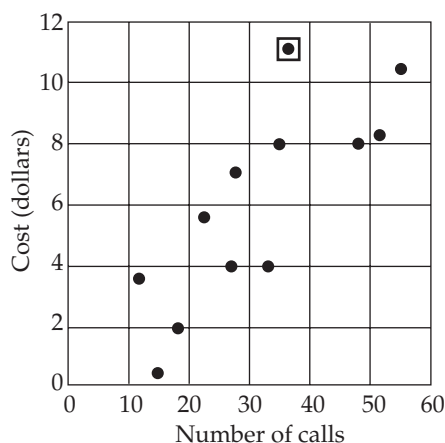


2003 Further Mathematics Written Examination 2 (Analysis task) Suggested answers and solutions

Core: Data analysis

Question 1

- a** mean = \$6.51
standard deviation = \$3.31
Both figures can be obtained from the calculator (to the required level of accuracy).
On the TI-83, enter data in List 1 then STAT, CALC, 1-VAR STATS, L1
- b** 75%
9 of the 12 workers made more than the mean number of calls.
 $\frac{9}{12} \times 100 = 75\%$
- c i** 0.19
On the calculator enter number of calls in List 1 and cost in List 2.
STAT, CALC, LinReg(a+bx), L1, L2
- c ii** 0.8054
Pearson's correlation coefficient, r , is displayed when the commands from **c i** are carried out.
- d** linear
The key assumption when performing a correlation analysis using Pearson's coefficient is that the relationship is linear.
- e** (36, 11.30)
This point must be plotted correctly on the given scatterplot.



- f** cost
Since we are predicting the cost from the number of calls made we say cost is dependent on number calls. The dependent variable is always on the y axis.
- g i** 19
The gradient of the linear regression line is a measure of the change in the dependent variable (cost) per unit change in the independent variable (number of calls).
The gradient is 0.19 (dollars) therefore 19 cents.
- g ii** 65
The coefficient of determination, r^2 , (when expressed as a percentage) gives the percentage of the variation in the dependent variable that can be explained by the variation in the independent variable.
 $0.8054^2 \times 100 = 65$ to the nearest whole percent.
- h** 1.38
A residual value is the observed value – the value predicted by the regression line.
From the table it is observed that a person who made 34 calls had a cost of \$8.50.
When 34 is substituted into the regression equation the predicted cost is \$7.12.
Therefore the residual is $\$8.50 - \$7.12 = \$1.38$

Question 2

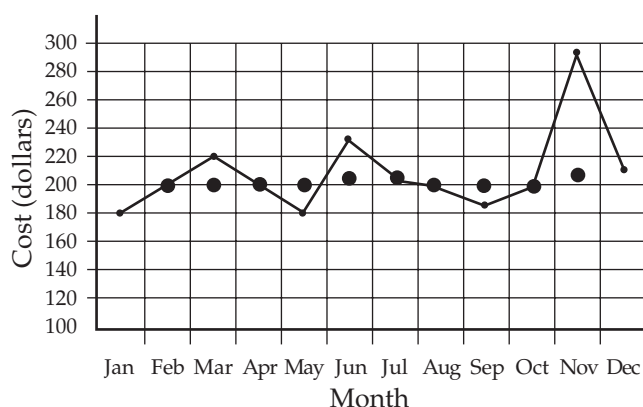
- a 10 points to be plotted are (Feb, 201) , (Mar, 201) , (Apr, 198) , (May, 198) , (Jun, 205) (Jul, 205) , (Aug, 199) , (Sep, 197) , (Oct, 197) , (Nov, 206).

Three median smoothing is done by taking the middle value in each group of 3 months.

For January, the middle value of 180, 201, 221 is 201.

For February the middle value of 201, 221, 198 is 201 and so on.

The points must be plotted in the middle of the columns (as the original points are).



- b No, as the pattern is basically flat.

There is little variation in the smoothed data with a difference of only \$9 between the lowest and highest values. On this basis we say that the smoothed data does not support the supervisor's belief that there has been an increasing trend.

- c Extreme value in November

Any possible outliers or extreme values will contribute significantly to the mean but will not affect the median.

Module 1 : Number patterns and applications

Question 1

- a 10 800

We have an arithmetic sequence where $a = 10\,000$ and $d = 800$.

One minute after pumping starts is the second term of the sequence.

$$t_n = a + (n - 1)d$$

$$t_2 = a + d$$

$$= 10\,000 + 800 = 10\,800 \text{ litres}$$

- b 14 000

$$t_n = a + (n - 1)d$$

$$t_6 = a + 5d$$

$$= 10\,000 + 5 \times 800 = 14\,000 \text{ litres}$$

- c $a = 10\,000$ $b = 800$

a is the initial volume in the tank.

b is the increase in volume as each minute passes.

- d 25 minutes

The time to empty the tank is the volume divided by the rate of outflow.

$$\frac{20\,000}{800} = 25$$

Question 2

- a 28 500 litres

The sequence is geometric with $a = 30\,000$ and $r = 0.95$.

The end of the first week is the second term of the sequence.

$$t_n = ar^{n-1}$$

$$t_2 = ar$$

$$= 30\,000 \times 0.95 = 28\,500$$

- b 24 435 litres

The end of the fourth week is the fifth term of the sequence.

$$t_n = ar^{n-1}$$

$$t_5 = ar^4$$

$$= 30\,000 \times 0.95^4$$

$$= 24\,435 \text{ (to the required level of accuracy)}$$

- c end of week 22

If the rule for the n th term is entered into the calculator this can be read off a table of values.

Alternatively we can solve algebraically as follows

$$30000 \times 0.95^n = 10000$$

$$0.95^n = 0.333$$

$$n = \frac{\log 0.333}{\log 0.95}$$

$$n = 21.42$$

which corresponds to the end of the 22nd week.

Question 3

- a 1.1

The water usage increases by 10% therefore is 110% of its original value.

- b 36 631 litres

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_5 = \frac{6000(1.1^5 - 1)}{1.1 - 1}$$

$$= 36\,631 \text{ (to the required level of accuracy)}$$

- c 7 881 litres

From the difference equation we see that each new term is found by multiplying the previous term by 0.98, then adding 100.

$$G_2 = 0.98 \times 8000 + 100 = 7940$$

$$G_3 = 0.98 \times 7940 + 100 = 7881 \text{ (to the required level of accuracy).}$$

- d In week 4

By tabulating the first few values for each group (to the nearest whole number) we find O'Callaghans

6000, 6600, 7260, 7986

Gergers

8000, 7940, 7881, 7824

So in week 4 is the first time the O'Callaghans use more water.

Module 2 : Geometry and trigonometry

Question 1

- a 8.50 metres

Using Pythagoras' theorem

$$BC^2 = BA^2 + AC^2$$

$$BC = \sqrt{2.20^2 + 8.21^2}$$

$$= 8.50 \text{ (to the required level of accuracy)}$$

- b 15°

$$\angle ACB = \tan^{-1}\left(\frac{2.20}{8.21}\right)$$

$$\angle ACB = 15^\circ \text{ (to the required level of accuracy)}$$

Question 2

- a 360° divided by 8 = 45°

The regular octagon has 8 sides so show this simple calculation.

- b 15m^2

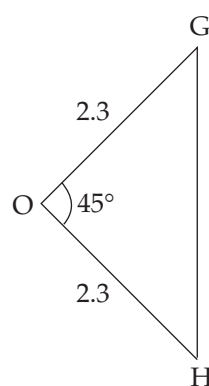
Using the formula $\frac{1}{2}bc \sin A$

Area of octagon = $8 \times$ area of triangle OGH

$$= 8\left(\frac{1}{2} \times 2.3 \times 2.3 \times \sin 45^\circ\right)$$

$$= 15\text{ m}^2 \text{ (to the required level of accuracy)}$$

- c



1.76 metres

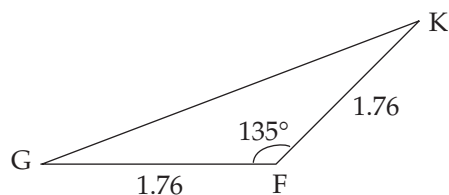
Using the cosine rule

$$GH^2 = 2.3^2 + 2.3^2 - (2 \times 2.3 \times 2.3 \times \cos 45^\circ)$$

$$GH = 1.76 \text{ (to the required level of accuracy)}$$

$$\text{Alternatively } GH = 2 \times 2.3 \times \sin 22.5^\circ$$

d i



3.25 metres

Using the cosine rule

$$GK^2 = 1.76^2 + 1.76^2 - (2 \times 1.76 \times 1.76 \times \cos 135^\circ)$$

$$GK = 3.25 \text{ (to the required level of accuracy)}$$

d ii 5.55 metres

$$OK = OG + GK$$

$$= 2.3 + 3.25$$

$$= 5.55 \text{ (to the required level of accuracy)}$$

e 5.05 metres

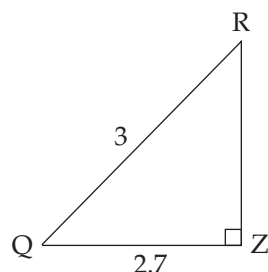
Using the sine rule

$$\frac{PR}{\sin 105^\circ} = \frac{3}{\sin 35^\circ}$$

$$PR = \frac{3 \sin 105^\circ}{\sin 35^\circ}$$

$$= 5.05 \text{ (to the required level of accuracy)}$$

f



4.81 metres (481 cm)

Form a right angled triangle with a line from Q to RZ.

Using Pythagoras the height of R above

$$Q = \sqrt{3^2 - 2.7^2} = 1.31$$

$$\text{Therefore } RZ = 3.5 + 1.31 = 4.81 \text{ m} = 481 \text{ cm}$$

Question 3

a 0.26m^3

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \times 0.6^2 \times 0.7$$

$$= 0.26 \text{ (to the required level of accuracy)}$$

b 1 : 8

The ratio of the lengths for the first pile to the second is 1 : 2

For these similar figures the volume ratio will be $1^3:2^3$ i.e. 1 : 8

Module 3 : Graphs and relations

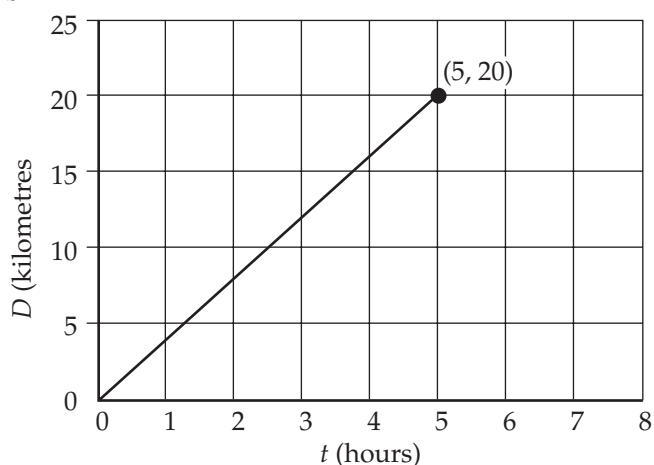
Question 1

a 6 km

She walks 4 km in 1 hour therefore will walk 2 km each half hour.

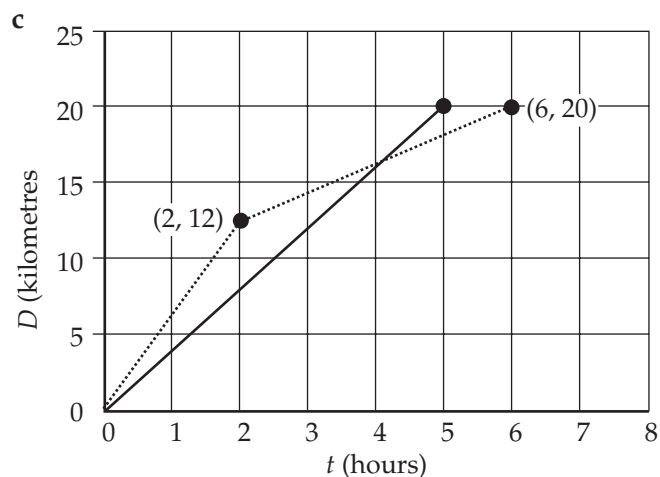
One and a half hour walk therefore $4 + 2 = 6$

b



Malinda – straight line from (0,0) to (5,20)

$D_m = 4t$ for $0 \leq t \leq 5$ therefore Malinda's line has gradient of 4 and intercept of 0.



Christos – 2 line segments $(0,0)$ to $(2,12)$ then $(2,12)$ to $(6,20)$

Christos walked for 2 hours at 6 km/hr therefore the first segment goes from $(0,0)$ to $(2,12)$. Walking at 2 km/hr from then on it will take him another 4 hours to travel the remaining 8 km.

d 4 hours

This is the intersection point of the 2 graphs. After 4 hours they have both covered 16 km.

e $a = 6$ $b = 2$ $h = 8$ $d = 6$

a is the gradient of the first segment (Christos' speed for the first 2 hours).

b is the gradient of the second segment (Christos' speed for the last 4 hours).

To determine h we need the equation of the second segment.

It is a straight line of the form $D = bt + h$.

We know $b = 2$ therefore $D = 2t + h$.

Substituting the point $(6,20)$ gives $20 = 12 + h$

d is the time that Christos' journey finishes as described earlier.

Question 2

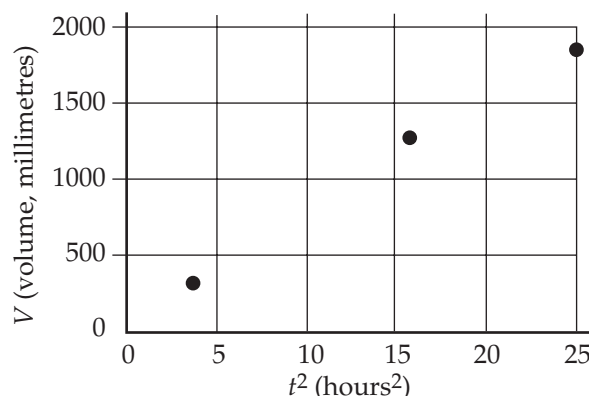
a i 4 16 25

These are the original t values squared i.e.

$$2^2 = 4, 4^2 = 16, 5^2 = 25$$

a ii Plot co-ordinates $(0,0)$, $(4,300)$, $(16,1200)$, $(25,1875)$

These are the co-ordinates (t^2, V) as read from the table.



b $k = 75$

The 4 points plotted form a straight line with a gradient of k .

Taking the first point $(0,0)$ and the final point $(25,1875)$

$$\begin{aligned} \text{Gradient} &= \frac{1875 - 0}{25 - 0} \\ &= 75 \end{aligned}$$

c 675 ml

The rule for volume is $V = 75t^2$.

Substituting $t = 3$ gives $V = 75 \times 3^2 = 675$.

d i 3.65 hours

The original volume is 2000 ml therefore we need to find the time when volume = 1000

$$1000 = 75t^2$$

$$t^2 = \frac{1000}{75}$$

$$t = 3.65 \text{ (to the required level of accuracy)}$$

d ii 0.7 km

Substituting $t = 3.65$ into each rule.

$$\text{Malinda : } D_m = 4 \times 3.65 = 14.6$$

$$\text{Christos : } D_c = 2 \times 3.65 + 8 = 15.3 \text{ (the second segment of Christos' rule)}$$

$$\text{Distance apart} = 15.3 - 14.6 = 0.7$$

Module 4 : Business-related mathematics

Question 1

a 10.1%

The discount given is $3450 - 3100 = \$350$.

This discount as a percentage of the original

price is $\frac{350}{3450} \times 100 = 10.1\%$

(to the required level of accuracy)

b i \$3500

$$200 + (12 \times 275) = \$3500$$

b ii 13.8%

The interest charged is $3300 - 2900 = \$400$

Rate of interest = $\frac{400}{2900} \times 100 = 13.8\%$ (to the required level of accuracy)

c i $n = 24$ $P = 3100$ $A = 0$

n is the number of time periods i.e.

$$2 \times 12 = 24 \text{ months}$$

P is the original amount owing of \$3100

A is the amount owing after 24 months i.e. 0

c ii \$141.63

Using the annuities formula

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

$$0 = 3100 \times 1.0075^{24} - \frac{Q(1.0075^{24} - 1)}{1.0075 - 1}$$

$Q = 141.63$ (rounded upwards to enable the loan to be fully paid off)

Alternatively:

Use TVM solver

$$N = 24$$

$$I = 9$$

$$PV = -3100$$

$$PMT = \square$$

$$FV = 0$$

$$P \setminus Y = 12$$

$$C \setminus Y = 12$$

So $FV = \$141.63$

So monthly payment required = \$141.63

c iii \$3 399

$$\$141.63 \times 24 = \$3399 \text{ (to required level of accuracy)}$$

d Discount King is cheaper by \$101

Total cost for Crazy Bill is \$3500

Total cost for Discount King is \$3399

$$\$3500 - \$3399 = \$101$$

Question 2

a \$1904

Reducing balance book value after t years

$$= \text{purchase price} \times \left(1 - \frac{\text{depreciation rate}}{100}\right)^t$$

$$= 3100 \times \left(1 - \frac{15}{100}\right)^3$$

$$= 3100 \times 0.85^3$$

$$= \$1904 \text{ (to the required level of accuracy)}$$

b \$1750

Depreciation per year is $15000 \times \$0.03 = \450

Depreciation for 3 years is $3 \times \$450 = \1350

The value of the machine after 3 years will be

$$3100 - 1350 = \$1750$$

c 2.7 cents

The amount to be depreciated is

$$3100 - 1904 = \$1196$$

The number of cups in 3 years is

$$3 \times 15000 = 45000$$

$$\text{Unit cost is } \frac{1196}{45000} = \$0.0266$$

$$= 2.7 \text{ cents (to the required level of accuracy)}$$

Module 5 : Networks and decision mathematics

Question 1

a cut A = 14 cut B = 23 cut C = 12

The capacity of each cut is the sum of the weights of the edges which flow from left to right across the cut.

For cut A the capacity is $4 + 7 + 3 = 14$

For cut B the capacity is $4 + 1 + 8 + 7 + 3 = 23$

For cut C the capacity is $4 + 1 + 4 + 3 = 12$

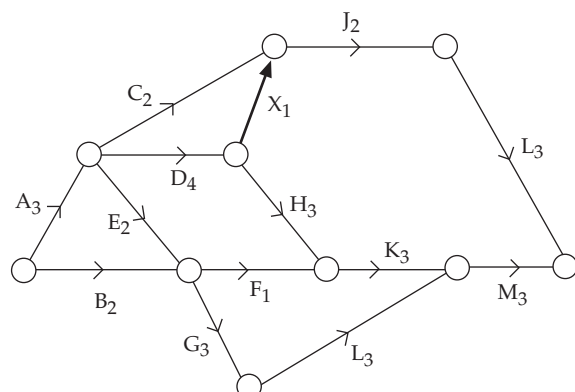
(The 8 is not included as the flow is right to left.)

- b Cut E does not isolate Bowen from Arlie.
Other acceptable comments are that “Cut E does not isolate the source from the sink” or “It is still possible to get from Arlie to Bowen”.
- c 12 seats
Maximum flow = the capacity of the ‘minimum cut’ (Cut C)

Question 2

- a** A (immediate predecessor of A)
8 (earliest start time of I)
13 (earliest start time of M)
- From the network diagram we see that before activity E can commence, activity A must finish therefore A is a predecessor to E. The earliest start time for I is the duration of the longest path leading to the start of I. This is the path A – E – G i.e. $3 + 2 + 3 = 8$
- The earliest start time for M is the duration of the longest path leading to the start of M. This is the path A – D – H – K i.e. $3 + 4 + 3 + 3 = 13$

b



X is drawn from the end of D to the beginning of J. It must have an arrow in this direction and a duration of 1.

In the table we see that X has D as predecessor therefore it begins where D finishes.

X is itself a predecessor to J therefore it must finish where J begins.

The earliest start time for J of 8 is the length of the path A – D – X (the longest path to J). Therefore $8 = 3 + 4 + \text{duration of X}$. The duration of X must be 1 hour.

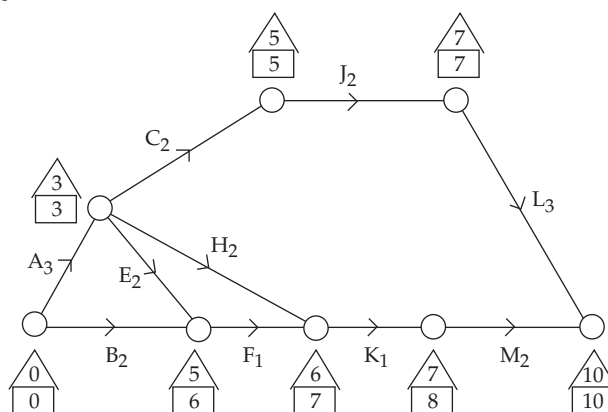
- c i** 16 hours
- Duration of A – D – H – K – M is $3 + 4 + 3 + 3 + 3 = 16$.
- c ii** Gives the minimum time for project completion.
- Or “any delay in the critical path will delay completion of the project”.
- d** 7 hours

The longest path involving B which runs for 11 hours is B – G – I – M .

Since the critical path is 16 hours, this means that B has a slack time of 5 hours.

Therefore the longest duration for B is its original 2 hours plus the slack time of 5 hours giving a maximum time of 7 hours.

Question 3



- a** 6 hours
- The earliest start time for K is the duration of the longest path leading to the start of K. This is the path A – E – F i.e. $3 + 2 + 1 = 6$
- b** A – C – J – L
- This is the longest path from beginning to completion hence giving the minimum time for project completion of 10 hours.
- c** 8 hours
- Since the earliest completion time is 10 hours and M has a duration of 2 hours, the latest time M can start is 8 hours.