

MATHEMATICAL METHODS (CAS)

Unit 1 – Written examination 2



2007 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: A

Explanation:

Gradient of 2, $c = 0$, so the equation is $y = 2x$

Question 2

Answer: E

Explanation:

Parallel lines have the same gradient, so $y = -3x - 5$

Question 3

Answer: B

Explanation:

Midpoint is $\left(\frac{0.5+1.5}{2}, \frac{3+8}{2}\right) = \left(1, \frac{11}{2}\right)$

Question 4

Answer: C

Explanation:

$$\text{Since } \tan \theta = \frac{y}{x} = \frac{1}{\sqrt{3}}, \theta = 30^\circ$$

Question 5

Answer: C

Explanation:

$$(x^{\frac{3}{2}} + 2)(x^{\frac{3}{2}} - 3) = x^3 - 3x^{\frac{3}{2}} + 2x^{\frac{3}{2}} - 6 = x^3 - x^{\frac{3}{2}} - 6$$

Question 6

Answer: C

Explanation:

$$x^3 + 3x^2 - 3.25x - 5.25 \text{ can be factorised to } (x + 3.5)(x + 1)(x - 1.5)$$

Question 7

Answer: D

Explanation:

Using the factor theorem, a factor of $f(x)$ must be $x - 2$

Question 8

Answer: D

Explanation:

$$3 \leq 4x - 5 \leq 7 \text{ becomes } 8 \leq 4x \leq 12, \text{ which simplifies to } 2 \leq x \leq 3$$

Question 9

Answer: A

Explanation:

Solving $x^2 + 6x - 4 = 0$ using the quadratic formula, $x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-4)}}{2(1)}$

Becomes $x = -3 \pm \frac{1}{2}\sqrt{52}$. Which simplifies to $x = -3 \pm \sqrt{13}$

Question 10

Answer: E

Explanation:

Check the discriminant of the quadratic equation: $\sqrt{1^2 - 4(1)(4)} = \sqrt{-15}$, so there are no real solutions (i.e. the graph does not touch the x -axis).

Question 11

Answer: A

Explanation:

$$0.5x + 1.5y - 0.3z = 1.1$$

$$1.0x - 0.6y + 0.2z = 0.8 \text{ simplifies to } x = -0.5, y = 5.5, z = 23$$

$$0.3x + 0.5y - 0.1z = 0.3$$

Question 12

Answer: E

Explanation:

Solve for $x^2 - 4x - 3 = x - 3$ to get $x = 0$ ($y = -3$) and $x = 5$ ($y = 2$).

Question 13

Answer: E

Explanation:

This hyperbola has vertical asymptote at $x = -4$, so its Domain is $\mathbb{R} \setminus \{-4\}$. It also has a horizontal asymptote at $y = -1$, so its Range is $\mathbb{R} \setminus \{-1\}$.

Question 14

Answer: D

Explanation:

This is a union question, so all negative real numbers, and 0 up to π (exclusive) are included:
 $(-\infty, \pi)$

Question 15

Answer: D

Explanation:

The vertical line $x = 4$ is not a function, as it fails the vertical line test.

Question 16

Answer: B

Explanation:

The function is dilated a units in the y -direction.

Question 17

Answer: B

Explanation:

$$4a - 2b + c = -4$$

Generate the following simultaneous equations $0.25a + 0.5b + c = 2.25$, and solve to

$$a + b + c = 8$$

get $a = 3, b = 7, c = -2$, so the equation is $3x^2 + 7x - 2$

Question 18

Answer: A

Explanation:

In factorised form, the equation is $y = (x + 3)(x + 1)(x - 1)$, but upon expansion, and multiplied through by 2, it becomes $2x^3 + 6x^2 - 2x - 6$

Question 19

Answer: E

Explanation:

Properly written, the correct answer is $\begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

Question 20

Answer: E

Explanation:

Solving the equation in **question 19**, we get the answer $\begin{bmatrix} 6 \\ -8 \end{bmatrix}$, which is the point (6, -8)

Question 21

Answer: A

Explanation:

$$f(5) = 25 + 15 - 5 = 35 \text{ and } f(0) = -5, \text{ so } \frac{35 - (-5)}{5} = 8$$

Question 22

Answer: C

Explanation:

The tangent has gradient $-\frac{5}{3} \approx -1.7$

Question 23

Answer: B

Explanation:

Solve $0.7 = 0.3 + x - 0.1$, to get $\Pr(B) = x = 0.5$

Question 24

Answer: D

Explanation:

$$\Pr(A') = 1 - 0.8 = 0.2$$

Question 25

Answer: D

Explanation:

$$\text{Solve } \frac{0.35}{0.55} = \frac{7}{11}$$

Question 26

Answer: C

Explanation:

They are independent events, so $\Pr = \frac{1}{6} \times \frac{1}{20} = \frac{1}{120}$

SECTION 2: Analysis questions**Question 1**

a. $\{(x, y) : x^2 - 4x + y^2 + y = 4.75\}$ becomes $(x^2 - 4x + 4) + \left(y^2 + y + \frac{1}{4}\right) - \frac{19}{4} - 4 - \frac{1}{4} = 0$

(M2 one for each of the two completion of the squares)

$$(x - 2)^2 + \left(y + \frac{1}{2}\right)^2 - 9 = 0$$

M1

Therefore, $(x - 2)^2 + \left(y + \frac{1}{2}\right)^2 = 9$

A1

b. Marking scheme: Centre identified at $\left(2, -\frac{1}{2}\right)$

M1

Appropriate radius of 3

M1

x-intercept at -0.96

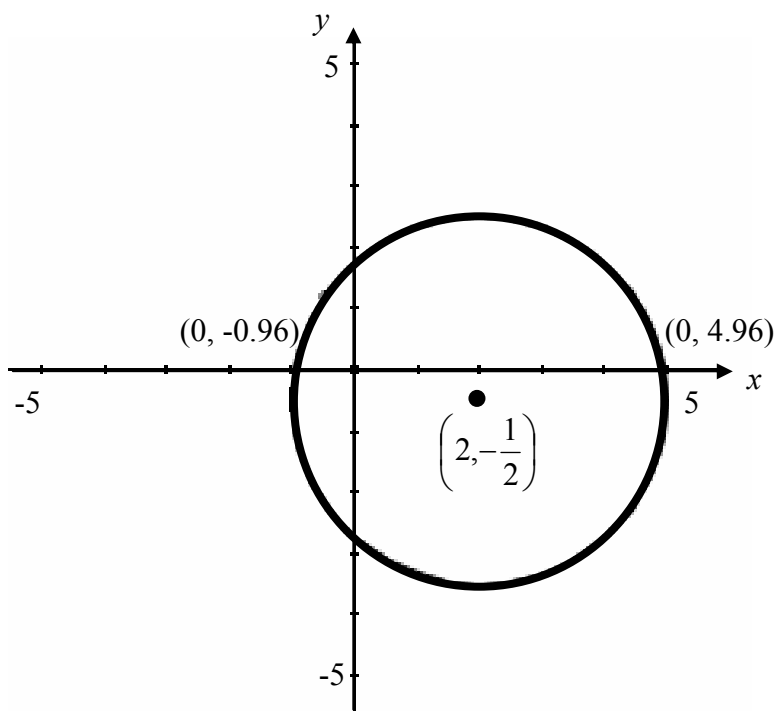
M1

x-intercept at 4.96

M1

Good attempt at drawing a circle

M1



c. Domain: $[-1, 5]$

A1

Range: $\left[-\frac{7}{2}, \frac{5}{2}\right]$

A1

d. Marking scheme: Centres labeled for both circles

M1

Radius of 4 for larger and 1 for smaller circle

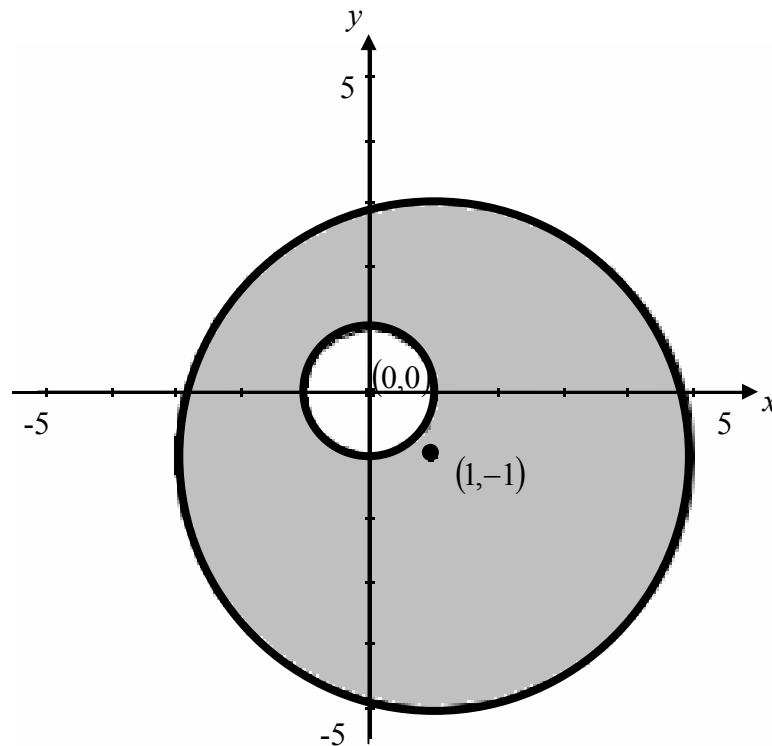
M1

Accurate graph of two circles

M1

The required region will be outside of the smaller circle, but inside the larger, forming a ring shape.

M1



Total 15 marks

Question 2

- a.** The point (3, -2) represents the 'centre' of the hyperbola (vertical asymptote at $x = 3$ and horizontal asymptote at $y = -2$).

Therefore, $h = 3$ **and** $k = -2$.

A2

Use the point (5, 0) to solve for a : $0 = \frac{a}{5-3} - 2$,

M1

where $a = 4$.

A1

Therefore, the equation is $y = \frac{4}{x-3} - 2$

- b.** Switch x and y to get: $x = \frac{4}{y-3} - 2$

M1

This simplifies down to $f^{-1}(x) = \frac{4}{x+2} + 3$

A1

- c.** Marking scheme: Vertical asymptote at $x = -2$

M1

Horizontal asymptote at $y = 3$

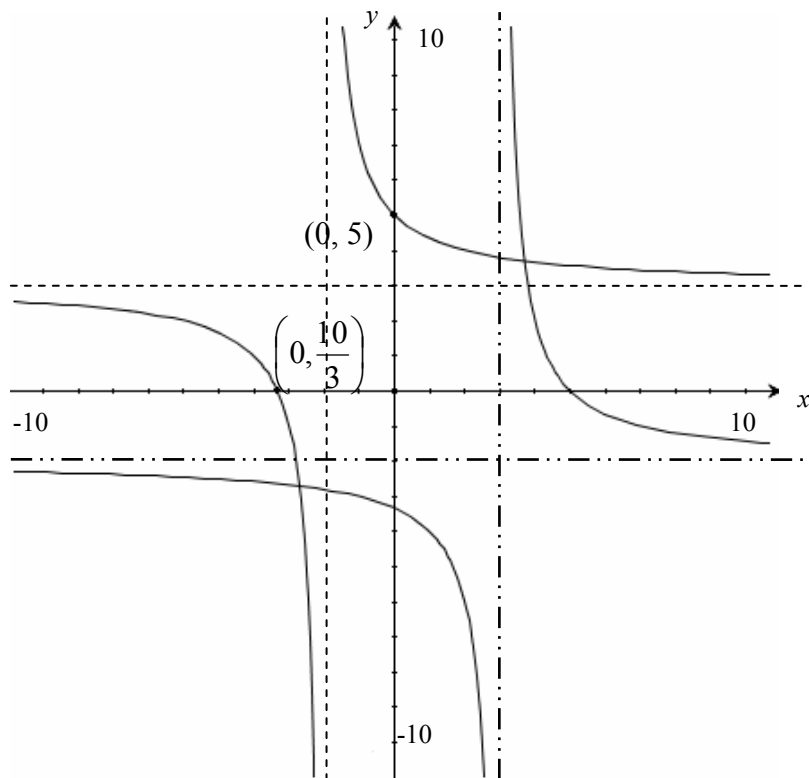
M1

x -intercept at $-\frac{10}{3}$

M1

y -intercept at (0, 5)

M1



Accurate graph

A1

- d. This implies that $h = -2$ and $k = 4$, yielding the equation $y = \frac{a}{x+2} + 4$

A1

Solve for $5 = \frac{a}{1+2} + 4$ to get $a = 3$

M1

Therefore, the equation is $y = \frac{3}{x+2} + 4$

A1

Total 14 marks

Question 3

a. $6 + 2 + 4 + 1 + 5 = 18 \text{ cm}$

M1, A1

b.

i. Gradient of OA is 3, so 3 cm/s

A1

ii. Gradient of AB is 1, so 1 cm/s

A1

iii. Gradient of BC is $-\frac{4}{3}$, so $-\frac{4}{3}$ cm/s

A1

c. Marking scheme: Horizontal lines **and** one open endpoint for each

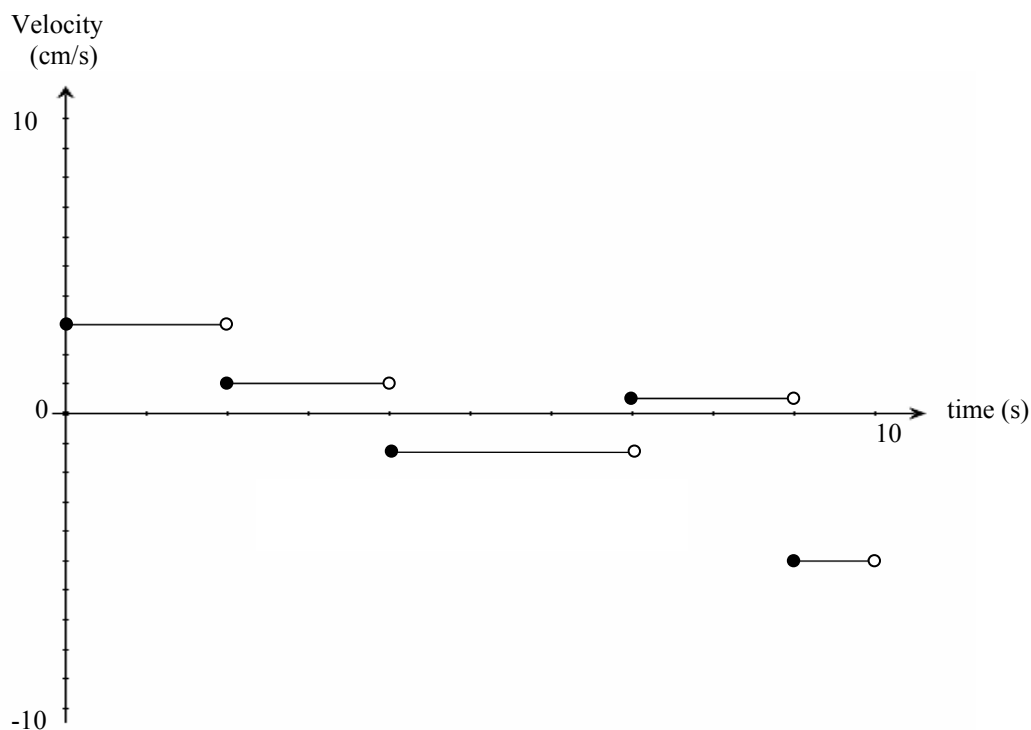
M2

Appropriate labeling of axes

M1

Accurate graph

M1



d. Calculate the area between the v - t graph and the x -axis:

$$A = (3 \times 2) + (1 \times 2) + \left(\left| \frac{4}{3} \right| \times 3 \right) + \left(\frac{1}{2} \times 2 \right) + (5 \times 1)$$

M2

$$= 6 + 2 + 4 + 1 + 5 = 18 \text{ cm/s}$$

A1

Total 12 marks

Question 4

a. Marking scheme: Key to terms and letters

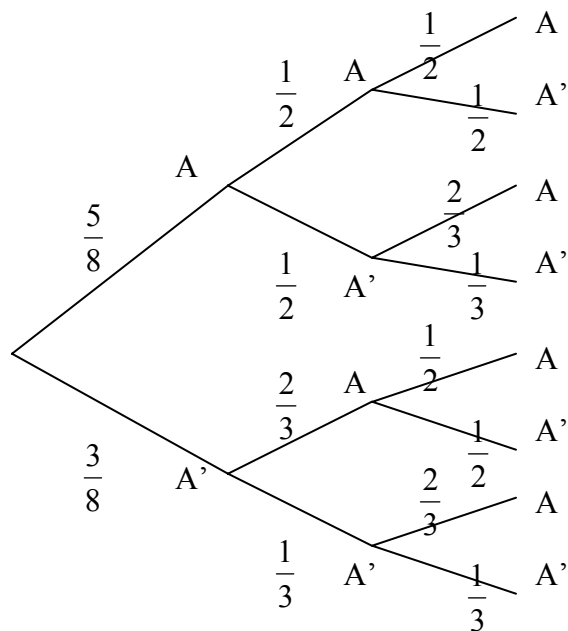
Correct construction of table

M1

Correct probabilities

M2

A1



KEY
A = Australia wins
A' = England wins

b. $\Pr(\text{Australia wins all three}) = \frac{5}{8} \times \frac{1}{2} \times \frac{1}{2}$

M1

$$= \frac{5}{32} \approx 0.16$$

A1

c. $\Pr(\text{England wins match one only}) = \frac{3}{8} \times \frac{2}{3} \times \frac{1}{2}$

M1

$$= \frac{6}{48} \approx 0.13$$

A1

d. $\Pr(\text{Tied 1-1}) = \left(\frac{5}{8} \times \frac{1}{2}\right) + \left(\frac{3}{8} \times \frac{2}{3}\right)$

M1

$$= \frac{5}{16} + \frac{6}{24} = \frac{9}{16} \approx 0.56$$

A1

e. $\Pr(\text{England lead 2-1}) = \left(\frac{3}{8} \times \frac{1}{3} \times \frac{2}{3}\right) + \left(\frac{3}{8} \times \frac{2}{3} \times \frac{1}{2}\right) + \left(\frac{5}{8} \times \frac{1}{2} \times \frac{1}{3}\right)$

M2

$$= \frac{6}{72} + \frac{6}{48} + \frac{5}{48} = \frac{5}{16} \approx 0.31$$

A1

Total 13 marks