

2003 Further Mathematics

Written Examination 1 (facts, skills and applications)

Suggested answers and solutions

Answers – Multiple Choice

SECTION A

Core: Data analysis

1. D 2. E 3. D 4. C 5. C
 6. B 7. B 8. B 9. A 10. C
 11. D 12. D 13. E

SECTION B

Module 1: Number patterns and relations

1. C 2. C 3. C 4. D 5. B
 6. A 7. B 8. B 9. E

Module 2: Geometry and trigonometry

1. E 2. C 3. C 4. D 5. A
 6. D 7. C 8. D 9. E

Module 3: Graphs and relations

1. C 2. A 3. C 4. D 5. B
 6. E 7. A 8. D 9. D

Module 4: Business related mathematics

1. C 2. C 3. D 4. C 5. B
 6. A 7. D 8. B 9. D

Module 5: Networks and decision mathematics

1. A 2. E 3. B 4. E 5. C
 6. C 7. D 8. B 9. B

Core

Question 1 [D]

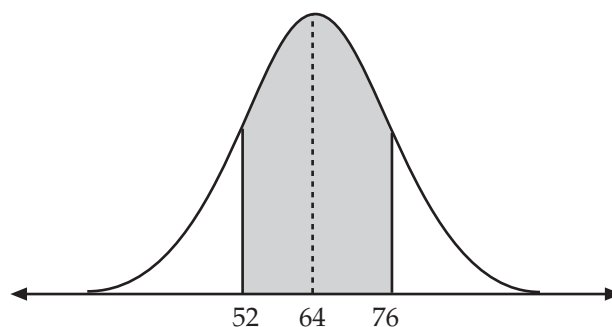
The median is the middle value when the 7 percentages are arranged in ascending order. The 4th value therefore is the median of -4.4%

Question 2 [E]

The range is the highest value – the lowest value.
 $2.9\% - (-5.5\%) = 8.4\%$

Question 3 [D]

68% of students would be within 1 standard deviation from the mean i.e. $64 - 12 = 52$ and $64 + 12 = 76$.



The **number** of students is therefore 68% of 2500 which is equal to 1700.

Question 4 [C]

The total weight is equal to the mean multiplied by the number of people i.e. $72 \times 12 = 864$.

Question 5 [C]

The additional 2 kg per person will raise the mean by 2 but the spread of data will be unaffected hence the standard deviation remains at 5.

Question 6 [B]

The number of standard size houses is $14 + 71 + 47 = 132$. Of these 132, 47 have a high level of water usage. $\frac{47}{132} \times 100 = 35.6\%$

Question 7 [B]

Each of the variables has been divided up into 3 distinct categories therefore they will be regarded as categorical rather than numerical.

Question 8 [B]

Substituting a study time of 35 into the regression equation

$$\begin{aligned}\text{Number of errors} &= 8.8 - 0.120 \times 35 \\ &= 4.6\end{aligned}$$

Question 9 [A]

We are given the coefficient of determination value $r^2 = 0.8198$. Taking the square root we obtain $r = \pm 0.91$. From the scatterplot we see that errors **decrease** as study time increases therefore there is a negative association.

Question 10 [C]

To linearise this data, the upper end of the x scale needs to be stretched (i.e. the points need to move right). This can only be done with the x^2 transformation.

Question 11 [D]

We use parallel boxplots when investigating the relationship between a numerical variable and a 2 or more level categorical variables. Resting pulse rate is numerical and fitness level has been divided into 3 categories.

Question 12 [D]

Column 3 below shows the first 2 moving averages. The centred figure in column 4 is the average of the 2 figures in column 3.

Year	Accidents	4 point moving	4 point centred
1993	13		
1994	7	8	
1995	3	7.25	7.625
1996	9		
1997	10		

Question 13 [E]

Seasonal indices always average out to 1 therefore the four indices must add up to 4.

$$\text{Index for quarter 4} = 4 - (1.05 + 0.84 + 0.92) = 1.19$$

Module 1: Number patterns and applications**Question 1 [C]**

The sequence is arithmetic with $a = 4$ and $d = 6$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\begin{aligned}S_{10} &= 5[2 \times 4 + 9 \times 6] \\ &= 310\end{aligned}$$

Question 2 [C]

$$t_n = a + (n-1)d$$

$$t_2 : a + d = 36 \quad (1)$$

$$t_4 : a + 3d = 20 \quad (2)$$

Solving simultaneously

$$(2) - (1) \quad 2d = -16 \Rightarrow d = -8$$

Substitute in (1) to give $a = 44$

Question 3 [C]

The sequence is arithmetic with $a = 47$ and $d = -4$

$$t_n = a + (n-1)d$$

$$\begin{aligned}t_{11} &= 47 + (10 \times -4) \\ &= 7\end{aligned}$$

Question 4 [D]

There are 480 adults consisting of 180 men and 300 women

Ratio of men to women is 180:300

In simplest form 3:5

Question 5 [B]

$$\text{To be geometric} \quad \frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$$\text{For the sequence in B} \quad \frac{t_2}{t_1} = \frac{1.1}{1} = 1.1 \text{ whereas}$$

$$\frac{t_3}{t_2} = \frac{1.01}{1.1} \approx 0.918$$

Question 6 [A]

Rate of outflow is 120 litres per minute.

3 hours is 180 minutes therefore possible for $180 \times 120 = 21600$ litres to flow out.

Since tank holds less than this initially, no water will be left after 3 hours.

Question 7 [B]

The growth amount per month forms a geometric sequence with $a = 8$ and $r = \frac{1}{2}$

Since the growth continues indefinitely we use the formula for infinite series

$$\begin{aligned} S_{\infty} &= \frac{a}{1-r} \\ &= \frac{8}{1-\frac{1}{2}} = 16 \end{aligned}$$

Adding the growth of 16 cm to the original height of 20 cm gives the maximum height of 36 cm.

Question 8 [B]

The terms generated by this difference equation are 100, 40, 16, 6.4, 2.56

This is a geometric sequence as it has a common ratio of 0.4

The numbers are decreasing but will approach 0 and will therefore always be positive

Question 9 [E]

For the difference equation $t_{n+1} = 30 - t_n$, each new term after the first is 30 minus the previous term.

$$\begin{aligned} t_2 &= 30 - t_1 \\ &= 30 - 20 \\ &= 10 \\ t_3 &= 30 - t_2 \\ &= 30 - 10 \\ &= 20 \quad \text{etc.} \end{aligned}$$

Module 2: Geometry and trigonometry**Question 1 [E]**

Angle QPR is equal to $180^\circ - (58^\circ + 73^\circ) = 49^\circ$

Therefore angle SPQ is equal to $180^\circ - 49^\circ = 131^\circ$

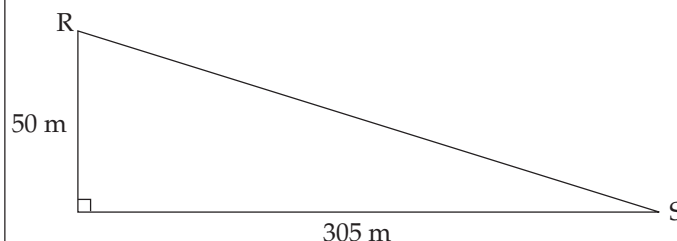
Question 2 [C]

Using the sine rule

$$\begin{aligned} \frac{PQ}{\sin 73^\circ} &= \frac{36}{\sin 58^\circ} \\ PQ &= \frac{36 \sin 73^\circ}{\sin 58^\circ} \\ &= 40.6 \text{ cm (to 1 decimal place)} \end{aligned}$$

Question 3 [C]

Forming a right angled triangle



$$\begin{aligned} RS^2 &= 50^2 + 305^2 \\ RS &= \sqrt{95525} \\ &= 309 \text{ m (to the nearest metre)} \end{aligned}$$

Question 4 [D]

$$\begin{aligned} \tan S &= \frac{50}{305} \\ S &= 9^\circ \text{ (to the nearest degree)} \end{aligned}$$

The bearing of R from S is $270^\circ + 9^\circ = 279^\circ$

Question 5 [A]

The scale is 1 : 10 000 therefore 5 cm on the map corresponds to 50 000 cm on the ground.

To convert from cm to km we divide by 100 000.

$$\frac{50\,000}{100\,000} = 0.5 \text{ km}$$

Question 6 [D]

As you move along through each of the sections along line PU, the elevation increases by 50 metres. Since ST is the smallest segment of the line it will be steepest as it has a 'rise' of 50 to the smallest 'run'.

Question 7 [C]

The ratio of the lengths of the enlarged model to the original container is 5 : 1

Since they are similar figures the area ratio is

$$5^2:1^2 \text{ i.e. } 25:1$$

$$25 \times 220 = 5500 \text{ cm}^2$$

Question 8 [D]

Using the cosine rule

$$\cos TUV = \frac{2.1^2 + 3.5^2 - 4.2^2}{2 \times 2.1 \times 3.5}$$

angle $TUV = 93.8^\circ$ (to 1 decimal place)

Question 9 [E]

Let the diameter of the smaller cone be x .

Using similar figures

$$\frac{x}{9} = \frac{12}{15}$$

$$x = \frac{12 \times 9}{15} = 7.2 \text{ cm}$$

Module 3: Graphs and relations**Question 1** [C]

The engine speed was above 2200 rpm between time = 4 and 8 and again between time = 20 and 28. This makes a total of 12 seconds.

Question 2 [A]

Substituting $x = 3$ and $y = -2$ into the equation

$$y = \frac{k}{x^2}$$

$$-2 = \frac{k}{9}$$

$$k = -18$$

Question 3 [C]

Find the gradient of the interval from (0,8) to (3,6)

$$m = \frac{6-8}{3-0} = -\frac{2}{3}$$

Therefore the gradient for the interval from (3,6) to (c,0)

$$-\frac{2}{3} = \frac{6-0}{3-c}$$

cross multiplying

$$-6 + 2c = 18$$

$$2c = 24 \Rightarrow c = 12$$

Question 4 [D]

The revenue from selling 1000 copies of the guide is \$25000 **not** \$15000. Therefore D is not a true statement.

Question 5 [B]

Profit = Revenue – Cost

$$P = 25x - (15000 + 15x)$$

$$= 25x - 15000 - 15x$$

$$= 10x - 15000$$

Question 6 [E]

Simultaneous equations

$$2x - 3y = 7 \quad (1)$$

$$3x + y = 5 \quad (2)$$

$$(2) \times 3 \quad 9x + 3y = 15 \quad (3)$$

$$(1) + (3) \quad 11x = 22 \Rightarrow x = 2$$

$$\text{sub } x = 2 \text{ in } (2) \quad 6 + y = 5 \Rightarrow y = -1$$

Question 7 [A]

Taking each of the constraints individually ;

The region must be to the right of the y axis, to the left of the line

$x = 2$, above the x axis, to the right of the line

$3x - y = 3$ and below the line $x + y = 3$.

Question 8**[D]**

Test each of the extreme points of the feasible region.

$$\text{at } (0,80) \quad Z = 5 \times 0 + 3 \times 80 = 240$$

$$\text{at } (40,60) \quad Z = 5 \times 40 + 3 \times 60 = 380$$

$$\text{at } (70,30) \quad Z = 5 \times 70 + 3 \times 30 = 440$$

$$\text{at } (85,0) \quad Z = 5 \times 85 + 3 \times 0 = 425$$

The maximum is therefore at (70,30)

Question 9**[D]**

At least 4 hours in the nursery : $x \geq 4$

At most 16 hours in the restaurant : $y \leq 16$

A total of at least 18 hours : $x + y \geq 18$

Restaurant hours at least twice nursery hours :
 $y \geq 2x$

Module 4: Business-related mathematics**Question 1****[C]**

The balance after the deposit is made is
 $960 - 120 = \$840$.

This amount is paid in 6 equal monthly payments
of $\frac{840}{6} = \$140$

Question 2**[C]**

Using the simple interest formula $I = \frac{PRT}{100}$
where $T = 1.5$ years (18 months)

$$975 = \frac{26\,000 \times R \times 1.5}{100}$$

$$39\,000R = 97500$$

$$R = 2.5$$

Question 3**[D]**

Using the rule for compound interest

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

$$A = 45000 \times 1.04^5$$

$$A = \$54749.38$$

This is the value of the investment after 5 years.

The interest earned is therefore

$$\$54749.38 - \$45000 = \$9749.38$$

which is closest to \$9750

Question 4**[C]**

Using the annuities formula

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

$$P = 150\,000$$

$$R = 1 + \frac{6.2}{100} = 1.0052$$

$$Q = 1100 \quad 5 \times 12 = 60$$

$$A = 150\,000 \times 1.0052^{60} - \frac{1100(1.0052^{60} - 1)}{0.0052}$$

$$A = \$127\,207.81 \text{ which is closest to } \$127\,000$$

Alternatively:

Use TVM solver

$$N = 60$$

$$I = 6.2$$

$$PV = -150000$$

$$PMT = 1100$$

$$FV = \square$$

$$P/Y = 12$$

$$C/Y = 12$$

$$\text{So } FV = \$127207.81$$

$$\approx \$127000$$

Question 5**[B]**

The amount to be depreciated over the 5 years is
 $6500 - 2000 = \$4500$

Since the straight line method is to be used for the 5 years, the equipment is depreciated annually by

$$\frac{4500}{5} = \$900$$

Question 6**[A]**

Interest is 3% per annum therefore $\frac{3}{12} = 0.25\%$
per month

$$R = 1 + \frac{0.25}{100} = 1.0025$$

After one month the balance is

$$12\,200 \times 1.0025 - 5 = \$12\,225.50$$

After two months the balance is

$$12\,225.5 \times 1.0025 - 5 = \$12\,251.06$$

Question 7**[D]**

Using the rule for compound interest

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

 $r\%$ per annum therefore $\frac{r}{4}$ per quarter.

$$t = 5 \times 4 = 20$$

$$A = 8000\left(1 + \frac{\frac{r}{4}}{100}\right)^{20}$$

$$A = 8000\left(1 + \frac{r}{400}\right)^{20}$$

Question 8**[B]**

The minimum monthly balances are

April : 6452.40

May : 5992.40

June : 5992.40

Interest for the 3 months =

$$\frac{0.25}{100}(6452.40 + 5992.40 + 5992.40) = \$46.09$$

The balance after interest is paid is

$$5992.40 + 46.09 = \$6038.49$$

Question 9**[D]**

This is best done by a process of elimination using the TVM solver.

The term of the loan can be worked out using N to be solved

I%	5.6
PV	-80000
PMT	555
FV	0
P/Y	12
C/Y	12
PMT:END	

Solving for N gives 239.87 months

Comparing this to weekly repayments of \$132 compounding weekly.

N to be solved

I%	5.6
PV	-80000
PMT	132
FV	0
P/Y	52
C/Y	52
PMT:END	

Solving for N gives 982.5 weeks

Converting to months ($\times \frac{12}{52}$) gives 226.62 months therefore a reduction in the period of the loan.

Module 5: Networks and decision mathematics**Question 1****[A]**

Tina is unable to do task 1 which eliminates B.

Peter is unable to do task 2 which eliminates C.

Peter is unable to do task 1 which eliminates D and E.

Question 2**[E]**

No other vertex can be reached from vertex Q.

P, M and L all are directed towards Q but no arrows go the other way.

Question 3**[B]**

An Euler path includes every edge just once but does not start and finish at the same vertex. It will exist if the only two vertices which have an odd degree are the start and the finish.

In the original network, vertices S, U, W and Z are odd therefore 2 of them need to be joined to create 2 more even degree vertices. The only option that allows this is B.

Question 4**[E]**

The degree of a vertex is the number of edges incident with it.

The sum for the 6 vertices (from left to right) is $3 + 3 + 2 + 3 + 4 + 1 = 16$.

Question 5

[C]

Using Euler's formula $v - e + f = 2$

$$v - 20 + f = 2$$

$$v + f = 22$$

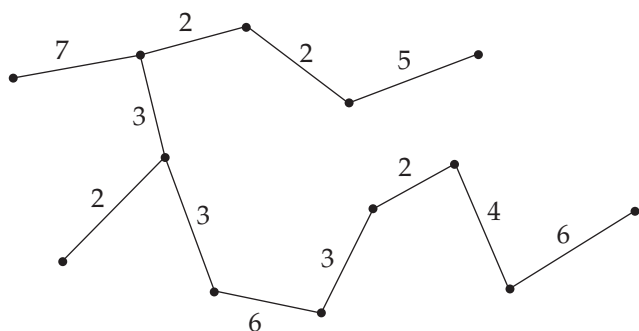
Since there is an equal number of vertices and faces they must both be 11.

Question 6

[C]

The minimum spanning tree can be constructed using Prim's algorithm.

Firstly take a vertex and connect it to a second vertex so that weight of the edge is as small as possible. For each subsequent step take the edge with the lowest weight to produce a tree with the edges already selected.



The answer is

$$7 + 2 + 2 + 5 + 3 + 2 + 3 + 6 + 3 + 2 + 4 + 6 = 45$$

Question 7

[D]

The critical path is the longest path from start to finish. The length of time of the critical path gives the minimum time for the project to be completed. The times for each path are

$$J - P - U \quad 3 + 6 + 1 = 10$$

$$K - R - T - U \quad 5 + 5 + 9 + 1 = 20$$

$$J - M - O - S - U \quad 3 + 8 + 6 + 1 + 1 = 19$$

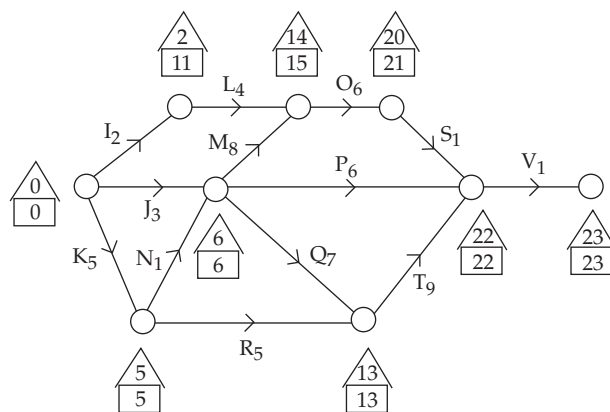
$$K - N - Q - T - U \quad 5 + 1 + 7 + 9 + 1 = 23$$

$$K - N - M - O - S - U \quad 5 + 1 + 8 + 6 + 1 + 1 = 22$$

Therefore $K - N - Q - T - U$ is the critical path.

Alternative

Use Forward and Backward Tracking



So critical path is K N Q T U (where $\triangle = \square$)

Question 8

[B]

The shortest time is the path from home that has times

$$7 + 2 + 2 + 2 + 3 + 7 = 23.$$

Question 9

[B]

A graph is connected if every pair of distinct vertices is joined by an edge, that is there is no isolated vertices.

When drawn we see that the graph described by the matrix N has no isolated vertices.

