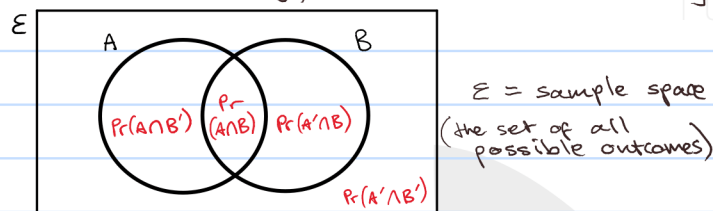


Probability Cheat Sheet

$P(A)$ = event occurring, $P(A)'$ = event NOT occurring



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For mutually exclusive events:

$$P(A \cup B) = P(A) + P(B) \quad \text{since } P(A \cap B) = 0$$

For independent events:

$$P(A \cap B) = P(A) \times P(B) \quad \text{and } P(A|B) = P(A)$$

Karnaugh probability table

	B	B'	
A	$P(A B)$	$P(A B')$	$P(A)$
A'	$P(A' B)$	$P(A' B')$	$P(A')$
	$P(B)$	$P(B')$	1

Conditional probability

$$P(A) \text{ given } B = P(A|B) = \frac{P(A \cap B)}{P(B)}$$

where $P(B) \neq 0$

Combinations

nC_r = the number of selections of n different objects taken r at a time

$$= \frac{n!}{(n-r)!r!} = \binom{n}{r}$$

Discrete probability distributions criteria:

$$\textcircled{1} \quad 0 \leq P(X=x) \leq 1$$

$$\textcircled{2} \quad \sum P(X=x) = 1$$

eg

x	outcome 1	outcome 2	outcome 3
$P(X=x)$	0.5	0.3	0.2

Expected value = average value of X

= mean of X = expectation = $E(X) = \mu$

$$E(X) = \sum_{\text{all } x} x P(X=x)$$

Expectation theorems

- $E(aX) = a E(X)$ $a, b = \text{constants}$
- $E(aX+b) = a E(X) + b$ $X, Y = \text{random variables}$
- $E(b) = b$
- $E(X+Y) = E(X) + E(Y)$
- $E(X^2) \neq [E(X)]^2$

Variance = measure of spread from the mean

$$\begin{aligned} \text{Var}(X) &= \sigma^2 = E(X - \mu)^2 \\ &= \sum (X - \mu)^2 P(X=x) \\ &= E(X^2) - [E(X)]^2 \end{aligned}$$

Variance of a linear function:

$$\text{Var}(aX+b) = a^2 \text{Var}(X)$$

Standard deviation

$$= SD(X) = \sqrt{\text{Var}(X)} = \sqrt{\sigma^2} = \sigma$$

Usually approximately 95% of the spread of the population lies between 2 standard deviations of the mean:

$$P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 0.95$$

Binomial distribution

$$X \sim B(n, p) \quad n = \text{number of trials}$$

$$P = P(\text{success})$$

$$P(X=x) = {}^nC_x P^x q^{n-x} \quad q = P(\text{failure}) = 1-P$$

$$x = 1, 2, 3, \dots, n$$

* If the probability of a sequence of events in a particular order is required then the binomial probability distribution should not be used.

Markov chains and transition matrices

$$\text{Transition matrix} = T = \begin{matrix} & \begin{matrix} X_1 & Y_1 \end{matrix} \\ \begin{matrix} X_2 \\ Y_2 \end{matrix} & \begin{bmatrix} x & y \\ 1-x & 1-y \end{bmatrix} \end{matrix}$$

$$\text{Initial system} = S_0 = \begin{matrix} X_0 & n \\ Y_0 & n_1 \end{matrix}$$

$$\text{In general } S_n = T^n \times S_0$$

$$\text{For } X \sim B(n, p)$$

$$E(X) = np \quad \text{Var}(X) = npq \quad SD(X) = \sqrt{npq}$$

* This applies only to binomial distributions

Dawood Miakhel
Senior Methods Tutor
LGT Tutoring

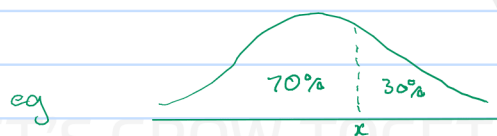
Probability density function criteria:

① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x) dx = 1$

* $\Pr(X > a) = \Pr(X \geq a)$

Quantiles and Percentiles
↓
decimal ↓
 percentage



eg $x = 70^{\text{th}}$ percentile or 0.7 quantile

Measures of spread

• Variance = $E(X^2) - [E(X)]^2$
 $= \int_a^b x^2 f(x) dx - \left[\int_a^b x f(x) dx \right]^2$

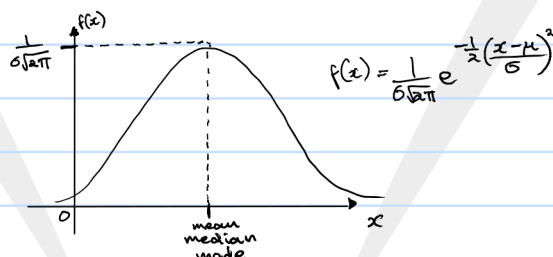
• Standard deviation = $\sqrt{\text{Var}(X)}$

• Interquartile range (IQR):

$\text{IQR} = 75^{\text{th}} \text{ percentile} - 25^{\text{th}} \text{ percentile}$
 $= Q_3 - Q_1$

The normal distribution

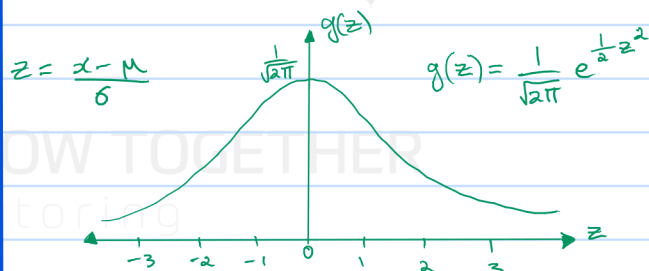
$X \sim N(\mu, \sigma^2)$ $\mu = \text{mean}$, $\sigma^2 = \text{SD}(X)$



- $\Pr(\mu - \sigma \leq X \leq \mu + \sigma) \approx 68\%$
- $\Pr(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 95\%$
- $\Pr(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx 99.7\%$

The standard normal distribution

$Z = N(0, 1^2)$ $\mu = 0$ $\text{S.D} = 1$

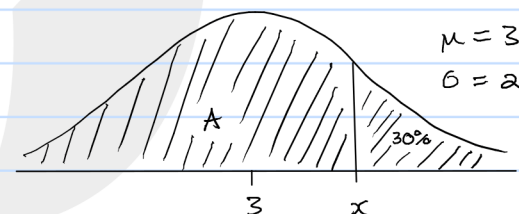


The inverse cumulative normal distribution

$\text{invNorm}(A, \mu, \sigma)$ on calculator

* $A = \text{area to the LEFT of the required value}$

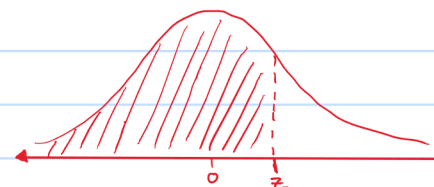
eg.



$A = 1 - 0.3 = 0.7$

$\therefore x = \text{invNorm}(0.7, 3, 2)$

Symmetry of normal curves



- $\Pr(Z > z) = 1 - \Pr(Z < z)$
- $<$ equivalent to $\leq$ and $>$ equivalent to $\geq$

Dawood Miakhel
Senior Methods Tutor
LGT Tutoring