

The Mathematical Association of Victoria

Specialist Mathematics

2005 Written Examinations

Solutions

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2005 Specialist Mathematics
Written Examination 1 (Facts, skills, and applications)
Suggested answers and solutions

Part 1 (Multiple-choice) Answers

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. E | 2. D | 3. C | 4. D | 5. E |
| 6. D | 7. A | 8. C | 9. B | 10. B |
| 11. A | 12. C | 13. A | 14. A | 15. D |
| 16. E | 17. D | 18. A | 19. B | 20. C |
| 21. B | 22. B | 23. C | 24. E | 25. A |
| 26. C | 27. E | 28. A | 29. C | 30. B |

Part 1 (Multiple-choice) Solutions**Question 1****[E]**

$$\frac{3(x+3)^2}{5} + \frac{(y-4)^2}{6} = 3$$

$$\frac{(x+3)^2}{5} + \frac{(y-4)^2}{18} = 1$$

Maximum y-value occurs when $x + 3 = 0$

$$(y - 4)^2 = 18$$

$$y - 4 = 3\sqrt{2}$$

$$y = 4 + 3\sqrt{2}$$

Question 2**[D]**

For there to be no vertical asymptote

$$x^2 + mx - n \neq 0$$

This means that $b^2 - 4ac < 0$, where $a = 1$,
 $b = m$ and $c = -n$

$$\therefore m^2 + 4n < 0$$

$$\Rightarrow m^2 < -4n$$

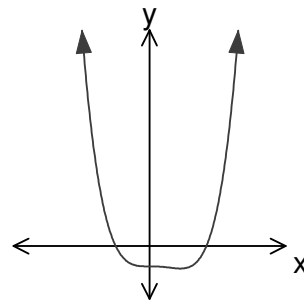
Question 3**[C]**

$$x^4 - x^3 = \operatorname{cosec}^2 x - \cot^2 x$$

$$x^4 - x^3 = 1$$

$$x^4 - x^3 - 1 = 0$$

Use a graphics calculator to sketch the graph.



We have two points of intersection

Question 4**[A]**The graph has the same shape as $y = \cot(x)$

$$y = \cot(x)$$

Period length is $\frac{\pi}{2}$

$$y = -\cot(2x)$$

Phase shift is $\frac{\pi}{12}$

$$y = -\cot\left(2\left(x - \frac{\pi}{12}\right)\right)$$

$$= -\cot\left(2x - \frac{\pi}{6}\right)$$

Question 5**[E]**

$$y = \tan^{-1}(\sqrt{3x})$$

$$\text{Let } u = \sqrt{3x} \text{ and } y = \tan^{-1}(u)$$

$$\frac{du}{dx} = \frac{\sqrt{3}}{2\sqrt{x}} \text{ and } \frac{dy}{du} = \frac{1}{1+u^2}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{1+3x} \times \frac{\sqrt{3}}{2\sqrt{x}} \\ &= \frac{\sqrt{3}}{2\sqrt{x}(1+3x)} \end{aligned}$$

Question 6**[D]**

$$\begin{aligned} z &= \frac{3-6i}{2+i} \times \frac{2-i}{2-i} \\ &= -\frac{15i}{5} \\ &= -3i \end{aligned}$$

$$|z| = 3 \text{ and } \text{Arg}(z) = -\frac{\pi}{2}$$

Question 7**[A]**

$$u = 7\text{cis}\left(\frac{\pi}{4}\right) \text{ and } v = a\text{cis}(b)$$

$$uv = 7a\text{cis}\left(\frac{\pi}{4} + b\right)$$

$$7a = 42 \text{ and } \frac{\pi}{4} + b = \frac{\pi}{20}$$

$$a = 6 \text{ and } b = \frac{\pi}{20} - \frac{5\pi}{20} = -\frac{\pi}{5}$$

Question 8**[C]**

$$\Delta = b^2 - 4ac$$

$$a = 1+i, b = 4i \text{ and } c = -2(1-i)$$

$$\begin{aligned} \Delta &= 16i^2 + 8(1-i)(1+i) \\ &= -16 + 16 \\ &= 0 \end{aligned}$$

Question 9**[B]**

$$u = \sqrt{2} \text{cis}\left(\frac{\pi}{16}\right)$$

$$z = u^4$$

$$z = (\sqrt{2})^4 \text{cis}\left(\frac{4\pi}{16}\right)$$

$$= 4\text{cis}\left(\frac{\pi}{4}\right)$$

$$z^{-1} = \frac{1}{4} \text{cis}\left(-\frac{\pi}{4}\right)$$

Question 10**[B]**

$$|z-1| + |z+1| = 3$$

This locus is an ellipse with foci at (1,0) and (-1,0). This sum of the distances from these points is 3.

Question 11**[A]**

$$\begin{aligned} &\int \frac{6}{\sqrt{1-4x^2}} dx \\ &= \int \frac{6}{2\sqrt{\frac{1}{4}-x^2}} dx \\ &= \int \frac{3}{\sqrt{\frac{1}{4}-x^2}} dx \\ &= 3\sin^{-1}(2x) \end{aligned}$$

Question 12**[C]**

$$\int_{\frac{\pi}{2}}^{\pi} \sin^2(2x) \sin(2x) dx$$

$$= \int_{\frac{\pi}{2}}^{\pi} (1 - \cos^2(2x)) \sin(2x) dx$$

$$\text{Let } u = \cos(2x)$$

$$\frac{du}{dx} = -2\sin(2x)$$

$$dx = \frac{du}{-2\sin(2x)}$$

For the terminals

$$u = \cos(2x)$$

$$\text{When } x = \pi, \cos(2\pi) = 1$$

$$\text{When } x = \frac{\pi}{2}, \cos(\pi) = -1$$

Substituting appropriate values we gain

$$\int_{-1}^1 (1 - u^2) \sin(2x) \times \frac{du}{-2\sin(2x)}$$

$$= -\frac{1}{2} \int_{-1}^1 (1 - u^2) du$$

Question 13**[A]**

$$V = \pi \int y^2 dx$$

$$\text{For } y = \frac{5}{x^2 + 1}$$

$$V_1 = \pi \int_0^2 \left(\frac{5}{x^2 + 1} \right)^2 dx$$

$$\text{For } y = 1$$

$$V_2 = \pi \int_0^2 1^2 dx$$

$$V = V_1 - V_2$$

$$= \pi \int_0^2 \left(\frac{5}{x^2 + 1} \right)^2 - 1 dx$$

Question 14**[A]**

Using a graphics calculator for example

fnInt((x+3)/(2sin(x)),x,4,5) we gain the result - 4.014 correct to three decimals places.**Question 15****[D]**

$$\int x\sqrt{3-x} dx$$

$$\text{Let } u = 3 - x$$

$$\Rightarrow x = 3 - u$$

$$\frac{du}{dx} = -1$$

$$dx = -du$$

Substituting appropriate values into original equation

$$\int x\sqrt{3-x} dx$$

$$= \int (3 - u)\sqrt{u} - du$$

$$= - \int 3\sqrt{u} - u^{\frac{3}{2}} du$$

$$= - \left[2u^{\frac{3}{2}} - \frac{2}{5} u^{\frac{5}{2}} \right]$$

$$= -2u^{\frac{3}{2}} + \frac{2}{5} u^{\frac{5}{2}}$$

Substituting $u = 3 - x$

$$-2(3-x)^{\frac{3}{2}} + \frac{2}{5} (3-x)^{\frac{5}{2}}$$

Question 16**[E]**

$$\int_0^{\frac{\pi}{8}} \sec^2(2x) e^{2\tan(2x)}$$

$$\text{Let } u = 2\tan(2x)$$

$$\frac{du}{dx} = 4\sec^2(2x)$$

$$dx = \frac{du}{4\sec^2(2x)}$$

Evaluating terminals

$$u_1 = 2\tan\left(\frac{\pi}{4}\right) = 2$$

$$u_2 = 2\tan(0) = 0$$

Substituting into the original equation to gain

$$\begin{aligned} & \int_0^2 \sec^2(2x) e^u \times \frac{du}{4\sec^2(2x)} \\ &= \frac{1}{4} \int_0^2 e^u du \\ &= \frac{1}{4} [e^u]_0^2 \\ &= \frac{1}{4} (e^2 - e^0) \\ &= \frac{1}{4} (e^2 - 1) \end{aligned}$$

Question 17**[D]**

$$\frac{dy}{dx} = f(x) = e^{-x}$$

$$x_0 = 2, y_0 = 1, \text{ and } h = 0.1$$

$$x_1 = 2.1 \quad y_1 = 1 + 0.1e^{-2} = 1.01353$$

$$x_2 = 2.2 \quad y_2 = 1.01353 + 0.1e^{-2.1} = 1.02578$$

$$y_2 \approx 1.0258$$

Question 18**[A]**

$$\frac{dS}{dt} = 10$$

$$S = \pi r^2$$

$$\frac{dS}{dt} = 2\pi r$$

$$\begin{aligned} \frac{dr}{dt} &= \frac{dr}{ds} \times \frac{ds}{dt} \\ &= \frac{1}{2\pi r} \times 10 \\ &= \frac{5}{\pi r} \end{aligned}$$

Question 19**[B]**

$$\frac{dy}{dx} = y^2 + 1$$

$$\frac{dx}{dy} = \frac{1}{y^2 + 1}$$

$$x = \tan^{-1} y + c$$

$$\text{At } y = 1, x = 0$$

$$0 = \tan^{-1} 1 + c$$

$$0 = \frac{\pi}{4} + c$$

$$c = -\frac{\pi}{4}$$

$$x = \tan^{-1}(y) - \frac{\pi}{4}$$

$$x + \frac{\pi}{4} = \tan^{-1}(y)$$

$$y = \tan\left(x + \frac{\pi}{4}\right)$$

Question 20**[C]**

$$a = v \frac{dv}{dx}$$

$$\frac{dv}{dx} = -\frac{1}{(1-x^2)^{\frac{3}{2}}} \times -\frac{2x}{1} = \frac{2x}{(1-x^2)^{\frac{3}{2}}}$$

$$a = \frac{2}{(1-x^2)^{\frac{1}{2}}} \times \frac{2x}{(1-x^2)^{\frac{3}{2}}} \\ = \frac{4x}{(1-x^2)^2}$$

Alternatively

$$a = \frac{d\left(\frac{1}{2}v^2\right)}{dx}$$

$$v = \frac{2}{\sqrt{1-x^2}}$$

$$v^2 = \frac{4}{1-x^2}$$

$$\frac{v^2}{2} = \frac{2}{1-x^2}$$

$$\frac{d\left(\frac{1}{2}v^2\right)}{dx} = \frac{4x}{(1-x^2)^2}$$

Question 21**[B]**

$$\frac{dv}{dt} = \frac{3}{v^2 - 9}$$

$$dt = \frac{v^2 - 9}{3} dv$$

$$t = \int_2^1 \frac{v^2 - 9}{3} dv$$

The terminals are reversed because the curve is below the x -axis for the interval $[1, 2]$

Question 22**[B]**For $PQRS$ to be a parallelogram

$$\overrightarrow{PQ} = \overrightarrow{SR}$$

$$\overrightarrow{PQ} = \mathbf{q} - \mathbf{p} \\ = \mathbf{i} + y\mathbf{j} - 3\mathbf{k} \\ = \mathbf{i} + y\mathbf{j} + 3\mathbf{k}$$

$$\overrightarrow{SR} = \mathbf{r} - \mathbf{s} \\ = 5\mathbf{i} + 2x\mathbf{j} + \mathbf{k} - y\mathbf{j} + 2\mathbf{k} \\ = (5-y)\mathbf{i} + 2x\mathbf{j} + 3\mathbf{k}$$

$$5 - y = 1$$

$$\Rightarrow y = 4$$

$$2x = y$$

$$\Rightarrow x = 2$$

Question 23**[C]**

Let the point $(2, 2, -1)$ be represented by the vector $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and the point $(-4, 0, -3)$ be represented by the vector $\mathbf{b} = -4\mathbf{i} - 3\mathbf{k}$

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

$$\mathbf{a} \times \mathbf{b} = -8 + 0 + 3$$

$$|\mathbf{a}| = \sqrt{4 + 4 + 1} = 3$$

$$|\mathbf{b}| = \sqrt{16 + 9} = 5$$

$$\cos\theta = \frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

$$= \frac{-5}{3 \times 5}$$

$$= -\frac{1}{3}$$

Question 24

[E]

$(x + 1)^2 + y^2 + 1$ is a circle with centre $(-1, 0)$ and radius 1

$$\text{Let } x + 1 = \cos t$$

$$\Rightarrow x = \cos t - 1$$

This possibility is not evident in the answers.

$$\text{Let } x + 1 = \sin t$$

$$\Rightarrow x = \sin t - 1$$

$$\text{and } y = \cos t$$

$$\underline{r}(t) = (\sin(t) - 1)\underline{i} + \cos(t)\underline{j}$$

Question 25

[A]

$$\underline{r}(t) = (3t^2 - 2)\underline{i} - (7 - 5t)\underline{j} - 4\underline{k}$$

$$\dot{\underline{r}}(t) = 6t\underline{i} - 5\underline{j}$$

$$= 6t\underline{i} + 5\underline{j}$$

Question 26

[C]

$$\underline{a} = \frac{d\underline{v}}{dt} = e^{-0.1t}\underline{i} + (6t)\underline{j}$$

$$\underline{v} = 10e^{-0.1t}\underline{i} + 3t^2\underline{j} + \underline{c}$$

$$\text{At } t = 0, \underline{v} = 0$$

$$0 = -10\underline{i} + \underline{c}$$

$$\underline{c} = 10\underline{i}$$

$$\begin{aligned}\underline{v} &= -10e^{-0.1t}\underline{i} + 3t^2\underline{j} + 10\underline{i} \\ &= 10(1 - e^{-0.1t})\underline{i} + 3t^2\underline{j}\end{aligned}$$

Question 27

[E]

$$F = ma$$

$$= 5(20 - 10\cos(2t))$$

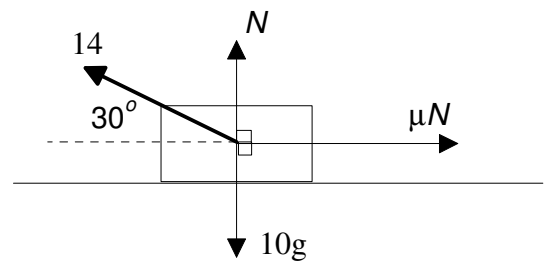
$$\text{Max value occurs when } \cos(2t) = -1$$

$$F = 5(20 + 10)$$

$$= 150$$

Question 28

[A]



$$7\sqrt{3} = \mu N$$

$$\mu = \frac{7\sqrt{3}}{N}$$

$$7 + N = 10g$$

$$N = 10g - 7$$

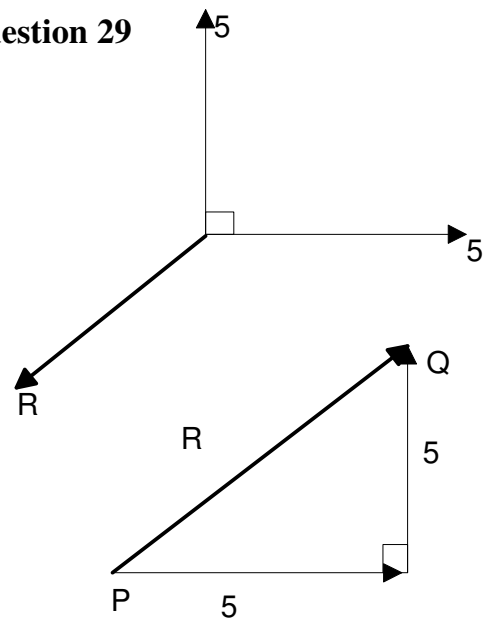
$$= 98 - 7 = 91$$

$$\mu = \frac{7\sqrt{3}}{91}$$

$$= \frac{\sqrt{3}}{13}$$

Question 29

[C]



$$\underline{Q} + \underline{P} = 5\sqrt{2} \text{ North East}$$

$\underline{R}$ must act with equal and opposite force, which is $5\sqrt{2}$ Southwest

Question 30**[B]**

$$F_r = 200a = 1000 - 200g$$

$$a = \frac{1000}{200} - \frac{200g}{200}$$

$$= 5 - 9.8$$

$$= -4.8$$

Part II (Short-answer) Solutions**Question 1****a**Equation of line: $y - y_1 = m(x - x_1)$

$$m = \frac{2}{3}$$

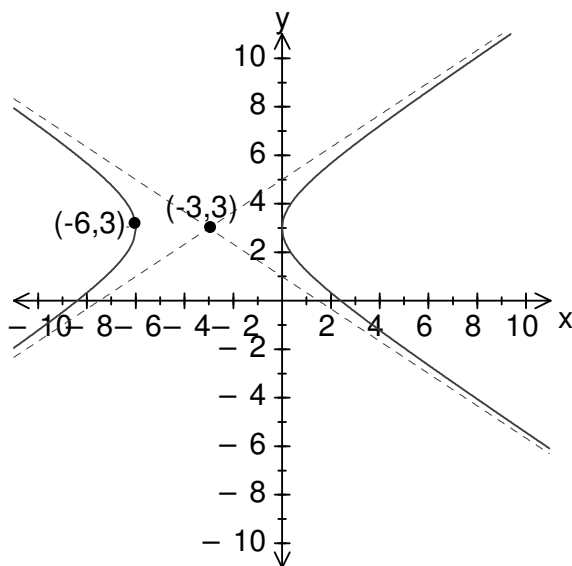
Point on straight line: $(c, 3)$

$$y - 3 = \frac{2}{3}(x - c)$$

$$y = \frac{2}{3}x - \frac{2}{3}c + 3$$

$$-\frac{2}{3}c + 3 = 5$$

$$c = -3$$

b**Question 2**

$$y = e^{2x} \cos(x)$$

$$\frac{dy}{dx} = 2e^{2x} \cos(x) - e^{2x} \sin(x)$$

$$= e^{2x} (2\cos(x) - \sin(x))$$

$$\frac{d^2y}{dx^2} = 2e^{2x} (2\cos(x) - \sin(x)) + e^{2x} (-2\sin(x) - \cos(x))$$

$$= e^{2x} (4\cos(x) - 2\sin(x) - 2\sin(x) - \cos(x))$$

$$= e^{2x} (3\cos(x) - 4\sin(x))$$

$$\frac{d^2y}{dx^2} + k \frac{dy}{dx} + y = -2e^{2x} \sin(x)$$

$$LHS = e^{2x} (3\cos(x) - 4\sin(x)) + ke^{2x} (2\cos(x) - \sin(x))$$

$$+ e^{2x} \cos(x)$$

$$= e^{2x} (4\cos(x) + 2k\cos(x) - 4\sin(x) - k\sin(x))$$

$$\cos(x): 3 + 2k + 1 = 0$$

$$k = -2$$

$$\sin(x): -4 - k = -2$$

$$k = -2$$

Question 3**a**Let $\underline{a} = 3\underline{i} - 2\underline{j} + \underline{k}$ and $\underline{b} = 2\underline{i} + x\underline{j} + 2\underline{k}$

We know that the two vectors are perpendicular.

$$\underline{a} \cdot \underline{b} = 6 - 2x + 2 = 0$$

$$2x = 8$$

$$x = 4$$

b

$$\underline{u} = \underline{a} + \underline{b}$$

$$= 3\underline{i} - 2\underline{j} + \underline{k} + 2\underline{i} + 4\underline{j} + 2\underline{k}$$

$$= 5\underline{i} + 2\underline{j} + 3\underline{k}$$

Question 4**a**

$$v = (t - 4)\tan\left(\frac{\pi t}{48}\right)$$

$$\text{at } t = 12$$

$$v = (12 - 4)\tan\left(\frac{12\pi}{48}\right)$$

$$= 8\tan\left(\frac{\pi}{4}\right)$$

$$= 8$$

b

Cyclist B passes Cyclist A when they are the same distance from the starting point.

$$\text{At } t = 12$$

$$\text{Cyclist A: Dist} = 72 \text{ m}$$

Cyclist B:

$$\text{Dist} = \int_4^{12} (t - 4)\tan\left(\frac{\pi t}{48}\right) dt = 22.8922$$

$$\text{Cyclist B remains } 72 - 22.89222 = 49.1078$$

Cyclist B is traveling 2 m/s faster than Cyclist A

He will now catch Cyclist A in 24.55389 seconds

Therefore it takes him $12 + 24.55389$ seconds to catch Cyclist A, which is 36.6 seconds to the nearest tenth of a second.

Question 5

$$135 - \mu N = 0.5(12 + 18)$$

$$N = 30g$$

$$135 - \mu 30g = 15$$

$$120 = \mu 30g$$

$$\mu = \frac{120}{30g}$$

$$= 0.4081$$

$$\approx 0.41$$

b

$$T - 0.41 \times 18g = 0.5 \times 18$$

$$T = 9 + 0.41 \times 18g$$

$$= 81.324$$

2005 Specialist Mathematics
Written Examination 2 (Analysis task)
Suggest answers and solutions

Question 1

a Concentration = $\frac{\text{Mass}}{\text{Volume}}$

Mass = x

Volume = $20t + 10 - 10t$

= $10t + 10$

Concentration = $\frac{x}{10t + 10}$

b Rate of Increase = Inflow – Outflow

Inflow = $\frac{20 \times 2}{1 + t^2}$

= $\frac{40}{1 + t^2}$

Outflow = $\frac{10x}{10 + 10t}$

= $\frac{x}{1 + t}$

$\frac{dx}{dt} = \frac{40}{1 + t^2} - \frac{x}{1 + t}$

$\Rightarrow \frac{dx}{dt} + \frac{x}{1 + t} = \frac{40}{1 + t^2}$

ci

$x = \frac{40}{1 + t} \tan^{-1}(x) + \frac{20}{1 + t} \log(1 + t^2)$

$\frac{dx}{dt} = \frac{40}{(t^2 + 1)(1 + t)} - \frac{40 \tan^{-1} t}{(1 + t)^2}$

+ $\frac{20 \times 2t}{(t^2 + 1)(1 + t)} - \frac{20 \log_e(1 + t^2)}{1 + t^2}$

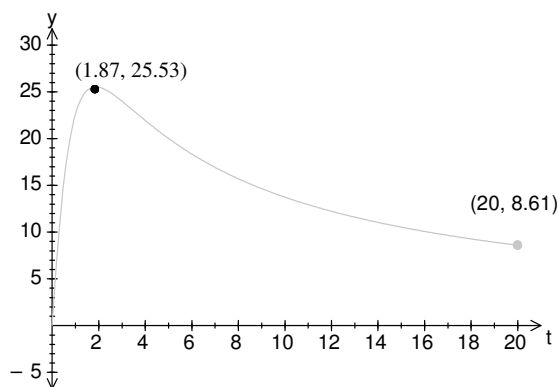
$$\begin{aligned} \frac{dx}{dt} &= \frac{40t + 40}{(t^2 + 1)(1 + t)} \\ &\quad - \frac{40 \tan^{-1} t}{(1 + t)^2} - \frac{20 \log_e(1 + t^2)}{1 + t^2} \\ &= \frac{40(1 + t)}{(t^2 + 1)(1 + t)} - \frac{40 \tan^{-1} t}{(1 + t)^2} - \frac{20 \log_e(1 + t^2)}{1 + t^2} \\ &= \frac{40}{1 + t^2} - \frac{40 \tan^{-1}(t)}{(1 + t)^2} - \frac{20 \log_e(1 + t^2)}{(1 + t)^2} \end{aligned}$$

cii

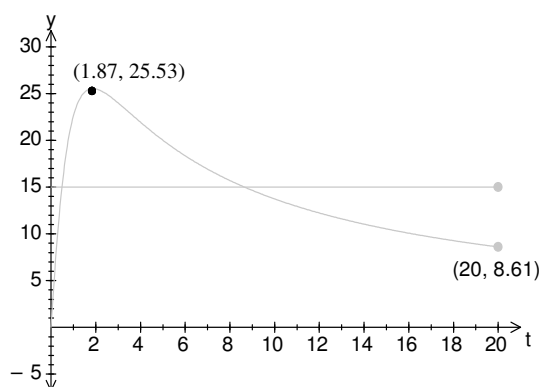
$$\begin{aligned} \frac{dx}{dt} &= \frac{40}{1 + t^2} - \frac{40 \tan^{-1}(t)}{(1 + t)^2} - \frac{20 \log_e(1 + t^2)}{(1 + t)^2} \\ &= \frac{40}{1 + t^2} - \frac{40 \tan^{-1}(t)}{(1 + t)^2} - \frac{20 \log_e(1 + t^2)}{(1 + t)^2} \\ &= \frac{40}{1 + t^2} - \frac{1}{1 + t} \left(\frac{40 \tan^{-1}(t)}{(1 + t)^2} - \frac{20 \log_e(1 + t^2)}{(1 + t)^2} \right) \\ &= \frac{40}{1 + t^2} - \frac{1}{1 + t} (x) \end{aligned}$$

$\Rightarrow \frac{dx}{dt} + \frac{x}{1 + t} = \frac{40}{1 + t^2}$

d



ei



Find the point of intersection using a graphics calculator
 $t = 0.485$

eii Second Point of Intersection

$$t = 8.655$$

Chemical remains effective

$$8.655 - 0.485 \approx 8.17 \quad 8.17 \text{ (2dp)}$$

Question 2

ai $u = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$$|u| = \frac{1}{4} + \frac{3}{4} = 1$$

$$\arg(u) = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = \tan^{-1}(\sqrt{3})$$

$$= \frac{\pi}{3}$$

$$u = \text{cis}\left(\frac{\pi}{3}\right)$$

aii

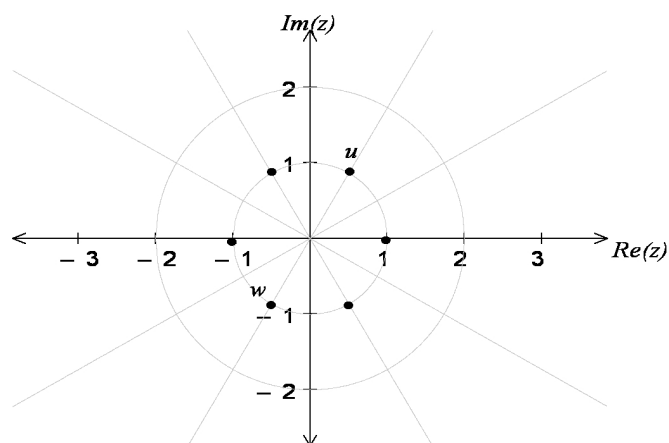
$$\begin{aligned} u^6 &= 1^6 \text{cis}\left(\frac{6\pi}{3}\right) \\ &= \text{cis}(2\pi) \\ &= 1 \end{aligned}$$

aiii

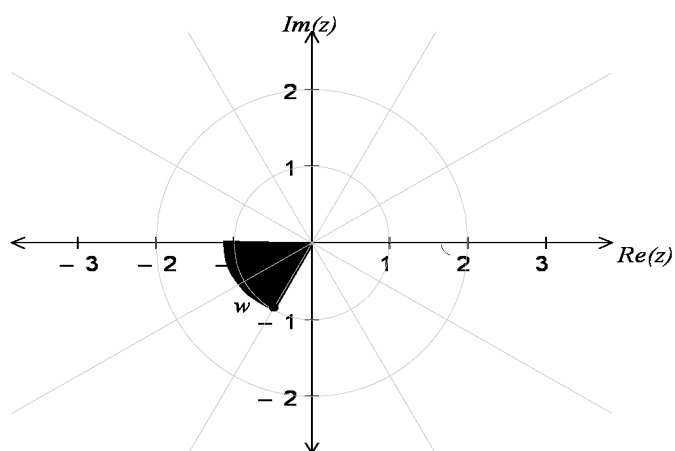
$$z^6 - 1 = 0$$

$$z^6 = \text{cis}(2\pi)$$

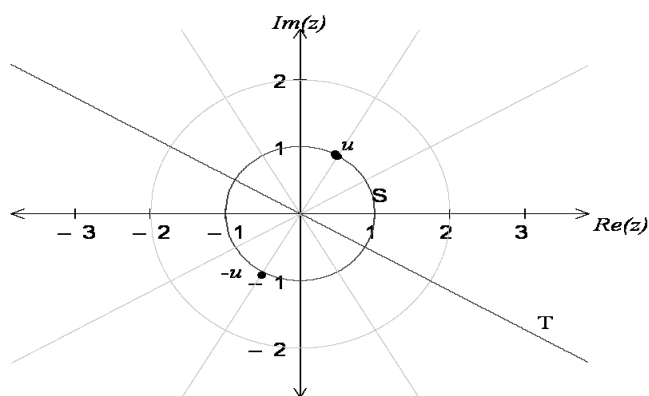
$$z = \text{cis}\left(\frac{2n\pi}{6}\right)$$



b



ci & cii



ciii

$$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right) \text{ and } \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

Question 3**a**

$$A \approx 0.25(2 \times 1.5 + 2 \times 1.25 + 2 \times 0.85 + 0.55) \\ = 1.9875$$

b

$$\frac{10x}{(x^2 + 1)(3x + 1)} = \frac{x + A}{x^2 + 1} + \frac{B}{3x + 1} \\ = \frac{(x + A)(3x + 1) + B(x^2 + 1)}{(x^2 + 1)(3x + 1)}$$

$$10x \equiv (x + A)(3x + 1) + B(x^2 + 1)$$

$$\text{Let } x = -\frac{1}{3}$$

$$-\frac{10}{3} = \frac{10B}{9}$$

$$B = -3$$

$$\text{Let } x = 0$$

$$0 = A + B$$

$$\Rightarrow A = 3$$

$$\therefore A = 3 \text{ and } B = -3$$

c

$$\int_0^2 \frac{x+3}{x^2+1} - \frac{3}{3x+1} dx \\ = \int_0^2 \frac{x}{x^2+1} + \frac{3}{x^2+1} - \frac{3}{3x+1} dx \\ = \left[\frac{1}{2} \log_e(x^2+1) + 3 \tan^{-1}(x) - \log_e(3x+1) \right]_0^2 \\ = \left(\frac{1}{2} \log_e(5) + 3 \tan^{-1}(2) - \log_e(7) \right) - (0 + 0 + 0) \\ \approx 2.18$$

d

$$\text{Using } h(x) = \frac{10x}{(x^2+1)(3x+1)}$$

$$\text{At } x = 2$$

$$h(2) = \frac{10 \times 2}{(2^2+1)(3 \times 2+1)} = \frac{20}{35}$$

$$\frac{10x}{(x^2+1)(3x+1)} = \frac{4}{7}$$

$$x \approx .0694 \text{ and } x = 2$$

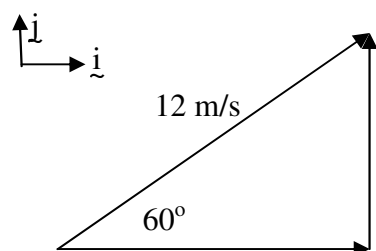
$$\text{Length of Usable Panel} = 2 - 0.0694 \\ = 1.9306$$

$$\text{Number of Panel Required} = \frac{100}{1.9306} = 51.79$$

Can't purchase part panel, therefore
52 required.

Question 4

a



$$\underline{v}_0 = 12\cos(60^\circ)\hat{i} + 12\sin(60^\circ)\hat{j}$$

$$= 12 \times \frac{1}{2}\hat{i} + 12 \times \frac{\sqrt{3}}{2}\hat{j}$$

$$= 6\hat{i} + 6\sqrt{3}\hat{j}$$

b

$$\ddot{\underline{r}}(t) = -0.1t\hat{i} - (g - 0.1t)\hat{j}$$

$$\dot{\underline{r}}(t) = -\frac{t^2}{20}\hat{i} - \left(gt - \frac{t^2}{20}\right)\hat{j} + \underline{c}$$

$$\dot{\underline{r}}(0) = 6\hat{i} + 6\sqrt{3}\hat{j} = \underline{c}$$

$$\dot{\underline{r}}(t) = \left(6 - \frac{t^2}{20}\right)\hat{i} - \left(6\sqrt{3} + gt - \frac{t^2}{20}\right)\hat{j}$$

$$\underline{r}(t) = \left(6t - \frac{t^3}{60}\right)\hat{i} + \left(6t\sqrt{3} - \frac{gt^2}{2} + \frac{t^3}{50}\right)\hat{j} + \underline{c}$$

$$\underline{r}(0) = 0 = \underline{c}$$

$$\underline{r}(t) = \left(6t - \frac{t^3}{60}\right)\hat{i} + \left(6t\sqrt{3} - \frac{gt^2}{2} + \frac{t^3}{50}\right)\hat{j}$$

c

To find T , we need to find

$$6t - \frac{t^3}{60} = -\left(6t\sqrt{3} - \frac{gt^2}{2} + \frac{t^3}{50}\right)$$

$$= -6t\sqrt{3} + \frac{gt^2}{2} - \frac{t^3}{50}$$

$$6t + 6t\sqrt{3} - \frac{1}{2}gt^2 = 0$$

$$t\left(6 + 6\sqrt{3} - \frac{1}{2}gt\right) = 0$$

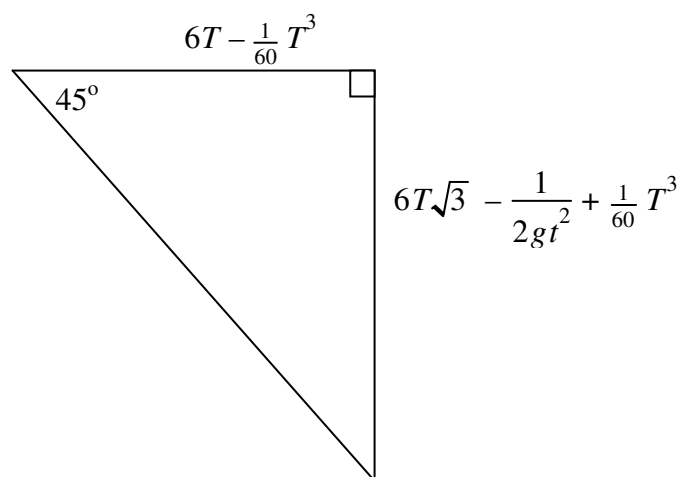
$$t = 0$$

and

$$6 + 6\sqrt{3} = \frac{gt}{2}$$

$$T = t = \frac{12(1 + \sqrt{3})}{g}$$

An alternative method



$$\tan(-45^\circ) = \frac{6T\sqrt{3} - \frac{1}{2}gT^2 + \frac{1}{60}T^3}{6T - \frac{1}{60}T^3}$$

$$-1 = \frac{6T\sqrt{3} - \frac{1}{2}gT^2 + \frac{1}{60}T^3}{6T - \frac{1}{60}T^3}$$

$$-6T + \frac{1}{60}T^3 = 6T\sqrt{3} - \frac{1}{2}gT^2 + \frac{1}{60}T^3$$

$$0 = 6T\sqrt{3} + 6T - \frac{1}{2}gT^2$$

$$= 12T\sqrt{3} + 12T - gT^2$$

$$= T(12\sqrt{3} + 12 - gT)$$

$$T = 0 \text{ or } T = \frac{12(\sqrt{3} + 1)}{g}$$

d

$$\dot{\mathbf{r}}(t) = \left(6 - \frac{t^2}{20}\right)\mathbf{j} - \left(6\sqrt{3} + gt - \frac{t^2}{20}\right)\mathbf{i}$$

$$\text{at } t = \frac{12(1 + \sqrt{3})}{g}$$

$$\dot{\mathbf{r}}(t) \approx 5.44\mathbf{j} - 42.62\mathbf{i}$$

$$|\dot{\mathbf{r}}(t)| = \sqrt{5.44^2 + 42.62^2}$$

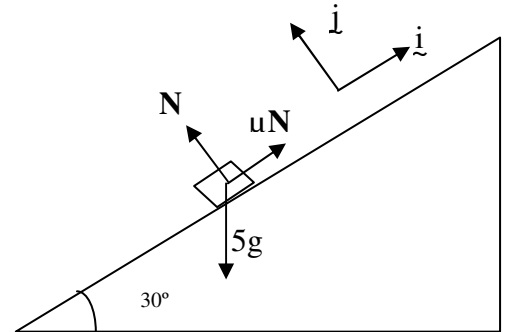
$$= 42.9658$$

$$\approx 43.0$$

Question 5

a

$$\begin{aligned}\mu N &= 5g \sin(30^\circ) \\ &= 2.5g\end{aligned}$$



b

$$\mu N = 5g \sin(30^\circ) + 0.5 \times 8$$

$$\mu = \frac{5g \sin(30^\circ) + 0.5 \times 8}{N}$$

$$= \frac{5g \sin(30^\circ) + 0.5 \times 8}{5g \cos(30^\circ)}$$

$$\approx 0.67$$

c

$$T - mg \sin(30^\circ) - \mu mg \cos(30^\circ) = 0.5m$$

$$T = mg \sin(30^\circ) + \mu mg \cos(30^\circ) + 0.5m$$

$$T = m(g \sin(30^\circ) + \mu g \cos(30^\circ) + 0.5)$$

$$m = \frac{T}{g \sin(30^\circ) + \mu g \cos(30^\circ) + 0.5}$$

$$= \frac{160}{g \sin(30^\circ) + \mu g \cos(30^\circ) + 0.5}$$

$$= \frac{160}{4.9 + 5.7 + 0.5}$$

$$\approx 14.4$$