

The Mathematical Association of Victoria
FURTHER MATHEMATICS
SOLUTIONS: Trial Exam 2014

Written Examination 2

CORE

Question 1

- (a) (i) The median occurs between values 11 and 12 in the ordered ranking.
 this value is 50. A1
- (ii) $Q1$ (6th value in ordered data) = 47,
 $Q3$ (17th value in ordered data) = 53,
 hence $IQR = 53 - 47 = 6$ A1
- (b) Upper fence limit = $Q3 + 1.5 \times IQR = 53 + 1.5 \times 6 = 53 + 9 = 62$ M1
 As the value of 63 > upper fence limit, it is an outlier A1

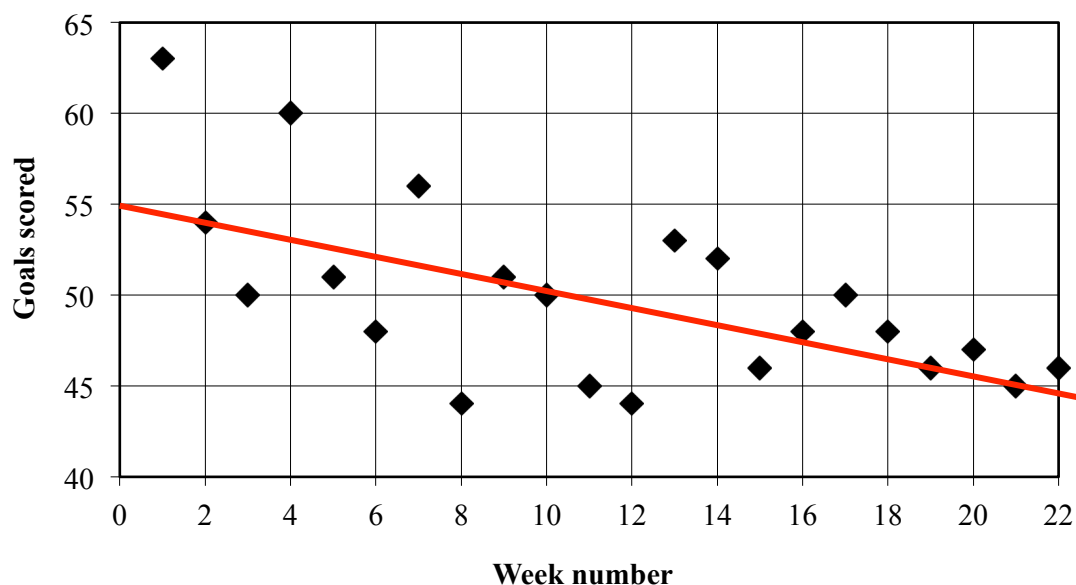
Question 2

No.	1	2	3	4	5	6	7	8	9	10
Value	1	2	3	3 or 4	4	5	5 or 6 or 7 or 8	8	9	10

Key values at numbers 3 (3), 5 (4), 6 (5), 8 (8) A1
 A CORRECT selection of the alternatives for the other values at numbers 4 and 7. A1

Question 3

- (a) A1



Key points (0, 55) (21, 45)

- (b) (i) 38.8% of the variation in **Goals scored** values can be accounted for the variation in the number of **Week number**. A1
- (ii) $r = \pm \sqrt{0.3883} = -0.623$ (slope of line !) A1
- (c) goals scored = $55 - 0.48 \times 12 = 49$ A1
- (d) Residual = actual – predicted = $44 - 49 = -5$ A1

Question 4

- (a) $(\text{fitness score})^2 = 13821.6 + 353.4 \times \text{age}$ A1
- (b) $(\text{fitness score})^2 = 13821.6 + 353.4 \times 27 = 4279.8$,
 $\text{fitness score} = \sqrt{4279.8} = 65.4$ A1

Question 5

mean + 2 standard deviations = $60.6 + 2 \times 14.7 = 90$

From normal distribution, mean $\pm$ 2 standard deviations encompasses 95 %
 leaving 2.5 % at each end.

2.5 % of 240 = 6

6 netballers had a fitness score above 90.

M1

A1

MODULE 1 : NUMBER PATTERNS**Question 1**

- (a) $a = 1.3$, $d = 1.65 - 1.3 = 0.35$
 $t_5 = 1.3 + (5 - 1) \times 0.35 = 1.3 + 4 \times 0.35 = 2.7$ A1
- (b) $4 = 1.3 + (n - 1) \times 0.35$, giving $n = \frac{4 - 1.3 \times 0.35}{0.35} = 8.7$
 Hence 9th day A1
- (c) Day 3 total = $1.3 + 1.65 + 2 = 4.95$
 Day 10 total = $\frac{10}{2}(2 \times 1.3 + 9 \times 0.35) = 28.75$ M1
 Required distance = $28.75 - 4.95 = 23.8$ H1
- OR
 Day 4 distance = $2 + 0.35 = 2.35$ M1
 Sum distances days (4 – 10) = $2.35 + 2.70 + 3.05 + 3.40 + 3.75 + 4.1 + 4.45 = 23.8$ H1
- (d) Day 31 total = $\frac{31}{2}(2 \times 1.3 + 30 \times 0.35) = 203.05$ A1

MODULE 2 : GEOMETRY & TRIGONOMETRY**Question 1**

$$(a) \quad \text{Volume} = 0.5 \times 2 \times 25 \times 110 = 2750 \quad \text{A1}$$

$$(b) \quad DE = \sqrt{25^2 + 2^2} = 25.08 \quad \text{M1}$$

$$\text{Total fence} = 110 + 110 + 25.08 + 25.08 = 270.16 = 270.2 \quad \text{A1}$$

$$(c) \quad \begin{aligned} \text{Volume} &= 110 \times 25 \times 0.1 & \text{M1} \\ &= 275 & \text{A1} \end{aligned}$$

$$(d) \quad \begin{aligned} (i) \quad 327^\circ + 90^\circ - 360^\circ &= 057^\circ & \text{A1} \\ (ii) \quad 327^\circ - 180^\circ &= 147^\circ & \text{A1} \end{aligned}$$

Question 2

$$(a) \quad \text{Area paint} = 3(\pi \times 0.45^2 + \pi \times 4.9^2) = 3(0.64 + 75.43) = 228.21 \quad \text{A1}$$

$$(b) \quad \begin{aligned} \text{Litres paint} &= \frac{2 \times 228.21}{12} = 38.04 \\ \text{Number of cans} &= \frac{38.04}{4} = 9.51 = 10 & \text{A1} \end{aligned}$$

Question 3

$$(a) \quad AB = \sqrt{7^2 + 17^2} = 18.38 = 18 \quad \text{A1}$$

$$(b) \quad AC = \sqrt{17^2 + 23^2 - 2 \times 17 \times 23 \times \cos 93^\circ} = 29.31 \quad \text{A1}$$

$$(c) \quad \text{angle} = \tan^{-1}\left(\frac{7}{23}\right) = 17^\circ \quad \text{A1}$$

$$(d) \quad \text{Height} = 32 \times \tan 15^\circ = 8.57 = 8.6 \quad \text{A1}$$

Question 4

$$\text{Ratio radii} = \frac{\sqrt[3]{6044}}{\sqrt[3]{4540}} = 1.1 \quad \text{M1}$$

$$\text{Hence multiplying factor for area} = 1.1^2 = 1.21 \quad \text{A1}$$

MODULE 3 : GRAPHS & RELATIONS**Question 1**

(a) $R = 3.5x + 2.2y$

A1

(b) 87 hotdogs

A1

$$579.50 = 3.5x + 2.2 \times 125$$

$$x = \frac{579.50 - 2.2 \times 125}{3.5} = 87 \text{ hotdogs}$$

(c) $P = 2.3x + 1.4y$

A1

$P = R - C$

$P = 3.5x + 2.2y - (1.2x + 0.8y)$

$P = 2.3x + 1.4y$

Question 2

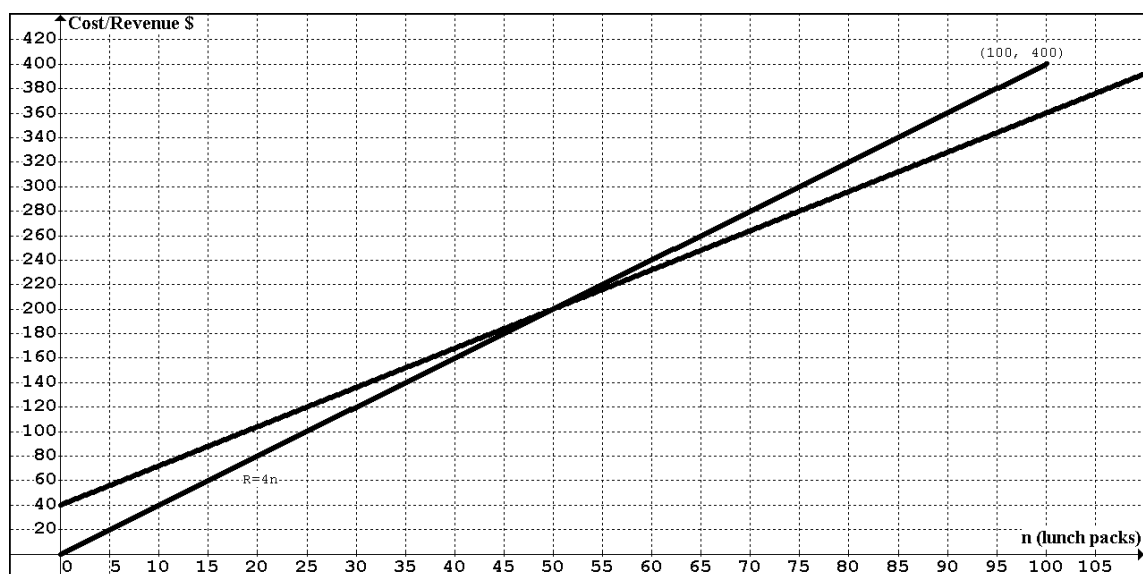
(a) $b = 40$

A1

(b) The cost of the delivery is \$40.

A1

(c)



Note: the line must not extend beyond the point (100, 400).

A1

(d) 51 lunch packs

A1

The break-even point is where $R = C$.

$4n = 3.5n + 40$

$n = 50$

Therefore a profit would first be made when the canteen sold 51 lunch packs.

Question 3

(a) \$20 if they have 2 deliveries of between 60 and 100 lunch packs (for example 75 in each delivery) A1

(b) $a = -0.5$ A1
 $b = 60$ A1

The line passes through (40, 40), (60, 30) and (100, 10).

$$m = \frac{10 - 40}{100 - 40} = -0.5 = a$$

$y = mx + c$ using $m = -0.5$ and point (40,40)

$$40 = -0.5 \times 40 + c$$

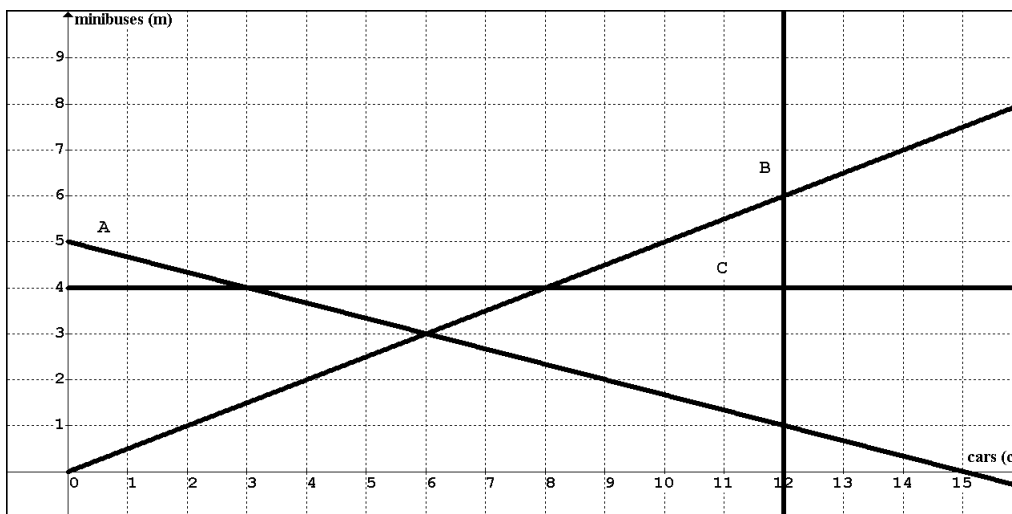
$$c = 60 = b$$

Question 4

(a) A1

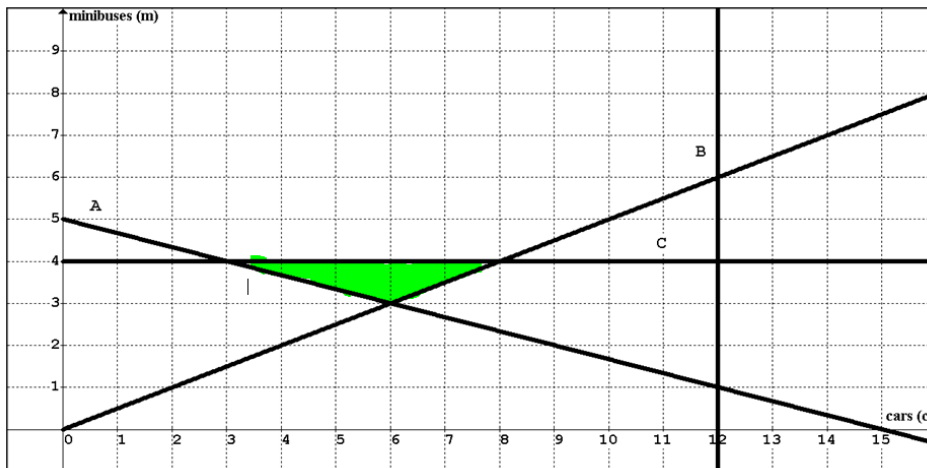
Line 1	C
Line 2	A
Line 3	B

(b) $c \leq 12$ A1



A1

(c) (3, 4), (4, 4), (5, 4), (6, 4), (7, 4), (8, 4) and (6, 3)



Accept either region shaded OR points OR both

A1

(d) A maximum of \$90

A1

The gradient of the objective function must be negative, but closer to zero than the gradient of $4c + 12m = 60$.

The gradient of $4c + 12m = 60$ is given by transposing the equation to $m = -\frac{c}{3} + 5$.

The gradient of this line is therefore, $\text{gradient} = -\frac{1}{3}$.

The objective function would be $C = ac + 270m$, where a is the cost of each car. This equation can be transposed to $m = -\frac{a}{270} + \frac{c}{270}$.

$$-\frac{a}{270} > -\frac{1}{3}$$

$$\frac{a}{270} < \frac{1}{3}$$

$$a < \frac{1}{3} \times 270 < 90 \quad \text{Therefore the amount available for each car must be less than \$90.}$$

- (e) \$140 A1

Activities B, H and I all need to be reduced. Activities C and D should not be paid for because reducing these activities has no effect on the length of time as neither of them are on the critical path before or after reduction.

MODULE 6 : Matrices

Question 1

- (a) $F = [40 \ 60 \ 80 \ 30]$ A1
- (b) The number of columns in F is 4, which is the same as the number of rows in N, so the product FN exists. $FN = (1 \times 4) \times (4 \times 2)$ results in a (1×2) matrix. A1
- (c) $FN = M = [6550 \ 5540]$ A1
- (d) The element m_{12} represents the total amount of money taken in annual fees from the female members of the club. A1
- (e) $G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ A1

Question 2

- (a) $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} C \\ P \\ H \end{bmatrix} = \begin{bmatrix} 226 \\ 206 \\ 139 \end{bmatrix}$ A1
- (b) $D = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 2 \\ 4 & -2 & -3 \end{bmatrix}$ A1
- (c) $-2 \times 226 + 1 \times 206 + 2 \times 139 = \32 A1

Question 3

- (a) Each week players move to a different training component and none repeat the same activity in consecutive weeks. A1

(b) 40 players will do football skills

A1

$$\begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.5 & 0 & 0.6 \\ 0.5 & 0.8 & 0 \end{bmatrix}^2 \times \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 30 \\ 30 \\ 40 \end{bmatrix}$$

(c) Must show 2 equal and subsequent state matrices rounding to $\begin{bmatrix} 23 \\ 36 \\ 41 \end{bmatrix}$

A1

$$\begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.5 & 0 & 0.6 \\ 0.5 & 0.8 & 0 \end{bmatrix}^{20} \times \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.5 & 0 & 0.6 \\ 0.5 & 0.8 & 0 \end{bmatrix}^{21} \times \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 23.42.. \\ 36.04.. \\ 40.54.. \end{bmatrix} \approx \begin{bmatrix} 23 \\ 36 \\ 41 \end{bmatrix}$$

Question 4

(a) 39 firsts, 56 seconds and 25 thirds

A1

$$P_3 = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.4 \\ 0 & 0.2 & 0.6 \end{bmatrix} \times \begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix} + \begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix} = \begin{bmatrix} 39 \\ 56 \\ 25 \end{bmatrix}$$

(b) 40 players at each level.

A1

$$\begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix} - \begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix} = \begin{bmatrix} 40 \\ 48 \\ 32 \end{bmatrix}$$

M1

$$\begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.4 \\ 0 & 0.2 & 0.6 \end{bmatrix}^{-1} \times \begin{bmatrix} 40 \\ 48 \\ 32 \end{bmatrix} = \begin{bmatrix} 40 \\ 40 \\ 40 \end{bmatrix}$$

(c) $x = -3$, $y = 2$, $z = 1$

A1

$$\begin{bmatrix} 39 \\ 56 \\ 25 \end{bmatrix} - \begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

Alternative solution:

$$\begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.4 \\ 0 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix} = \begin{bmatrix} 41 \\ 51 \\ 28 \end{bmatrix}$$

Hence

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix} - \begin{bmatrix} 41 \\ 51 \\ 28 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$