



The Mathematical Association of Victoria

Further Mathematics

2005 Written Examinations

Solutions

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2005 Further Mathematics
Written Examination 1 (Facts, skills and applications)
Suggested answers and solutions

SECTION A (Multiple-Choice)**Answers****Core**

1. **B** 2. **E** 3. **D** 4. **C** 5. **A**
 6. **C** 7. **D** 8. **C** 9. **B** 10. **B**
 11. **A** 12. **B** 13. **E**

SECTION B (Multiple-Choice)**Answers****Module 1 Number patterns and applications**

1. **C** 2. **D** 3. **D** 4. **C** 5. **E**
 6. **B** 7. **A** 8. **C** 9. **B**

Module 2 Geometry and trigonometry

1. **C** 2. **C** 3. **E** 4. **B** 5. **E**
 6. **A** 7. **B** 8. **B** 9. **A**

Module 3 Graphs and relations

1. **A** 2. **E** 3. **C** 4. **C** 5. **D**
 6. **E** 7. **A** 8. **E** 9. **A**

Module 4 Business-related mathematics

1. **D** 2. **B** 3. **B** 4. **A** 5. **B**
 6. **C** 7. **A** 8. **D** 9. **E**

Module 5 Networks and decision mathematics

1. **C** 2. **D** 3. **B** 4. **B** 5. **A**
 6. **C** 7. **D** 8. **C** 9. **B**

Core**Question 1****[B]**

Input data into 2 lists.

Speed into L_1 and Age into L_2

L_1	L_2	L_3	1
71.8	27		
68.3	38		
65.1	22		
63.2	64		
62.8	57		
62.6	37		
62.5	21		

 $L_1 = \{71.8, 68.3, 65.1, 63.2, 62.8, 62.6, 62.5\}$

STAT CALC

1-Var Stats L_1

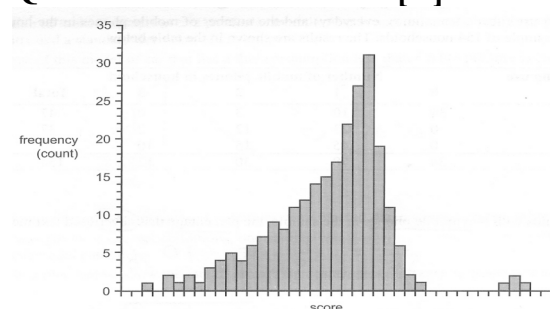
1-Var Stats

 $n=10$ $\min X = 59.8$ $Q_1 = 61.3$ $\text{Med} = 62.7$ $Q_3 = 65.1$ $\max X = 71.8$

The median speed (in km/h) of the ten cars is 62.7.

Question 2**[E]**

Looking down the Age column (L_2) on the calculator, there are 3 drivers (22, 21 and 19) under 25 years of age. Therefore 70% are over the age of 25 years.

Question 3**[D]**

The histogram is best described as negatively skewed with outliers.

Question 4 [C]

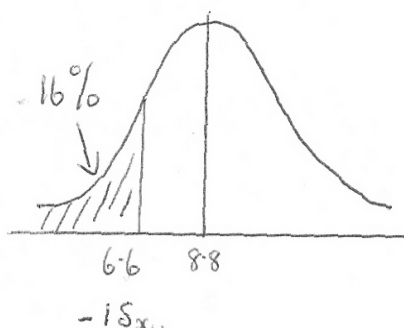
30 households have two mobile phones and 3 of those never use text messaging.

$$\text{So } \frac{3}{30} = 10\%$$

Question 5 [A]

$$\bar{x} = \frac{(0 \times 34) + (78 \times 1) + (30 \times 2) + (3 \times 12)}{154}$$

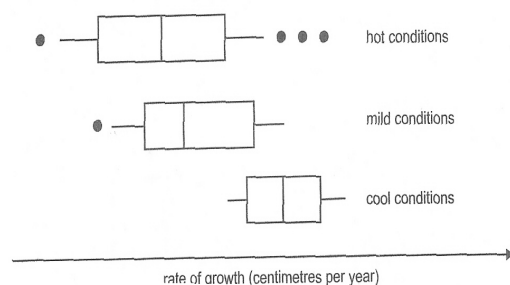
$$\bar{x} = 1.13$$

Question 6 [C]


$$S_x = 2.2$$

$$\bar{x} - 1.5S_x = 8.8 - 2.2 = 6.6$$

16% of this model of car has fuel consumption less than 6.6 km per litre.

Question 7 [D]


From hot to mild to cool, the rate of growth for these trees increases on average and becomes less variable.

Question 8 [C]

Input data into 2 lists.

Length (Independent) L_1

Wingspan (Dependent) L_2

STAT

CALC

LinReg (a+bx) L_1, L_2

LinReg

$y=a+bx$

$a=-2.993646793$

$b=.958253448$

$r^2=.9371355395$

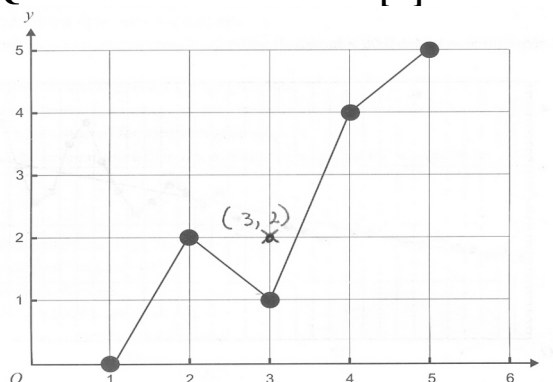
$r=.9680576117$

Pearson's product moment correlation coefficient r is 0.9681.

Question 9 [B]

$$\text{Wingspan} = 2.99 + 0.96 \times \text{length}$$

On average, for these aeroplanes, wingspan increases by 0.96 with each one metre increase in length.

Question 10 [B]


The coordinates of the median of these five points is (3, 2).

Question 11 [A]

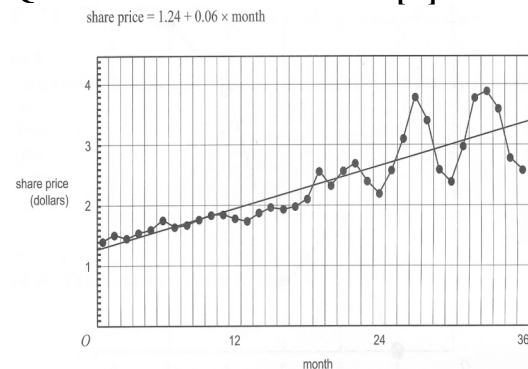
A seasonal index of 1.1 for Quarter 4 means that sales in Quarter 4 are typically 10% above the yearly Average.

Question 12 [B]

$$\begin{aligned} \text{Share Price} &= 1.24 + 0.06 \times 48 \\ &= \$4.12 \end{aligned}$$

Question 13

[E]



The share price shows an increasing linear trend with increasing variability.

Module 1 Number patterns and applications

Question 1

[C]

The sequence $-5, 1, 7, \dots$ is Arithmetic with $a = -5$ and $d = 6$.

The sequence follows as $13, 19, 25, 31, \dots$

So 19 also appears in the sequence.

Question 2

[D]

Geometric sequences have a common ratio r .

$1.1, 1.11, 1.111, 1.1111, \dots$

$$r = \frac{1.11}{1.1} \neq \frac{1.111}{1.11} \quad (\text{not geometric})$$

$3, 6, 12, 24, \dots$

$$r = \frac{6}{3} = \frac{12}{6} = \frac{24}{12} = 2 \quad (\text{geometric})$$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

$$r = \frac{1}{4} \div \frac{1}{2} = \frac{1}{8} \div \frac{1}{4} = \frac{1}{16} \div \frac{1}{8} = \frac{1}{2}$$

(geometric)

$27, 9, 3, 1, \dots$

$$r = \frac{9}{27} = \frac{3}{9} = \frac{1}{3} \quad (\text{geometric})$$

There are **3** geometric sequences.

Question 3

[D]

$$\begin{aligned} \frac{3}{5} &= \frac{1200}{x} \\ 3x &= 6000 \\ x &= \$2000 \end{aligned}$$

Question 4

[C]

The pattern is Arithmetic with

$a = 3$ and $d = 2$

$$\text{So } 3 + 5 + 7 + 9 + 11 + 13 = 48$$

The child can build a maximum of 6 towers.

Alternative

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$50 = \frac{n}{2}[2 \times 3 + (n-1)2]$$

Solve for n .

$$n = 6$$

Question 5

[E]

$a = 80$ and $r = 0.75$

The sum of an infinite sequence is

$$S_{\infty} = \frac{a}{1-r} = \frac{80}{1-0.75} = 320$$

Question 6

[B]

The difference equation described is a combination type.

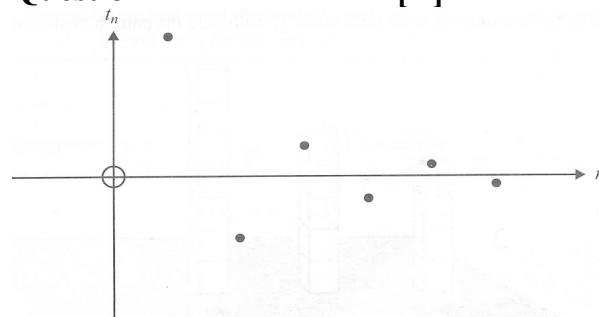
$a = 3, d = -2, r = 5$

The difference equation which generates this sequence is

$$t_{n+1} = 5t_n - 2, \text{ where } t_1 = 3$$

Question 7

[A]



The sequence is **best** described as Geometric with $-1 < r < 0$

Question 8 [C]

For the difference equation

$$t_{n+1} = at_n + b, \text{ where } t_1 = 3$$

The sequence is arithmetic if

$$a = 1 \text{ and } b = 2$$

Question 9 [B]

The difference equation is

$$P_n = 2P_{n-1} - 200$$

Solve

$$1000 = 2P_5 - 200$$

$$1200 = 2P_5$$

$$\text{So } P_5 = 600$$

To find

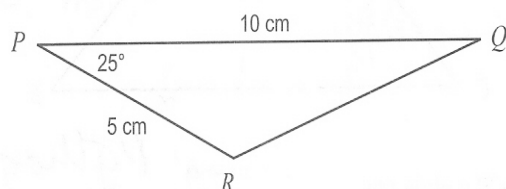
$$600 = 2P_4 - 200$$

$$800 = 2P_4$$

$$P_4 = 400$$

Module 2 Geometry and trigonometry

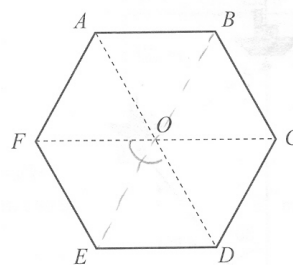
Question 1 [C]



Area of triangle PQR is

$$A = \frac{1}{2} \times 10 \times 5 \times \sin 25^\circ = 10.56 \approx 10.6 \text{ cm}$$

Question 2 [C]



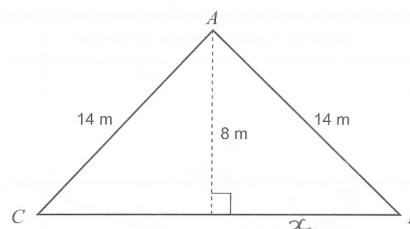
As $ABCDEF$ is a regular hexagon

$$\angle EOD = \frac{360}{6} = 60^\circ$$

$$\angle FOD = 2 \times \angle EOD$$

$$= 2 \times 60 = 120^\circ$$

Question 3 [C]



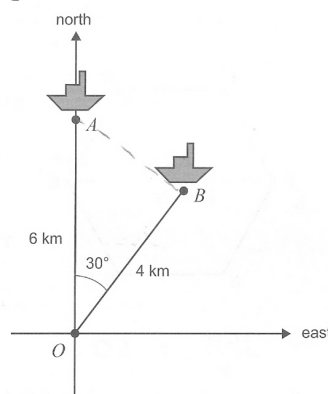
Using Pythagoras

$$x^2 = 14^2 - 8^2 = 196 - 64 = 132$$

$$\text{So } x = \sqrt{132} \approx 11.49$$

$$\text{So } CB = 2 \times 11.49 \approx 23.0 \text{ m}$$

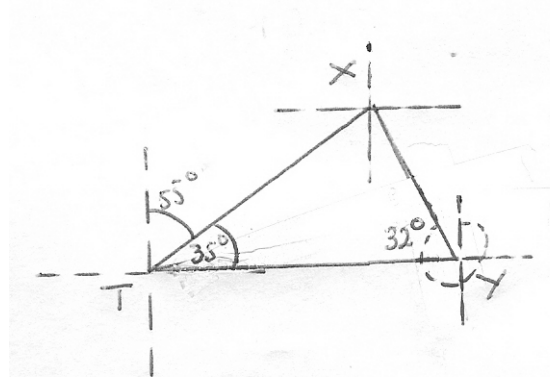
Question 4 [B]



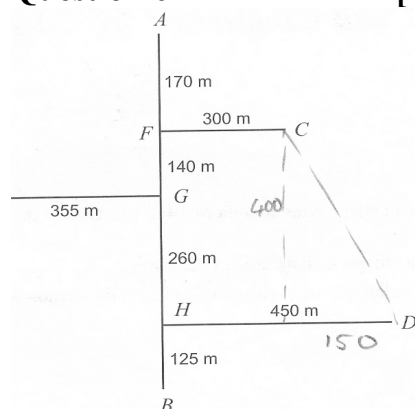
Using the Cosine Rule

$$AB^2 = 6^2 + 4^2 - 2 \times 6 \times 4 \cos 30^\circ$$

$$\text{So } AB = 3.2 \text{ km.}$$

Question 5 [E]


$$\angle TXY = 180^\circ - (35 + 32)^\circ = 113^\circ$$

Question 6 [A]


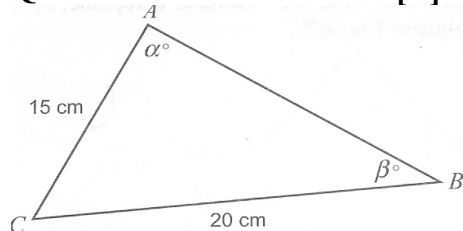
Using Pythagoras

$$CD^2 = 400^2 + 150^2$$

$$= 160000 + 22500$$

$$= 182500$$

$$CD = \sqrt{182500} \approx 427 \text{ m}$$

Question 7 [B]


Using the Sine Rule

$$\frac{15}{\sin \beta} = \frac{20}{\sin \alpha}$$

$$\sin \beta = \frac{15 \sin \alpha}{20} = \frac{15 \times 0.8}{20} = 0.6$$

Question 8 [B]

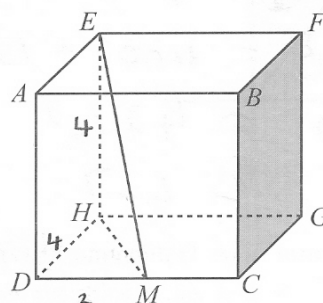
The area ratio is

$$\frac{2250}{1440} = 1.5625$$

To convert back to side length

$$\sqrt{1.5625} = 1.25$$

So the corresponding linear dimension will be $36 \times 1.25 = 45\text{m}$.

Question 9 [A]


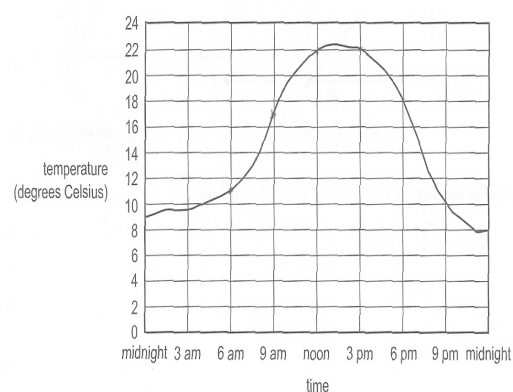
Using Pythagoras

$$HM^2 = 4^2 + 2^2 = 16 + 4 = 20$$

$$HM = \sqrt{20} \approx 4.47$$

$$\text{So } \tan \angle EMH = \frac{4}{4.47} = 0.8949$$

$$\angle EMH = \tan^{-1} 0.8949 = 41.8^\circ$$

Module 3 Graphs and relations
Question 1 [A]


The period of greatest temperature increase was from 6am-9am because it has the steepest positive gradient.

Question 2 [E]

Solve

$$-x + 5 = -5$$

$$-x = -10$$

$$x = 10$$

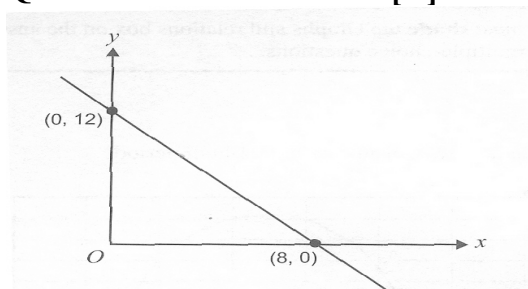
The point that lies on both lines is
(10, -5)

Question 3 [C]By transposing $3x + 5y = 0$

$$5y = -3x$$

$$y = \frac{-3x}{5}$$

The equation has a **negative** gradient.
One alternative (C) in the question said
that it had a positive gradient. This is
false.

Question 4 [C]Substituting (0, 12) into $3x + 2y = 4k$

$$3(0) + 2(12) = 4k$$

$$4k = 24, \text{ so } k = 6$$

Question 5 [D]

For n hours worked, the total charge in
dollars is $50 + 65n$.

Question 6 [E]

Let cost of ice-cream = \$a and cost of
drink = \$b

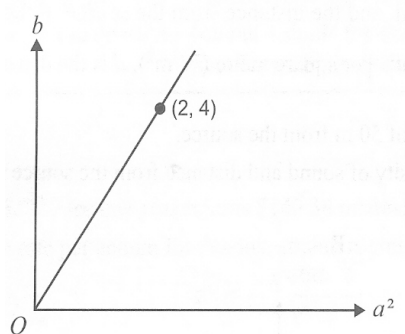
$$4a + 3b = 21.4 \text{ (i)}$$

$$5a + 2b = 20.8 \text{ (ii)}$$

Multiply (i) by 5 and (ii) by 4

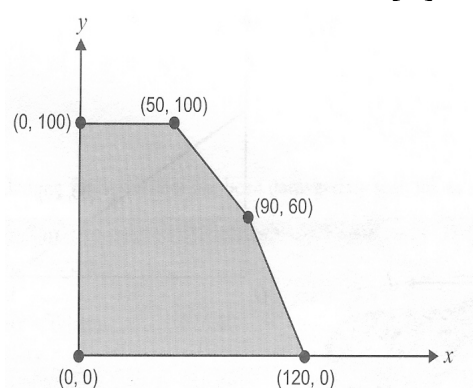
$$20a + 15b = 107 \text{ (iii)}$$

$$20a + 8b = 83.2 \text{ (iv)}$$

Subtracting (iv) from (iii): $7b = 23.8$ Therefore $b = \$3.40$ **Question 7** [A]The general equation is $b = ka^2$

Substituting (2, 4) into the equation

$$4 = k \times 2^2, \text{ so } k = 1$$

The equation is $b = a^2$ **Question 8** [E]

To find where the objective function

 $P = 3x - 4y$ is at a maximum,

substitute in all vertex points.

$$(0, 100) \quad P = 0 - 300 = -300$$

$$(50, 100) \quad P = 200 - 300 = -100$$

$$(0, 0) \quad P = 0$$

$$(90, 60) \quad P = 360 - 180 = 180$$

$$(120, 0) \quad P = 480 - 0 = 480$$

The maximum value occurs at (120, 0)

Question 9 [A]

The intensity $I = \frac{k}{d^2}$

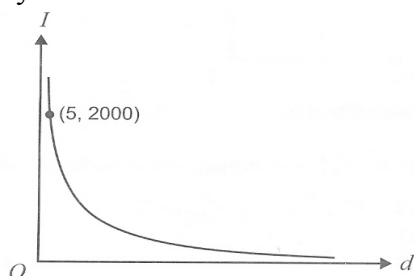
When $I = 20$, $d = 50$

So $k = Id^2 = 20 \times 50^2 = 50000$

For the point (5, 2000)

$$2000 = \frac{50000}{5^2}$$

The relationship between intensity of sound and distance from source is given by:

**Module 4 Business-related mathematics****Question 1 [D]**

$P = 1200$, $I = \$140.38$, $t = 1$

$$r = \frac{100I}{Pt} = \frac{100 \times 140.38}{1200 \times 1} \approx 11.70\%$$

Question 2 [B]

GST is 10% so solve

$$1.1x = 22$$

$$x = \frac{22}{1.1} = 20$$

The price before GST is added is \$20

Question 3 [B]

To calculate the value of the deposit solve

$$2143.50 + 2.45 - 616.40 + x = 1971.75$$

$$x = \$442.20$$

Question 4 [A]

Depreciation at the end of 2 years is
 $= 120000 \times 2 \times 0.05 = 12000$

Depreciated value of the machine at the end of 2 years is
 $= 45000 - 12000 = \$33000$

Question 5 [B]

Using TVM solver

```
N=60
I%=8.5
PV=200000
PMT=-1416.6666...
FV=-200000
P/Y=12
C/Y=12
PMT:[FV] BEGIN
```

His monthly repayment is \$1416.67.

Question 6 [C]

Year	Interest	Balance \$
0		3000
1	195.00	3195
2	207.68	3402.68
3	221.17	3623.85
4	235.55	

Interest (year 1)

$$I = \frac{3000 \times 6.5 \times 1}{100} = 195$$

Interest for years 2 to 4 are calculated as for year 1.

So interest for year 4 is \$235.55.

Question 7 [A]

```
N=16
I%=6
PV=10000
PMT=-500
FV=-3723.670555
P/Y=4
C/Y=4
PMT:[FV] BEGIN
```

The value of the remaining investment is closest to \$3720.

Question 8 [D]

Monday's price = 1

Friday's price

$$1.1 \times 0.9 \times 1.2 \times 0.8 = 0.95$$

So Friday's price is 5% lower than Monday's price.

Question 9 [E]

The planned payments of \$800 per month over 6 months are

$N=6$
 $I\%=6$
 $PV=4717.107525$
 $PMT=-800$
 $FV=0$
 $P/Y=12$
 $C/Y=12$
 $PMT:[END] BEGIN$

At end of 2 months

$N=2$
 $I\%=6$
 $PV=4717.107525$
 $PMT=-800$
 $FV=3160.396528$
 $P/Y=12$
 $C/Y=12$
 $PMT:[END] BEGIN$

At the end of 3 months without a payment

$N=1$
 $I\%=6$
 $PV=3160.4$
 $PMT=0$
 $FV=-3176.202$
 $P/Y=12$
 $C/Y=12$
 $PMT:[END] BEGIN$

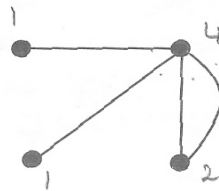
At the end of 4 months with \$1600 payment

$N=1$
 $I\%=6$
 $PV=3176.2$
 $PMT=-1600$
 $FV=-1592.081$
 $P/Y=12$
 $C/Y=12$
 $PMT:[END] BEGIN$

At the end of 6 months

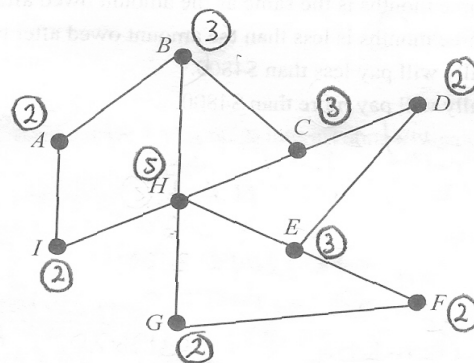
$N=2$
 $I\%=6$
 $PV=1592.081$
 $PMT=-800$
 $FV=-4.041612025$
 $P/Y=12$
 $C/Y=12$
 $PMT:[END] BEGIN$

There is still \$4.04 to pay after the 6 months. To fully repay the loan, Sally will pay more than \$4800.

Module 5 Networks and decision mathematics**Question 1 [C]**

The sum of the degrees of all the vertices in this network is

$$1 + 1 + 4 + 2 = 8$$

Question 2 [D]

An Euler path can be found if there are no more than 2 edges of odd degree.

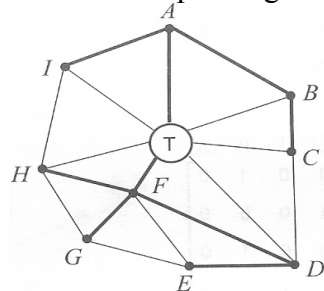
By removing the edge between B and C, there remain only 2 vertices of odd degree, ie H and E so an Euler path will exist.

Question 3 [B]

person	task	U	V	W	X
Kael		1	0	1	0
Lian		1	0	0	1
Minh		0	0	1	0
Nelly		0	1	1	1

Question 4 [B]

A Minimal Spanning Tree is required.



Question 5 [A]

The connected graph initially had 15 edges.

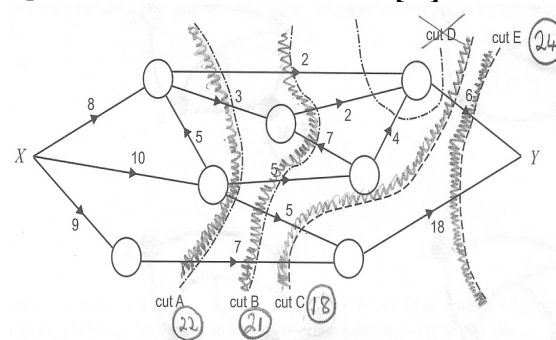
Using Euler's formula for planar graphs

$$V - E + F = 2$$

$$\text{So } E = V + F - 2 = 10 + 3 - 2 = 11$$

4 edges have to be removed.

Question 6 [C]



Maximum flow = minimum cut

Cut A – 22

Cut B – 21

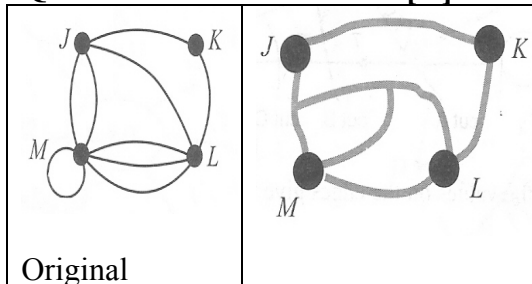
Cut C – 18

Cut D – is not a valid cut

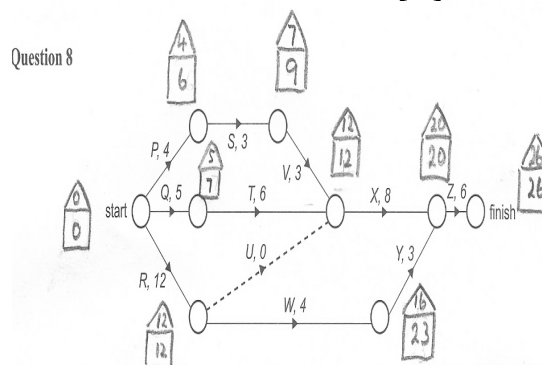
Cut E – 24

So Cut C allows you to find the maximum flow from point X to point Y.

Question 7 [D]



Question 8 [C]



The activities on the Critical path are R, U, X and Z.

If R is delayed, the project will be delayed

Question 9 [B]

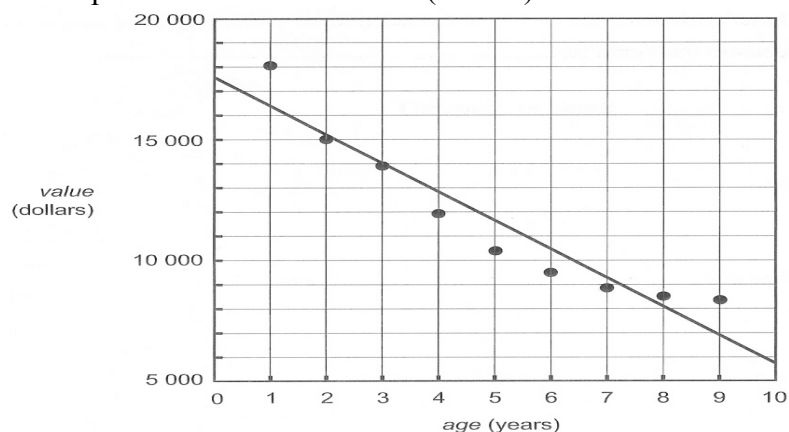
	Q	R	S	T	U
Q	0	8	15	–	4
R	8	0	5	–	–
S	15	5	0	10	7
T	–	–	10	0	–
U	4	–	7	–	0

The shortest distance in km between Q and T is QUST = 21 km.

**2005 Further Mathematics
Written Examination 2 (Analysis task)
Suggested answers and solutions**

Core**Question 1****a** Value (dollars)

The dependent variable is Value (dollars).

**b i** The correlation coefficient is the square root of the coefficient of determination.

$$r = -\sqrt{0.9058} = -0.952 \text{ (correct to 3 decimal places)}$$

ii 91% (r^2) of the variation in the value of a car can be accounted for by the variation in age .

c $value = 17500 - 1200 \times age$

Age (years)	Values (dollars)
1	18 100
2	15 050
3	13 900
4	11 900
5	10 400
6	9 600
7	8 900
8	8 500
9	8 400

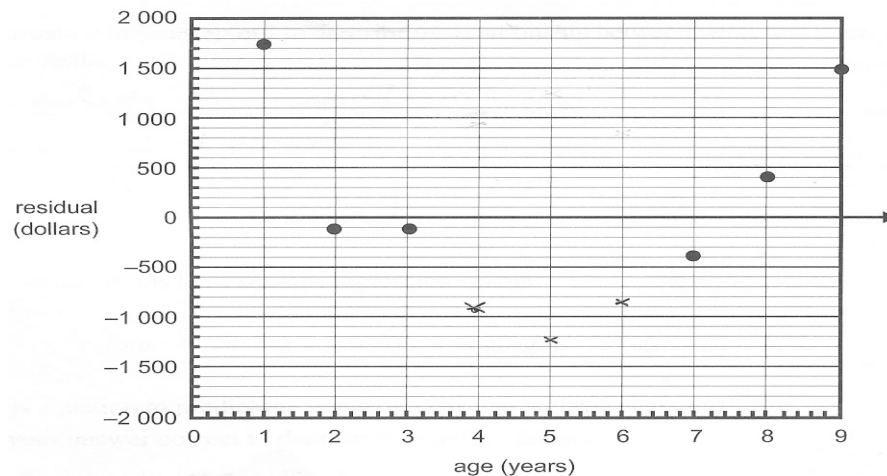
Input data into 2 lists L_1, L_2 and do a LinReg ($a+bx$).**LinReg** $y=a+bx$ $a=17534.72222$ $b=-1179.166667$ $r^2=.9058311416$ $r=-.9517516176$ 

The least squares regression line (to the nearest hundred) is

$$value = 17500 - 1200 \times age$$

d Residual = actual - estimate

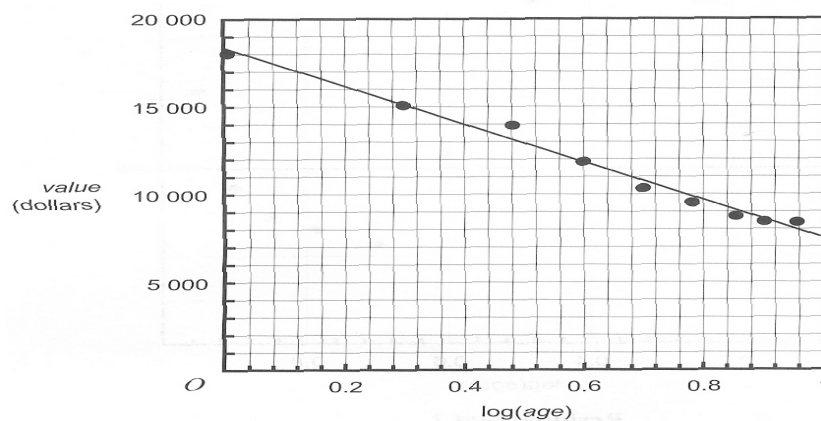
Year	Actual	Estimate	Residual
4	11 900	12818	-918
5	10 400	11639	-1239
6	9600	10460	-860



e Yes. Residual plot 1 above has a definite pattern suggesting that the relationship is non-linear.

f 0.95 as $\log(9) = 0.95$ (correct to 2 decimal places)

g



Direction – Negative

Form – Linear

Strength – Strong

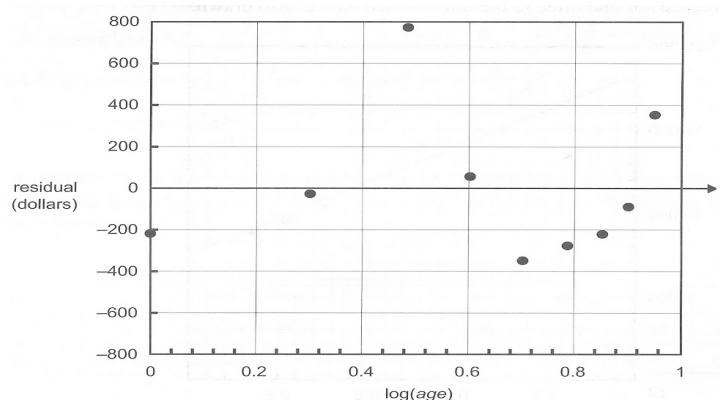
h

$$\text{value} = 18300 - 10800 \times \log(\text{age})$$

When $\text{age} = 3$ the $\text{value} = 18300 - 10800 \times \log(3) \approx 13100$

Answer: \$13 100 (to the nearest hundred dollars).

i



The log (age) transformation has been successful in linearising the data as the residual plot has no discernible pattern (or it has a random placement of dots.)

j A log (value) or $\frac{1}{\text{value}}$ would be suitable transformations as they both reduce the values and compress the values axis.

Module 1: Number patterns and applications

Question 1

Day	1	2	3
Distance (km)	4.2	4.5	4.8

a On Day 5 Noel will run 5.4 km.

$$t_n = a + (n - 1)d$$

$$a = 4.2, d = 0.3 \text{ and } t = 7$$

$$7 = 4.2 + (n - 1) \times 0.3$$

$$n = 10.33$$

On Day 11, Noel runs more than 7 km.

c For days 3 to 12

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$n = 10, a = 4.8, d = 0.3$$

$$S_{10} = \frac{10}{2}[(4.8 \times 2) + (9 \times 0.3)]$$

$$= 61.5 \text{ km}$$

d The n th term can be written as

$$t_n = 0.3n + b$$

The general n th term is $t_n = a + (n - 1)d$

$$= 4.2 + (n - 1) \times 0.3$$

$$= 0.3n + 3.9 \text{ and so } b = 3.9$$

Question 2

Day	1	2	3
Distance (km)	3	3.5	4.05

Arithmetic

The common difference from Day 1 to Day 2 should equal that from Day 3 to Day 4 if the sequence is Arithmetic.

$d = 3.5 - 3 \neq 4.05 - 3.5$ and so the sequence is not Arithmetic.

Geometric

The common ratio of term 2 to term 1 should be the same as term 3 to term 2 if the sequence is Geometric.

$$a = 3, \quad r = \frac{t_2}{t_1} = \frac{3.5}{3} = 1.167$$

$$r = \frac{t_3}{t_2} = \frac{4.05}{3.5} = 1.157$$

So the sequence is not geometric.

b The difference equation for this sequence is $w_{n+1} = 1.1w_n + c$

$$w_2 = 3.5, w_1 = 3, c = ?$$

$$3.5 = 1.1 \times 3 + c$$

$$c = 3.5 - 3.3 = 0.2$$

c $w_4 = 1.1w_3 + 0.2$
 $= 1.1 \times 4.05 + 0.2 = 4.655$

$$w_5 = 1.1w_4 + 0.2$$

$$= 1.1 \times 4.655 + 0.2 = 5.32 \text{ km correct to 2 decimal places}$$

Question 3

a Geometric sequence with $a = 10$ and $r = 1.05$

$$t_n = ar^{n-1} \text{ so } t_2 = 10 \times 1.05^1 = 10.5$$

Day	1	2	3
Distance km	10	10.5	11.025

b $S_n = \frac{a(r^n - 1)}{r - 1}$

$$S_{14} = 10 \left(\frac{1.05^{14} - 1}{1.05 - 1} \right) = 10 \times 19.599 \approx 196 \text{ km}$$

Total distance travelled in 14 days is 196km to the nearest kilometre.

c i 5 parts = 10 km, so each part = 2 km

Her running to walking ratio is 3:2 so she runs $2 \times 2 = 4$ km

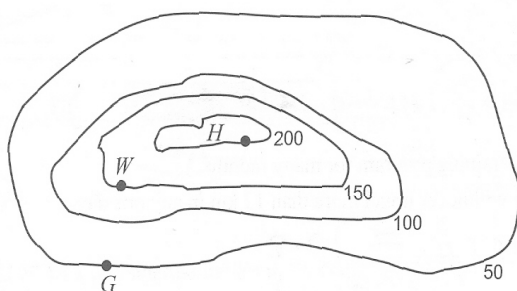
ii Running : Walking = 0.25 : 0.75 which is the same as 1 : 3

d i $t_{n+1} = 0.9t_n + 1.2$, $t_1 = 5$

ii As the distanced travelled approaches 12, the common ratio of 0.9 applied to any term will equal close to 1.2. As the value of n increases, she will never travel more than 12 km as $0.9 \times 12 + 1.2 = 12$ thus creating a converging sequence.

Module 2: Geometry and trigonometry

Question 1

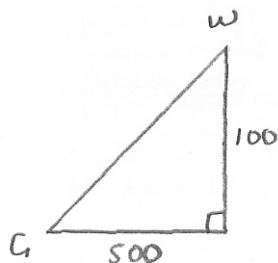


scale 1:40 000

a The difference in height (in metres) between hut H and the water source W is $200 - 150 = 50$ m .

Answer: 50 m

b



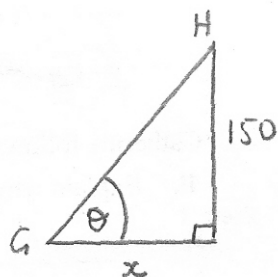
Using Pythagoras

$$GW^2 = 500^2 + 100^2$$

$$= 250000 + 10000$$

$$GW = \sqrt{260000} \approx 510 \text{ m (correct to the nearest metre.)}$$

c



$$\tan \theta = \frac{150}{x}$$

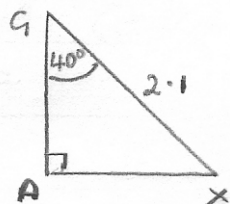
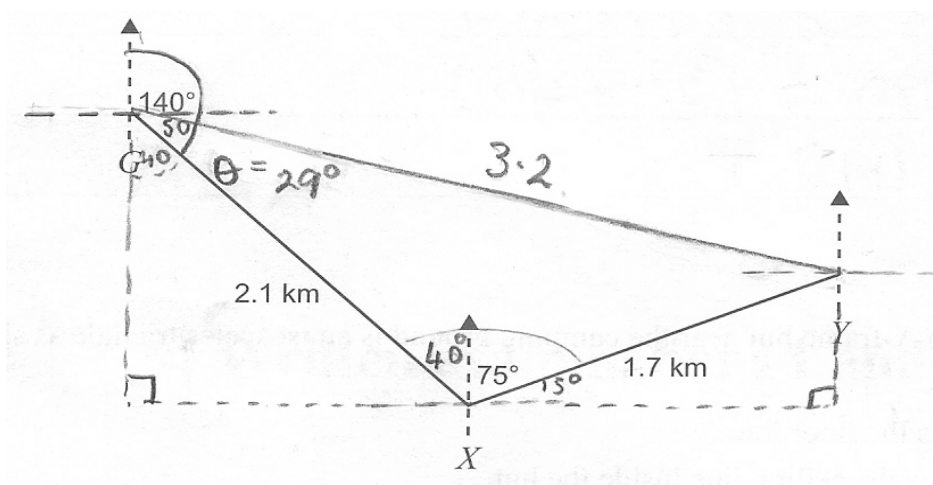
$$x = \frac{150}{\tan \theta} = \frac{150}{0.12} = 1250 \text{ m}$$

d 2 kilometres = $2 \times 1000 \times 100 \text{ cm}$, converting to centimetres.

$$\text{On the map: } \frac{1}{40000} \times 2 \times 1000 \times 100 = 5$$

It is represented by a line of length 5cm on the map.

Question 2

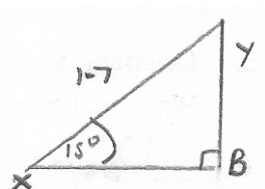


a i Find GA where A is the point due south of G and west of X.

$$\cos 40^\circ = \frac{GA}{2.1} \text{ and so } GA = 2.1 \times \cos 40^\circ$$

Therefore GA = 1.6 km, correct to 1 decimal place.

ii



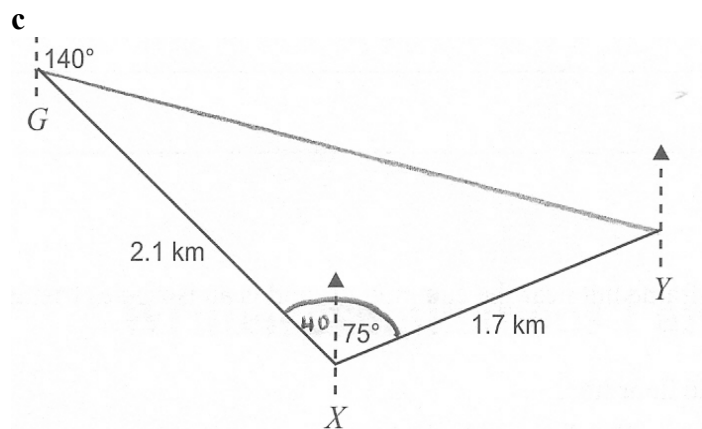
Find YB where B is the point due south of Y (and east of X).

$$\sin 15^\circ = \frac{YB}{1.7} \text{ and so } YB = 1.7 \times \sin 15^\circ$$

Therefore YB $\approx$ 0.44 km.

The distance required is $GA - YB = 1.6 - 0.44 \approx 1.2$ km correct to 1 decimal place.
Checkpoint Y is 1.2 km south of G .

b From the diagram at the start of Question 2, using alternate angles, $\angle GXY = 115^\circ$



Using the Cosine Rule

$$GY^2 = 2.1^2 + 1.7^2 - (2 \times 2.1 \times 1.7 \cos 115^\circ)$$

$$GY = 3.2 \text{ km (correct to 1 decimal place)}$$

d Using the Sine Rule

$$\frac{1.7}{\sin XGY} = \frac{3.2}{\sin 115^\circ}$$

$$\sin XGY = \frac{1.7 \sin 115^\circ}{3.2}$$

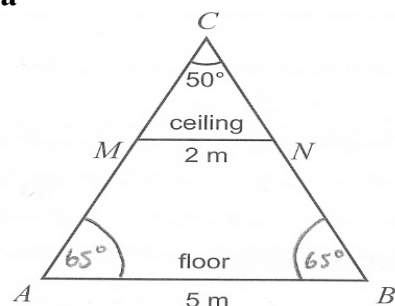
$$\angle XGP = 29^\circ$$

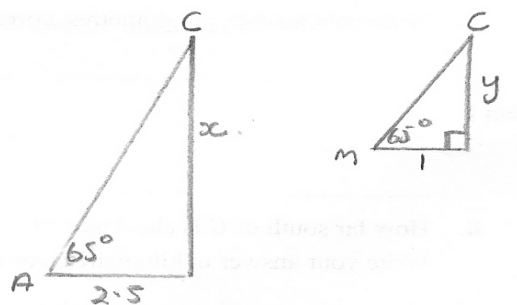
$$\text{The bearing of checkpoint } Y \text{ from } G = 180^\circ - (40 + 29)^\circ = 111^\circ$$

Answer: 111°T

Question 3

a





$$\tan 65^\circ = \frac{x}{2.5}$$

$$x = 2.5 \tan 65^\circ$$

$$x = 5.36 \text{ m}$$

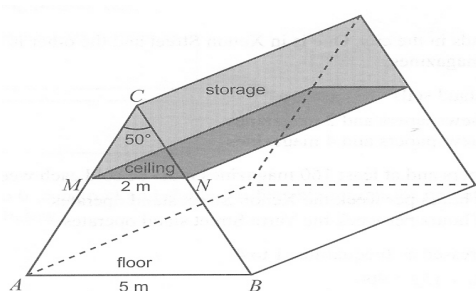
$$\tan 65^\circ = \frac{y}{1}$$

$$y = 2.14$$

Height of ceiling MN above floor AB

$$= 5.36 - 2.14 = 3.22 \approx 3.2 \text{ m (correct to 1 decimal place)}$$

b



The ratio of the *lengths* of the two prisms is 2: 5.

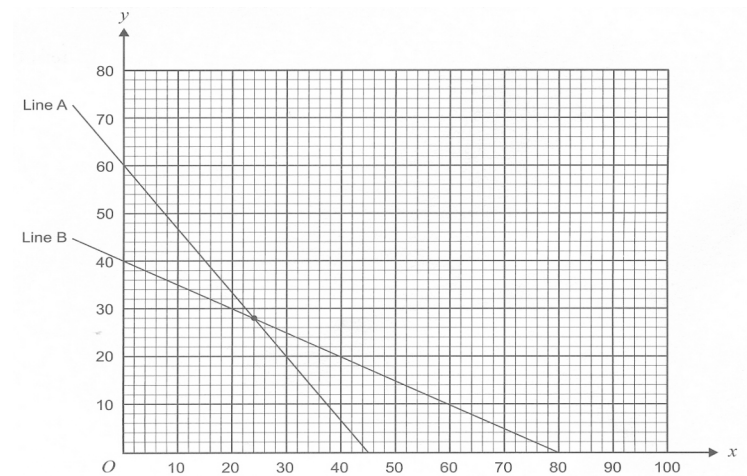
As the length of both prisms is the same, to calculate the fraction of V used for storage, use the ratios of the end area only.

$$2^2 : 5^2 \text{ which is } \frac{4}{25}$$

$$\text{Answer: } \frac{4}{25}$$

Module 3: Graphs and relations

Question 1



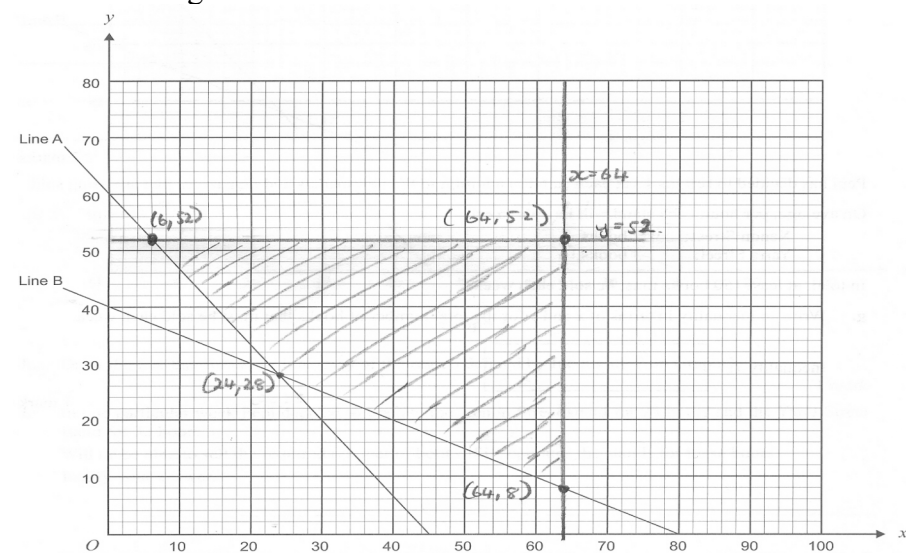
a Inequality 1 is $20x + 15y \geq 900$. This has an x intercept of (45, 0) and y intercept of (0, 60). These two points are on Line A.

b From the graph, the point of intersection of Line A and Line B is (24, 28)

c Inequality 5 is $x \leq 64$

Inequality 6 is $y \leq 52$

d Feasible region shaded



e $C = 100x + 70y$

- f** Using the cost equation $C = 100x + 70y$, substitute in the coordinates of the vertices from the feasible region.

$$(6, 52) \quad C = 100 \times 6 + 70 \times 52 = 4240$$

$$(24, 28) \quad C = 100 \times 24 + 70 \times 28 = 4360$$

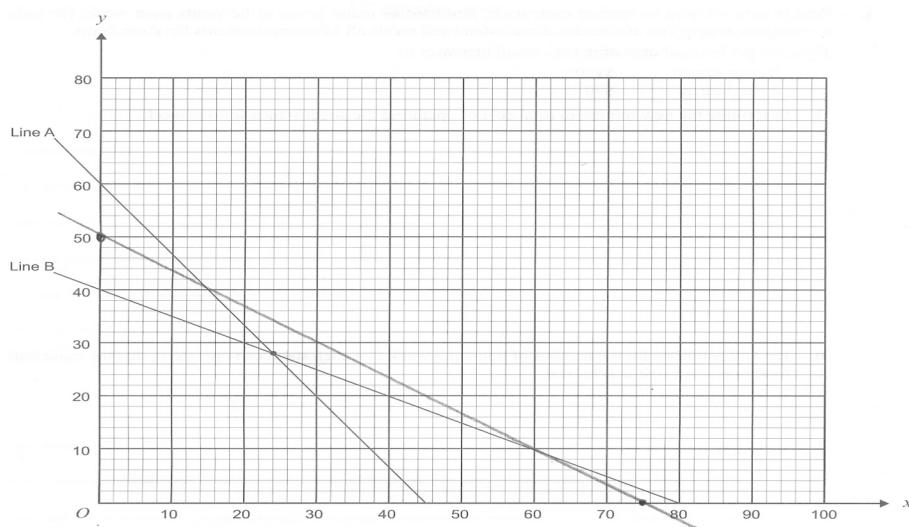
$$(64, 8) \quad C = 100 \times 64 + 70 \times 8 = 6960$$

$$(64, 52) \quad C = 100 \times 64 + 70 \times 52 = 10040$$

The minimum total cost of operating the street stands for one week is \$4240.

- g** Inequality 7 is $2x + 3y \geq 150$

- h** For $2x + 3y \geq 150$, the x intercept is $(75, 0)$ and the y intercept is $(0, 50)$.



- i** For the point $(25, 32)$ the number of books sold is $2 \times 25 + 3 \times 32 = 146$ and so he will not be able to sell the required number of 150.

- j** The Cost equation is now $C = 120x + 90y$

Substitute each of the corner points:

$$(0, 60) \quad C = 120 \times 0 + 90 \times 60 = 5400$$

$$(15, 40) \quad C = 120 \times 15 + 90 \times 40 = 5400$$

$$(60, 10) \quad C = 120 \times 60 + 90 \times 10 = 8100$$

$$(80, 0) \quad C = 120 \times 80 + 90 \times 0 = 9600$$

The new minimum total cost of operating the street stands for one week is \$5400.

- k** The minimum total operating cost is achieved for **all** the points on $20x + 15y = 900$ between $(0, 60)$ and $(15, 40)$ inclusive.

Module 4: Business-related mathematics

Question 1

a Total amount of interest = \$4560 – \$4000 = \$560

b

$$\begin{aligned} r_f &= \frac{100I}{Pt} \\ &= \frac{100 \times 560}{3500 \times 2} \\ &= 8\% \text{ as required} \end{aligned}$$

c

$$\begin{aligned} r_e &= \frac{2n}{n+1} r_f \\ &= \frac{2 \times 24}{24+1} \times 8 \\ &= 15.36\% \\ &= 15.4\% \text{ correct to one decimal place} \end{aligned}$$

d The effective interest rate differs from a flat interest rate as effective rate is calculated on the average amount owing over the period of the loan. Flat interest rate does not take into account reductions in the balance owing after repayments are made.

Question 2

a Using flat rate depreciation, the annual depreciation rate is:

$$\begin{aligned} 1000 &= 4000 - 5x \\ 5x &= 3000 \\ x &= \$600 \text{ per year} = 15\% \end{aligned}$$

b

$$\begin{aligned} 1000 &= 4000 \left(1 - \frac{r}{100}\right)^5 \\ \frac{1000}{4000} &= 0.25 = \left(1 - \frac{r}{100}\right)^5 \\ \left(1 - \frac{r}{100}\right) &= \sqrt[5]{0.25} = 0.7569 \end{aligned}$$

Therefore $\frac{r}{100} = 1 - 0.7579$ and so $r = 24.2$

Answer: 24.2%

Question 3**a i Investment Option A**

$$P = 10000$$

$$R = 1 + \frac{4.8}{100 \times 12} = 1.004$$

$$n = 60$$

$$\text{ii Investment Option A after 5 years} = 10000(1.004)^{60} = \$12\,706.41$$

b Using TVM solver (shown below) the Investment Option B is worth \$11 848.58 at the end of 5 years.

```

N=60
I%=4.8
PV=4000
PMT=100
▪ FV=-11848.58084
P/Y=12
C/Y=12
PMT:[END] BEGIN

```

c i Firstly, Investment Option B is followed for two years

```

N=24
I%=4.8
PV=4000
PMT=100
▪ FV=-6915.90071
P/Y=12
C/Y=12
PMT:[END] BEGIN

```

Therefore \$6915.90 is the PV amount for TVM solver for Investment Option C

This is used for the remaining three years.

```

N=36
I%=4.8
PV=6915.9
▪ PMT=129.8001124
FV=-13000
P/Y=12
C/Y=12
PMT:[END] BEGIN

```

Therefore, the new amount deposited at the end of each month for the remaining three years is \$129.80.

$$\text{ii Interest earned in the first two years} = 6915.90 - [4000 + (24 \times 100)] = \$515.90$$

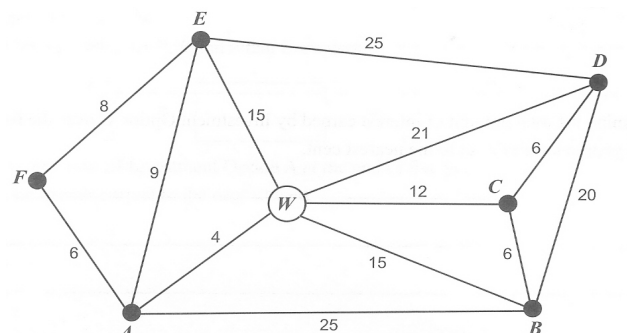
$$\text{Interest earned in years 3 to 5} = 13000 - [6915.90 + (36 \times 129.80)] = \$1411.30$$

$$\text{Total Interest} = \$515.90 + \$1411.30$$

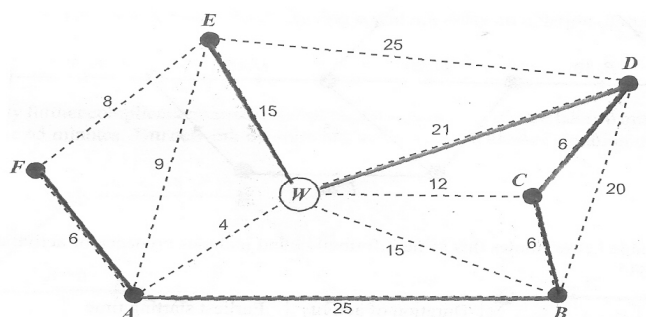
$$= \$1927.20$$

Module 5: Networks and decision mathematics

Question 1



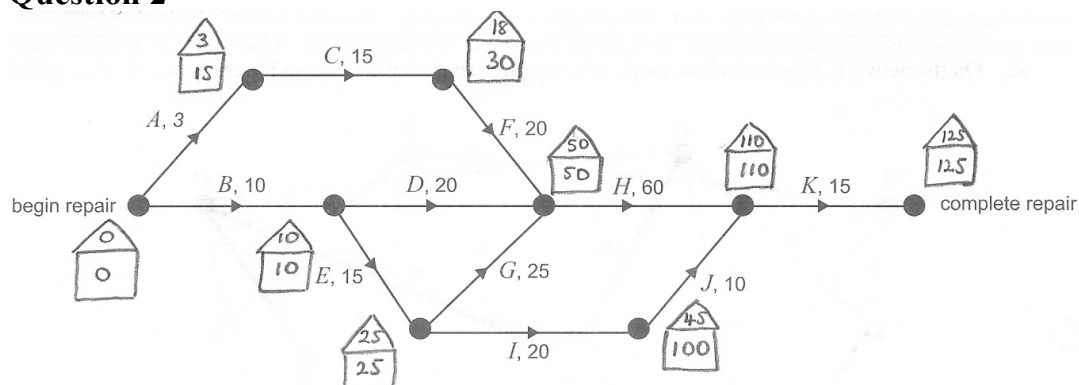
- a** The degree of vertex W is 5.
- b** The minimum distance travelled from factory B to factory D and then to warehouse W is along edges BCDCW = $6+6+6+12 = 30$ km.
- c i** Hamilton path
The salesman visits each location only once but does not return to Factory E is called a Hamilton path.
- ii** A complete route for his planned journey is



An alternative route is EWCDBAF.

- d i** An Euler circuit requires all vertices to be of even degree. As W and C are odd degree vertices, an Euler circuit is not possible.
- ii** An Euler path starts and finishes on an odd degree vertex. As it starts at W, it must finish at vertex C.

Question 2



a From the diagram

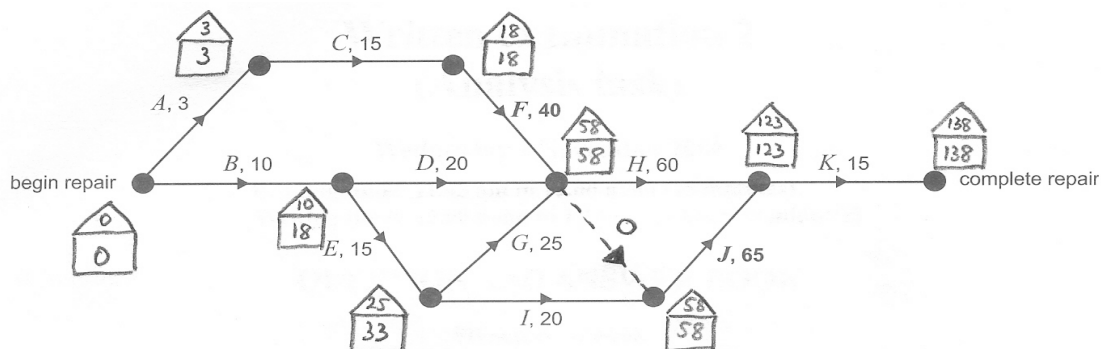
Activity	Predecessor(s)	Earliest starting time (EST)
<i>H</i>	D, F, G	50

b The minimum time needed to complete this repair is 125 minutes.

c i As Activity F is not on the critical path, its activity length can be increased without delaying this repair.

ii The maximum duration of activity F is $50 - 18 = 32$ minutes.

d i

ii The latest starting time (LST) for activity H is $123 - 60 = 63$ minutes.

iii From beginning to completion, the minimum time needed to repair this delivery van is 138 minutes.

iv Activities B, D, E, G and H are not on the critical path. Their float times are:

Activity	Float
B	$18 - 10 - 0 = 8$
D	$58 - 20 - 10 = 28$
E	$33 - 15 - 10 = 8$
G	$58 - 25 - 25 = 8$
H	$123 - 60 - 58 = 5$

Activity D can be delayed the longest as it has the greatest float time.