ | e $6f + 9$ | f $12c + 20$ |
| g $2d + 8$ | h $2x - 4$ | i $12g - 18$ |
| j $11h + 121$ | k $4s - 16$ | l $8x - 20$ |
| m $12g - 24$ | n $14 - 4b$ | o $16a + 64$ |
| p $48 - 12q$ | q $16 + 8f$ | r $12 - 12d$ |

- 4 **WE18** Factorise the following.

- | | | |
|---------------------|-------------------|------------------|
| a $3gh + 12$ | b $2xy + 6y$ | c $12pq + 4p$ |
| d $14g - 7gh$ | e $16jk - 2k$ | f $12eg + 2g$ |
| g $12k + 16$ | h $7mn + 6m$ | i $14ab + 7b$ |
| j $5a - 15abc$ | k $8r + 14rt$ | l $24mab + 12ab$ |
| m $4b - 6ab$ | n $12fg - 16gh$ | o $ab - 2bc$ |
| p $14x - 21xy$ | q $11jk + 3k$ | r $3p + 27pq$ |
| s $12ac - 4c + 3dc$ | t $4g + 8gh - 16$ | u $28s + 14st$ |
| v $15uv + 27vw$ | | |

eBookplus
Digital doc

 SkillSHEET
Highest common factor
doc-6928

eBookplus

Digital doc
Spreadsheet
Finding the HCF
doc-2289

UNDERSTANDING

- 5 Find the highest common factor of $4ab$, $6a^2b^3$ and $12a^3b$.
- 6 Find the lowest common multiple of $3ab$, $4a^3bc$ and $6a^2b^2$.
- 7 Simplify: $\frac{4x-4}{10x-20} \times \frac{15x+15}{3x-3} \times \frac{6x-12}{20x+20}$.
- 8 Simplify: $\frac{3x+6}{x-1} \div \frac{12x+24}{6(x+1)}$.
- 9 Factorise and hence simplify: $\frac{2x^2y-6xy^2}{a+2b} \times \frac{4a+8b}{3xy} \div \frac{2x-6y}{7}$.

REASONING

- 10 Simplify $(5ax^2y - 6bxy + 2ax^2y - bxy) \div (ax^2 - bx)$. Show your working.

PROBLEM SOLVING

- 11 A cuboid measures $(3x-6)$ cm by $(2x+8)$ cm by $(ax-5a)$ cm.
 - a Write an expression for its volume.
 - b If the cuboid weighs $(8x-16)$ g, find a factorised expression for its density in g/cm^3 .
- 12 A farmer's paddock is a rectangle of length $(2x-6)$ m and width $(3x+6)$ m.
 - a Find the area of the paddock in factorised form.
 - b State the smallest possible value of x . Explain your reasoning.



- 13 Factorise and hence simplify:

- a $\frac{3x^2y}{z^2} \times \frac{z^2}{12xy} \times \frac{6y^3}{z}$
- b $\frac{4x^2(y-1)}{(z-1)^2} \times \frac{z-1}{24x(y-1)} \times \frac{3(y-1)^2}{9z}$
- c $\frac{5xz^2}{y^2} \times \frac{y^2}{15xz} \div \frac{5y^3}{xz^2}$
- d $\frac{24z(x+1)}{(y-1)^2} \div \frac{6y^2(x+1)}{5x} \div \frac{15(y-1)}{z^2(x+1)^2}$

eBookplus

Digital doc
WorkSHEET 8.2
doc-2291

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8.11 Review



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The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search
int-2629



Crossword
int-2630



Sudoku
int-3188



Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

associative law

brackets

commutative law

distributive law

evaluate

expanded form

expanding

expression

factorised form

factorising

highest common factor

identity law

inverse law

like terms

pronomeral

substitution

unknowns

variable

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

assesson

Link to SpyClass, an exciting online game combining a comic book-style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

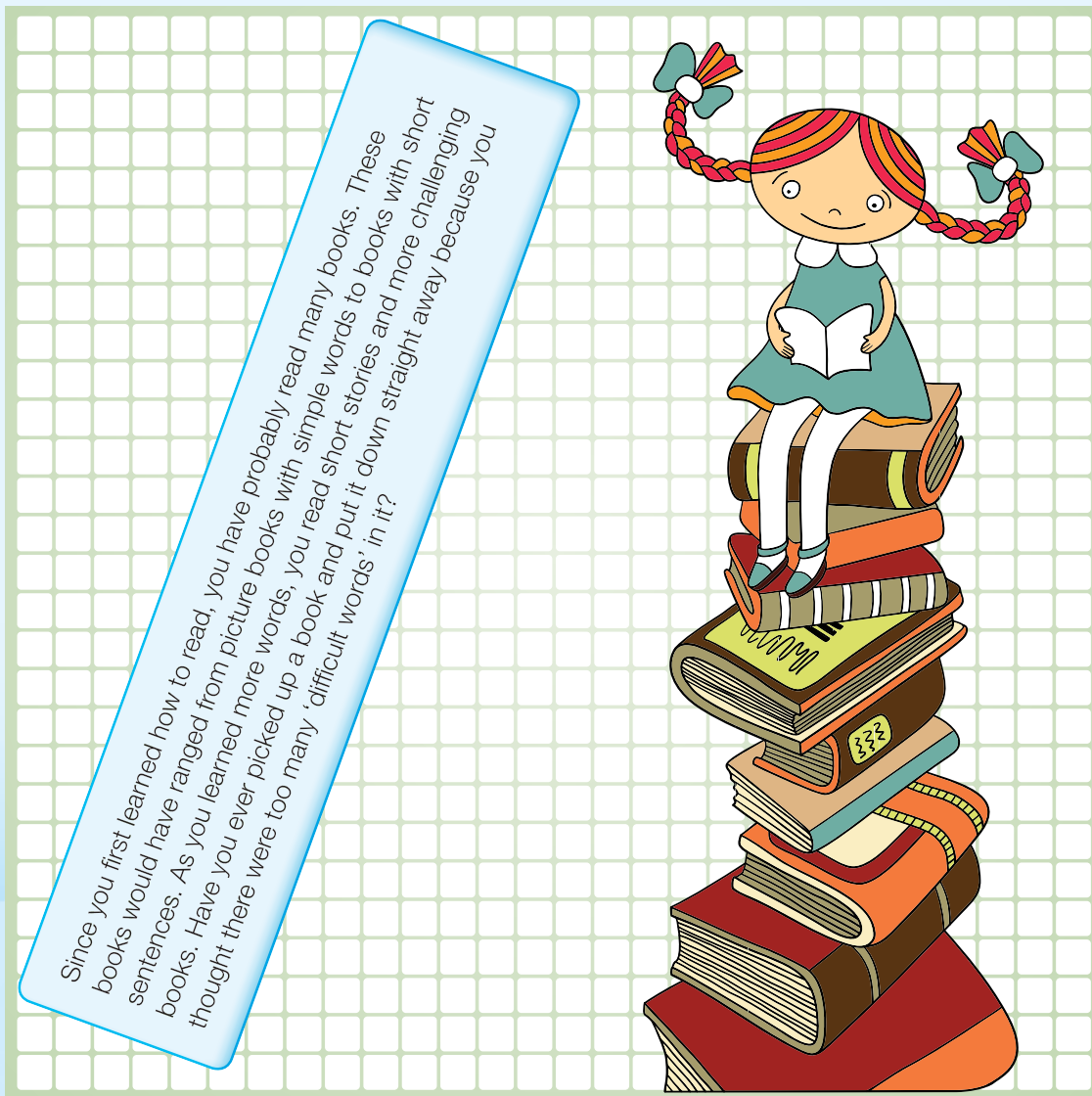
www.spyclass.com.au

SPY CLASS



RICH TASK

Readability index



The reading difficulty of a text can be described by a readability index. There are several different methods used to calculate reading difficulty, and one of these methods is known as the Rix index. The Rix index is obtained by dividing the number of long words by the number of sentences.

- 1 Use a variable to represent the number of long words and another to represent the number of sentences. Write a formula that can be used to calculate the Rix index.

When using the formula to determine the readability index, follow these guidelines:

- A long word is a word that contains seven or more letters.
- A sentence is a group of words that ends with a full stop, question mark, exclamation mark, colon or semi-colon.
- Headings and numbers are not included and hyphenated words count as one word.

Consider this passage from a Science textbook.

- 2 How many sentences and long words appear in this passage of text?
- 3 Use your formula to calculate the Rix index for this passage. Round your answer to 2 decimal places.

A fatal fall . . . or was it murder?

In 1991, some German hikers found a body preserved in ice near the Italy–Austria border. Scientists used radiometric dating and found that the body was about 5300 years old! They thought that the person, known now as the Iceman, had died of hypothermia (extreme cold). Ten years later, another group of scientists using high-tech X-rays found the remains of an arrowhead lodged near his left lung. Specialists have not yet confirmed whether the Iceman fell back onto his arrow, or if he was murdered. And without any witnesses to question, the truth may never be known!

Once you have calculated the Rix index, the table below can be used to work out the equivalent year level of the passage of text.

Rix index	Equivalent year level
Below 0.2	1
0.2–0.49	2
0.5–0.79	3
0.8–1.29	4
1.3–1.79	5
1.8–2.39	6
2.4–2.99	7

Rix index	Equivalent year level
3.0–3.69	8
3.7–4.49	9
4.5–5.29	10
5.3–6.19	11
6.2–7.19	12
Above 7.2	University level

- 4 What year level is the passage of text equivalent to?

When testing the reading difficulty of a book, it is not necessary to consider the entire book. Choose a section of text with at least 10 sentences and collect the required information for the formula.

- 5 Choose a passage of text from one of your school books, a magazine or newspaper. Calculate the Rix index and use the table above to determine the equivalent year level.
- 6 Repeat question 5 using another section of the book, magazine or newspaper. Did the readability level change?
- 7 On a separate page, rewrite the passage from question 5 with minimum changes, so that it is now suitable for a higher or lower year level. Explain the method you used to achieve this. Provide a Rix index calculation to prove that you changed the level of reading difficulty.

CODE PUZZLE

Doctor, I've swallowed the film out of my camera!

The factorised form of the expressions and the letter beside each of them gives the puzzle code.



$8y(x - 9)$	$2(4 - x)$	$2y(3x - 5)$	$-3(4x + 7)$				
$-(2x - 1)$	$-7(7 + 4x)$	$2y(x - 4)$	$2(3x - 7)$				
$6(3 - 7x)$	$-y(x - 1)$	$x(y - 2)$	$-5(4x + 5)$	$4(3x + 5)$	$-(x - 2)$	$5(3x + 2)$	
$3(2x - 1)$	$-8(x + 3)$	$7(8x - 5)$	$3x(2y + 1)$	$y(2x - 3)$	$-2(x + 1)$	$3(x - 4)$	$y(3x - 2)$

$$D = 6x - 3 =$$

$$E = 6xy + 3x =$$

$$E = 8 - 2x =$$

$$H = -20x - 25 =$$

$$G = 15x + 10 =$$

$$L = 8xy - 72y =$$

$$H = -2x + 1 =$$

$$N = -x + 2 =$$

$$I = 12x + 20 =$$

$$E = 6x - 14 =$$

$$L = 2xy - 3y =$$

$$O = -xy + y =$$

$$N = 18 - 42x =$$

$$P = 3x - 12 =$$

$$O = -2x - 2 =$$

$$E = -8x - 24 =$$

$$P = 2xy - 8y =$$

$$S = 3xy - 2y =$$

$$S = -12x - 21 =$$

$$T = 6xy - 10y =$$

$$T = xy - 2x =$$

$$V = 56x - 35 =$$

$$O = -49 - 28x =$$

8.2 Using variables**Digital doc**

- SkillSHEET (doc-6922) Alternative expressions used to describe the four operations

eLesson

- Using variables (eles-0042)

Interactivity

- IP interactivity 8.2 (int-4429) Using variables

8.3 Substitution**Digital doc**

- Spreadsheet (doc-2287) Substitution

Interactivity

- IP interactivity 8.3 (int-4430) Substitution

8.4 Working with brackets**Interactivity**

- IP interactivity 8.4 (int-4431) Working with brackets

8.5 Substituting positive and negative numbers**Digital docs**

- SkillSHEET (doc-6923) Order of operations II
- SkillSHEET (doc-6924) Order of operations with brackets
- SkillSHEET (doc-6925) Operations with directed numbers
- WorkSHEET 8.1 (doc-2290)

Interactivity

- IP interactivity 8.5 (int-4432) Substituting positive and negative numbers

8.6 Number laws and variables**Interactivities**

- The Associative Law (int-2370)
- IP interactivity 8.6 (int-4433) Number laws and variables

8.7 Simplifying expressions**Digital docs**

- SkillSHEET (doc-6926) Combining like terms
- SkillSHEET (doc-6927) Simplifying fractions

Interactivity

- IP interactivity 8.7 (int-4434) Simplifying expressions

8.8 Multiplying and dividing expressions with variables**Interactivity**

- IP interactivity 8.8 (int-4435) Multiplying and dividing expressions with variables

8.9 Expanding brackets**Digital doc**

- Spreadsheet (doc-2288) Expanding brackets

Interactivity

- IP interactivity 8.9 (int-4436) Expanding brackets

8.10 Factorising**Digital docs**

- SkillSHEET (doc-6928) Highest common factor
- Spreadsheet (doc-2289) Finding the HCF
- WorkSHEET 8.2 (doc-2291)

Interactivity

- IP interactivity 8.10 (int-4437) Factorising

8.11 Review**Interactivities**

- Word search (int-2629)
- Crossword (int-2630)
- Sudoku (int-3188)

Digital docs

- Topic summary (doc-10760)
- Concept map (doc-10773)

To access eBookPLUS activities, log on to



www.jacplus.com.au

Answers

TOPIC 8 Algebra

8.2 Using variables

- 1 a $x + 420$
 b $3x$
 c $x - 130$
 d The nearby nest has 60 more ants.
 e The nearby nest has 90 fewer ants.
 f The nest is one quarter of the size of the original nest.
- 2 a $x + y$ b $x + y + 260$
 c $x + y + 90$ d $x + y - 260$
- 3 Between 9.00 am and 9.15 am one Danish pastry was sold. In the next hour-and-a-half, a further 11 Danish pastries were sold. No more Danish pastries had been sold at 12.30 pm, but in the next half-hour 18 more were sold. No Danish pastries were sold after 1.00 pm.
- 4 a $a + b$ b $a + b + c$
 c $b + 4$ d $a - 6$
- 5 a $y + 7$ b $y - 8$
 c $5y$ d $14 - y$
 e $\frac{y}{3}$ f $8y + 3$
- 6 a $a + b$ b $a - b$
 c $2b - 3a$ d ab
 e $2ab$ f $3a + 7b$
 g a^2 h $\frac{a^2}{5}$
- 7 a $\$27y$ b $\$14d$ c $\$(27r + 14h)$
- 8 a $t + 2$ b $t + g$
 c $t - 5$ d $2t$
- 9 a Various answers are possible; an example is shown. The number of passengers doubled at the next stop and continued to increase, more than quadrupling in the first nine minutes. At 7.22 pm, 5 people alighted the train, and by 7.25 pm the same number of passengers were on the train as there were at the beginning. By 7.34 pm there were 12 fewer passengers than there were at the beginning.
 b 7.22 pm c 7.19 pm d 7.34 pm
- 10 a The number of bacteria in each of these intervals is double the number of bacteria in the previous interval.
 b The bacteria could be dividing in two.
 c It is lower than expected, based on the previous pattern of growth.
 d Some of the bacteria may have died, or failed to divide and reproduce.
- 11 a Odd b Even
 c i $n + 2$, $n + 4$ and $n + 6$
 ii $n - 2$
- 12 $TSA = 2x^2 + 4xh$; $V = x^2h$
- 13 $TSA = 5xh + 3x^2$; $V = \frac{3}{2}x^2h$
- 14 $TSA_{\text{square box}} = 2000 \text{ cm}^2$; $V_{\text{square box}} = 6000 \text{ cm}^3$
 $TSA_{\text{rectangular box}} = 2700 \text{ cm}^2$; $V_{\text{rectangular box}} = 9000 \text{ cm}^3$

Challenge 8.1

Number is 4; Bill's is age 15.

8.3 Substitution

- 1 a 6 b 14 c 30 d 1
 e 9 f 1 g 7 h 3
 i 6 j 15 k 19 l 4
 m 5 n 10 o 20 p 5
 q 22 r 2 s $\frac{89}{10}(8\frac{9}{10})$ t $\frac{39}{10}(3\frac{9}{10})$

- 2 a 42 b 3 c 54 d 1
 e 9 f 15 g 15 h 21
 i 24.3 j 21 k 1 l 4.8
 m 18.3 n 8.1 o 16.2 p $\frac{7}{4}(1\frac{3}{4})$
- 3 a 7 b 7 c -3 d 3
 e 4 f 10 g 50 h 1
 i -15 j -4 k 37 l 15
 m 40 n $2\frac{1}{3}$ o 19 p 1
- 4 150 m
 5 10 cm
 6 $C = \$64$
- 7 a 48 cm^2 b 8400 m^2 c 4.472 m^2 or $44\,720 \text{ cm}^2$
- 8 a $F = 212^\circ\text{F}$ b $28^\circ\text{C} = 82.4^\circ\text{F}$ c 32°F
- 9 a $100 \text{ km} \approx 60 \text{ miles}$ b $248 \text{ km} \approx 148.8 \text{ miles}$
 c $12.5 \text{ km} \approx 7.5 \text{ miles}$
- 10 If $x = 0$, then the expression becomes $\frac{0}{0}$, which is indeterminate.
- 11 172.625
- 12 a $V = x(x + 5)(x - 2)$
 b 1200
 c Because $1.5 - 2 < 0$
- 13 a $13p - 10$
 b 2590

8.4 Working with brackets

- 1 a 36 b 4 c 84 d 18
 e 56 f 35 g 90 h 350
 i 55 j 20 k 133 l 147
 m 784 n 250 o 9800 p 200
- 2 a 90 b 16 c 6 d 32
 e 72 f 90 g 7 h 36
 i 60 j 58 k 21 l 180
 m 576 n 32 o -26 p 33
- 3 a 62 cm b 97.8 cm
- 4 B
- 5 a 720° b 540° c 180°
 d 360° e 3240°
- 6 a $CD = m + 4n$ b $BC = 3m + n$
 c Perimeter $= 8m + 18n$
- 7 a $A = \frac{1}{2}(m + n)(m - n)$
 b $A = \frac{1}{2}(21 \times 9) = 94.5 \text{ cm}^2$
 c If $m < n$, then $(m - n) < 0$. Negative height is impossible.
 d If m and n move closer in value, then the length of the base of the triangle gets closer to $2m$ (or $2n$) and the height gets closer to zero.
- 8 Check with your teacher.

8.5 Substituting positive and negative numbers

- 1 a 3 b 9 c -9 d 3
 e -9 f -12 g -12 h 2
 i 5 j -13 k 9 l 3
 m -2 n -3 o -8 p -2
 q 0 r -109 s 6 t 16
- 2 a 18 b 32 c -15 d 12
 e 16 f 24 g 22 h 155
 i 32 j 3 k 6 l 21
- 3 a -9 b 1 c 3 d 40
 e -8 f 2 g -3 h 12
 i $-\frac{4}{5}$ j -2 k -1 l -6
 m 125 n 175 o -5 p -12.5
 q -74 r -117 s -104 t -15.5
- 4 $\frac{30}{7}$ or $4\frac{2}{7}$

5 If x is negative then $5x$ will also be a negative integer (less than or equal to -5). Subtracting this number is equivalent to adding a positive integer. The result will be positive.

6 Check with your teacher.

7 a Negative

b Positive

8 a $\approx 0.804x^d$

b ≈ 3.216

c $\frac{x}{2}$

8.6 Number laws and variables

1 a i 11 ii 11, same

b i 25 ii 25, same

c i 31 ii 31, same

d i 32 ii 32, same

e i -5 ii 5, different

f i -18 ii 18, different

g i -28 ii 28, different

h i 1 ii -1 , different

2 a i -10 ii -10 , same

b i -180 ii -180 , same

c i -40 ii -40 , same

d i -350 ii -350 , same

e i $-\frac{2}{5}$ ii $-\frac{5}{2}$, different

f i -1 ii -1 , same

g i $-\frac{4}{5}$ ii $-\frac{5}{4}$, different

h i $-\frac{14}{45}$ ii $-\frac{45}{14}$, different

3 a True b False c False

d True e True f True

g False h False i False

j True k False l True

4 a i 13 ii 13, same

b i 24 ii 24, same

c i 40 ii 40, same

d i -3 ii -7 , different

e i -35 ii -71 , different

f i -43 ii -67 , different

5 a i -64 ii -64 , same

b i 768 ii 768, same

c i -1536 ii -1536 , same

d i -4 ii -1 , different

e i -6 ii $-\frac{1}{6}$, different

f i $\frac{8}{5}$ ii $\frac{1}{10}$, different

6 a False b False c False

d True e True f False

7 C

8 D

9 a i 6 ii 6

b The answers are equal because of the use of the Associative Law.

10 a -10 b -12

11 a $(-3p + 4q)$ b $\frac{1}{(3p - 4q)}$

c $15, -\frac{1}{15}$ d The result is the identity, which is 1.

8.7 Simplifying expressions

1 a $6c$ b $-3c$ c $4a$

d q e $-3h$ f $2x$

g $-6a$ h $4f$ i $-3p$

j h k $18b$ l $3t$

m $13m$ n $-x$ o $20z$

p $10p$ q $17g$ r $3b$

s $14t$ t $-7j$ u $-15l$

v $8m$ w 0 x $10t$

2 a $10x - 2y$ b $7x - 12$ c $11 - 2f$

d $6 - u$ e $7m + 3p$ f $4r - 5h$

g $17a - 5b$ h $9t - 2$ i $17 - 3g$

j $10m - 2n$ k $7k - 12$ l $4n - 9$

m $12 - 2b$ n $20 - 12h$ o $9y - 2g - 6$

p $11h - 8$ q $4s - 2t$ r $14l - 5m$

s $3k - 13h + 7$ t $5 - 4t$ u $7g - 2$

v $19f - 10k$

3 a $3x^2$ b $5y^2$ c $4a^3$

d $7d^2$ e $-g^2$ f $10y^3$

g $7b^2$ h a^2 i $-g^2$

j $4a^2 + 9$ k $23x^2$ l $11s^2 + 4$

m $8a^2 + 5a$ n $b^2 + 23b$ o $t^2 - 4g - 7$

p $8g^3 - g^2 + 22$ q $18ab + 3$ r $11xy$

s $3fg + 3s$ t $12ab - 5$ u $20ab^2 - 14ac$

4 Check with your teacher.

5 Check with your teacher.

6 a $14t$ b $12t$ c $13t$ d $39t$

7 a $2x + 15$ b $9x + 4$ c $15x$ d $7.5x - 2$

Challenge 8.2

$x = 3$

8.8 Multiplying and dividing expressions with variables

1 a $12g$ b $21h$ c $24d$ d $15z$

e $30r$ f $35t$ g $12u$ h $42p$

i $21gy$ j $22ht$ k $24gx$ l $70ah$

m $36dm$ n $15ch$ o $18gx$ p $12.5bt$

q $156mn$ r $72ad$ s $6abc$ t $12fgh$

u $48wx$ v $231abd$ w $24xy$ x $10.5xy$

y $132qs$ z $24abc$

2 a $4f$ b $2h$ c $5x$ d $3g$

e $2r$ f 2 g 2 h 2

i $\frac{2}{3}$ j $\frac{1}{2}$ k $\frac{6}{7}$ l $\frac{2}{3}$

m $\frac{1}{3}$ n $\frac{1}{2}$ o $\frac{2}{3}$ p $\frac{1}{34}$

q 9 r $\frac{5}{12}$ s $\frac{16}{11}(1\frac{5}{11})$ t 3

3 a $5fg$ b $3cd$ c $\frac{2xy}{3}$ d cg

e y f $\frac{p}{2}$ g $\frac{3a}{4}$ h $\frac{3d}{4}$

i 5 j $\frac{5r}{7}$ k $\frac{2y}{7}$ l $\frac{4b}{3}$

m $13y$ n $\frac{2}{5}$ o $2a$ p $\frac{g}{2}$

q $\frac{11m}{5}$ r $\frac{1}{6}$ s $\frac{2d}{5}$ t $\frac{b}{7}$

4 a $-15f$ b $12d$ c $-33ag$

d $27gt$ e $20dht$ f $-18st$

g $42dw$ h $24abce$ i $-33abh$

j $18absx$ k $-75hqt$ l $144pw$

m $-21abg$ n $-51abgh$ o $-49gh$

p $-40hjk$ q $112.5xy$ r $-144prt$

s $30abc$ t $-408wx$ u $-90abcdf$

5 a $-\frac{a}{2}$ b $-\frac{a}{3}$ c $-12j$ d $\frac{1}{2d}$

e $-\frac{4}{5l}$ f $-\frac{x}{4}$ g $-\frac{6}{7}$ h $\frac{f}{5j}$

i $-\frac{2x}{3}$ j $-\frac{1}{6}$ k $-\frac{m}{4}$ l $\frac{st}{2}$

m -2 n $\frac{b}{3}$ o $-\frac{d}{5h}$ p $-\frac{12}{11p}$

q $\frac{14ef}{9}$ r $-\frac{18x}{7}$ s $\frac{3}{2r}$ t $-\frac{11}{12t}$

6 a $2a^2$ b $25p^2$ c $-30x^2$
d $7a^2b$ e $6b^2cd$ f $-160x^2y$
g $42p^2q^2$ h $-30mn^2t^2$ i $-18xy^2z^2$
j $-42abc^2$ k 0 l $-18w^2x^2y^2z^2$
m $6a^{11}$ n $2x^5y^3$ o $10m^9$
p $\frac{5p^{10}q^5}{6}$ q $\frac{4x^2yz^2}{3}$ r $\frac{3b}{5}$
7 a $\frac{6}{a^2}$ b $\frac{10b^2}{3}$ c $\frac{5}{w}$ d $\frac{9k}{5}$ e $\frac{3g}{5a}$
f $\frac{16h^2}{9d^2}$ g $\frac{5t}{n}$ h $\frac{-27h^2}{2g^2}$ i $\frac{8y}{w}$ j $6fz$

8 The pronumeral must be a common factor of every term of the numerator and every term of the denominator before it can be cancelled.

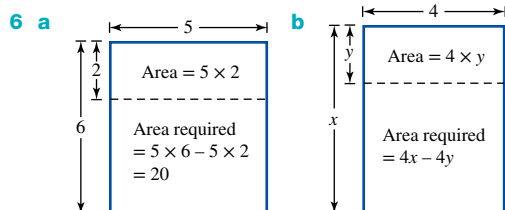
9 -5

10 $\frac{7}{6}$

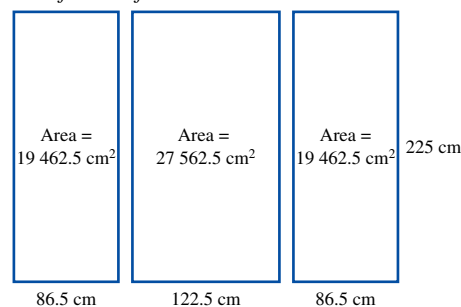
11 a $V_{\text{small container}} = ab^2$, $V_{\text{large container}} = 4ab^2$ b 4 times

8.9 Expanding brackets

1 a $3d + 12$ b $2a + 10$
c $4x + 8$ d $5r + 35$
e $6g + 36$ f $2t - 6$
g $7d + 56$ h $18x - 54$
i $48 + 12c$ j $42 + 21x$
k $90g + 135$ l $1.5t + 9$
m $11t - 22$ n $6t - 18$
o $t^2 + 3t$ p $x^2 + 4x$
q $g^2 + 7g$ r $2g^2 + 10g$
s $3fg + 9f$ t $6mn - 12m^2$
2 a $9x - 6$ b $3x^2 - 18xy$
c $15xy - 45y^2$ d $100y - 250$
e $-3c - 9$ f $-15x - 20$
g $-5x^2 - 30x$ h $-12y - 2y^2$
i $-6t + 18$ j $-20f + 8f^2$
k $27xy - 18x$ l $-6bh + 18h^2$
m $20ab + 12ac$ n $-6ag + 21a^2$
o $15ab + 30ac$ p $-18w^2 + 10wz$
q $48m^2 + 120m$ r $6k^2 - 15k$
3 a $35x + 49$ b $3c - 4$
c $22c - 2c^2$ d $6v + 30$
e $5d^2 - 12d$ f $11y + 12$
g $26r + r^2$ h $9g - 37$
i $11f - 12g - 7$ j $9x$
k $-5k - 10$ l $18x + 6rx$
m $8r - 13$ n $18gh - 24g$
o $11t + 1$ p $24 + 11r - 11r^2$
4 a $5x + 8$ b $9x + 23$
c $6y + 26$ d $d + 22$
e $14h$ f $21m - 4$
g $28f - 1$ h $4a - 3a^2 - 35$
i $6 - t^2 + 2t$ j $7m$
5 a $10hk + 21h + 20k$ b $15n - 6ny$
c $8gm + 24g - 18$ d $18ab + 67b$
e $5a^2 - 35a - 35$ f $11cf + 3c$
g $28x - 5xy - 29$ h $37vw + 55v - 24$
i $6xy - 45x$ j $77mn - 16m + 12n$



7 a $x^2 + 3x + 2$ b $a^2 + 7a + 12$
c $c^2 - c - 6$ d $y^2 - 16$
e $u^2 - 5u + 6$ f $k^2 - 7k + 10$
8 a $\$50 - d$ b i Sale price (P) = $3(50 - d)$ ii $P = 150 - 3d$
c Amount of change (C) = $200 - (150 - 3d) = 50 + 3d$
9 a $A_{\text{left}} = fm - 36f$; $A_{\text{centre}} = fm$; $A_{\text{right}} = fm - 36f$
b $A = 3fm - 72f$ c $A = 3m^2 + 235.5m - 7380$



8.10 Factorising

1 a 2 b 3 c 6 d 13
e 7 f 2 g 3 h 4
2 a $2g$ b $3m$ c 11 d $2m$
e $2a$ f $12g$ g $2g$ h $11l$
i $4mn$ j $4c$ k $4c$ l x
3 a $3(x + 2)$ b $2(y + 2)$ c $5(g + 2)$
d $4(2x + 3)$ e $3(2f + 3)$ f $4(3c + 5)$
g $2(d + 4)$ h $2(x - 2)$ i $6(2g - 3)$
j $11(h + 11)$ k $4(s - 4)$ l $4(2x - 5)$
m $12(g - 2)$ n $2(7 - 2b)$ o $16(a + 4)$
p $12(4 - q)$ q $8(2 + f)$ r $12(1 - d)$
4 a $3(gh + 4)$ b $2y(x + 3)$
c $4p(3q + 1)$ d $7g(2 - h)$
e $2k(8j - 1)$ f $2g(6e + 1)$
g $4(3k + 4)$ h $m(7n + 6)$
i $7b(2a + 1)$ j $5a(1 - 3bc)$
k $2r(4 + 7t)$ l $12ab(2m + 1)$
m $2b(2 - 3a)$ n $4g(3f - 4h)$
o $b(a - 2c)$ p $7x(2 - 3y)$
q $k(11j + 3)$ r $3p(1 + 9q)$
s $c(12a - 4 + 3d)$ t $4(g + 2h - 4)$
u $14s(2 + t)$ v $3v(5u + 9w)$
5 $2ab$
6 $12a^3b^2c$
7 $\frac{3}{5}$
8 $\frac{3(x + 1)}{2(x - 1)}$
9 $\frac{28}{3}$ or $9\frac{1}{3}$
10 $7y$
11 a $(3x - 6)(2x + 8)(ax - 5a)$ b $\frac{4}{3a(x + 4)(x - 5)}$
12 a $6(x - 3)(x + 2)$ b $x > 3$
13 a $\frac{3xy^3}{2z}$ b $\frac{x(y - 1)^2}{18z(z - 1)}$ c $\frac{xz^3}{15y^3}$ d $\frac{4xz^3(x + 1)^2}{3y^2(y - 1)^3}$

Investigation — Rich task

- Let l represent the number of long words and s represent the number of sentences. The Rix index = $\frac{l}{s}$.
- Six sentences and 19 long words
- 3.17
- Grade 8
- to 7 Check with your teacher.

Code puzzle

Let's hope nothing develops.

