



The Mathematical Association of Victoria SPECIALIST MATHEMATICS

Trial written examination 2

2007

Reading time: 15 minutes

Writing time: 2 hours

Student's Name:

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	4	4	59
			Total 81

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Working space

MULTIPLE CHOICE ANSWER SHEET

Student Name:

Circle the letter that corresponds to each correct answer

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E

Working space

SECTION 1**Instructions for Section 1**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

The range of the graph with equation $y = a \cos^{-1}(3x) + \frac{\pi}{2}$ is:

- A. $\left(\frac{\pi}{2}, \frac{\pi}{2} + a\right)$
- B. $\left[\frac{\pi a}{2}, \frac{3\pi a}{2}\right]$
- C. $\left[-\frac{1}{3}, \frac{1}{3}\right]$
- D. $\left[\frac{\pi}{2}, (2a + 1)\frac{\pi}{2}\right]$
- E. $[2\pi a, 6\pi a]$

Question 2

The distance between vertices, and the equations of the asymptotes for the following hyperbola with equation $9(x - 1)^2 - 4y^2 = 36$ are, respectively:

- A. 2; $3x - 2y - 3 = 0$, $-3x - 2y + 3 = 0$
- B. 4; $3x - 2y - 3 = 0$, $-3x - 2y + 3 = 0$
- C. 3; $3x - 2y = 0$, $3x + 2y = 0$
- D. 2; $3x - 2y = 0$, $3x + 2y = 0$
- E. 4; $3x - 2y - 2 = 0$, $3x + 2y + 2 = 0$

Question 3

If $8\text{cis}\left(-\frac{5\pi}{6}\right)$ is one of the cube roots of the complex number z , then $|z|$ and $\text{Arg}(z)$ are:

- A. 2 and $-\frac{5\pi}{18}$
- B. 2 and $-\frac{\pi}{2}$
- C. 512 and $-\frac{\pi}{2}$
- D. 512 and $\frac{\pi}{2}$
- E. $\frac{8}{3}$ and $-\frac{\pi}{2}$

Question 4

The following equation $z^4 - 4z^3 + 8z^2 - 8z + 16 = 0$, where $z \in C$, has:

- A. three complex and one real solution
- B. two real and two complex solutions
- C. three real and one complex solution
- D. no complex solutions
- E. four complex solutions

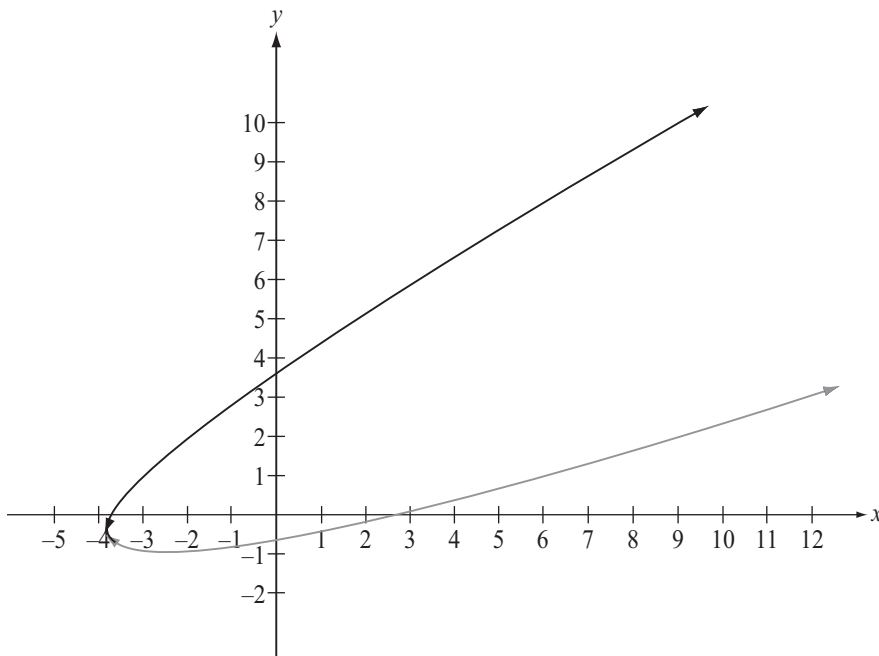
Question 5

The unit vector in the direction of vector $\underline{a} = \sqrt{2}\underline{i} - \underline{j} + 7\underline{k}$ is:

- A. $\frac{1}{2\sqrt{13}}(\sqrt{2}\underline{i} - \underline{j} + 7\underline{k})$
- B. $\frac{1}{2\sqrt{13}}(-\sqrt{2}\underline{i} + \underline{j} - 7\underline{k})$
- C. $\frac{1}{2\sqrt{3}}(-\sqrt{2}\underline{i} + \underline{j} - 7\underline{k})$
- D. $\frac{1}{\sqrt{10}}(\sqrt{2}\underline{i} - \underline{j} + \sqrt{7}\underline{k})$
- E. $\frac{1}{\sqrt{54}}(-2\underline{i} + \underline{j} + 7\underline{k})$

Question 6

The graph below shows the implicit relation $x^2 - 4xy + 4y^2 + x - 12y - 10 = 0$.



The gradient of the tangent at the point $(6, 1)$ is equal to:

- A. $\frac{19}{28}$
- B. $\frac{9}{28}$
- C. $\frac{31}{4}$
- D. $-\frac{9}{28}$
- E. $-\frac{19}{28}$

Question 7

The set of points in the complex plane defined by $|z| = |z + 4|$ is:

- A. the circle with centre $(-2, 0)$ and radius 2
- B. the circle with centre $(2, 0)$ and radius 2
- C. the line $\operatorname{Re}(z) = 2$
- D. the line $\operatorname{Re}(z) = -2$
- E. the line $\operatorname{Im}(z) = -2$

Question 8

Using a suitable substitution, $\int_0^{\frac{\pi}{6}} (\cos(3x)e^{\sin(3x)})dx$ is equal to:

- A. $\int_0^{\frac{\pi}{6}} e^u du$
- B. $\int_0^1 e^u du$
- C. $3 \int_0^1 e^u du$
- D. $\frac{1}{3} \int_0^1 e^u du$
- E. $-\frac{1}{3} \int_0^1 e^u du$

Question 9

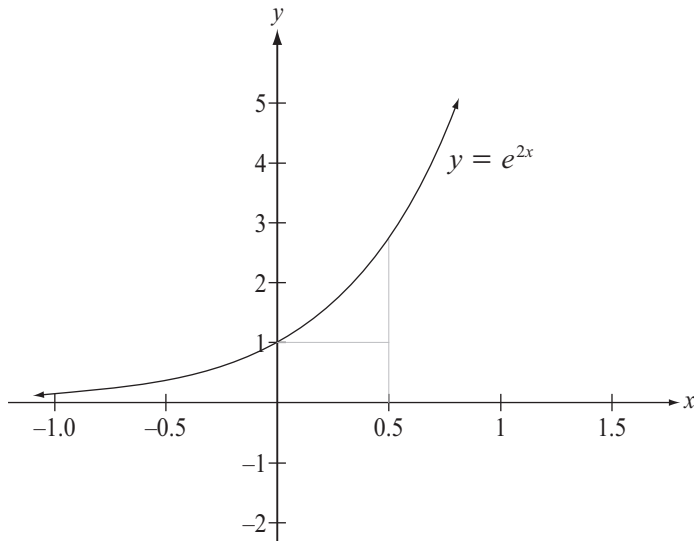
Points A , B and C have position vectors $2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $3\mathbf{i} + \mathbf{j}$ respectively, with respect to an origin O . The cosine of $\angle ABC$ is:

- A. $\frac{8}{9}$
- B. $-\frac{8}{9}$
- C. $\frac{4}{\sqrt{17}}$
- D. $\frac{5}{\sqrt{30}}$
- E. $\frac{8}{81}$

Question 10

Euler's method, with a size step of 0.1, is used to solve the differential equation $\frac{dy}{dx} = \log_e(x)$ with $y = 3$ at $x = 1$. The value of y at $x = 1.3$, correct to four decimal places, is:

- A. 3.0095
- B. 3.0277
- C. 3.0278
- D. 3.2602
- E. 3.2603

Question 11

The region enclosed by the curve $y = e^{2x}$ and the lines $x = \frac{1}{2}$ and $y = 1$ is rotated about the y -axis. The volume of revolution is given by:

- A. $\pi \int_1^e \left(\frac{1}{2} - \frac{1}{2} \log_e y \right)^2 dy$
- B. $\frac{\pi}{4} \int_1^e (1 - \log_e^2 y) dy$
- C. $\pi \int_0^{\frac{1}{2}} (e^{4x} - 1) dx$
- D. $\frac{\pi}{4} \int_1^e (\log_e y)^2 dy$
- E. $\int_1^e \frac{1 - (\log_e y)^2}{4} dy$

Question 12

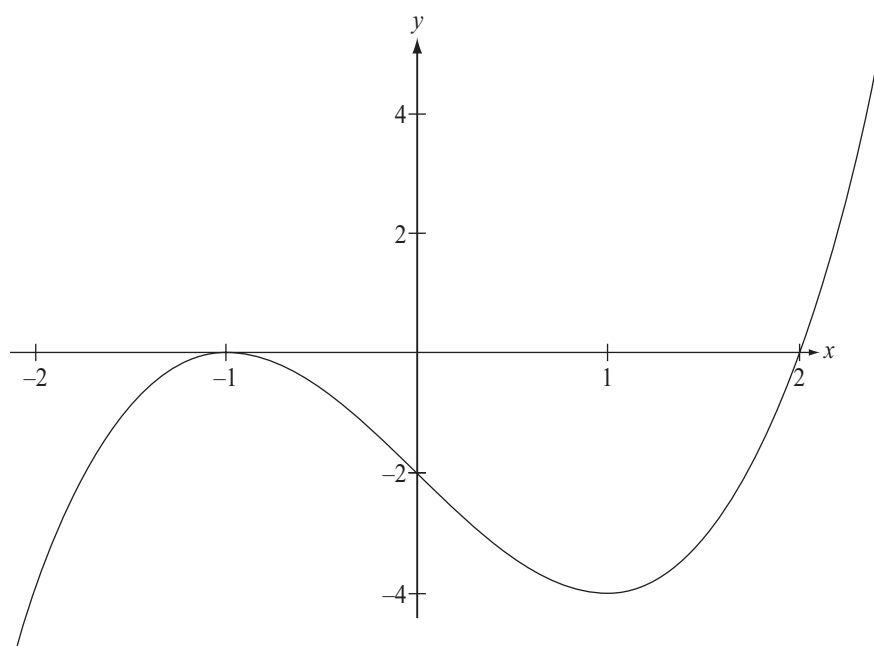
The graph of the function $y = \frac{x^2 + 9}{3x}$ has:

- A. no asymptotes and no turning points
- B. no asymptotes and one turning point
- C. a single asymptote $x = 0$ and no turning points
- D. a single asymptote $x = 0$ and two turning points
- E. two asymptotes and two turning points

Question 13

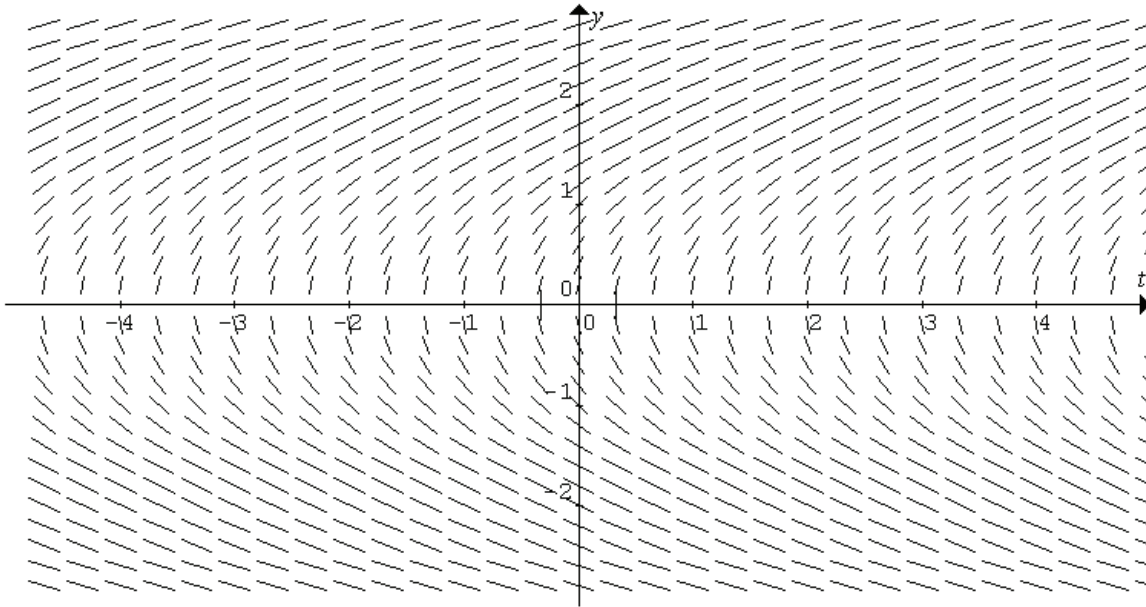
The necessary and sufficient conditions for a point P to be a point of inflection are that, as the curve passes through P :

- A. $\frac{d^2y}{dx^2} = 0$ and $\frac{dy}{dx} = 0$
- B. $\frac{d^2y}{dx^2} = 0$ and $\frac{dy}{dx}$ changes sign
- C. $\frac{d^2y}{dx^2} = 0$ and $\frac{dy}{dx} > 0$
- D. $\frac{d^2y}{dx^2}$ changes sign and $\frac{dy}{dx}$ does not change sign
- E. $\frac{d^2y}{dx^2} = 0$ and $\frac{dy}{dx} < 0$

Question 14

The graph of $y = f'(x)$ is shown above. Which one of the following statements is true for the graph of $y = f(x)$?

- A. The graph has a local maximum at $x = -1$ and a local minimum at $x = 1$.
- B. The graph has a stationary point of inflection at $x = -1$, a local minimum at $x = 1$ and a local minimum at $x = 2$.
- C. The graph has a stationary point of inflection at $x = -1$, a point of inflection at $x = 1$ and a local minimum at $x = 2$.
- D. The graph has a stationary point of inflection at $x = -1$ as well as a local maximum at $x = 2$.
- E. The graph has a stationary point of inflection at $x = 2$ as well as a local maximum at $x = -1$.

Question 15

Given the slope field diagram shown above, the possible differential equation could be:

- A. $\frac{dy}{dt} = \frac{1}{y}$
- B. $\frac{dy}{dt} = t^2 - 2$
- C. $\frac{dy}{dt} = \sin(t)$
- D. $\frac{dy}{dt} = t$
- E. $\frac{dy}{dt} = e^t$

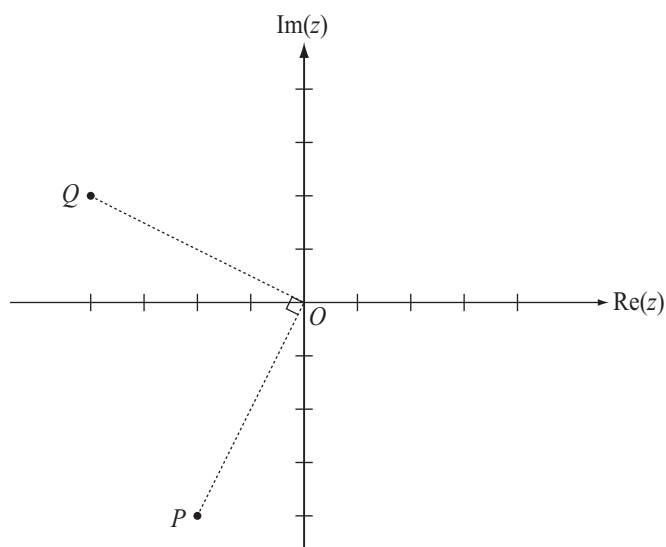
Question 16

The position vector of a particle at time t seconds, where $t \geq 0$, is given by

$$\underline{r}(t) = (2 + 3t - 2t^2)\underline{i} + 6t\underline{j} + \sqrt{t}\underline{k}.$$

The direction of motion at $t = 2$ is:

- A. $-4\underline{i} + 6\underline{j} + \frac{1}{4}\underline{k}$
- B. $-5\underline{i} + 6\underline{j} + \frac{1}{4}\underline{k}$
- C. $-5\underline{i} + 6\underline{j} + \frac{1}{2}\underline{k}$
- D. $-5\underline{i} + 6\underline{j} + \frac{\sqrt{2}}{4}\underline{k}$
- E. $12\underline{j} + \sqrt{2}\underline{k}$

Question 17

The point P on the Argand plane represents a complex number z . If $OP = OQ$ and the angle POQ is 90° , then point Q represents:

- A. iz
- B. $-iz$
- C. $-\bar{z}$
- D. $\bar{z}$
- E. $-z$

Question 18

A particle starts from rest at $t = 0$ and moves in a straight line with acceleration a given by the equation $a = 5 \sin(2t) - 1$. The velocity of the particle at $t = 1$, correct to 2 decimal places, is closest to:

- A. -14.16
- B. -4.54
- C. -3.54
- D. 1.54
- E. 2.54

Question 19

Jimmy, whose mass is 75 kg, is skiing down a hill which is inclined to the horizontal at an angle of $\arctan\left(\frac{1}{5}\right)$. He starts from rest on the top of the hill and skis down the total distance of 250 metres. Assuming that friction can be neglected, the magnitude of his momentum at the bottom of the hill is closest to:

- A. 2326 kgms^{-1}
- B. 2325 kgms^{-1}
- C. 2324 kgms^{-1}
- D. 333 kgms^{-1}
- E. 34 kgms^{-1}

Question 20

A particle is acted upon by two forces, one of magnitude 8 newton acting due north and the other of magnitude 6 newton acting S60°W. The magnitude of the resultant force (in newton) acting on the particle is:

- A. $2\sqrt{13}$
- B. $2\sqrt{37}$
- C. $\sqrt{34}$
- D. 14
- E. 52

Question 21

A bullet of mass m kg is shot vertically up with an initial speed of 400 m/s. It is subjected to an air resistance of $\frac{mv^2}{1000}$ newton where v m/s is the speed of the bullet. The maximum height (in metres) that the bullet reaches is closest to:

- A. 8163
- B. 1426
- C. 1417
- D. 1396
- E. 785

Question 22

A balloon is ascending with a uniform velocity of 20 m/s. When the balloon is 60 m above the ground, a ball is released from it. Assume that air resistance can be neglected. The time (in seconds) after which the ball will reach the ground is closest to:

- A. 2.01
- B. 3.50
- C. 6.09
- D. 2.04
- E. 5.71

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

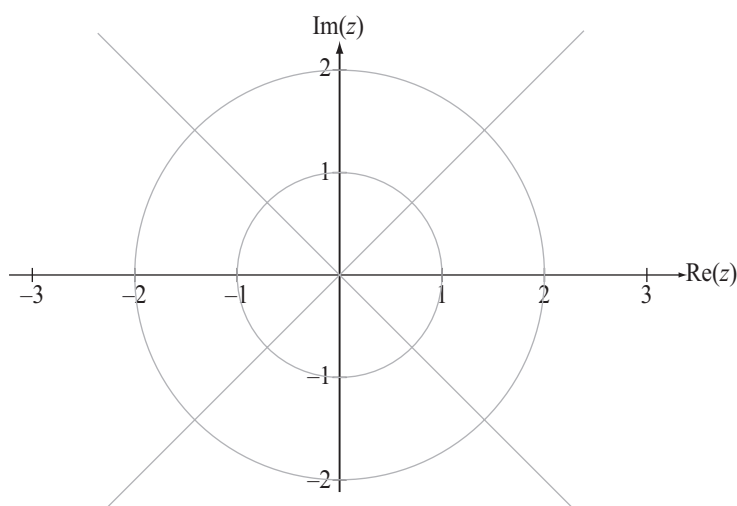
Question 1

Let u and v be the square roots of the complex number $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$, where $\text{Arg}(u) > \text{Arg}(v)$.

- a. Use de Moivre's theorem to find u and v in polar form.

3 marks

- b. Mark u and v on the Argand diagram below.



2 marks

- c. On the same diagram, draw and clearly label the following region T where $T = \{z: |z - u| = |z - v|\} \cap \{z: |z| \leq 2\}$.

3 marks

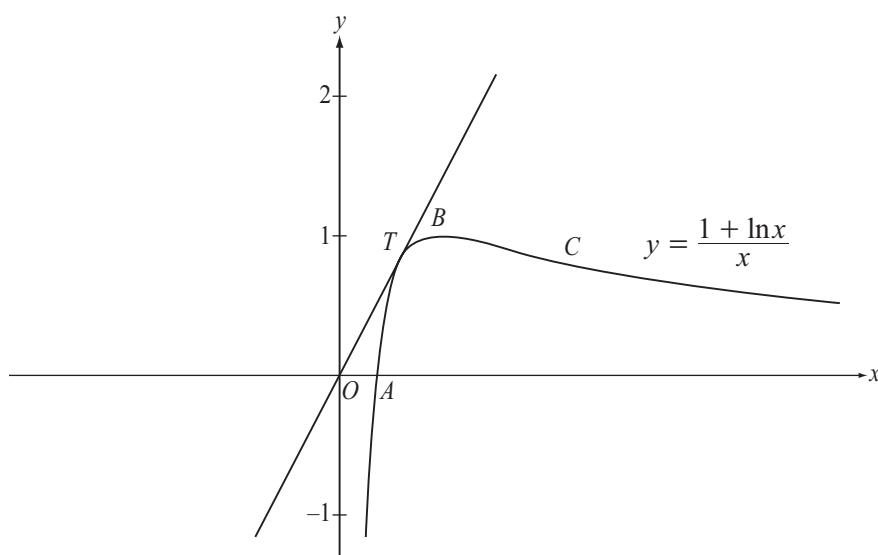
- d. For what values of n is u^n purely imaginary?

3 marks

Total 11 marks

Question 2

The graph of $y = f(x) = \frac{1 + \log_e x}{x}$ is given below.



The line OT , where O is the origin, is the tangent to the graph at T . The graph of $f(x)$ meets the x -axis at A .

- a.** Find the **exact** coordinates of the x -intercept A .

1 mark

- b. i.** Find $f'(x)$.

1 mark

- ii.** Hence find the coordinates of the maximum turning point B .

1 mark

- c. Find the **exact** coordinates of the point of inflection C .

3 marks

- d. Verify that point C is a point of inflection.

1 mark

- e. Show that the x -coordinate of point T is $x_T = \frac{1}{\sqrt{e}}$.

2 marks

- f. Using calculus, find the **exact** area between the tangent OT , the graph of $f(x)$ and the x -axis.

4 marks

Total 13 marks

Question 3

A triangle has its vertices at $A(-1, 3, 2)$, $B(3, 6, 1)$ and $C(-4, 4, 3)$.

- a. Show that $\overrightarrow{BC} \cdot \overrightarrow{BA} = 36$.

2 marks

Let D be a point on the side BC and let $\overrightarrow{BC} = \underline{\underline{p}}$, $\overrightarrow{BA} = \underline{\underline{q}}$ and $\overrightarrow{BD} = k\underline{\underline{p}}$ where $k \in \mathbb{R}$.

- b. Express $\overrightarrow{AD}$ in terms of $\underline{\underline{p}}$, $\underline{\underline{q}}$ and k .

1 mark

- c. Find the value of k if $\overrightarrow{AD}$ is perpendicular to $\overrightarrow{BC}$.

3 marks

- d. Hence, or otherwise, find the shortest distance from vertex A to the side BC , correct to 2 decimal places.

3 marks

Total 9 marks

Question 4

A large tank initially contains 800 litres of water in which is dissolved 10 kg of salt. A salt solution of concentration 0.1 kg per litre flows into the tank at the rate of 5 litres per minute. The mixture, which is kept uniform by stirring, flows out of the tank at the rate of 5 litres per minute. After t minutes the tank contains x kg of salt.

- a. i.** Show that the differential equation for x in terms of t is of the form

$$\frac{dx}{dt} = \frac{a - x}{b}$$

where a and b are positive constants.

[illegible]

3 marks

- ii.** Hence, write down the values of a and b .

1 mark

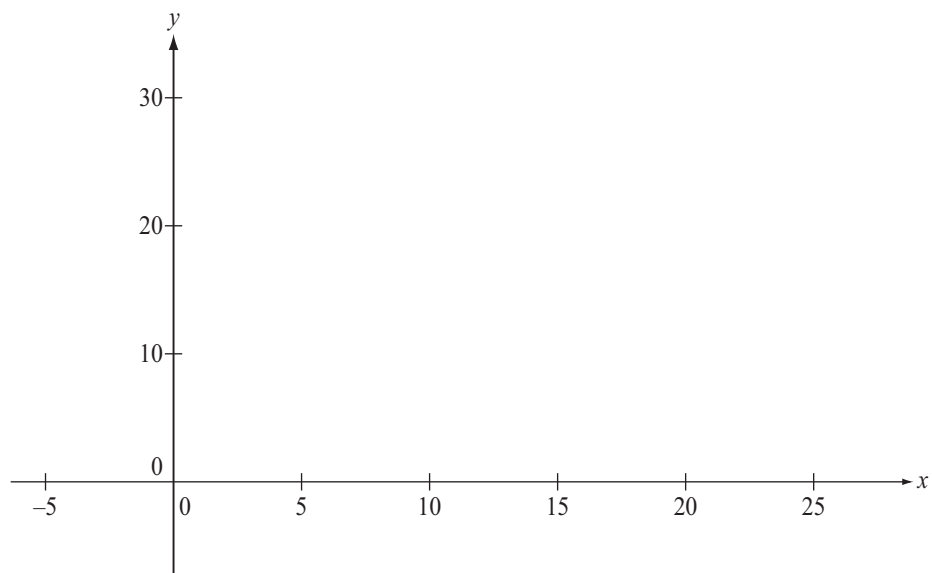
This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

SECTION 2 – Question 4 – continued

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- This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

Total 11 marks

- c. Find the Cartesian equation of the path of the toy plane and sketch its path on the axes below, showing all relevant features.

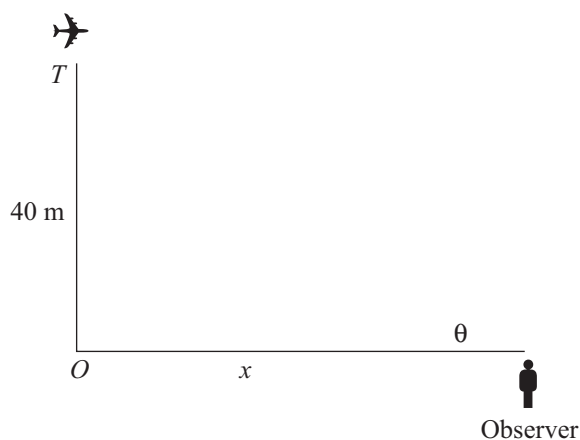


4 marks

- d. Find the magnitude of the momentum of the toy plane when it hits the ground. Give your answer to 3 significant figures.

3 marks

Another toy plane is battery operated so it can maintain a constant altitude. It also starts from the top of the cliff but flies horizontally at a constant speed of 5 m/s at a constant height of 40 metres in a straight line so that it will eventually fly directly over an observer who is at ground level.



- e. Differentiate implicitly the equation $\tan(\theta) = \frac{40}{x}$ to find $\frac{d\theta}{dt}$.

2 marks

- f. Hence evaluate $\frac{d\theta}{dt}$, in radian per second, when $x = 10$ metres, expressing the answer in **exact** form.

2 marks

Total 15 marks

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse:	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	hyperbola:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
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Circular (trigometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x) \cos(y) - \cos(x) \sin(y)$$

$$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$

END OF FORMULA SHEET