

TOPIC 11

Linear equations

11.1 Overview

Why learn this?

Linear equations are a form of algebra, and they are often used to describe everyday situations using mathematics. One of the most useful skills that you learn in algebra is how to solve equations. Solving equations means finding numbers whose values you do not know. Once you learn the techniques of solving equations, you will find that many problems can be expressed and solved using linear equations.

What do you know?

assessment

- 1 THINK** List what you know about equations. Use a thinking tool such as a concept map to show your list.
- 2 PAIR** Share what you know with a partner and then with a small group.
- 3 SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge equations.

Learning sequence

- 11.1** Overview
- 11.2** Identifying patterns
- 11.3** Backtracking and inverse operations
- 11.4** Keeping equations balanced
- 11.5** Using algebra to solve problems
- 11.6** Equations with the unknown on both sides
- 11.7** Review

eBook plus
ONLINE ONLY



11.2 Identifying patterns

- Mathematics is used to describe *relationships* in the world around us.
- Mathematicians study *patterns* in numbers and shapes found in nature to discover rules.
- These rules can then be applied to other, more general situations.
- Looking at the number pattern 1, 4, 7, 10, ... we can see that by adding 3 to any of these numbers, we obtain the next number.
- This number pattern is called a **sequence**.
- Each number in the sequence is called a **term**.
- Each sequence has a *rule* that describes the pattern. For the sequence above, the rule is 'add 3'.

WORKED EXAMPLE 1

Describe the following number patterns in words then write down the next three numbers in the pattern.

a 4, 8, 12, ...

THINK

a 1 The next number is found by adding 4 to the previous number.

2 Add 4 each time to get the next three numbers.

3 Write down the next three numbers.

b 1 The next number is found by multiplying the previous number by 2.

2 Multiply by 2 each time to get the next three numbers.

3 Write the next three numbers.

b 4, 8, 16, ...

WRITE

a Next number = previous number + 4

Next three numbers are 16, 20, 24.

b Next number = previous number $\times$ 2

Next three numbers are 32, 64, 128.

WORKED EXAMPLE 2

Using the following rules, write down the first five terms of the number pattern.

a Start with 32 and divide by 2 each time.

b Start with 2, multiply by 4 and subtract 3 each time.

THINK

a 1 Start with 32 and divide by 2.

2 Keep dividing the previous answer by 2 until five numbers have been calculated.

3 Write the answer.

WRITE

a $32 \div 2 = 16$

$16 \div 2 = 8$

$8 \div 2 = 4$

$4 \div 2 = 2$

$2 \div 2 = 1$

The first five numbers are 16, 8, 4, 2 and 1.

- b** **1** Start with 2 then multiply by 4 and subtract 3.
- 2** Continue to apply this rule to the answer until five numbers have been calculated.
- 3** Write the five numbers.
- b** $2 \times 4 - 3 = 5$
 $5 \times 4 - 3 = 17$
 $17 \times 4 - 3 = 65$
 $65 \times 4 - 3 = 257$
 $257 \times 4 - 3 = 1025$
 The first five numbers are 5, 17, 65, 257, 1025.

WORKED EXAMPLE 3

Describe the pattern that occurs in the final digit of the number set represented by: $7^1, 7^2, 7^3, \dots$

THINK

- 1** Write the powers as basic numerals.
- 2** Continue until a pattern in the last digit is noticed. When the number becomes large, be concerned with only the last digit.
- 3** Write the pattern.

WRITE

$7^1 = 7$
 $7^2 = 49$
 $7^3 = 49 \times 7 = 343$
 $7^4 = 343 \times 7 = 2401$
 $7^5 = 2401 \times 7 = 16\,807$
 $7^6 = \dots 7 \times 7 = \dots 9$
 The pattern in the last digit is 7, 9, 3, 1 repeated.

Geometric patterns

- Patterns can be found in geometric shapes.
- If we examine the three shapes below, we can see patterns by investigating the changes from one shape to the next. For example, look at the number of matchsticks in each set of triangles.



By using a table of values we can see a number pattern developing:

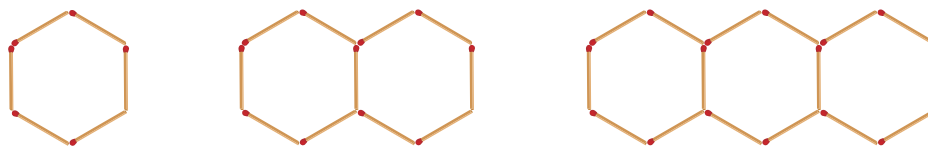
Number of triangles	Number of matchsticks
1	3
2	6
3	9
4	12
5	15
6	18

The pattern in the final row is 3, 6, 9, ...; we can see that the rule here is 'add 3'.

We can also look for a relationship between the number of triangles and the number of matchsticks in each shape. If you examine the table, you will see that a relationship can be found. In words, the relationship is 'the number of matchsticks equals 3 times the number of triangles'.

WORKED EXAMPLE 4

Consider a set of hexagons constructed according to the pattern shown below.



- a Using matches, pencils or similar objects, construct the above figures. Draw the next two figures in the series.
- b Draw up a table showing the relationship between the number of hexagons in the figure and the number of matches used to construct the figure.
- c Devise a rule to describe the number of matches required for each figure in terms of the number of hexagons in the figure.
- d Use your rule to determine the number of matches required to make a figure consisting of 20 hexagons.

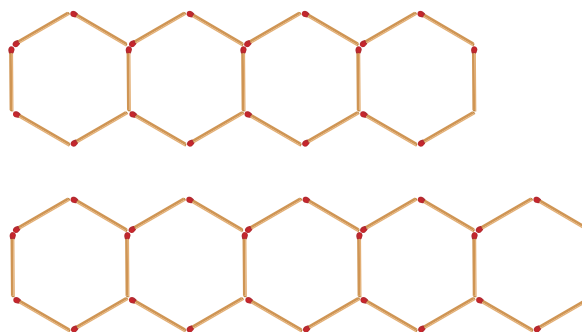
THINK

- 1 Construct the given figures with matches. Note the number of additional matches it takes to progress from one figure to the next — 5 in this case.
- 2 Draw the next two figures, adding another 5 matches each time.

- b Draw up a table showing the number of matches needed for each figure in terms of the number of hexagons. Fill it in by looking at the figures.
- c 1 Look at the pattern in the number of matches going from one figure to the next. It is increasing by 5 each time.
2 If we take the number of hexagons and multiply this number by 5, it does not give us the number of matches. However, if we add 1 to this number, it does give us the number of matches in each shape.
- d 1 Use the rule to find the number of matches to make a figure with 20 hexagons.
2 Work out the answer and write it down.

WRITE

- a The next two figures are:



- b

Number of hexagons	1	2	3	4	5
Number of matches	6	11	16	21	26

- c The number of matches increased by 5 in going from one figure to the next.

$$\text{Number of matches} = \text{number of hexagons} \times 5 + 1$$

- d Number of matches for 20 hexagons

$$= 20 \times 5 + 1$$

$$= 101$$

So 101 matches would be required to construct a figure consisting of 20 hexagons.

Exercise 11.2 Identifying patterns

assess on

INDIVIDUAL PATHWAYS

PRACTISE

 Questions:
1–7

CONSOLIDATE

 Questions:
1–8

MASTER

 Questions:
1–9

Individual pathway interactivity int-4446 eBookplus

REFLECTION

What should you look for when trying to determine number patterns?

FLUENCY

- 1 **WE1** Copy the patterns below, describe the pattern in words and then write down the next three numbers in the pattern.

a 2, 4, 6, 8, ...

b 3, 8, 13, 18, ...

c 27, 24, 21, 18, ...

d 1, 3, 9, 27, ...

e 128, 64, 32, 16, ...

f 1, 4, 9, 16, ...

- 2 Fill in the missing numbers in the following number patterns.

a 3, ..., 9, 12, ..., ...

b 8, ..., ..., 14, ...

c 4, 8, ..., 32, ...

d ..., ..., 13, 15, ...

e 66, 77, ..., 99, ... 121

f 100, ..., ..., 85, 80, ...

- 3 **WE2** Using the following rules, write down the first five terms of the number patterns.

a Start with 1 and add 4 each time.

b Start with 5 and multiply by 3 each time.

c Start with 50 and take away 8 each time.

d Start with 64 and divide by 2 each time.

e Start with 1, multiply by 2 and add 2 each time.

f Start with 1, add 2 then multiply by 2 each time.

eBookplus

Digital docs

SkillSHEET

Number patterns

doc-6978

SkillSHEET

Using tables to show

number patterns

doc-6979

SkillSHEET

Describing a number

pattern from a table

doc-6980

UNDERSTANDING

- 4 Each of the following represents a special number set. What is common to the numbers in the set?

a 2, 4, 6, 8, 10

b 1, 8, 27, 64

c 2, 3, 5, 7, 11

d 1, 1, 2, 3, 5, 8

e 1, 2, 3, 4, 6, 12

f 3, 9, 27, 81

- 5 **WE3** Investigate the pattern that occurs in the final digit of the following sets. Describe the pattern in each case.

 a $2^1, 2^2, 2^3, \dots$

 b $3^1, 3^2, 3^3, \dots$

 c $4^1, 4^2, 4^3, \dots$

 d $5^1, 5^2, 5^3, \dots$

 e $8^1, 8^2, 8^3, \dots$

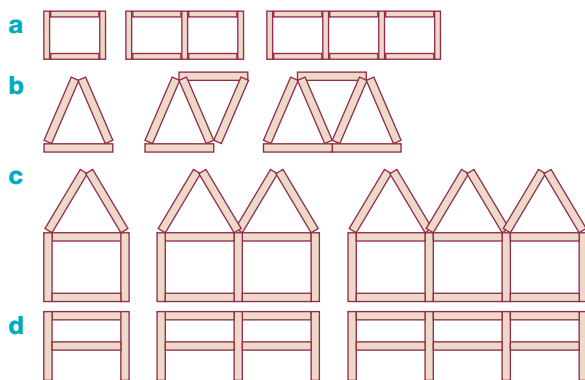
- 6 **WE4** For each of the sets of shapes below, follow these instructions to investigate the pattern.

i Construct the shapes using matches. Draw the next two shapes in the series.

ii Construct a table to show the relationship between the number of shapes in each figure and the number of matchsticks used to construct it.

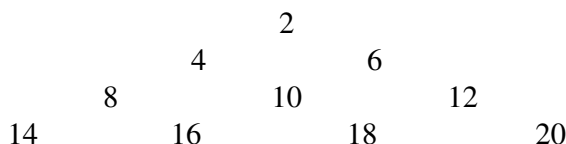
iii Devise a rule in words that describes the pattern relating the number of shapes in each figure and the number of matchsticks used to construct it.

- iv Use your rule to work out the number of matchsticks required to construct a figure made up of 20 such shapes.



REASONING

- 7 Consider the triangular pattern of even numbers shown below.



- Complete the next two lines of the triangle using this pattern.
- Complete the triangle as far as necessary to find the position of the number 60.
- Explain how, without completing any more of the triangle, you could find the position of the number 100.
- Create a similar triangle using odd numbers. Look for the patterns in this triangle. Are they the same as or different from those for the triangle of even numbers? Justify your answers by illustrations from the triangle.

PROBLEM SOLVING

- 8 A grain of rice is placed on the first square of a chess board, three grains of rice on the next square, nine grains of rice on the next square, and so on.

- Write a rule in index form to give the number of grains of rice on the n th square.
- Which square has 2187 pieces of rice on it?



- 9 0, 1, 1; 1, 1, 2; 1, 2, 3; 2, 3, 5; 3, 5, 8; 5, 8, 13...

- What is the next term in this sequence of numbers?
- Which term would follow the term a, b, c ?

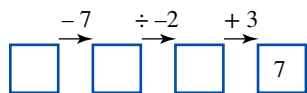
11.3 Backtracking and inverse operations

- An equation links two expressions with an equals sign.
- Adding and subtracting are inverse operations.
- Multiplying and dividing are inverse operations.
- A flowchart can be used to represent a series of operations.

- In a flowchart, the starting number is called the input number and the final number is called the output number.
- By using inverse operations, it is possible to reverse the flowchart and work from the output number to the input number.

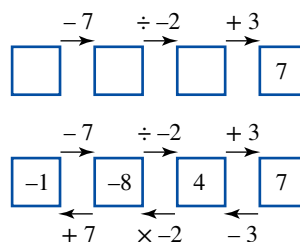
WORKED EXAMPLE 5

Find the input number for this flowchart.


THINK

- 1 Copy the flowchart.
- 2 Backtrack to find the input number.
The inverse operation of $+3$ is -3
($7 - 3 = 4$).
The inverse operation of $÷ -2$ is $× -2$
($4 × -2 = -8$).
The inverse operation of -7 is $+7$
($-8 + 7 = -1$).
Fill in the missing numbers.

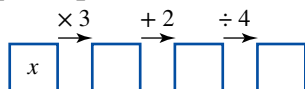
- 3 State the input number.

WRITE/DRAW


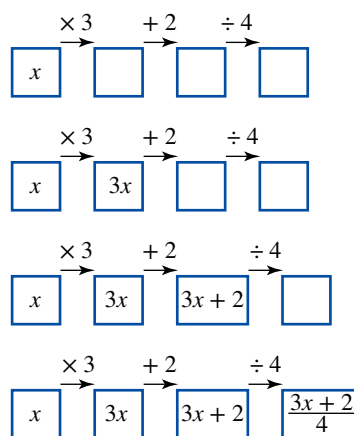
The input number is -1 .

WORKED EXAMPLE 6

Find the output expression for this flowchart.


THINK

- 1 Copy the flowchart and look at the operations that have been performed.
- 2 Multiplying x by 3 gives $3x$.
- 3 Adding 2 gives $3x + 2$.
- 4 Now place a line beneath all of $3x + 2$ and divide by 4.
- 5 State the output expression.

WRITE/DRAW


The output expression

is $\frac{3x + 2}{4}$.

WORKED EXAMPLE 7

Starting with x , draw the flowchart whose output number is given by the expressions:

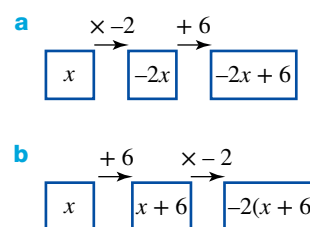
a $6 - 2x$

b $-2(x + 6)$.

THINK

- a 1 Rearrange the expression.
Note: $6 - 2x$ is the same as $-2x + 6$.
- 2 Multiply x by -2 , and then add 6.
- b 1 The expression $x + 6$ is grouped in a pair of brackets, so we must obtain this part first. Therefore, add 6 to x .
- 2 Multiply the whole expression by -2 .

WRITE/DRAW



assessment

Exercise 11.3 Backtracking and inverse operations

INDIVIDUAL PATHWAYS

PRACTISE

Questions:
1, 2, 3a, d, e, m, 4, 5

CONSOLIDATE

Questions:
1a, c, e, g, i, k, 2a, c, e, g, i, k
3b, e, h, k, n, 4, 5

MASTER

Questions:
1b, d, f, h, j, l, 2b, f, j, l, p, 3a, b,
e, i, k-p, 4-6

Individual pathway interactivity int-4447

eBookplus

REFLECTION

What do you need to be careful of when you are backtracking equations?

eBookplus

Digital docs

SkillsSHEET

Flowcharts

doc-6981

SkillsSHEET

Inverse operations

doc-6982

SkillsSHEET

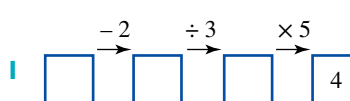
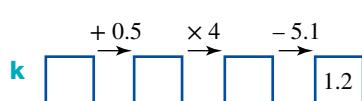
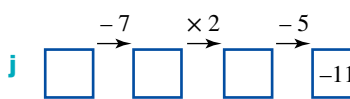
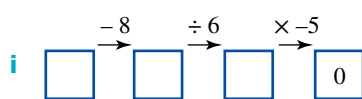
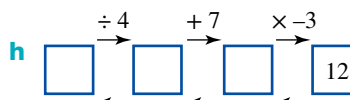
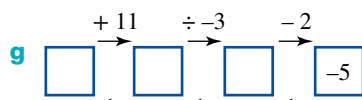
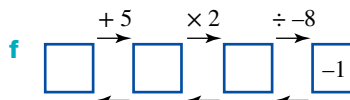
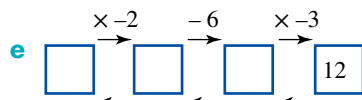
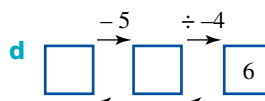
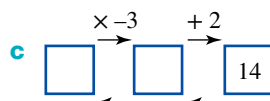
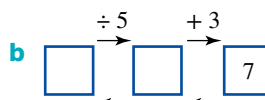
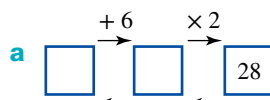
Solving equations by

backtracking

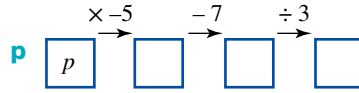
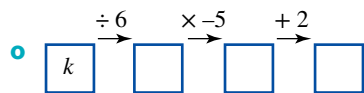
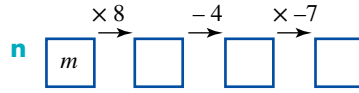
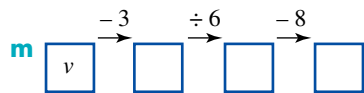
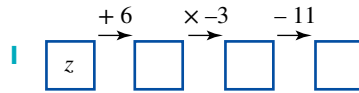
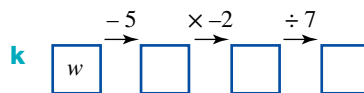
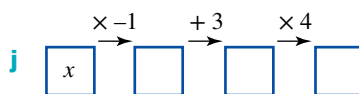
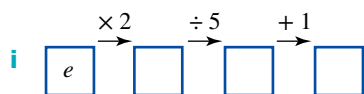
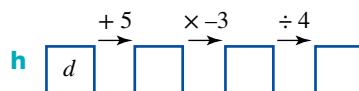
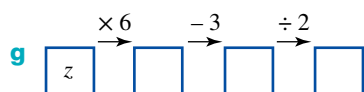
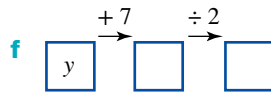
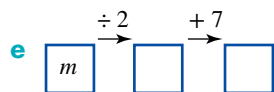
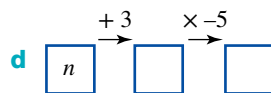
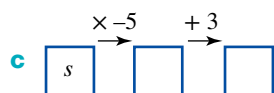
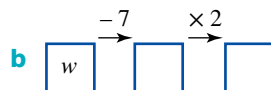
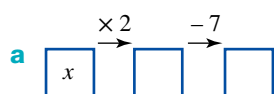
doc-6983

FLUENCY

- 1 WE5 Find the input number for each of the following flowcharts.



2 WE6 Find the output expression for each of the following flowcharts.



UNDERSTANDING

3 WE7 Starting with x , draw the flowchart whose output expression is:

a $2(x + 7)$

b $-2(x - 8)$

c $3m - 6$

d $-3m - 6$

e $\frac{x - 5}{8}$

f $\frac{x}{8} - 5$

g $-5x + 11$

h $-x + 11$

i $-x - 13$

j $5 - 2x$

k $\frac{3x - 7}{4}$

l $\frac{-3(x - 2)}{4}$

m $\frac{x + 5}{8} - 3$

n $-7\left(\frac{x}{5} - 2\right)$

o $3\left(\frac{2x}{7} + 4\right)$

p $\frac{1}{4}\left(\frac{6x}{11} - 3\right)$

REASONING

4 a Draw a flowchart to convert from degrees Fahrenheit (F) to degrees Celsius (C) using the formula $C = \frac{5}{9}(F - 32)$.

b Use the flowchart to convert 50°F to degrees Celsius by substituting into the flowchart.

c Backtrack through the flowchart to find what 35°C is in degrees Fahrenheit.

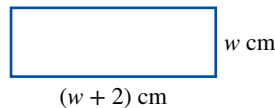


PROBLEM SOLVING

5 Consider the following puzzle.

*Think of a number.
Double it.
Add 10.
Divide by 2.
Subtract the number you first thought of.*

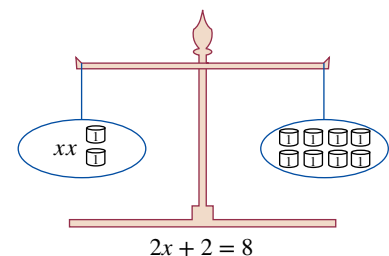
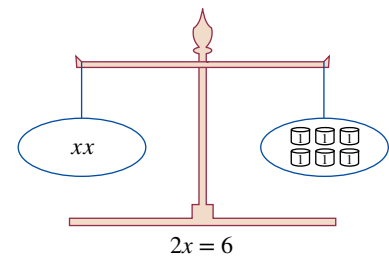
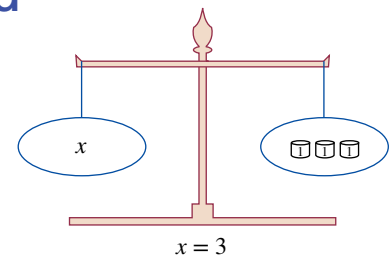
- a Represent this puzzle as a flowchart, with n representing the unknown number.
 - b What do you notice about the final number?
 - c Repeat this with different starting numbers.
- 6 The rectangle shown has an area of 255 cm^2 .



- a Discuss what strategies you might use to find the value of w . Can you use backtracking?
- b Find the value of w and explain the method you use.

11.4 Keeping equations balanced

- As an equation can be thought of as two expressions with an equals sign between them, an equation can be thought of as a balanced scale. The diagram at right represents the simple equation $x = 3$.
- If the amount of the left-hand side (LHS) is doubled, the scale will stay balanced provided that the amount on the right-hand side (RHS) is doubled.
- Similarly, the scale will stay balanced if we add a quantity to both sides.
- The scales will remain balanced as long as we do the same to both sides.



WORKED EXAMPLE 8

Starting with the equation $x = 4$, write the new equation when we:

a multiply both sides by 4

b take 6 from both sides

c divide both sides by $\frac{2}{5}$.

THINK

- a**
- 1 Write the equation.
 - 2 Multiply both sides by 4.
 - 3 Simplify by removing the multiplication signs.
Write numbers before variables.

- b**
- 1 Write the equation.
 - 2 Subtract 6 from both sides.
 - 3 Simplify.

- c**
- 1 Write the equation.
 - 2 Dividing by a fraction is the same as multiplying by its reciprocal. Multiply both sides by $\frac{5}{2}$.
 - 3 Simplify.

WRITE

a $x = 4$
 $x \times 4 = 4 \times 4$
 $4x = 16$

b $x = 4$
 $x - 6 = 4 - 6$
 $x - 6 = -2$

c $x = 4$
 $x \div \frac{2}{5} = 4 \div \frac{2}{5}$
 $x \times \frac{5}{2} = 4 \times \frac{5}{2}$
 $\frac{5x}{2} = \frac{20}{2}$
 $\frac{5x}{2} = 10$

Exercise 11.4 Keeping equations balanced

assess on

INDIVIDUAL PATHWAYS

PRACTISE

Questions:
1–7, 10–13

CONSOLIDATE

Questions:
1–13

MASTER

Questions:
1–14

Individual pathway interactivity int-4448 eBookplus

REFLECTION

How does using inverse operations keep the equations balanced?

FLUENCY

1 WE8 Starting with the equation $x = 6$, write the new equation when we:

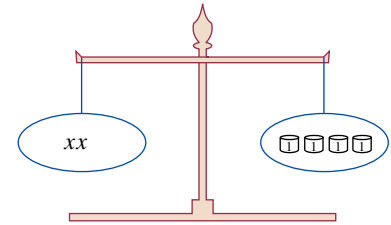
- | | |
|---|---|
| a add 5 to both sides | b multiply both sides by 7 |
| c take 4 from both sides | d divide both sides by 3 |
| e multiply both sides by -4 | f multiply both sides by -1 |
| g divide both sides by -1 | h take 9 from both sides |
| i multiply both sides by $\frac{2}{3}$ | j divide both sides by $\frac{2}{3}$ |
| k take $\frac{2}{3}$ from both sides. | |

eBookplus

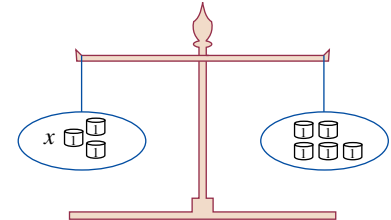
Interactivity
Balancing equations
int-0077

UNDERSTANDING

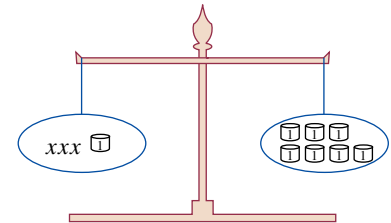
- 2 a** Write the equation that is represented by the diagram at right.
b Show what happens when you halve the amount on both sides. Write the new equation.



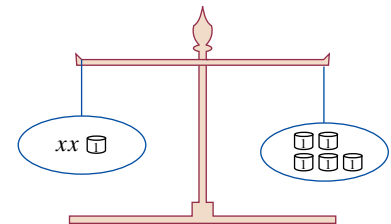
- 3 a** Write the equation that is represented by the diagram at right.
b Show what happens when you take three from both sides. Write the new equation.



- 4 a** Write the equation that is represented by the diagram at right.
b Show what happens when you add three to both sides. Write the new equation.



- 5 a** Write the equation that is represented by the diagram at right.
b Show what happens when you double the amount on each side. Write the new equation.



- 6 MC** If we start with $x = 5$, which of these equations is not true?

A $x + 2 = 7$

B $3x = 8$

C $-2x = -10$

D $\frac{x}{5} = 1$

E $x - 2 = 3$

- 7 MC** If we start with $x = 3$, which of these equations is not true?

A $\frac{2x}{3} = 2$

B $-2x = -6$

C $2x - 6 = 0$

D $\frac{x}{5} = \frac{3}{5}$

E $x - 5 = 2$

- 8 MC** If we start with $x = -6$, which of these equations is not true?

A $-x = 6$

B $2x = -12$

C $x - 6 = 0$

D $x + 4 = -2$

E $x - 2 = -8$

- 9 MC** If we start with $2x = 12$, which of these equations is not true?

A $\frac{2x}{3} = 4$

B $-2x = -12$

C $2x - 6 = 2$

D $4x = 24$

E $2x + 5 = 17$

REASONING

- 10** Make an equivalent equation for each of the equations listed below by performing the operation given in brackets to both sides.

a $m + 8 = 9$ (+ 8)

b $n - 3 = 5$ (- 2)

c $2p + 3 = 9$ (+ 3)

d $3m = 12$ ($\times 4$)

e $12n = 36$ ($\div 6$)

f $\frac{p}{3} = 5$ ($\times 6$)

g $3p + 7 = 2$ ($\times 4$)

h $5m - 3 = -1$ ($\div 3$)

i $x + 5 = 7$ ($\times a$)

PROBLEM SOLVING

- 11** Some simple equations are hard to represent on a balance scale. What would you do to balance the following equations?

a $x - 4 = 10$

b $x - 6 = 3$

c $\frac{w}{3} = 7$

d $\frac{e}{2} = 9$

e $\frac{q}{4} - 2 = 10$

f $\frac{q - 2}{4} = 10$

- 12 a** Why is it difficult to represent the problems in question **11** on a balance scale?

- b** What is the difference between parts **e** and **f** in question **11**?

- 13** Solve the equations in question **11**.

- 14** Represent the following problems on balance scales and solve for the unknown quantity.

- a** Five bags of sugar weigh three kilos. What does one bag of sugar weigh?
- b** Four chocolate bars and an ice-cream cost \$9.90. If the ice-cream costs \$2.70, what does a chocolate bar cost?
- c** A shopkeeper weighed three apples at 920 g. He thought this was heavy, and realised somebody had left a 500-g weight on his balance scale. What was the average weight of one apple?



CHALLENGE 11.1

You have eight \$1 coins, one of which is heavier than the rest. Using a set of balance scales, describe how you can find the heavy coin in the least number of weighings.



eBookplus

Digital doc
WorkSHEET 11.1
doc-2351

11.5 Using algebra to solve problems

- A **linear equation** is an equation where the variable has an index (power) of 1. This means that it never contains terms like x^2 or $\sqrt{x}$.
- To solve a linear equation, perform the same operations on both sides until the variable or unknown is left by itself.
- Sometimes the variable or unknown is called a pronumeral.
- A flowchart is useful to show you the order of operations applied to x , so that the reverse order and inverse operation can be used to solve the equation. As you become confident with solving equations algebraically, you can leave out the flowchart steps.

WORKED EXAMPLE 9

Solve these one-step equations by doing the same to both sides.

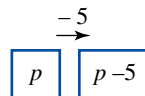
a $p - 5 = 11$

b $\frac{x}{16} = -2$

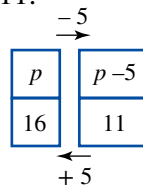
THINK

WRITE

- a** **1** Write the equation.
- 2** Draw a flowchart and fill in the arrow to show what has been done to p .



- 3** Backtrack from 11.

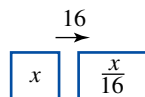


- 4** Add 5 to both sides.
- 5** Give the solution.

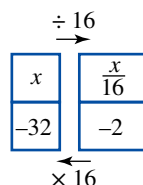
$$p - 5 + 5 = 11 + 5$$

$$p = 16$$

- b** **1** Write the equation.
- 2** Draw a flowchart and fill in the arrow to show what has been done to x .



- 3** Backtrack from -2 .



- 4** Multiply both sides by 16.
- 5** Give the solution.

$$\frac{x}{16} \times 16 = -2 \times 16$$

$$x = -32$$

- The equations in Worked example 9 are called one-step equations because only one operation needs to be undone to obtain the value of the unknown.

WORKED EXAMPLE 10

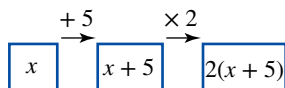
Solve these two-step equations by doing the same to both sides.

a $2(x + 5) = 18$

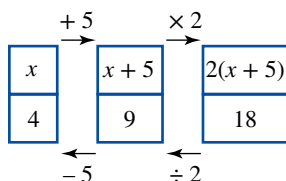
b $\frac{x}{3} + 1 = 7$

THINK

- a** **1** Write the equation.
- 2** Draw a flowchart and fill in the arrow to show what has been done to x .



- 3** Backtrack from 18.



- 4** Divide both sides by 2.

$$\frac{2(x + 5)}{2} = \frac{18}{2}$$

$$x + 5 = 9$$

- 5** Subtract 5 from both sides.

$$x + 5 - 5 = 9 - 5$$

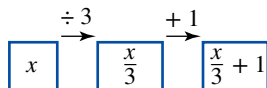
- 6** Give the solution.

$$x = 4$$

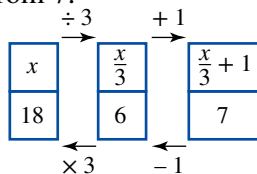
- b** **1** Write the equation.

b $\frac{x}{3} + 1 = 7$

- 2** Draw a flowchart and fill in the arrows to show what has been done to x .



- 3** Backtrack from 7.



- 4** Subtract 1 from both sides.

$$\frac{x}{3} + 1 - 1 = 7 - 1$$

$$\frac{x}{3} = 6$$

- 5** Multiply both sides by 3.

$$\frac{x}{3} \times 3 = 6 \times 3$$

- 6** Give the solution.

$$x = 18$$

- The equations in Worked example 10 are called two-step equations because two operations need to be undone to obtain the value of the unknown.

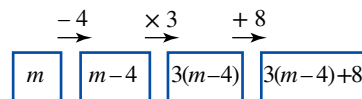
WORKED EXAMPLE 11

Solve the following equations by doing the same to both sides.

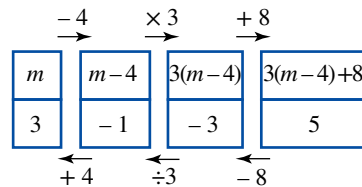
a $3(m - 4) + 8 = 5$ **b** $6\left(\frac{x}{2} + 5\right) = -18$

THINK

- a** **1** Write the equation.
2 Draw a flowchart and fill in the arrows to show what has been done to m .



- 3** Backtrack from 5.



- 4** Subtract 8 from both sides.

$$3(m - 4) + 8 - 8 = 5 - 8$$

$$3(m - 4) = -3$$

- 5** Divide both sides by 3.

$$\frac{3(m - 4)}{3} = \frac{-3}{3}$$

$$m - 4 = -1$$

- 6** Add 4 to both sides.

$$m - 4 + 4 = -1 + 4$$

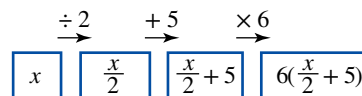
- 7** Give the solution.

$$m = 3$$

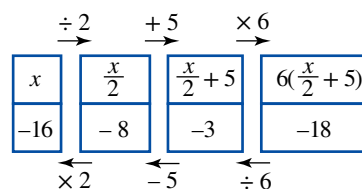
- b** **1** Write the equation.

b $6\left(\frac{x}{2} + 5\right) = -18$

- 2** Draw a flowchart and fill in the arrows to show what has been done to x .



- 3** Backtrack from -18.



- 4 Divide both sides by 6.

$$\frac{6\left(\frac{x}{2} + 5\right)}{6} = \frac{-18}{6}$$

$$\frac{x}{2} + 5 = -3$$

- 5 Subtract 5 from both sides.

$$\frac{x}{2} + 5 - 5 = -3 - 5$$

$$\frac{x}{2} = -8$$

- 6 Multiply both sides by 2.

$$\frac{x}{2} \times 2 = -8 \times 2$$

- 7 Give the solution.

$$x = -16$$

Exercise 11.5 Using algebra to solve problems

assess 

INDIVIDUAL PATHWAYS

PRACTISE

Questions:

- 1, 2a, b, e, f, i, j, 3a, e, i, m, q,
r, 4: columns 1 and 2;
5: columns 1 and 2;
6, 7: column 1; 8, 11, 12

CONSOLIDATE

Questions:

- 1, 2, 3a–l, 4, 5a–l, 6–12

MASTER

Questions:

- 1i–l, 2i–l, 3i–l, 4i–l, 6–14

REFLECTION

How do you decide on the order to undo the operations?

 **Individual pathway interactivity** int-4449 

FLUENCY

- 1 **WE9a** Solve these one-step equations by doing the same to both sides.

a $x + 8 = 7$

b $12 + r = 7$

c $31 = t + 7$

d $w + 4.2 = 6.9$

e $\frac{5}{8} = m + \frac{1}{8}$

f $\frac{2}{7} = j + 3$

g $q - 8 = 11$

h $-16 + r = -7$

i $21 = t - 11$

j $y - 5.7 = 8.8$

k $-\frac{11}{7} = z - \frac{2}{3}$

l $-\frac{9}{13} = f - 1$

- 2 **WE9b** Solve these one-step equations by doing the same to both sides.

a $11d = 88$

b $7p = -98$

c $5u = 4$

d $2.5g = 12.5$

e $8m = \frac{1}{4}$

f $-\frac{3}{5} = 9j$

g $\frac{t}{8} = 3$

h $\frac{k}{5} = -12$

i $-5.3 = \frac{l}{4}$

j $\frac{v}{6} = \frac{2}{3}$

k $\frac{c}{9} = -\frac{5}{27}$

l $-\frac{7}{12} = \frac{h}{5}$

- 3 **WE10a** Solve these two-step equations by doing the same to both sides.

a $3m + 5 = 14$

b $-2w + 6 = 16$

c $-5k - 12 = 8$

d $4t - 3 = -15$

eBookplus

Digital doc
Spreadsheet
2-step equations
doc-2353

e $2(m - 4) = -6$	f $-3(n + 12) = 18$
g $5(k + 6) = -15$	h $-6(s + 11) = -24$
i $2m + 3 = 10$	j $40 = -5(p + 6)$
k $5 - 3g = 14$	l $11 - 4f = -9$
m $2q - 4.9 = 13.2$	n $7.6 + 5r = -8.4$
o $13.6 = 4t - 0.8$	p $-6k + 7.3 = 8.5$
q $-4g - \frac{1}{5} = \frac{4}{5}$	r $-\frac{3}{8} = 2f - \frac{18}{8}$

4 WE10b Solve these two-step equations by doing the same to both sides.

a $\frac{x}{3} + 2 = 9$	b $\frac{x - 5}{4} = 1$
c $\frac{m + 3}{2} = -7$	d $\frac{h}{-3} + 1 = 5$
e $\frac{-m}{5} - 3 = 1$	f $\frac{2w}{5} = -4$
g $\frac{-3m}{7} = -1$	h $\frac{c - 7}{3} = -2$
i $\frac{-5m}{4} = 10$	j $\frac{t + 2}{7} = -5$
k $\frac{c - 21}{9} = -4.5$	l $\frac{x}{8} - 3.2 = -5.8$

5 WE11 Solve these equations by doing the same to both sides. They will need more than two steps.

a $2(m + 3) + 7 = 3$	b $\frac{-2(x + 5)}{5} = 6$	c $\frac{5m + 6}{3} = 4$
d $\frac{3x - 2}{7} = 1$	e $\frac{4 - 2x}{3} = 6$	f $\frac{-x + 3}{2} = -4$
g $\frac{3x}{7} - 2 = 1$	h $\frac{4b}{5} - 3 = 5$	i $\frac{7f}{9} + 2 = -5$
j $6 - \frac{4z}{3} = -2$	k $8 - \frac{6m}{5} = 2$	l $-9 - \frac{5u}{11} = -4$
m $\frac{3m - 5}{-2} = 7$	n $-7(5w + 3) = 35$	o $5\left(\frac{x}{2} - 6\right) = -10$
p $\frac{d - 7}{2} + 10 = 8$	q $\frac{3n + 1}{4} - 5 = 2$	r $\frac{3(t - 5)}{7} + 9 = 6$

UNDERSTANDING

6 Below is Alex's working to solve the equation $2x + 3 = 14$.

$$\begin{aligned}
 2x + 3 &= 14 \\
 \frac{2x}{2} + 3 &= \frac{14}{2} \\
 x + 3 &= 7 \\
 x + 3 - 3 &= 7 - 3 \\
 x &= 4
 \end{aligned}$$

- a** Is the solution correct?
b If not, can you find where the error is and correct it?

eBookplus

Digital doc
 Spreadsheet
 3-step equations
 doc-2354

- 7 Simplify the left-hand side of the following equations by collecting like terms, and then solve.

a $3x + 5 + 2x + 4 = 19$

c $-3m + 6 - 5m + 1 = 15$

e $-3y - 7y + 4 = 64$

g $5w + 3w - 7 + w = 13$

i $7 - 3u + 4 + 2u = 15$

b $13v - 4v + 2v = -22$

d $-3y + 7 + 4y - 2 = 9$

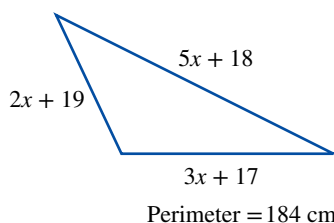
f $5t + 4 - 8t = 19$

h $w + 7 + w - 15 + w + 1 = -5$

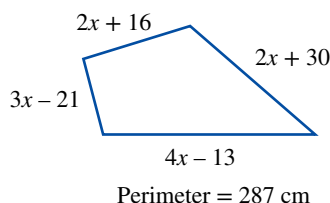
j $7c - 4 - 11 + 3c - 7c + 5 = 8$

REASONING

- 8 A repair person calculates his service fee using the equation $F = 40t + 55$, where F is the service fee in dollars and t is the number of hours spent on the job.
- a** How long did a particular job take if the service fee was \$155?
- b** Explain what the numbers 40 and 55 could represent as costs in the service fee equation.
- 9 Lyn and Peta together raised \$517 from their cake stalls at the school fete. If Lyn raised l dollars and Peta raised \$286, write an equation that represents the situation and determine the amount Lyn raised.
- 10 **a** Write an equation that represents the perimeter of the figure below and then solve for x .



- b** Write an equation that represents the perimeter of the figure below and then solve for x .



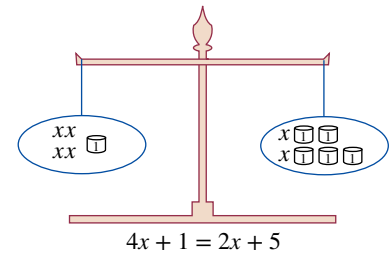
PROBLEM SOLVING

- 11 If four times a certain number equals nine minus a half of the number, find the number.
- 12 Tom is 5 years old and his dad is 10 times his age, being 50 years old. Is it possible, at any stage, for Tom's dad to be twice the age of his son? Explain your answer.
- 13 Lauren earns the same amount for mowing four of the neighbours' lawns every month. Each month she saves all her pay except \$30, which she spends on her mobile phone. If she has \$600 at the end of the year, how much did she earn each month? (Write an equation to solve for this situation first.)

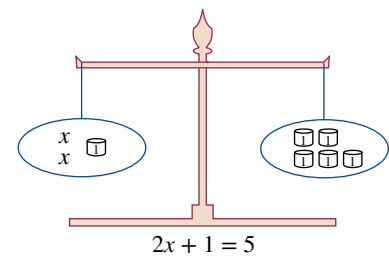


11.6 Equations with the unknown on both sides

- Some equations have unknowns on both sides of the equation.
- If an equation has unknowns on both sides, eliminate the unknowns from one side and then solve as usual.
- Consider the equation $4x + 1 = 2x + 5$.
 - Drawing the equation on a pair of scales looks like this:



- The scales remain balanced if $2x$ is eliminated from both sides:



- Writing this algebraically, we have:

$$\begin{aligned}
 4x + 1 &= 2x + 5 \\
 4x + 1 - 2x &= 2x + 5 - 2x \\
 2x + 1 &= 5 \\
 2x + 1 - 1 &= 5 - 1 \\
 2x &= 4 \\
 \frac{2x}{2} &= \frac{4}{2} \\
 x &= 2
 \end{aligned}$$

WORKED EXAMPLE 12

Solve the equation $5t - 8 = 3t + 12$ and check your solution.

THINK

- 1 Write the equation.
- 2 Subtract the smaller unknown (that is, $3t$) from both sides and simplify.
- 3 Add 8 to both sides and simplify.
- 4 Divide both sides by 2 and simplify.

WRITE

$$\begin{aligned}
 5t - 8 &= 3t + 12 \\
 5t - 8 - 3t &= 3t + 12 - 3t \\
 2t - 8 &= 12 \\
 2t - 8 + 8 &= 12 + 8 \\
 2t &= 20 \\
 \frac{2t}{2} &= \frac{20}{2} \\
 t &= 10
 \end{aligned}$$

- 5 Check the solution by substituting $t = 10$ into the left-hand side and then the right-hand side of the equation.

$$\begin{aligned}\text{If } t = 10, \\ \text{LHS} &= 5t - 8 \\ &= 50 - 8 \\ &= 42\end{aligned}$$

$$\begin{aligned}\text{If } t = 10, \\ \text{RHS} &= 3t + 12 \\ &= 30 + 12 \\ &= 42\end{aligned}$$

- 6 Comment on the answers obtained.

Since the LHS and RHS are equal, the equation is true when $t = 10$.

WORKED EXAMPLE 13

Solve the equation $3n + 11 = 6 - 2n$ and check your solution.

THINK

- 1 Write the equation.
- 2 The inverse of $-2n$ is $+2n$.
Therefore, add $2n$ to both sides and simplify.
- 3 Subtract 11 from both sides and simplify.
- 4 Divide both sides by 5 and simplify.
- 5 Check the solution by substituting $n = -1$ into the left-hand side and then the right-hand side of the equation.

WRITE

$$3n + 11 = 6 - 2n$$

$$\begin{aligned}3n + 11 + 2n &= 6 - 2n + 2n \\ 5n + 11 &= 6\end{aligned}$$

$$\begin{aligned}5n + 11 - 11 &= 6 - 11 \\ 5n &= -5\end{aligned}$$

$$\begin{aligned}\frac{5n}{5} &= \frac{-5}{5} \\ n &= -1\end{aligned}$$

$$\begin{aligned}\text{If } n = -1, \\ \text{LHS} &= 3n + 11 \\ &= -3 + 11 \\ &= 8\end{aligned}$$

$$\begin{aligned}\text{If } n = -1, \\ \text{RHS} &= 6 - 2n \\ &= 6 - -2 \\ &= 6 + 2 \\ &= 8\end{aligned}$$

- 6 Comment on the answers obtained.

Since the LHS and RHS are equal, the equation is true when $n = -1$.

WORKED EXAMPLE 14

Expand the brackets and then solve the following equation, checking your solution.

a $3(s + 2) = 2(s + 7) + 4$

b $4(d + 3) - 2(d + 7) + 4 = 5(d + 2) + 7$

THINK

- a 1 Write the equation.
- 2 Expand the brackets on each side of the equation first and then simplify.

WRITE

$$\begin{aligned}\text{a } 3(s + 2) &= 2(s + 7) + 4 \\ 3s + 6 &= 2s + 14 + 4 \\ 3s + 6 &= 2s + 18\end{aligned}$$





- 3 Subtract the smaller unknown term (that is, $2s$) from both sides and simplify.
- 4 Subtract 6 from both sides and simplify.
- 5 Check the solution by substituting $s = 12$ into the left-hand side and then the right-hand side of the equation.

$$\begin{aligned} 3s + 6 - 2s &= 2s + 18 - 2s \\ s + 6 &= 18 \\ s + 6 - 6 &= 18 - 6 \\ s &= 12 \end{aligned}$$

If $s = 12$,

$$\begin{aligned} \text{LHS} &= 3(s + 2) \\ &= 3(12 + 2) \\ &= 3(14) \\ &= 42 \end{aligned}$$

If $s = 12$,

$$\begin{aligned} \text{RHS} &= 2(s + 7) + 4 \\ &= 2(12 + 7) + 4 \\ &= 2(19) + 4 \\ &= 38 + 4 \\ &= 42 \end{aligned}$$

Since the LHS and RHS are equal, the equation is true when $s = 12$.

- b**
- 1 Write the equation.
 - 2 Expand the brackets on each side of the equation first, and then simplify.
 - 3 Subtract the smaller unknown term (that is, $2d$) from both sides and simplify.
 - 4 Rearrange the equation so that the unknown is on the left-hand side of the equation.
 - 5 Subtract 17 from both sides and simplify.
 - 6 Divide both sides by 3 and simplify.
 - 7 Check the solution by substituting $d = -5$ into the left-hand side and then the right-hand side of the equation.

$$\begin{aligned} \text{b } 4(d + 3) - 2(d + 7) + 4 &= 5(d + 2) + 7 \\ 4d + 12 - 2d - 14 + 4 &= 5d + 10 + 7 \\ 2d + 2 &= 5d + 17 \\ 2d - 2d + 2 &= 5d - 2d + 17 \\ 2 &= 3d + 17 \end{aligned}$$

$$3d + 17 = 2$$

$$\begin{aligned} 3d + 17 - 17 &= 2 - 17 \\ 3d &= -15 \end{aligned}$$

$$\begin{aligned} \frac{3d}{3} &= \frac{-15}{3} \\ d &= -5 \end{aligned}$$

If $d = -5$,

$$\begin{aligned} \text{LHS} &= 4(d + 3) - 2(d + 7) + 4 \\ &= 4(-5 + 3) - 2(-5 + 7) + 4 \\ &= 4(-2) - 2(2) + 4 \\ &= -8 - 4 + 4 \\ &= -8 \end{aligned}$$

If $d = -5$,

$$\begin{aligned} \text{RHS} &= 5(-5 + 2) + 7 \\ &= 5(-3) + 7 \\ &= -15 + 7 \\ &= -8 \end{aligned}$$

Since the LHS and RHS are equal, the equation is true when $d = -5$.

- 8 Comment on the answers obtained.

- *Note:* When solving equations with the unknown on both sides, it is good practice to remove the unknown with the smaller coefficient from the relevant side.

Exercise 11.6 Equations with the unknown on both sides

assessment

INDIVIDUAL PATHWAYS

PRACTISE

Questions:
1–4, 6, 8, 9, 13, 15

CONSOLIDATE

Questions:
1, 2, 3a–l, 4–10, 13–15

MASTER

Questions:
1g–i, 2g–i, 3k–p, 4–16

Individual pathway interactivity int-4450 eBookplus

REFLECTION

Does it matter to which side of the equation the unknowns are moved?

FLUENCY

- 1 **WE12** Solve the following equations and check your solutions.

- a $8x + 5 = 6x + 11$
- b $5y - 5 = 2y + 7$
- c $11n - 1 = 6n + 19$
- d $6t + 5 = 3t + 17$
- e $2w + 6 = w + 11$
- f $4y - 2 = y + 9$
- g $3z - 15 = 2z - 11$
- h $5a + 2 = 2a - 10$
- i $2s + 9 = 5s + 3$
- j $k + 5 = 7k - 19$
- k $4w + 9 = 2w + 3$
- l $7v + 5 = 3v - 11$

- 2 **WE13** Solve the following equations and check your solutions.

- a $3w + 1 = 11 - 2w$
- b $2b + 7 = 13 - b$
- c $4n - 3 = 17 - 6n$
- d $3s + 1 = 16 - 2s$
- e $5a + 12 = -6 - a$
- f $7m + 2 = -3m + 22$
- g $p + 7 = -p + 15$
- h $3 + 2d = 15 - 2d$
- i $5 + m = 5 - m$
- j $7s + 3 = 15 - 5s$
- k $3t - 7 = -17 - 2t$
- l $16 - 2x = x + 4$

eBookplus

Digital docs

SkillSHEET
Combining like terms
doc-6984
SkillSHEET
Expanding expressions
containing brackets
doc-6985
SkillSHEET
Checking solutions by
substitution
doc-6986
SkillSHEET
Writing equations from
worded statements
doc-6987

eBook plus

Digital doc

Spreadsheet

Unknowns on both sides

doc-2355

3 WE14 Expand the brackets and then solve the following equations, checking your solutions.

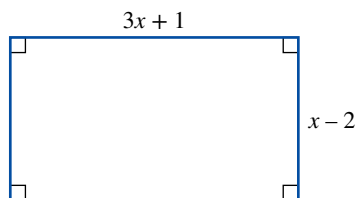
- a** $3(2x + 1) + 3x = 30$
- b** $2(4m - 7) + m = 76$
- c** $3(2n - 1) = 4(n + 5) + 1$
- d** $t + 4 = 3(2t - 7)$
- e** $3d - 5 = 3(4 - d)$
- f** $4(3 - w) = 5w + 1$
- g** $2(k + 5) - 3(k - 1) = k - 7$
- h** $4(2 - s) = -2(3s - 1)$
- i** $-3(z + 3) = 2(4 - z)$
- j** $5(v + 2) = 7(v + 1)$
- k** $2m + 3(2m - 7) = 4 + 5(m + 2)$
- l** $3d + 2(d + 1) = 5(3d - 7)$
- m** $4(d + 3) - 2(d + 7) + 5 = 5(d + 12)$
- n** $5(k + 11) + 2(k - 3) - 7 = 2(k - 4)$
- o** $7(v - 3) - 2(5 - v) + 25 = 4(v + 3) - 8$
- p** $3(l - 7) + 4(8 - 2l) - 7 = -4(l + 2) - 6$

UNDERSTANDING

4 Solve the equation $\frac{-3(x + 4)}{7} - \frac{5(1 - 3x)}{3} = 3x - 4$.

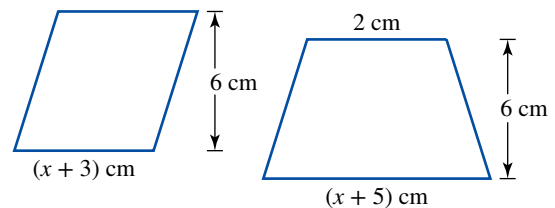
5 Solve the equation $\frac{(x - 2)}{3} + 5 = 2x$.

6 Find the value of x given the perimeter of the rectangle is 48 cm.



7 The two shapes shown have the same area.

- a** Write an equation to show that the parallelogram and the trapezium have the same area.
- b** Solve the equation for x .
- c** State the dimensions of the shapes.



REASONING

- 8** A maths class has equal numbers of boys and girls. Eight of the girls left early to play in a netball match. This left 3 times as many boys in the class as girls. How many students are in the class? Show your working.
- 9** Judy is thinking of a number. First she doubles it and adds 2. She realizes that if she multiplies it by 3 and subtracts 1, she gets the same result. Find the number.



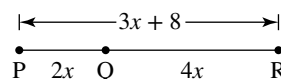
- 10 Mick's father was 28 years old when Mick was born. If his father is now 3 times as old as Mick is, how old are they both now?
- 11 Given the following number line for a line segment PR, determine the length of PR?



eBookplus

 Interactivity
Solving equations
int-2373

- 12 In 8 years' time, Tess will be 5 times as old as her age 8 years ago. How old is Tess now?

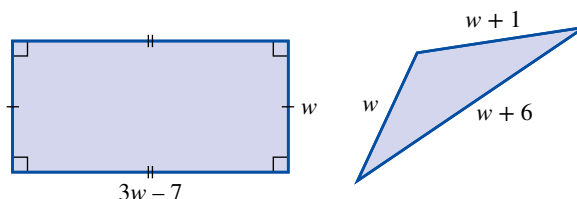


PROBLEM SOLVING

- 13 You have 12 more than three times the number of marker pens in your pencil case than your friend has in his pencil case. The teacher has 5 more than four times the number of marker pens in your friend's pencil case.


 Could this
be your
friend's
pencil
case?

- a Write an expression for the number of marker pens in:
- your pencil case
 - your teacher's pencil case.
- b You have the same number of marker pens as the teacher. Write an equation to show this.
- c Calculate the number of marker pens in your friend's pencil case by solving the equation from part b.
- 14 The length of the rectangle below is 7 centimetres less than three times its width. If the perimeter of the rectangle is the same as the perimeter of the triangle, calculate the side lengths of the rectangle and triangle.



- 15 a How many rides do you need so that option 1 and option 2 cost the same?
- b If you were planning on having only two rides, explain which option you would choose and why.



eBookplus

Digital doc
WorkSHEET 11.2
doc-2352

- 16** On her birthday today, a mother is three times as old as her daughter will be in eight years time. It just so happens that the mother's age today is the same as their house number, which is four times the value of thirteen minus the daughter's age.
- Write each sentence above as an expression. (*Hint: Make the daughter's age the unknown variable.*)
 - Use these two expressions to make an equation and solve it.
 - How old are the mother and the daughter?
 - What is the number of their house?



CHALLENGE 11.2

A balanced scale containing boxes of Smarties and loose Smarties in the pans. There are three full boxes of Smarties and another box with four Smarties missing in one pan. The other pan contains four empty boxes and 48 loose Smarties. How many Smarties are in a full box?



eBookplus
ONLINE ONLY

11.7 Review



www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search
int-2633



Crossword
int-2634



Sudoku
int-3190



Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

backtracking

equation

flowchart

geometric pattern

input number

inverse operations

linear equation

output number

patterns

relationships

rule

sequence

term

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assesson

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

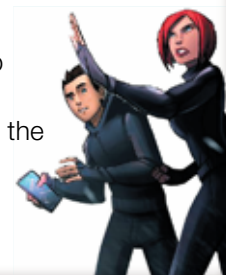
www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book-style story with problem-based learning in an immersive environment.

SPY CLASS

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

www.spyclass.com.au



RICH TASK

The Olympic freestyle final

Swimming is one of the most popular sports of the Olympic Games. Over the years the Olympics have been held, competitors have been swimming more quickly and their times have been correspondingly reduced.

The table below displays the winning times for the men's and women's 100-metre freestyle final for the Olympic Games from 1956 to 2008.



Men's and women's 100-metre freestyle results — Olympic Games 1956–2008

Year	Men's time (seconds)	Women's time (seconds)
1956	55.4	62.0
1960	55.2	61.2
1964	53.4	59.5
1968	52.2	60.0
1972	51.2	58.6
1976	50.0	55.7
1980	50.4	54.8
1984	49.8	55.9
1988	48.6	54.9
1992	49.0	54.7
1996	48.7	54.5
2000	48.3	53.8
2004	48.2	53.8
2008	47.2	53.1

Note: All times have been rounded to 1 decimal place.

The women's times have been graphed on the set of axes at right. There is no straight line that passes through all the points, so a line of best fit has been selected to approximate the swimming times. The equation of this line is given by $t = 62 - 0.1864x$, where t represents the time taken and x represents the number of years after 1956.

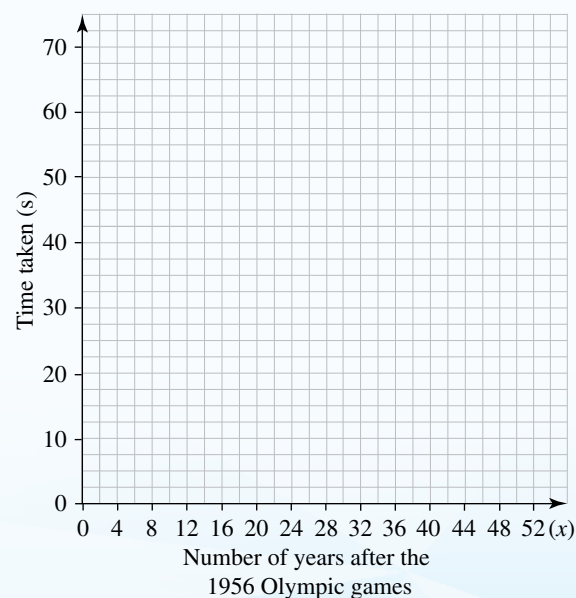
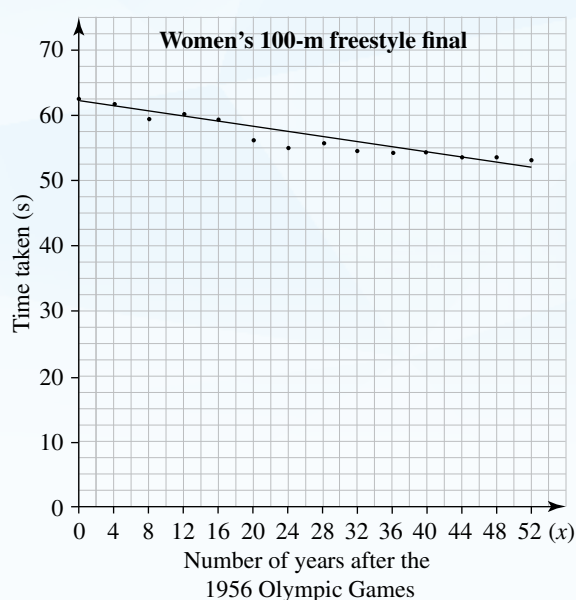
- 1 What year is represented by $x = 16$?
- 2 Substitute $x = 16$ into the equation to find an approximation for the time taken. How close is this approximation to the actual time given in the table?
- 3 What x -value would you use for the 2012 Olympics? Use the equation to predict the women's 100-metre freestyle final time for the 2012 Olympics.
- 4 The women's time in 2012 was 53.0 seconds. How does your prediction compare with the actual time?
- 5 Use backtracking to solve the equation to the nearest whole number when $t = 56$ seconds. What year does your solution represent?

An equation to approximate the men's times is given by $t = 54 - 0.1364x$. Use this equation to answer the following questions.

- 6 The men's time in 1984 was 49.8 seconds. How does this compare with the time obtained from the equation?
- 7 Use the equation to predict the men's time for the 2012 Olympic Games.
- 8 Plot the men's times on the set of axes provided. Between the points, draw in a line of best fit.
- 9 Extend your line of best fit so that it passes the x -value that represents the year 2012. How does this value compare with the prediction you obtained in question 7 above?
- 10 The men's time in 2012 was 47.52 seconds. How does your prediction compare with the actual time?
- 11 Use your graphs to compare both the men's and women's times with the actual results obtained during the 2004 Athens Olympics.

How long will it be before the men and women are swimming identical times?

- 12 Investigate this by plotting the times given on the same set of axes and drawing a line of best fit. Extend both lines until they intersect. The point of intersection represents the time when the swimming times are identical. Present your findings on graph paper.



CODE PUZZLE

Kidneys . . .

Solve the equations to discover the puzzle's answer code.



		4	9	-8	-6	-3	3	2	13	13		
-8	1	-6	3	-3	15	13	4	4	-8	-16	9	2
2	5	1	-8	-1	5	-5	2	6	5	-7	-3	2
-8	9	-6	3	-3	15	13	4	-16	3	1	-5	-3

$$2A - 5 = A$$

$$3D = 10 - 2D$$

$$7 + 5E = 2E - 2$$

$$15 - F = F + 7$$

$$6I + 11 = 74 - I$$

$$3K + 8 = K - 6$$

$$L = 4L + 24$$

$$M + 6 = 3M - 6$$

$$6(N + 3) = 2N - 2$$

$$2O + 4 = 3O - 9$$

$$12 - 4I = 8I$$

$$3(2R - 1) = 5R$$

$$3S - 6 - S = S + 9$$

$$2(T - 6) = 3(2T + 4)$$

$$U = 48 + 4U$$

$$9 - 2Y = 5Y + 16$$

11.2 Identifying patterns**Digital docs**

- SkillSHEET (doc-6978) Number patterns
- SkillSHEET (doc-6979) Using tables to show number patterns
- SkillSHEET (doc-6980) Describing a number pattern from a table

Interactivity

- IP interactivity 11.2 (int-4446) Identifying patterns

11.3 Backtracking and inverse operations**Digital docs**

- SkillSHEET (doc-6981) Flowcharts
- SkillSHEET (doc-6982) Inverse operations
- SkillSHEET (doc-6983) Solving equations by backtracking

Interactivity

- IP interactivity 11.3 (int-4447) Backtracking and inverse operations

11.4 Keeping equations balanced**Digital doc**

- WorkSHEET 11.1 (doc-2351)

Interactivities

- Balancing equations (int-0077)
- IP interactivity 11.4 (int-4448) Keeping equations balanced

11.5 Using algebra to solve problems**Digital docs**

- Spreadsheet (doc-2353) 2-step equations
- Spreadsheet (doc-2354) 3-step equations

Interactivity

- IP interactivity 11.5 (int-4449) Using algebra to solve problems

11.6 Equations with the unknown on both sides**Digital docs**

- SkillSHEET (doc-6984) Combining like terms
- SkillSHEET (doc-6985) Expanding expressions containing brackets
- SkillSHEET (doc-6986) Checking solutions by substitution
- SkillSHEET (doc-6987) Writing equations from worded statements
- Spreadsheet (doc-2355) Unknowns on both sides
- WorkSHEET 11.2 (doc-2352)

Interactivities

- Solving equations (int-2373)
- IP interactivity 11.6 (int-4450) Equations with the unknown on both sides

11.7 Review**Interactivities**

- Word search (int-2633)
- Crossword (int-2634)
- Sudoku (int-3190)

Digital docs

- Topic summary (doc-10762)
- Concept map (doc-10775)

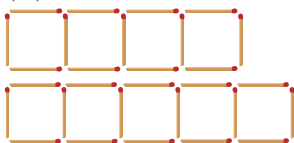
To access eBookPLUS activities, log on to



www.jacplus.com.au

11.2 Identifying patterns

- 1**
 - a** Add 2; 10, 12, 14
 - b** Add 5; 23, 28, 33
 - c** Subtract 3; 15, 12, 9
 - d** Multiply by 3; 81, 243, 729
 - e** Divide by 2; 8, 4, 2
 - f** Squares of numbers 1; 25, 36, 49
- 2**
 - a** 6, 15, 18
 - b** 10, 12, 16
 - c** 16, 64
 - d** 9, 11, 17
 - e** 88, 110
 - f** 95, 90, 75
- 3**
 - a** 1, 5, 9, 13, 17
 - b** 5, 15, 45, 135, 405
 - c** 50, 42, 34, 26, 18
 - d** 64, 32, 16, 8, 4
 - e** 1, 4, 10, 22, 46
 - f** 1, 6, 16, 36, 76
- 4**
 - a** Even numbers/multiples of 2
 - b** Cubes
 - c** Prime numbers
 - d** Fibonacci sequence
 - e** Factors of 12
 - f** Multiples/powers of 3
- 5**
 - a** 2, 4, 8, 6
 - b** 3, 9, 7, 1
 - c** 4, 6
 - d** 5
 - e** 8, 4, 2, 6



Number of squares	1	2	3	4	5
Number of matches	4	7	10	13	16

- iii Number of matches = number of squares $\times 3 + 1$
- iv 61



Number of triangles	1	2	3	4	5
Number of matches	3	5	7	9	11

- iii Number of matches = number of triangles $\times 2 + 1$
- iv 41



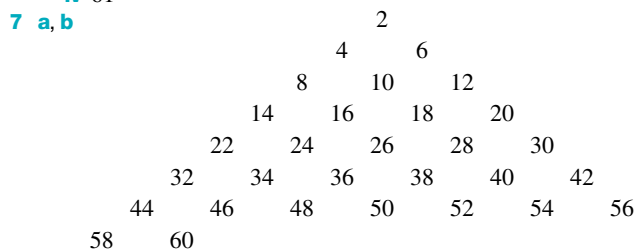
Number of houses	1	2	3	4	5
Number of matches	6	11	16	21	26

- iii Number of matches = number of houses $\times 5 + 1$
- iv 101



Number of fence units	1	2	3	4	5
Number of matches	4	7	10	13	16

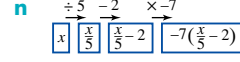
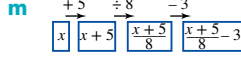
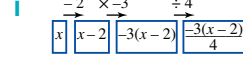
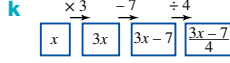
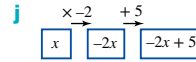
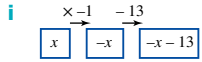
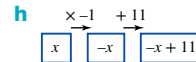
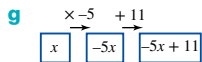
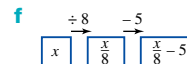
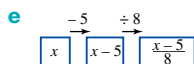
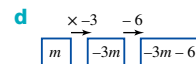
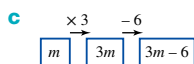
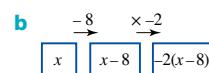
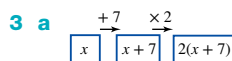
- iii Number of matches = number of fence units $\times 3 + 1$
iv 61



- c–d** Check with your teacher.
- 8 a** Grains of rice = 3^{n-1}
- b** 8th square
- 9 a** 8, 13, 21
- b** $b, c, b + c$

11.3 Backtracking and inverse operations

- | | | |
|--------------------------------|--------------------------------|-----------------------------|
| 1 a 8 | b 20 | c -4 |
| d -19 | e -1 | f -1 |
| g -2 | h -44 | i 8 |
| j 4 | k 1.075 | l 4.4 |
| 2 a $2x - 7$ | b $2(w - 7)$ | c $-5s + 3$ |
| d $-5(n + 3)$ | e $\frac{m}{2} + 7$ | f $\frac{y + 7}{2}$ |
| g $\frac{6z - 3}{2}$ | h $\frac{-3(d + 5)}{4}$ | i $\frac{2e}{5} + 1$ |
| j $4(3 - x)$ | k $\frac{-2(w - 5)}{7}$ | l $-3(z + 6) - 11$ |
| m $\frac{v - 3}{6} - 8$ | n $-7(8m - 4)$ | |
| o $\frac{-5k}{6} + 2$ | p $\frac{-5p - 7}{3}$ | |



$$\begin{array}{c} \times 2 \quad \div 7 \quad + 4 \quad \times 3 \\ \boxed{x} \rightarrow \boxed{2x} \rightarrow \boxed{\frac{2x}{7}} \rightarrow \boxed{\frac{2x}{7} + 4} \rightarrow \boxed{3(\frac{2x}{7} + 4)} \end{array}$$

$$\begin{array}{c} \times 6 \quad \div 11 \quad - 3 \quad \times \frac{1}{4} \\ \boxed{x} \rightarrow \boxed{6x} \rightarrow \boxed{\frac{6x}{11}} \rightarrow \boxed{\frac{6x}{11} - 3} \rightarrow \boxed{\frac{1}{4}(\frac{6x}{11} - 3)} \end{array}$$

$$4 \quad a \quad \boxed{F} \xrightarrow{-32} \boxed{F-32} \xrightarrow{\times 5} \boxed{5(F-32)} \xrightarrow{\div 9} \boxed{\frac{5(F-32)}{9}}$$

$$b \quad 10$$

$$c \quad 95$$

$$5 \quad a \quad \boxed{n} \xrightarrow{\times 2} \boxed{2n} \xrightarrow{+10} \boxed{2n+10} \xrightarrow{\div 2} \boxed{\frac{2n+10}{2}}$$

$$b \quad \text{It is 5.}$$

$$c \quad \text{It is always 5.}$$

6 a Strategies include 'guess and check'. Backtracking will not work as the pronumeral w appears more than once in the equation and cannot be simplified.

$$b \quad w = 15 \text{ cm}$$

11.4 Keeping equations balanced

$$1 \quad a \quad x + 5 = 11$$

$$b \quad 7x = 42$$

$$c \quad x - 4 = 2$$

$$d \quad \frac{x}{3} = 2$$

$$e \quad -4x = -24$$

$$f \quad -x = -6$$

$$g \quad -x = -6$$

$$h \quad x - 9 = -3$$

$$i \quad \frac{2x}{3} = 4$$

$$j \quad \frac{3x}{2} = 9$$

$$k \quad x - \frac{2}{3} = 5\frac{1}{3}$$

$$2 \quad a \quad 2x = 4$$

$$b \quad x = 2$$

$$3 \quad a \quad x + 3 = 5$$

$$b \quad x = 2$$

$$4 \quad a \quad 3x + 1 = 7$$

$$b \quad 3x + 4 = 10$$

$$5 \quad a \quad 2x + 1 = 5$$

$$b \quad 4x + 2 = 10$$

$$6 \quad B$$

$$7 \quad E$$

$$8 \quad C$$

$$9 \quad C$$

$$10 \quad a \quad m + 16 = 17$$

$$b \quad n - 5 = 3$$

$$c \quad 2p + 6 = 12$$

$$d \quad 12m = 48$$

$$e \quad 2n = 6$$

$$f \quad 2p = 30$$

$$g \quad 12p + 28 = 8$$

$$h \quad \frac{5m + 3}{3} = \frac{-1}{3}$$

$$i \quad ax + 5a = 7a$$

11 a Add 4 to both sides.

b Add 6 to both sides.

c Multiply both sides by 3.

d Multiply both sides by 2.

e Add 2 to both sides, then multiply both sides by 4.

f Multiply both sides by 4, then add 2 to both sides.

12 a They have subtraction and division in the expressions and these cannot be represented on a pan balance scale.

b In part e, the pronumeral is divided by 4 and then 2 is subtracted from the result. In part f, 4 is subtracted from the pronumeral and then the result is divided by 2.

$$13 \quad a \quad x = 14 \quad b \quad x = 9 \quad c \quad w = 21$$

$$d \quad e = 18 \quad e \quad q = 48 \quad f \quad q = 42$$

$$14 \quad a \quad 5x = 3$$

$$x = \frac{3}{5}. \text{ One bag of sugar weighs } \frac{3}{5} \text{ kg.}$$

$$b \quad 4x + 2.70 = 9.90$$

$$x = 1.8. \text{ One chocolate bar costs \$1.80.}$$

$$c \quad 3x + 500 = 920$$

$$x = 140. \text{ One apple weighs 140 g.}$$

Challenge 11.1

Weigh 1:

The least number of weighings is two. Take 6 coins and place 3 coins on each side of the balance. There is a remainder of 2 unweighed coins. If the 6 coins balances, the heavy coin must be one of the 2 unweighed coins.

Weigh 2:

If the 6 coins do not balance, the heavy coin must be one of the 3 coins on the heavy side of the balance (the side that drops lower than the other). Take the 3 coins, place one coin on each side of the empty balance, so that there are 2 coins on the scale at a time. If the balance balances, the heavy coin is the coin not on the balance. If the balance does not balance, the heavy coin is on the heavy side of the balance.

11.5 Using algebra to solve problems

$$1 \quad a \quad x = -1$$

$$b \quad r = -5$$

$$c \quad t = 24$$

$$d \quad w = 2.7$$

$$e \quad m = \frac{1}{2}$$

$$f \quad j = -\frac{19}{7} \text{ or } -2\frac{5}{7}$$

$$g \quad q = 19$$

$$h \quad r = 9$$

$$i \quad t = 32$$

$$j \quad y = 14.5$$

$$k \quad z = -\frac{19}{21}$$

$$l \quad f = \frac{4}{13}$$

$$2 \quad a \quad d = 8$$

$$b \quad p = -14$$

$$c \quad u = \frac{4}{5} \text{ or } 0.8$$

$$d \quad g = 5$$

$$e \quad m = \frac{1}{32}$$

$$f \quad j = -\frac{1}{15}$$

$$g \quad t = 24$$

$$h \quad k = -60$$

$$i \quad l = -21.2$$

$$j \quad v = 4$$

$$k \quad c = -\frac{5}{3} \text{ or } -1\frac{2}{3}$$

$$l \quad h = -\frac{35}{12} \text{ or } -2\frac{11}{12}$$

$$3 \quad a \quad m = 3$$

$$b \quad w = -5$$

$$c \quad k = -4$$

$$d \quad t = -3$$

$$e \quad m = 1$$

$$f \quad n = -18$$

$$g \quad k = -9$$

$$h \quad s = -7$$

$$i \quad m = 3.5$$

$$j \quad p = -14$$

$$k \quad g = -3$$

$$l \quad f = 5$$

$$m \quad q = 9.05$$

$$n \quad r = -3.2$$

$$o \quad t = 3.6$$

$$p \quad k = -0.2$$

$$q \quad g = -\frac{1}{4} \text{ or } -0.25$$

$$r \quad f = \frac{15}{16}$$

$$4 \quad a \quad x = 21$$

$$b \quad x = 9$$

$$c \quad m = -17$$

$$d \quad h = -12$$

$$e \quad m = -20$$

$$f \quad w = -10$$

$$g \quad m = 2\frac{1}{3}$$

$$h \quad c = 1$$

$$i \quad m = -8$$

$$j \quad t = -37$$

$$k \quad c = -19.5$$

$$l \quad x = -20.8$$

$$5 \quad a \quad m = -5$$

$$b \quad x = -20$$

$$c \quad m = 1\frac{1}{5}$$

$$d \quad x = 3$$

$$e \quad x = -7$$

$$f \quad x = 11$$

$$g \quad x = 7$$

$$h \quad b = 10$$

$$i \quad f = -9$$

$$j \quad z = 6$$

$$k \quad m = 5$$

$$l \quad u = -11$$

$$m \quad m = -3$$

$$n \quad w = -1\frac{3}{5}$$

$$o \quad x = 8$$

$$p \quad d = 3$$

$$q \quad n = 9$$

$$r \quad t = -2$$

6 a The solution is not correct.

b Alex should have subtracted 3 from both sides first before dividing both sides by 2. The solution should be $x = 5\frac{1}{2}$. The error made was that he didn't divide all parts of the equation by 2.

$$7 \quad a \quad x = 2$$

$$b \quad v = -2$$

$$c \quad m = -1$$

$$d \quad y = 4$$

$$e \quad y = -6$$

$$f \quad t = -5$$

$$g \quad w = 2\frac{2}{9}$$

$$h \quad w = \frac{2}{3}$$

$$i \quad u = -4$$

$$j \quad c = 6$$

$$8 \quad a \quad 2\frac{1}{2} \text{ hours}$$

b 40 could be the charge per hour; 55 could be the flat fee covering travel and other expenses.

$$9 \quad l + 286 = 517, l = \$231$$

$$10 \quad a \quad 10x + 54 = 184, x = 13 \text{ cm} \quad b \quad 11x + 12 = 287, x = 25 \text{ cm}$$

- 11 2
 12 Yes, when Tom is 45 and his dad is 90.
 13 \$80 per month

11.6 Equations with the unknown on both sides

- 1 a $x = 3$ b $y = 4$ c $n = 4$
 d $t = 4$ e $w = 5$ f $y = 3\frac{2}{3}$
 g $z = 4$ h $a = -4$ i $s = 2$
 j $k = 4$ k $w = -3$ l $v = -4$
 2 a $w = 2$ b $b = 2$ c $n = 2$
 d $s = 3$ e $a = -3$ f $m = 2$
 g $p = 4$ h $d = 3$ i $m = 0$
 j $s = 1$ k $t = -2$ l $x = 4$
 3 a $x = 3$ b $m = 10$ c $n = 12$
 d $t = 5$ e $d = 2\frac{5}{6}$ f $w = 1\frac{2}{9}$
 g $k = 10$ h $s = -3$ i $z = -17$
 j $v = 1\frac{1}{2}$ k $m = 11\frac{2}{3}$ l $d = 3\frac{7}{10}$
 m $d = -19$ n $k = -10$ o $v = 2$
 p $l = 18$
 4 $x = -\frac{13}{33}$
 5 $x = \frac{13}{5}$ or $2\frac{3}{5}$
 6 6.25 cm
 7 a $6(x + 3) = \frac{1}{2} \times 6 \times (x + 5 + 2)$
 b $x = 1$ cm
 c Parallelogram: base 4 cm, height 6 cm, Trapezium: base 6 cm, top 2 cm, height 6 cm.
 8 24
 9 3
 10 Mick is 14, his father is 42.
 11 16
 12 12
 13 a i $3x + 12$ ii $4x + 5$
 b $3x + 12 = 4x + 5$
 c $x = 7$

- 14 $2(3w - 7) + 2w = 3w + 7$
 $6w - 14 + 2w = 3w + 7$
 $5w = 21$
 $w = 4.2$
 Rectangle lengths are 4.2 cm and 5.6 cm; triangle lengths are 4.2 cm, 5.2 cm and 10.2 cm.
 15 a $2.5x + 10 = 3.5x + 5$
 $x = 5$
 Both options cost the same for 5 rides.
 b Option 1: $2.5 \times 2 + 10 = \$15$; option 2: $3.5 \times 2 + 5 = \$12$.
 Option 2 is cheaper for 2 rides.
 16 a Let daughter's age = x . Mother's age = $3(x + 8)$ and $= 4(13 - x)$.
 b $3(x + 8) = 4(13 - x)$
 $3x + 24 = 52 - 4x$
 $x = 4$
 c The daughter is 4 and the mother is 36.
 d The number of their house is 36.

Challenge 11.2

13 Smarties

Investigation — Rich task

- 1 1972
 2 $t = 59.0$ s; difference of 0.4 s
 3 $x = 56$, $t = 51.6$ s
 4 Teacher to check.
 5 $x = 32$; year is 1988
 6 $t = 50.2$ s; difference of 0.4 s
 7 $t = 46.4$ s
 8–12 Teacher to check.

Code puzzle

Filter 200 litres of fluid daily and make 2 litres of urine.

