

**The Mathematical Association of Victoria**  
**Trial Exam 2014**  
**FURTHER MATHEMATICS**  
**Written Examination 2**

**STUDENT NAME** \_\_\_\_\_

**Reading time: 15 minutes**  
**Writing time: 1 hour 30 minutes**

**QUESTION AND ANSWER BOOK**

**Structure of Book**

| Core                       |                                           |                        |
|----------------------------|-------------------------------------------|------------------------|
| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
| 5                          | 5                                         | 15                     |
| Module                     |                                           |                        |
| <i>Number of modules</i>   | <i>Number of modules to be answered</i>   | <i>Number of marks</i> |
| 6                          | 3                                         | 45                     |
|                            |                                           | Total 60               |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

- Question and answer book of 49 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

**Instructions**

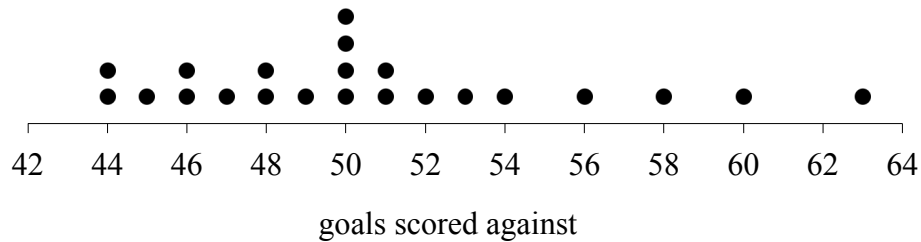
- Detach the formula sheet from the centre of this book during reading time.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

## Core

### Question 1(4 marks)

In her end-of-season review, the coach of the Senior B netball team of the Wombat Creek Sports and Social Club produced the following dot plot. The dot plot displays the goals scored against the Senior B netball team in every one of their 22 Home and Away games last season.



- (a) Using the information in the dot plot, determine the following :
- (i) median.

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- (ii) Interquartile Range

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1 + 1 = 2 marks

- (b) Write down an appropriate calculation and use it to explain why the goals against score of 63 is an outlier for this group of scores.

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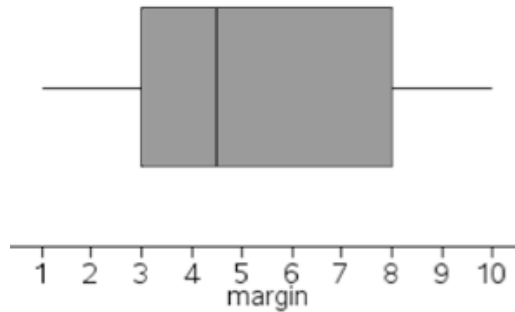
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2 marks

Core - continued

**Question 2** (2 marks)

Unfortunately, during the season, the Senior B team lost ten games. When their coach constructed a boxplot of the losing margins (the number of goals that the team lost by in those ten games), she obtained the graph as shown below.



The five number summary was :

Minimum = 1       $Q_1 = 3$       Median = 4.5       $Q_3 = 8$       Maximum = 10

Given that all margins were integer values (whole numbers), complete the table below to give one set of losing margin values that would give the boxplot shown above.

(Note : there is more than one correct set of values, you only have to find one of them)

|       |   |   |   |   |   |   |   |   |   |    |
|-------|---|---|---|---|---|---|---|---|---|----|
| No.   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Value | 1 | 2 |   |   |   |   |   |   | 9 | 10 |

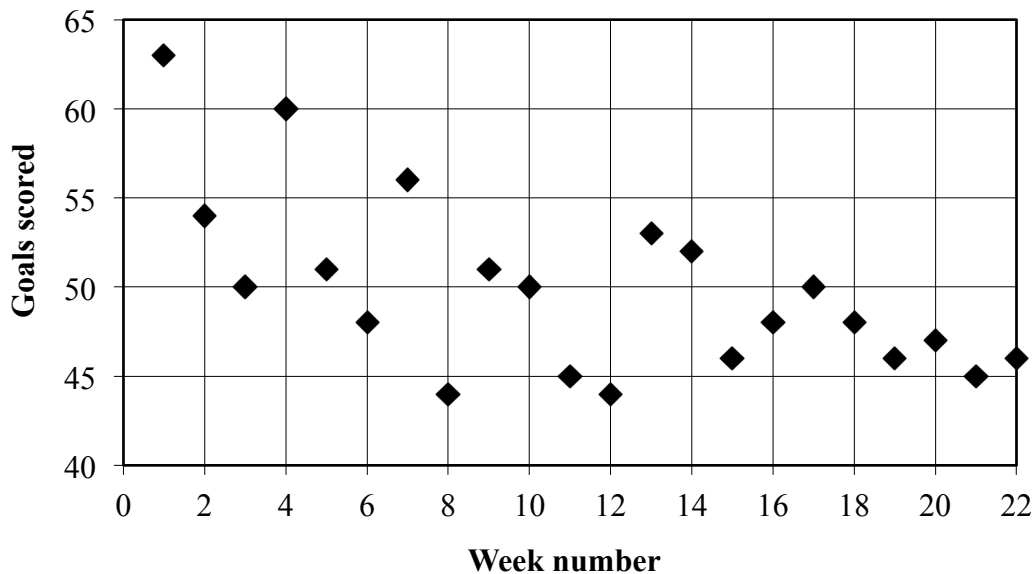
2 marks

**Core – continued**

**TURN OVER**

**Question 3** (5 marks)

The Senior B netball team started their season strongly scoring over 60 goals in their first game, but as injuries mounted (their star goal shooter missed a large number of games), their ability to score winning totals diminished greatly. The scatterplot of goals scored in each game is shown below.



The least squares regression equation for this data is

$$\text{Goals scored} = 55 - 0.48 \times \text{week number}$$

- (a) Draw the line of this regression on the graph above.

1 mark

- (b) The coefficient of determination for this data set is 38.8%.

- (i) Interpret the coefficient of determination in terms of the variables goals scored and week number.

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- (ii) Determine the value of Pearson's product moment correlation coefficient. Write your answer correct to three decimal places.

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1 + 1 = 2 marks

**Core** – continued

- (c) Use the least squares regression equation to predict the number of goals scored in the twelfth week, correct to the nearest whole number.

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1 mark

- (d) Given that the team actually scored 44 goals in the twelfth week, calculate the residual value.

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1 mark

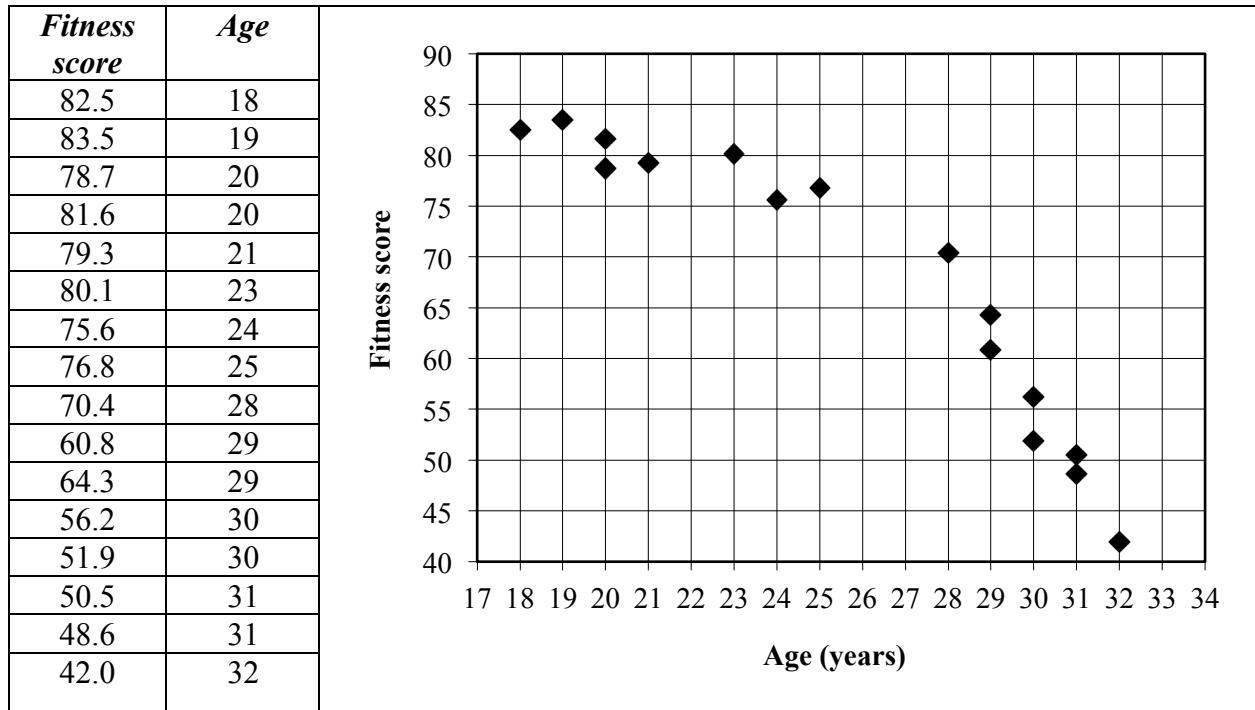
**Core – continued**

**TURN OVER**

**Question 4** (2 marks)

The senior netball coach ran a fitness clinic, in which players undertook a series of activities. From their efforts a “fitness score” was derived, being a number between 0 (very unfit) and 100.0 (super-fit).

The *fitness score* and *age* of sixteen players are given in the table below. A scatterplot showing fitness score versus age of these results is also shown below.



The relationship between *fitness score* and *age* shown is nonlinear.

A **squared transformation** can be applied to the variable *fitness score* to linearise the data in the scatterplot.

- (a) Apply the squared transformation to the variable *fitness score* and determine the equation of the least squares regression line that allows  $(\text{fitness score})^2$  to be predicted from *age*. Write the coefficients for this equation, correct to one decimal place, in the spaces provided below.

$$(\text{fitness score})^2 = \dots\dots\dots = \dots\dots\dots \times \text{age}$$

1 mark

**Core** - continued

- (b) Use this equation to predict the fitness score of a netballer who is 27 years old.  
Write your answer, correct to one decimal place.

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1 mark

**Question 5 (2 marks)**

Over the season, all 240 members of the Wombat Creek Sports and Social Club netball teams undertook the fitness analysis. The *fitness score* values were approximately normally distributed with a mean score value of 60.6 and a standard deviation value of 14.7.

Calculate the number of netballers who had a fitness score above 90.

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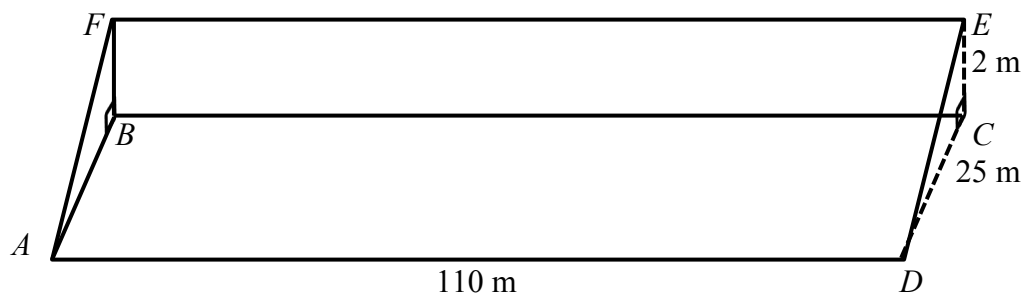
2 marks

**END OF CORE**

**Module 2: Geometry and Trigonometry****Question 1** (7 marks)

The Wombat Creek Sports and Social Club will build a new set of netball courts on a site which will be carved into the side of a hill.

The site has the shape of a right-triangular prism  $ABCDEF$ , with a length of 110 m, a width of 25 m and a height on the uphill side of 2 m. This is shown in the diagram below.



- (a) Calculate the volume of earth to be removed, writing your answer in  $\text{m}^3$ . 1 mark

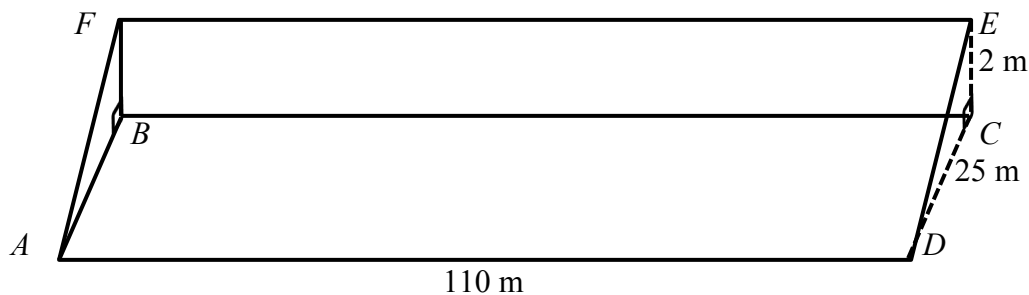
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**Module 2 - continued**

Once the hole has been dug, a safety fence will be erected around the site – along the two sloping sides AF and DE, and along the edges AD and FE.



- (b) Calculate the total length of safety fencing that will be required, writing your answer, in metres, correct to one decimal place.

2 marks

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The site will accommodate three netball courts end-to-end, with a 5 m gap between adjacent courts. The entire horizontal surface (ABCD) will be surfaced with a special black-coloured rubber material that is 0.1 m thick.

- (c) Find the volume in cubic metres, of special rubber material that is needed to surface the site.

2 marks

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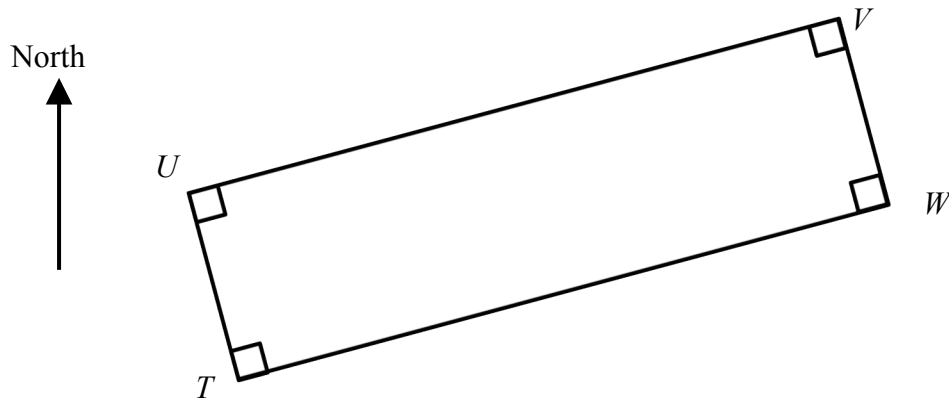
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**Module 2 – continued**  
**TURN OVER**

During the construction process, four key survey points forming a rectangle were established.



- (d) The bearing of point U from point T is  $327^\circ$ .

Determine the bearing of :

- (i) point V from point U

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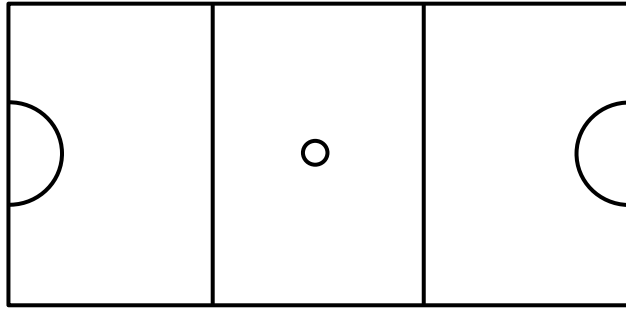
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- (ii) point W from point V

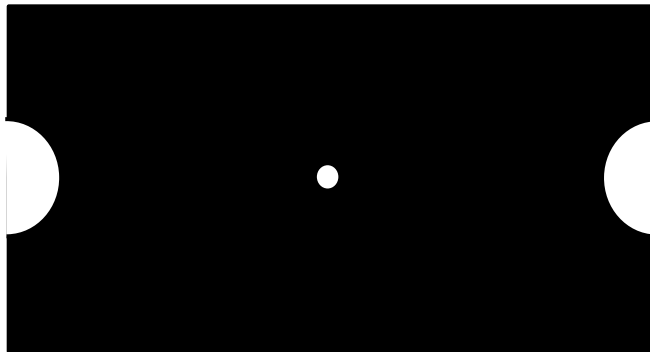
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1 + 1 = 2 marks

**Question 2** (2 marks)

The 0.9 m diameter centre circle and the two 4.9 m radius semi-circular “shooting circles” or “goal circles” on **each** court will be painted white to aid players and umpires. This is shown in the diagram below.



- (a) Calculate the total area to be painted white on **all three courts**.  
Give your answer in square metres correct to two decimal places.

1 mark

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The special white paint required only comes in four-litre cans, each litre of paint will cover  $12 \text{ m}^2$  of the rubber material, and to ensure durability, two coats of paint will required.

- (b) Calculate the number of four-litre paint cans needed to paint all three courts as required.

1 mark

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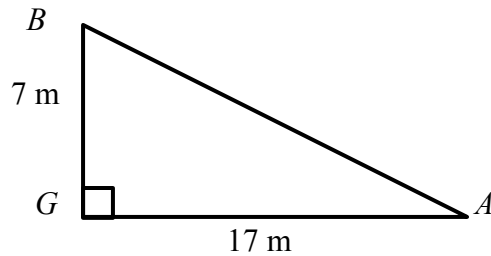


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**Module 2 – continued**  
**TURN OVER**

**Question 3** (4 marks)

Briony (B) is sitting in the newly completed seating 7 m above the ground level (G) of the new netball courts. One of her friends, Ally (A), is playing on one of the courts, at a horizontal distance of 17m away from point G. This is shown in the diagram below.



- (a) Find the distance  $BA$ , correct to the nearest metre.

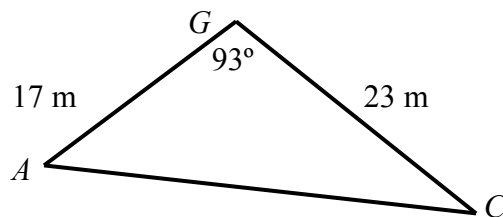
1 mark

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Another of Briony's friends, Chloe (C), is playing on another court at a distance of 23m from G, and the top view of their relative positions is shown below.



- (b) Show that the distance between Ally and Chloe is 29.3 m, correct to one decimal place.

1 mark

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Chloe fell over where she was standing, and when Briony shrieked in fear of Chloe hurting herself, Chloe looked up at Briony, who is still sitting above point G.

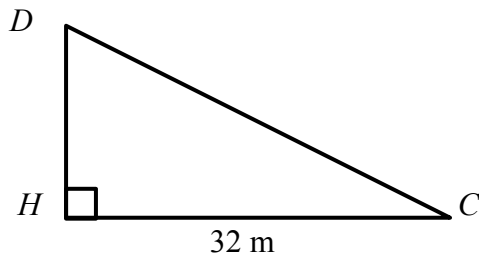
- (c) At what elevation was her line of sight to Briony, correct to the nearest whole degree ?  
1 mark

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Delia, ( $D$ ), in another part of the grandstand above the court level  $H$  which is 32m from  $C$ , also shrieked when Chloe fell over.



- (d) If Chloe looked up at Delia at an angle of  $15^\circ$ , how high was Delia above court level, in metres correct to one decimal place?  
1 mark

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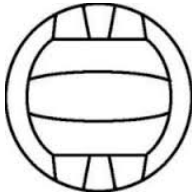
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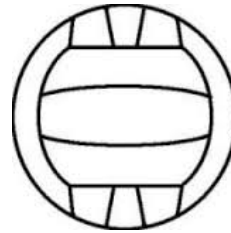
**Module 2 – continued**  
**TURN OVER**

**Question 4** (2 marks)

The netball used in senior competition has a volume of  $6044 \text{ cm}^3$ . The junior netballers at Wombat Creek (the Under 12s and below) train with a ball whose volume is  $4540 \text{ cm}^3$ .



Junior netball



Senior netball

By what value can the surface area of the junior netball be multiplied to give the surface area of the senior netball? Give your answer correct to two decimal places.

2 marks

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**END OF MODULE 2**

**Module 3: Graphs and Relations****Question 1** (3 marks)

During the football season, the Wombat Creek Sports and Social Club has a canteen that is run by volunteers. The canteen sells hotdogs for \$3.50 and cans of soft drink for \$2.20.

- (a) Write an equation for the revenue ( $R$ ) obtained from selling  $x$  hotdogs and  $y$  cans of soft drink.

1 mark

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- (b) One Saturday the canteen took in a revenue of \$579.50. If they sold 125 cans of soft drink, how many hotdogs were sold?

1 mark

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The cost to make each hotdog is \$1.20 and the cost of each can of soft drink is \$0.80.

An equation for the canteen's cost ( $C$ ) for  $x$  hotdogs and  $y$  cans of soft drink would be:

$$C = 1.2x + 0.8y$$

- (c) The canteen's profits ( $P$ ) is given by the difference between the revenue and the costs. Write an equation for the profit made from selling  $x$  hotdogs and  $y$  cans of soft drink.

1 mark

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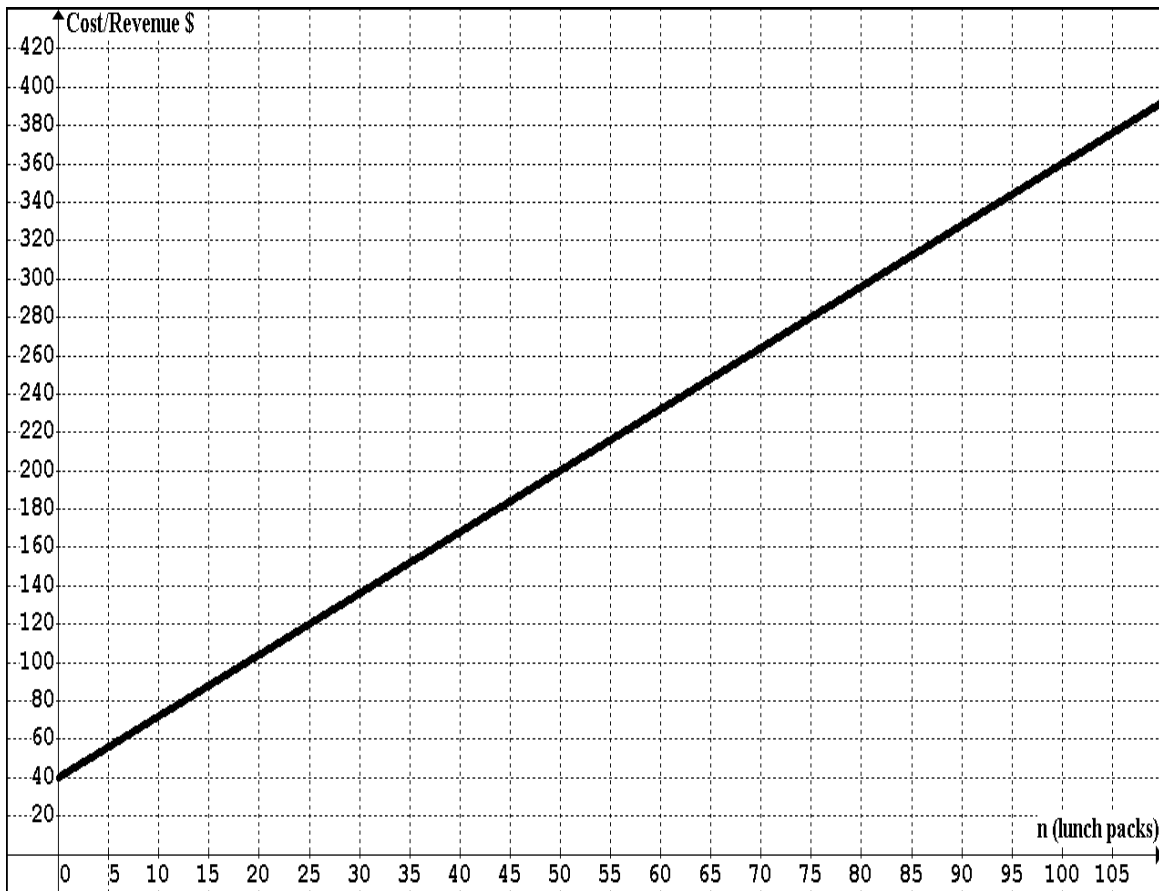
**Module 3 - continued**

**Question 2** (5 marks)

It has been suggested that the canteen should start selling healthier food.

One suggestion is that they could sell salad rolls and orange juice as a lunch pack. The packs can be purchased for \$3.20 each, but the supplying company charges a delivery fee each time that they deliver the packs.

A graph is shown below for the cost of supplying and delivering  $n$  lunch packs:



- (a) The equation for the cost of supplying and delivering  $n$  lunch packs is  $C = 3.2n + b$ .

State the value of  $b$  in this equation.

1 mark

**Module 3** - continued

- (b) What does the value of  $b$  in this equation represent? 1 mark

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The suggested sale price for the lunch packs is \$4.00. This means that the equation for the revenue ( $R$ ) would be:

$$R = 4n$$

- (c) Add the line  $R = 4n, 0 < n \leq 100$  to the graph above. 1 mark

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- (d) What is the minimum number of lunch packs that the canteen would need to sell in order to make a profit? 2 marks

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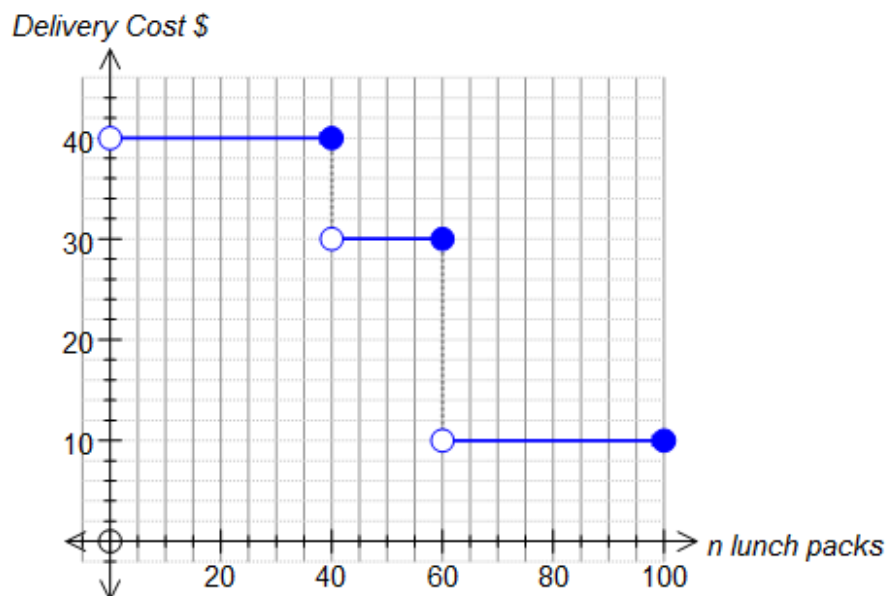
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**Module 3 – continued**  
**TURN OVER**

**Question 3** (3 marks)

The Wombat Creek Sports and Social Club have decided to negotiate with another supplier for their lunch packs. This supplier agrees to reduce the delivery fee if a greater number of packs is delivered. A graph showing the relationship between the delivery fee and the number of packs delivered is shown below for up to 100 lunch packs, the maximum number allowed per delivery:



- (a) On one particular day the club has 150 lunch packs delivered in more than one delivery. What is the minimum cost for delivery on this day? 1 mark

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A relationship connecting the maximum number of lunch packs for the three delivery costs is to be investigated.

The rule is found to be of the form  $cost = a \times \text{number of lunch packs} + b$

- (b) Determine the values of  $a$  and  $b$ . 2 marks

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**Module 3 – continued**

**TURN OVER**

**Question 4** (4 marks)

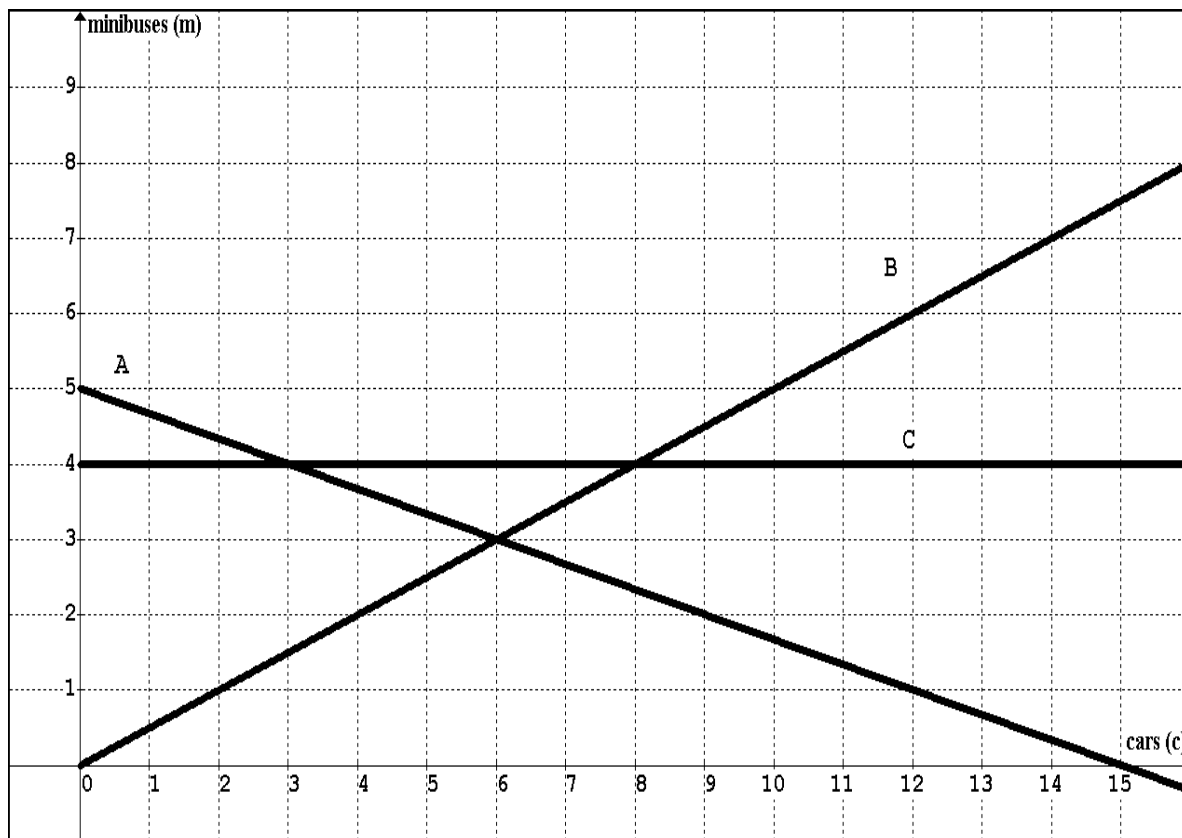
The Wombat Creek Sports and Social Club are planning for the annual football clash at the home ground of their great rivals, Platypus Flat. They plan to take a combination of cars (c) and minibuses (m) to transport the participants.

There are a number of factors that may affect the number of each of the vehicles used including that  $m \geq 0$  and  $c \geq 0$ .

These can be summarised in a number of constraint inequations:

|        |                                                                                                                                |                       |
|--------|--------------------------------------------------------------------------------------------------------------------------------|-----------------------|
| Line 1 | There are only 4 minibuses available                                                                                           | $m \leq 4$            |
| Line 2 | Each car can take 4 passengers and each minibus can take 12 passengers. There are at least 60 people who are going on the trip | $4c + 12m \geq 60$    |
| Line 3 | For every two cars at least one minibus should be taken                                                                        | $m \geq \frac{1}{2}c$ |

These three lines are shown on the graph below and labelled A, B and C.



- (a) Write down the letter associated with each of these lines in the table below. 1 mark

|        |  |
|--------|--|
| Line 1 |  |
| Line 2 |  |
| Line 3 |  |

- (b) Another constraint inequation applies to this situation. There are only 12 people who have offered to drive cars. Write down the associated inequation below **and** add the line that borders the associated region to the graph above. 1 mark

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- (c) Shade the feasible region for this problem **and** state the value of points in this region, given that the number of minibuses and the number of cars must each be whole numbers. 1 mark

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- (d) The cost of transport must be minimised.  
Given that hiring a minibus (including fuel) for the day is \$270, what is the maximum amount of money that could be allowed for each car's costs, if the cost is minimised only when there are 6 cars and 3 minibuses? 1 mark

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**END OF MODULE 3**

**Module 6: Matrices****Question 1** (5 marks)

The Wombat Creek Sports and Social Club have members in a number of categories; Juniors(J), Intermediates(I), Adults(A) and Social (S) members.

The numbers of each type of member and their gender, male (M) and female (F), is given in the matrix, N, below:

$$N = \begin{matrix} & \begin{matrix} M & F \end{matrix} \\ \begin{matrix} J \\ I \\ A \\ S \end{matrix} & \begin{bmatrix} 30 & 21 \\ 40 & 15 \\ 20 & 25 \\ 45 & 60 \end{bmatrix} \end{matrix}$$

- (a) The annual club fees are \$40 for Juniors, \$60 for Intermediates, \$80 for Adults and \$30 for Social club members. Write this information in a  $1 \times 4$  matrix, F. 1 mark

- (b) Explain why the matrix product,  $FN$ , is defined for multiplication. 1 mark

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**Module 6 - continued**

(c) Write down the matrix product,  $FN$ . Call this matrix  $M$ . 1 mark

(d) Explain the meaning of the element,  $m_{12}$ . 1 mark

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(e) A matrix showing the total number of each of the Juniors, Intermediates, Adults and Social club members is required. It is obtained using the matrix product,  $NG$ .

Write down the matrix  $G$  that would produce the required matrix. 1 mark

**Module 6 – continued**  
**TURN OVER**

**Question 2** (3 marks)

The Wombat Creek Sports and Social Club sell caps (C), polo shirts (P) and hoodies (H) with their logo design.

One family buys 1 cap, 2 polo shirts and 2 hoodies, costing \$226.

Another family buys 2 caps and 1 polo shirt and 2 hoodies, costing \$206.

A third family buy 2 polo shirts and 1 hoodie and this costs \$139.

- (a) Complete the entries in the matrix equation below that could represent the situation described. 1 mark

$$\begin{bmatrix} & & \end{bmatrix} \times \begin{bmatrix} C \\ P \\ H \end{bmatrix} = \begin{bmatrix} 226 \\ 206 \\ 139 \end{bmatrix}$$

- (b) The values of C, P and H are calculated using a matrix multiplication,  $D \times \begin{bmatrix} 226 \\ 206 \\ 139 \end{bmatrix}$ . 1 mark  
Complete matrix D below.

$$D = \begin{bmatrix} & & \end{bmatrix}$$

- (c) Show, using a calculation, that the cost of each polo shirt is \$32. 1 mark

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**Module 6 – continued**  
**TURN OVER**

**Question 3** (3 marks)

The gym at the Wombat Creek Sports and Social Club has a number of fitness activities. Players are advised to vary their training each week to maximise their fitness outcomes.

The football players can choose between the treadmills (T), the weight circuit (W) or football skills training (F).

During the first week of the season, 100 players from various levels attended training. All of the players started on the treadmill. The state matrix for week 1, is:

$$W_1 = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} T \\ W \\ F \end{matrix}$$

The transition matrix showing the movement between training regimes each week is:

$$T = \begin{bmatrix} \mathbf{0} & 0.2 & 0.4 \\ 0.5 & \mathbf{0} & 0.6 \\ 0.5 & 0.8 & \mathbf{0} \end{bmatrix}$$

- (a) Three elements in the transition matrix have been highlighted. Explain the meaning of these three elements. 1 mark

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- (b) Determine the number of players who will do football skills training in the third week of training. 1 mark

- (c) The pattern continues. Show that in the long run, 41 players will do football skills training each week if values are rounded to the nearest whole number. 1 mark

**Module 6 – continued**  
**TURN OVER**

**Question 4** (4 marks)

One year the Wombat Creek Sports and Social Club has football teams playing at each of three levels; firsts, second and thirds.

Some players are promoted or dropped a level during the season and others are unavailable due to injury or return from injury.

The number of players at each level each week is modelled by the equation:

$$P_{n+1} = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.4 \\ 0 & 0.2 & 0.6 \end{bmatrix} \times P_n + \begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}, P_2 = \begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix}$$

- (a) How many players would be available at each level during the third week? 1 mark

- (b) How many players were available at each level during week 1? 2 marks

**Module 6-** continued

- (c) The club would like to retain the same number of players that they had at each level during week 2.

$\begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}$  represents the number of players lost or returned due to injury at that level each week.

Using the matrix  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  instead of  $\begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}$  will ensure that the numbers do not change after week 2.

What are the values of  $x, y$  and  $z$ ?

1 mark

**END OF QUESTION AND ANSWER BOOK**